Prelab 8 *6.S02 – Spring 2013*

MRI – Projection Imaging

--L.L. Wald, April 2013

1. Nomenclature

"spin" In the lectures and labs, you will hear the word "spin" or "nuclear" spin to refer to the proton of the hydrogen atom. The term "spin" refers to a particular type of intrinsic angular momentum possessed by the proton (and other nuclei). This intrinsic angular momentum plays an important role, as we have seen, in the gyroscopic equations of motion that tell us how an ensemble of spins behaves. The gyroscopic procession which we detect is the classical limit of the behavior of a large ensemble of spins. An individual spin behaves quantum mechanically in very interesting ways, but since we will always have at least 10^{15} spins in any given imaging voxel, we can use the classical limit. This is because quantum mechanical behavior always reduces to the classical equations of motion when a macroscopic ensemble is considered. Sometimes, we will also refer to "the spins" as meaning this macroscopic ensemble, described by the classical magnetization vector; $\vec{M}(x, y, z, t)$.

"phantom" The medical imaging community uses the word phantom to mean any object or sample to be imaged that is not a human or animal. These are often specially designed to calibrate some aspect of the system, but can also be a simple spherical or cylindrical sample. In MRI, they are almost always water filled containers doped with salt to mimic the body's conductivity, and some relaxation agent to make the magnetization return to the z axis (equilibrium state) in a reasonable time (say 0.5s).

2. The Spin Echo

The spin echo, discovered accidently in 1949, has become a standard way to collect NMR signals. Part of the appeal is practical; it gives a way to move the signal away from the RF pulses used to generate, say the FID. Part of the appeal is biological; it sensitizes the signal to useful biological effects and has thus become the most useful type of clinical acquisition (if you have just one type of MRI image acquired, it will likely be a spin echo).

After a standard 90 degree excitation, all of the magnetization that was previously at equilibrium along the z axis is tipped down into the xy plane and starts processing. We can pick up the induced voltage thru the Faraday effect, and this is called the FID signal. Figure 1 is a time domain recording of the FID and Spin Echo from the lab's magnet. At t=0 the 90 degree excitation pulse creates the oscillating M_{xy} which produces the FID. After about 0.13s, the dephasing of Mxy has caused the signal to

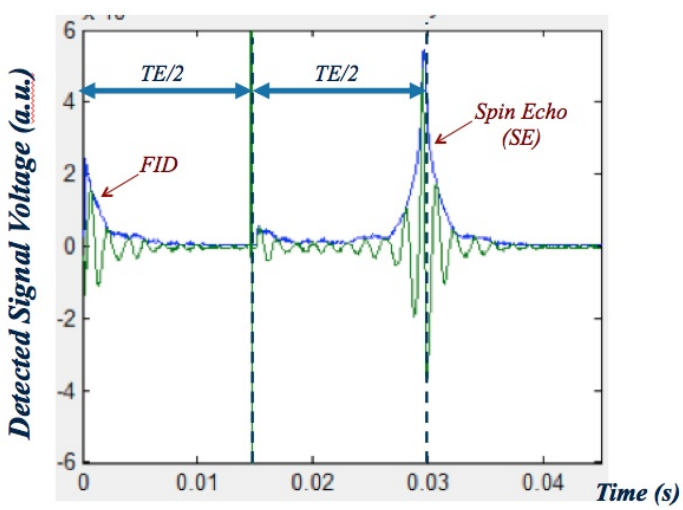
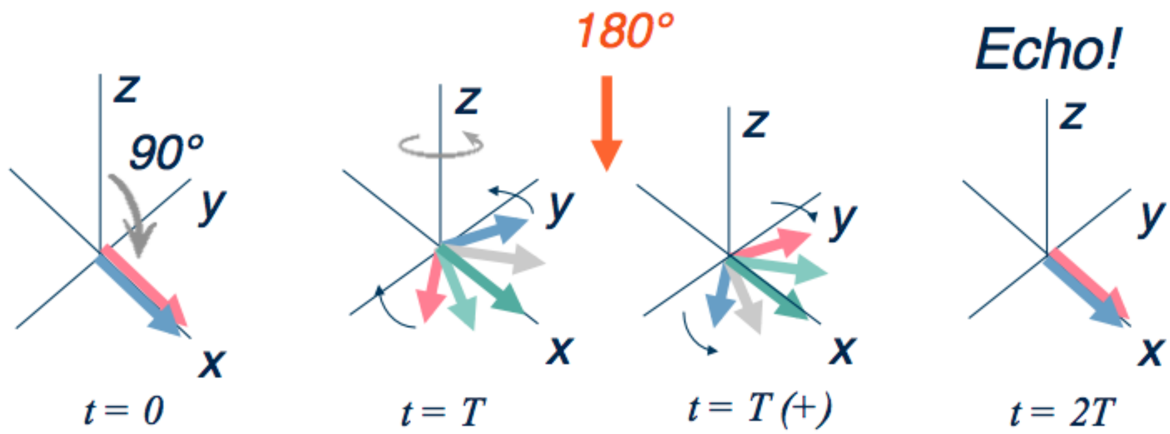


