Name :- Kirat Bir Singh
Class :- 10th
Roll Number :- 57

Subject :- Mathematics
Submitted To :- Vishal Sir
School Name :- Shri Guru Harkrishan Public School, Patiala

Chapter :- 1
Real Numbers
Important Terms

R = Real Numbers:
All rational and irrational numbers are called real numbers.

I = Integers:
All numbers from (…-3, -2, -1, 0, 1, 2, 3…) are called integers.

Q = Rational Numbers:
Real numbers of the form pq, q ≠ 0, p, q ∈ I are rational numbers.

All integers can be expressed as rationals, For example , 5 = 5/1

Q’ = Irrational Numbers:

Real numbers which cannot be expressed in the form pq and whose decimal expansions are non-terminating and non-recurring.

N = Natural Numbers:
Counting numbers are called natural numbers. N = {1, 2, 3, …}

W = Whole Numbers:
Zero along with all natural numbers are together called whole numbers. {0, 1, 2, 3,…}

Even Numbers:
Natural numbers of the form 2n are called even numbers. (2, 4, 6, …}

Odd Numbers:
Natural numbers of the form 2n -1 are called odd numbers. {1, 3, 5, …}

Prime Numbers:
The natural numbers greater than 1 which are divisible by 1 and the number itself are called prime numbers, Prime numbers have two factors i.e., 1 and the number itself For example, 2, 3, 5, 7 & 11 etc.

Composite Numbers:
The natural numbers which are divisible by 1, itself and any other number or numbers are called composite numbers. For example, 4, 6, 8, 9, 10 etc.

Remember This!

All natural numbers are whole numbers.

All whole numbers are integers.

All integers are rational numbers.

All rational numbers are real numbers.

Euclid's Division lemma

Given two positive integers a and b, there exist unique integers q and r satisfying a = bq + r, 0 ≤ r ≤ b.

Notice this. Each time 'r' is less than b. Each 'q' and 'r' is unique.

Application of lemma

Euclid's Division lemma is used to find HCF of two positive integers. Example: Find HCF of 56 and 72 ?

Steps:

Apply lemma to 56 and 72.

Take bigger number and locate 'b' and 'r'. 72 = 56 × 1 + 16

Since 16 ≠ 0, consider 56 as the new dividend and 16 as the new divisor. 56 = 16 × 3 + 8

Again, 8 ≠ 0, consider 16 as new dividend and 8 as new divisor. 16 = 8 × 2 + 0

Since remainder is zero, divisor (8) is HCF.

Although Euclid's Division lemma is stated for only positive integers, it can be extended for all integers except zero. i.e.. b ≠ 0.

What does this mean

a		b		q		r
18	=	2	×	9	+	0
18	=	3	×	5	+	3
18	=	5	×	3	+	3
18	=	7	×	2	+	4
18	=	11	×	1	+	7

Constructing a factor tree :-

Steps

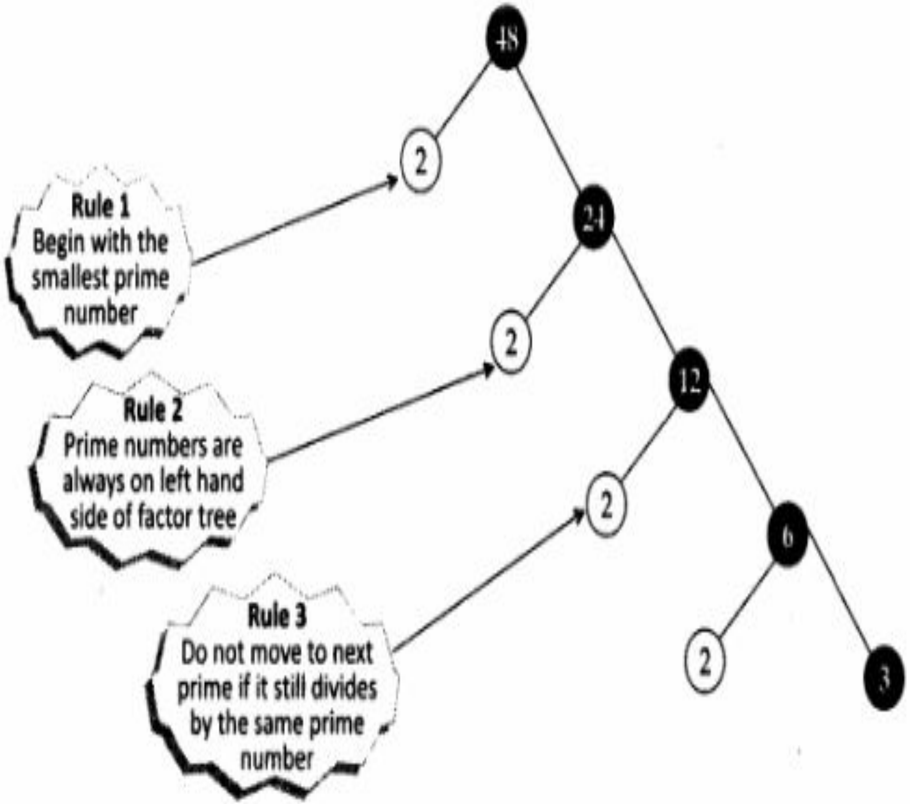
Write the number as a product of prime number and a composite number

