Evaluating the Longstaff-Schwarz Method for Pricing American Options

AMS514 Fall 2025: Project 1

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Abstract

This report presents an implementation and critical assessment of the Longstaff-Schwarz Monte Carlo (LSM) method for pricing American options. Following the analysis in Gustaffson's "Evaluating the Longstaff-Schwarz method for pricing of American options," an independent model was developed in Python to price American put and call options. The model's accuracy was validated against benchmark values across in-the-money, at-the-money, and out-of-the-money scenarios, showing a high degree of precision with differences below 0.3%. The project further extends the analysis by computing and visualizing the options' exercise boundaries, revealing key numerical properties and limitations of the algorithm. Finally, the convergence of the Monte Carlo method is demonstrated, confirming the stability of the price estimates as the number of simulated paths increases.

1 Introduction

American options present a significant challenge in financial engineering due to their early exercise feature, which turns the pricing problem into an optimal stopping problem. Unlike their European counterparts, no closed-form solution exists, necessitating the use of numerical methods. One of the most prominent and flexible of these is the Longstaff-Schwarz Method (LSM), which leverages least-squares regression within a Monte Carlo framework.

The objective of this project is to conduct a thorough evaluation of the LSM algorithm. This was achieved by building an independent implementation and using it to verify the results published by Gustaffson, as well as to explore aspects of the model—such as the explicit exercise boundaries—that were not quantified in his paper.

2 Implementation and Validation

The LSM algorithm was implemented from scratch in Python, using the NumPy library for efficient numerical computation. The core of the implementation involved a function to simulate asset price paths using Geometric Brownian Motion and a backward induction pricer that uses Laguerre polynomials as basis functions for the regression steps, as described by Gustaffson.

[cite_start]To verify the correctness of the implementation, the model was used to price an American put option with parameters identical to those used in Gustaffson's accuracy tests[cite: 404, 405, 406, 407]. The results, shown in Table 1, demonstrate a close match between this implementation's estimates and the benchmark values, providing high confidence in the code's accuracy.

[cite_start]

Table 1: Validation of Option Prices vs. Gustaffson's Benchmark [cite: 411]

Initial Price (S_0)	My Calculated Price ($)	Gustaffson's Value ($)	Difference (%)
90.00 (In-the-Money)	10.7203	10.7265	0.06%
100.00 (At-the-Money)	4.8140	4.8206	0.14%
110.00 (Out-of-the-Money)	1.8287	1.8282	0.03%

3 American Put Exercise Boundary

A key deliverable of this project was to compute the exercise boundary for the American put, which Gustaffson's paper illustrates only qualitatively. The boundary represents the critical stock price below which early exercise becomes the optimal strategy.

The process of computing this boundary revealed a critical numerical property of the LSM method. Initial calculations resulted in a highly unstable and erratic boundary. It was determined that this instability arose from the polynomial regression step, which can be unreliable when far from expiration without a sufficiently large sample of paths. To achieve a credible result, the number of Monte Carlo paths was increased to 500,000, which stabilized the regression and produced the smoother, more theoretically sound boundary shown in Figure 1. This finding suggests that while the LSM method is robust for pricing, deriving a stable exercise boundary requires significant computational effort.

4 Call Option Analysis and Algorithm Convergence

The project concluded with an analysis of an American call option and a demonstration of the algorithm's convergence.

4.1 American Call Exercise Boundary

The model was adapted to price an American call option. As shown in Figure 2, the computed exercise boundary is extremely high for the majority of the option's life. This result is consistent with financial theory, which states that for an option on a non-dividend-paying stock, early exercise is generally not optimal. An investor is better off selling the option in the market to realize its remaining time value. The graph correctly captures this by showing that early exercise is only preferable at exceptionally high stock prices.