Figure 1: Time domain spin echo recorded from classroom MRIs

become nearly invisible. This dephasing is due to the distribution of frequencies present for the many spins (time constant = T2*). Namely some spins will have a slightly higher precession frequency than their neighbors due to a higher local magnetic field. At time = "TE/2" or about t=0.015s, a 180 degree rotation pulse is delivered. This flips the spins 180 degrees about the x axis, "flipping the pancake" of dephased spins. This starts the rephrasing process. Spins that "got ahead" in phase will now be behind, and vice-versa. The spins come back together in phase at time TE.



Blue arrows precesses slower due to local field inhomogeneity than red arrow

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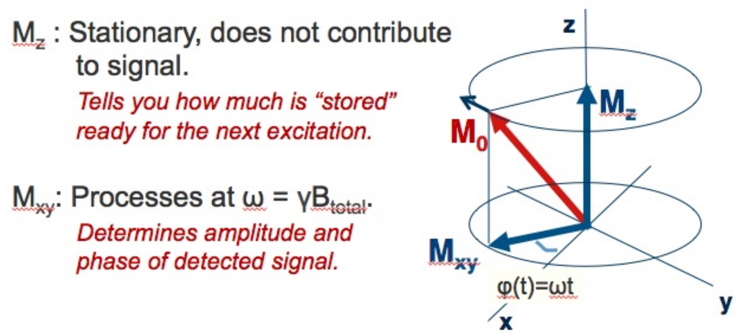
3. Encoding the Magnetic Resonance Image

Recap: The proton has both intrinsic spin angular momentum and a magnetic moment. A macroscopic ensemble of such moments add vectorally to produce $\vec{M}$(x,y,z) in the body. Because $\vec{M}$ and the macroscopic angular momementum $\vec{L}$ are collinear, the torque on M from an external applied field $B_0\ \hat{z}$ causes a change in the direction of the angular momentum (but not its magnitude). The resulting motion of the magnetic moment in an external field is precession, similar to a top and is called magnetic resonance. $\frac{d\vec{M}(t)}{dt} = \gamma\vec{M}(t)\times\vec{B}$ which are the equations of motion of a precessing gyroscope. Since $\frac{dM_z(t)}{dt} = 0$ then M_z is a constant of motion and the solutions to the equations of motion are $M_{xy}(t) = M_0e^{-i\omega_0 t}$. The M_{xy} component is rotating clockwise at the Larmor frequency $\omega_0 = \gamma B_0$ in the xy plane where B_0 is the applied magnetic field along the z axis.



We will make a special point of tracking the phase of the M_{xy} vector as a function of time:

$$\varphi(t) = \int_0^t \omega(\tau)d\tau = \gamma\int_0^t \omega B(x,y,z,\tau)d\tau \qquad \text{Eq.1}$$

If B is uniform through the sample, then $\vec{B}(x,y,z) = B_0\hat{z}$ and no interesting spatial information is learned from observing the frequency or phase of the spins precession. Since we can experimentally control B(x,y,z), we can introduce a spatial dependence to phase and frequency by making B vary across the object. The easiest way to do this is to apply a gradient to the static magnetic field.

External magnetic field gradients

A linear gradient is the simplest form of variation of the static field; it is a linear increase in the static field's $\hat{z}$ component as a function of position. The gradient field is added to the uniform field by injecting current into an wire coil wound to produce a spatial field variation. The coil is called the gradient coil, and the current can be switched on and off to switch between a uniform magnetic field and the gradient field. When an "x gradient" is applied the field as a function of position is:

$$\vec{B}(x,,y,z) = B_0\,\hat{z} + G_x x\,\hat{z} \quad \text{where } G_x \text{ is defined as } G_x = \frac{\partial B_z}{\partial x} \quad \text{Eq. 2}$$

