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A Theorem on Numbers of the Form 10^X

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Abstract

Number theory is one of the core branches of pure mathematics. It has played an important role in the study of natural numbers. In this paper, we are presenting a theorem on the numbers of form 10^X, where $x \in \mathbb{Z}^+$. The proposed theorem have a major application in computer science. It can be used to predict 'n' bits which will always represent more than 10^X total numbers. We proved that the nature of the 'n' bits is always one of the forms $10i$, $10i + 4$, or $10i + 7$, where $i \in W$.

Keywords: *Number theory, Binary Number System, Modular Arithmetic,* 10^X

1. Introduction

Number theory is one of the oldest fields of pure mathematics. It covers board topics dealing with theories focused on subsets of real numbers such as positive integers, rational numbers, and natural numbers (Kraft and Washington, 2018). Number theory also deals with the diverse subtopics of modular arithmetic (Inam and Büyükaþýk, 2019), and prime numbers (Flath, 2018).

In this paper, along with the mathematical proof, an application of our proposed theorem is also discussed in computer science.

2. Proposed Theorem

For any number of the form 10^X, where $x \in \mathbb{Z}^+$ the following mathematical expression is always true.

$$2^{\left(\left(10\left\lfloor \frac{x}{3} \right\rfloor\right)+4\left((x \bmod 3) \bmod 2\right)+7\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor\right)} > 10^x \text{, where } x \in \mathbb{Z}^+.$$

2.1 Mathematical Proof

In a decimal number system, we know that:

$$2^{10} > 10^3 \qquad \text{...(1)}$$

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$2^4 > 10$ ...(2)

$2^7 > 10^2$ ...(3)

From Equations (1), (2) and (3), it can be concluded that,

$2^{10i} > 10^{3i}$ where $i \in \mathbb{Z}^+$ ...(4)

$2^{4j} \geq 10^{j}$ where $j \in \{0,1\}$ ...(5)

$2^{7k} \geq 10^{2k}$ where $k \in \{0,1\}$ ...(6)

On combining Equations (4), (5) and (6), a new combined form is obtained.

$2^{10i+4j+7k} > 10^{3i+j+2k}$ ...(7)

Since, $0 \leq (x \bmod 3) \leq 2$, it can be said that:

$0 \leq ((x \bmod 3) \bmod 2) \leq 1$, where $x \in \mathbb{Z}^+$ ...(8)

$$0 \leq \left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor \leq 1, \text{ where } x \in \mathbb{Z}^+ \qquad ...(9)$$

In Equation (7), we substitute i with $\left\lfloor \frac{x}{3} \right\rfloor$, j is substituted with $((x \bmod 3) \bmod 2)$ using Equation (8), and k is substituted with $\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor$ using Equation (9). After performing all these substitutions, following equation is obtained.

$$2^{\left(\left(10\left\lfloor \frac{x}{3} \right\rfloor\right)+4((x \bmod 3) \bmod 2)+7\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor\right)} > 10^{\left(\left(3\left\lfloor \frac{x}{3} \right\rfloor\right)+((x \bmod 3) \bmod 2)+2\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor\right)}, \ x \in \mathbb{Z}^+ \qquad ...(10)$$

Equation (10) can further be simplified using a well-known relationship (Meidânis, 1990) of modular arithmetic.

$$x \bmod y = x - y\left\lfloor \frac{x}{y} \right\rfloor \qquad ...(11)$$

RHS of equation 10 can further be simplified using Equation (11).

$$3\left\lfloor \frac{x}{3} \right\rfloor = x - (x \bmod 3) \qquad ...(12)$$

$$2\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor = (x \bmod 3) - ((x \bmod 3) \bmod 2) \qquad ...(13)$$

Now simplifying Equation (10), using Equations (12) and (13).

$$2^{\left(\left(10\left\lfloor \frac{x}{3} \right\rfloor\right)+4((x \bmod 3) \bmod 2)+7\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor\right)} > 10^{(x-(x \bmod 3)+((x \bmod 3) \bmod 2)+(x \bmod 3)-((x \bmod 3) \bmod 2))}$$

i.e.,

$$2^{\left(\left(10\left\lfloor \frac{x}{3} \right\rfloor\right)+4((x \bmod 3) \bmod 2)+7\left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor\right)} > 10^{x}, \text{ where } x \in \mathbb{Z}^+ \qquad ...(14)$$

This proves that our proposed theorem on numbers of the form 10^x is mathematically correct.

3. Applications in Computer Science

In digital computers, our proposed theorem proves that 'n' bits of the form $10i$, $10i+4$, or $10i+7$, where $i \in W$ can always represent numbers greater than 10^x.

$$2^n > 10^x \text{, where } x \in \mathbb{Z}^+ \quad \text{...(15)}$$

In Equation (15), for a given value of *x*, value of *n* bits can be found using the proposed theorem presented in Equation (14).

$$n = \left(\left(10 \left\lfloor \frac{x}{3} \right\rfloor \right) + 4\left((x \bmod 3) \bmod 2 \right) + 7 \left\lfloor \frac{(x \bmod 3)}{2} \right\rfloor \right) \text{, where } x \in \mathbb{Z}^+$$

It showed that in a digital computer, for a given x, "*n*" bits will always be able to represent a number greater than 10^X, where $x \in \mathbb{Z}^+$. For ease of implementation, a computer program of the proposed theorem is provided open source in the github repository (Github Repository, 2019).

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