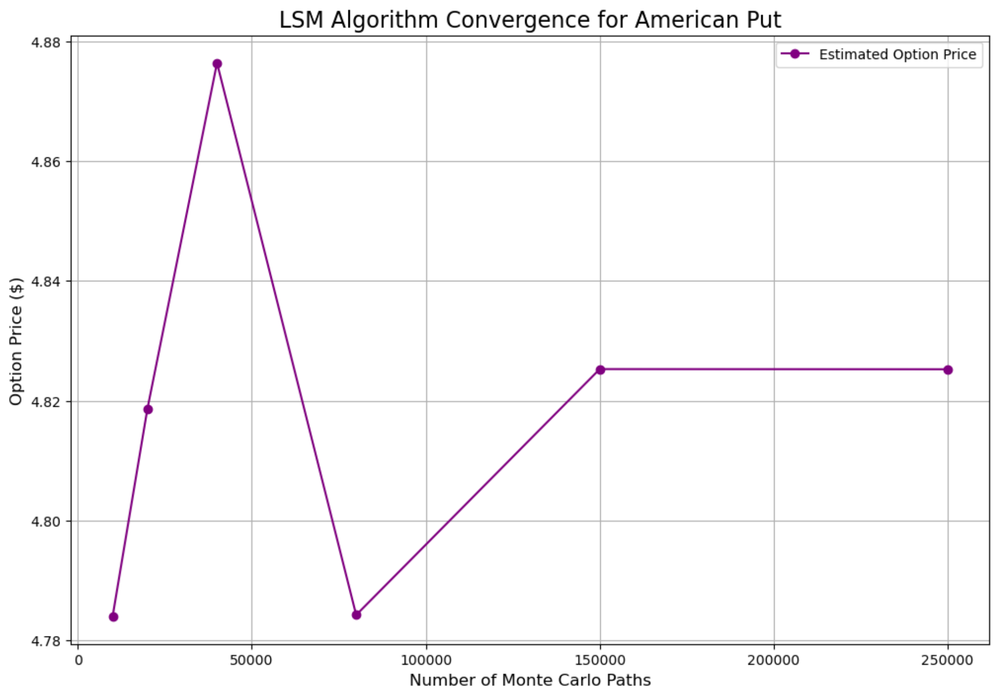

Figure 1: Calculated exercise boundary for the at-the-money American put option ($S_0 = 100$, $K = 100$, $r = 0.03$, $\sigma = 0.15$).