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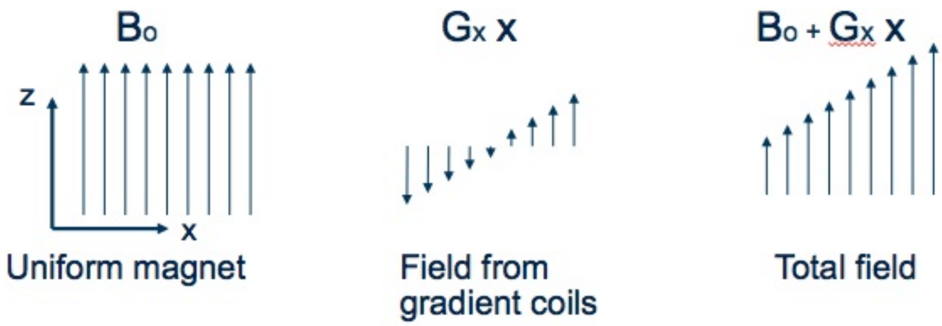

Then $\omega(x,y,z) = \gamma B_o + \gamma G_x x$. Its pretty easy to see that the gradient can be in x, y or z. In fact it can be a linear variation in any direction. The general gradient:

$$\vec{G} = \left(G_x, G_y, G_z\right) = \vec{\nabla} \cdot \vec{B} = \left(\frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial z}\right) = \frac{\partial B_z}{\partial x}\hat{x} + \frac{\partial B_z}{\partial y}\hat{y} + \frac{\partial B_z}{\partial z}\hat{z} \quad \text{Eq.3}$$

is a vector that points in the direction of increasing B. Two gradients applied simultaneously add vectorally since B_z fields superimpose. The total field with a gradient on can be written as a function of position, $\vec{r}$, as $\vec{B}(\vec{r}) = \left(B_0 + \vec{G} \cdot \vec{r}\right)\hat{z}$. Note the B field always point in the z direction, but varies in magnitude linearly in x, y and z. Thus, the frequency (now position dependent) of the signal is:

$$\omega(\vec{r}) = \gamma B_0 + \gamma \vec{G} \cdot \vec{r}\,. \quad \text{Eq. 4}$$

Consider the phase: $$\varphi(\vec{r}, t) = \int_0^t \omega(\vec{r}, \tau)\, d\tau = \gamma B_0 t + \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}\, d\tau \quad \text{Eq. 5}$$

We mentioned that the system demodulates the carrier, removing the frequency component γB_0. This is the same as always measuring the phase relative to the spin that is at r=0. We call this the "rotating frame" since this reference spin is precessing in the xy plane at a frequency of γB_0. In this case the relative phase is:

$$\Delta\varphi(\vec{r}, t) = \gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}\, d\tau \quad \text{Eq. 6}$$

And the equations of motion for M_{xy} in this frame is: $M_{xy}(t) = M_0 e^{-j\Delta\varphi(\vec{r},t)}$ Eq. 7

If we define a new variable; $$\vec{k}(t) = \frac{1}{2\pi}\gamma \int_0^t \vec{G}(\tau) d\tau \quad \text{Eq. 8}$$

since r is not a function of time (the spins are stationary), all if the time dependence is in k and we can write:

$$\Delta\varphi(\vec{r}, t) = 2\pi\, \vec{k}(t) \cdot \vec{r} \quad \text{Eq. 9}$$

$$M_{xy}(t) = M_0 e^{-2\pi j\, \vec{k}(t) \cdot \vec{r}} \quad \text{Eq. 10}$$

Its worth physically interpreting $\vec{k}(t)$. Since we control $\vec{G}(t)$ thru the current in the x, y and z gradient coils, $\vec{k}(t)$ is under complete user control. If we plot the gradient waveforms such as $G_x(t)$ as a time-plot, then k_x at time, t, is simply proportional to the area under the G_x curve from time 0 to t. Note if $G_x(t)$ = constant = G_x, then $k_x(t) = \frac{1}{2\pi}\gamma G_x\, t$ and k is just a simple linear change of variables for t.

<u>One dimensional projection imaging</u> Consider a magnetization distribution $M_{xy}(x,y,z)$ present in the body and the signal it produces when a constant gradient, G_x, is applied. The signal voltage detected is proportional to the sum total of the magnetization in the coil phased by the position dependent phase-factor in Eq. 10 giving:

$$S(t) = C \iiint M_{xy}\,(x,y,z)e^{-2\pi j\, \vec{\mathrm{k}}(t)\cdot\vec{r}}\, dV \qquad \text{Eq. 11}$$

$$= C \iiint M_{xy}\,(x,y,z)e^{-2\pi j\, \mathrm{k}_x\, \mathrm{x}}\, dV$$

$$= C \iiint M_{xy}\,(x,y,z)e^{-j\gamma \mathrm{G}_x\, \mathrm{x\, t}}\, dV$$

where the integral is over the volume of the detector coil and C is a constant that has to do with the peculiarities of how efficient a generator the coil geometry is. Note the detector coil does not localize signal but just sums up all the signal in its volume. Here we have assumed the detector efficiency is constant over the volume, but often its spatial variation is just lumped in with M_{xy}, and produces a shading on the reconstructed image of the true M_{xy}. Since only a constant G_x gradient is on, $\vec{\mathrm{k}}(t)\cdot\vec{r} = k_x\, x = \frac{1}{2\pi}\gamma G_x\, x\, t$. Since the exponential does not have any y or z dependence, we can do the y and z integral. Let:

$$\widetilde{M}_0(x) = \iint M_{xy}(x,y,z)\,\mathrm{dy\, dz} \qquad \text{Eq. 12}$$

be the integral over these other two directions. Let's take a second to interpret $\widetilde{M}_{xy}(x)$. We call $\widetilde{M}_{xy}(x)$ the "1D projection image" of the object because it represents the total amount of water contributing to the signal at each x location. If we can calculate $\widetilde{M}_{xy}(x)$, we will have successfully encoded the amount of water as a function of x, but have not resolved where it comes from in y and z. Going back to the signal, then Eq. 11 becomes:

$$S(t) = \int \widetilde{M}_{xy}(x)\, e^{-j\gamma \mathrm{G}_x\, \mathrm{x\, t}}\, dx \qquad \text{Eq. 13}$$

Now we make a significant change in how we view the problem. I will switch back to using the variable k, instead of t to write the signal as a function of k instead of t:

$$S(k) = \int \widetilde{M}_{xy}(x)\, e^{-2\pi j \mathrm{k}_x\, \mathrm{x}}\, dx \qquad \text{Eq. 14}$$

Note that for the constant gradient case, k and t are proportional, so it's just an innocent change of variables. Or is it? Actually it's a paradigm shift in how we view our signal. We will view it not as a discrete vector in time (although it was originally sampled in time), but change variables to k, and view it as a discrete sampling in k. For constant amplitude gradients this is just a scaling.

Lets interpret this as a discrete time sampled signal, and the integral as a discrete summation over a discrete sampled; Δx . As before, let n be the integer index for our N time samples (t = nΔt) but let u be the integer index of k. Then S[u] is the recorded discrete voltage signal, trivially related to the discrete time signal for constant applied gradient. For the discrete vector representing the object, we define the integer index a= -N/2, -N/2 +1, … -2, -1, 0, 1, 2, 3,… (N/2 -1) as the corresponding integer index for x. x = aΔx. Apologies that "u" and "a" don't usually represent integer indexes, but the usual ones, I, j, k, l, m, n are taken. Then:

$$S[u] = \sum_{a=0}^{N-1} \widetilde{M}_{xy}[a] e^{-\frac{2\pi j}{N} \mathrm{a\,u}} \qquad \text{Eq. 15}$$

This equation should be recognizable. The recorded signal, recast as a discrete time signal as a function of k, is represented as a DFS. We know how to calculate $\widetilde{M}_{xy}[a]$, which is the discrete representation of the function related to our object, $M_{xy}(x, y, z)$ by Eq. 12:

$$\widetilde{M}_{xy}[a] = \sum_{u=-N/2}^{N/2-1} S[u] e^{\frac{2\pi j}{N} \mathrm{a\,u}} \qquad \text{Eq.16}$$

We have now calculated, $\widetilde{M}_0[a]$, the discrete representation of the "1D projection image" of the object. It represents the total amount of water contributing to the signal at each x location (where a is the integer index for x location).

Notation for the MRI section Lets take a step back and organize how we are going to notate the different variables and functions we are interested in, paying special attention to pairs of variables and functions that are related through the DFS. We have already gotten into trouble since PreLab 1 used the notation of previous labs; x[n] for the time domain FID signal and X[k] for the frequency domain spectrum where n and k were integer indexes for time and temporal frequency respectively. Then in this PreLab we are using k for a completely different thing.