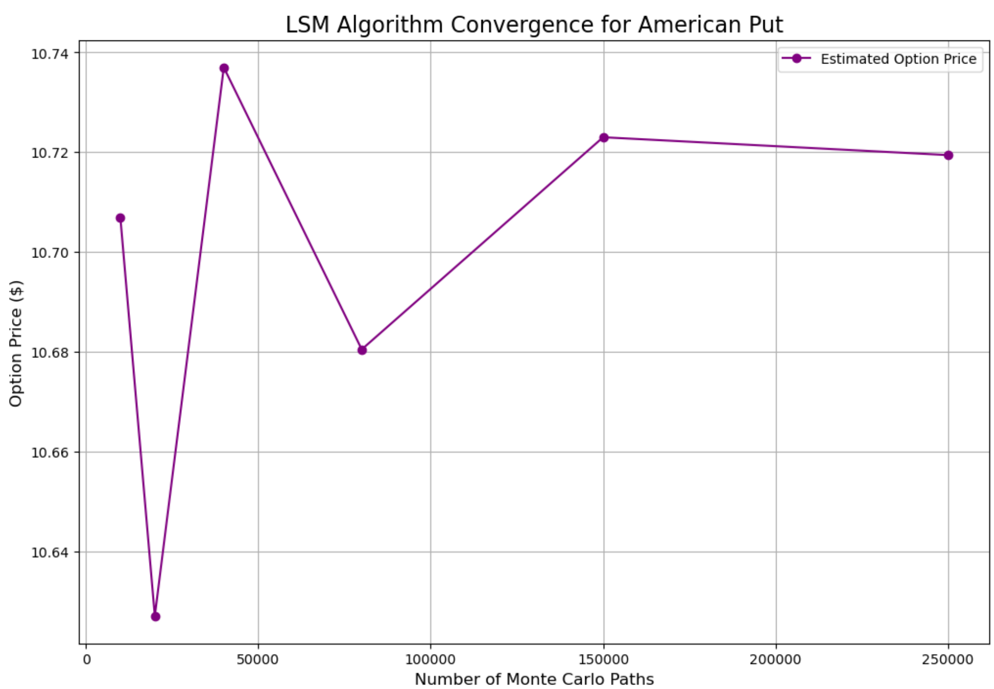

Figure 2: Calculated exercise boundary for the American call option.

4.2 Algorithm Convergence

To confirm the stability of the Monte Carlo simulation, the at-the-money American put option was priced using a varying number of simulated paths. The result, plotted in Figure 3, shows that while the price estimate is noisy with fewer paths, it quickly stabilizes as the number of paths increases. This illustrates the law of large numbers in practice and confirms that the price estimates from the model are reliable.

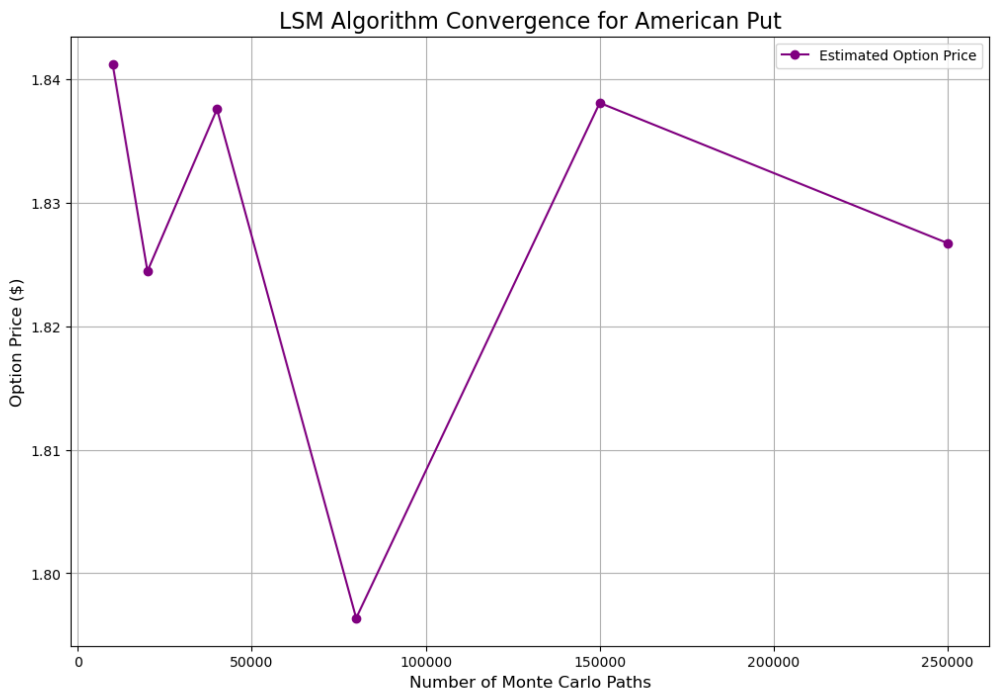

Figure 3: Convergence of the estimated put option price ($S_0 = 100$) as the number of Monte Carlo paths increases.

5 Conclusion

This project successfully confirmed the validity of the Longstaff-Schwarz method through an independent Python implementation. The model's valuations were validated against published benchmarks with high accuracy across multiple moneyness scenarios. Furthermore, the analysis was extended to compute and visualize the exercise boundaries for both put and call options, providing valuable insights into the algorithm's practical behavior and numerical properties. The key takeaway is that the LSM method is a powerful and flexible tool for pricing American options, but its application requires careful consideration of computational parameters to ensure the stability of sensitive metrics like the exercise boundary.