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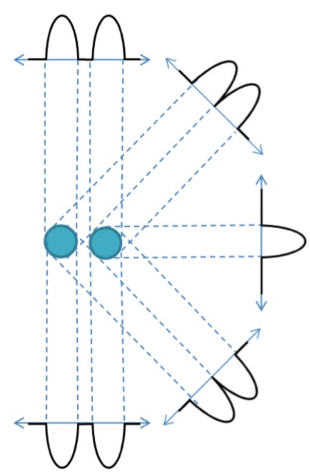
Going forward we will use:

Cont. function	**Cont. variable**	**Description**	**Discrete function**	**Corr. Integer index**
f(t)	Time, t	Free Induction Decay (FID) of precessing M_0	f[n]	n
F(ω)	Temp. angular frequency, ω	Spectrum of the MR signal	F[l]	l
$S(k_x, k_y, k_z)$	k-space location, k_x, k_y, k_z	Complex MR signal (phase and ampl) as a function of k-space location (as determined by the gradient history.	S[u,v,p]	u, v, p
$M_{xy}(x,y,z)$	Spatial location; x,y,z	Complex magnetization vector (ampl. And phase) in the x,y plane (moving component) detected by RF coil	M_{xy}[a,b,c]	a,b,c

Then f[n] and F[l] are related thru the DFS, as are S[u] and M_{xy}[a]. The measured quantities in MRI are either f[n] or S[u] and then we determine F[l] or M_{xy}[a] from the DFS.

Figure 1: Example of 5 different projection images of a phantom containing two spheres. For a sparse object like this, the actual 2D object can be readily determined from just a few projection images.



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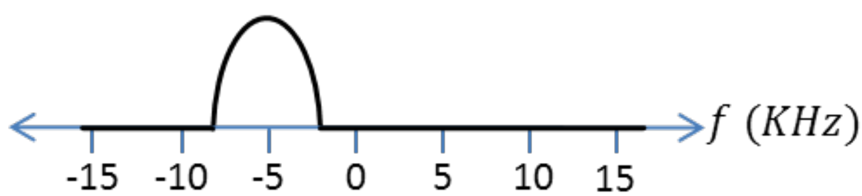
Prelab problems:

Prelab problem #1:

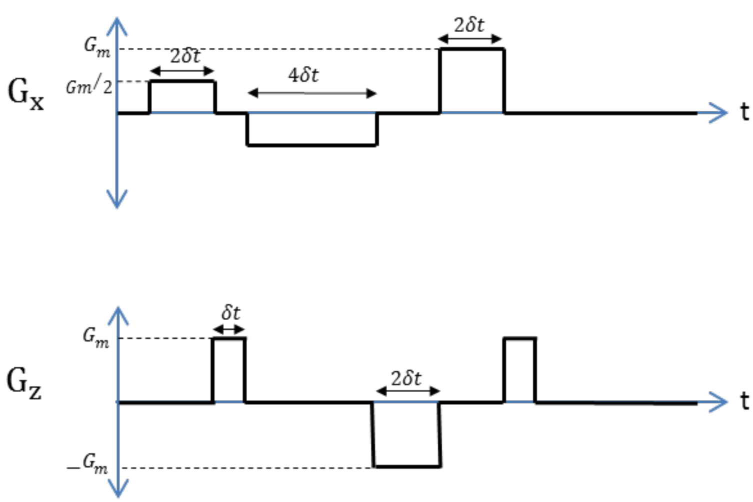
Below is a 1D projection image (frequency domain representation of a spin echo when a gradient G_r is present) of a 5mm diameter tube of water. From the width of the spectrum, estimate G_r in mT/m. From the offset of the spectrum from 0 KHz, estimate the offset of the phantom from the magnet's isocenter (in m).

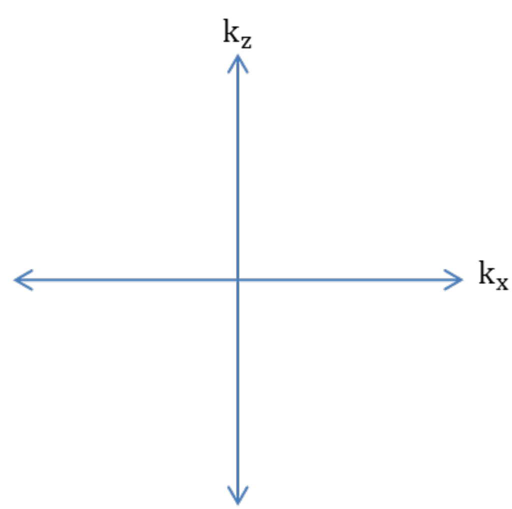
(Hint: gamma = 42.577 kHz/mT, 0 KHz represents the Larmor frequency present at the isocenter of the magnet.)



Prelab problem #2:

Using the G_x and G_z waveforms below, plot the k_x and k_z trajectory as a path in the k_x, k_z plane. Assume





that at time t=0, $k_x = k_z = 0$.