Example:

Factorize 48

Repeat the process till all the primes are obtained

∴ Prime factorization of 48 = 24 x 3



1. Algorithm to locate HCF and LCM of two or more positive integers:

Step I:
Factorize each of the given positive integers and express them as a product of powers of primes in ascending order of magnitude of primes.
Step II:
To find HCF, identify common prime factor and find the least powers and multiply them to get HCF.
Step III:
To find LCM, find the greatest exponent and then multiply them to get the LCM.

Alert!
2^3 can be written as:
$2^3 = 2^3\,5^0$
5^2 can be written as:
$5^2 = 2^0\,5^2$

2. To prove Irrationality of numbers:

The sum or difference of a rational and an irrational number is irrational.
The product or quotient of a non-zero rational number and an irrational number is irrational.
3. To determine the nature of the decimal expansion of rational numbers:

Let x = p/q, p and q are co-primes, be a rational number whose decimal expansion terminates. Then the prime factorization of'q' is of the form 2m5n, m and n are non-negative integers.
Let x = p/q be a rational number such that the prime factorization of 'q' is not of the form 2m5n, 'm' and 'n' being non-negative integers, then x has a non-terminating repeating decimal expansion.

Problem Sums

- An army contingent of 616 members is to March behind an army band of 32 members in a parade. The two groups are 2 march in the same number of column what is the maximum number of columns in which they can march.

Answer:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

- Use euclid's vision lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Let a be any positive integer and b = 3.

Then a = 3q + r for some integer q ≥ 0

And r = 0, 1, 2 because 0 ≤ r < 3

Therefore, a = 3q or 3q + 1 or 3q + 2

Or,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$
$$a^2 = (9q^2) \text{ or } 9q^2+6q+1 \text{ or } 9q^2+12q+4$$
$$= 3\times(3q^2) \text{ or } 3(3q^2+2q)+1 \text{ or } 3(3q^2+4q+1)+1$$
$$= 3k_1 \text{ or } 3k_2+1 \text{ or } 3k_3+1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m + 1.

- Use Euclid 's division algorithm to find the HCF of 196 and 38220.

(i) 196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

38220 = 196 × 195 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

Chapter :- 2
Polynomials
Important Terms

- "Polynomial" comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)-so it means many terms.
- A polynomial is made up of terms that are only added, subtracted or multiplied.
- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Degree – The highest exponent of the variable in the polynomial is called the degree of polynomial. Example: 3x^3 + 4, here degree = 3.
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomial respectively.
- A polynomial can have terms which have Constants like 3, -20, etc., Variables like x and y and Exponents like 2 in y^2.
- These can be combined using addition, subtraction and multiplication but NOT DIVISION.
- The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x-axis.
- If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

Sum of zeroes, $\alpha+\beta = -b/a$
Product of zeroes, $\alpha\beta = c/a$

If α, β, γ are the zeroes of the cubic polynomial ax3 + bx2 + cx + d = 0, then

$\alpha+\beta+y = -b/a$
$\alpha\beta+\beta y+y\alpha = c/a$
and, $\alpha\beta y = -d/a$

Zeroes (α, β, γ) follow the rules of algebraic identities, i.e.,

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$\therefore (\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$$

DIVISION ALGORITHM:
If p(x) and g(x) are any two polynomials with g(x) ≠ 0, then
$p(x) = g(x) \times q(x) + r(x)$
Dividend = Divisor x Quotient + Remainder

Remember this!
If r (x) = 0, then g (x) is a factor of p (x).
If r (x) ≠ 0, then we can subtract r (x) from p (x) and then the new polynomial formed is a factor of g(x) and q(x).

Problem Sums

Find the zeros of the following quadratic polynomial and verify the relationship between the zero and the coefficients.

1. x^2 - 2x - 8
2. 4s^2 - 4s + 1

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x - 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = -2

Therefore, the zeroes of x^2-2x-8 are 4 and -2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

Chapter :- 3
Pair Of Linear Equations In Two Variables
Important Terms

- For any linear equation, each solution (x, y) corresponds to a point on the line. General form is given by $ax + by + c = 0$.
- The graph of a linear equation is a straight line.
- Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is: $a1x + b1y + c1 = 0$; $a2x + b2y + c2 = 0$
- where a1, a2, b1, b2, c1 and c2 are real numbers, such that $a12 + b12 \neq 0$, $a22 + b22 \neq 0$.
- A pair of values of variables 'x' and 'y' which satisfy both the equations in the given system of equations is said to be a solution of the simultaneous pair of linear equations.
- A pair of linear equations in two variables can be represented and solved, by
- (i) Graphical method
- (ii) Algebraic method

- (i) Graphical method. The graph of a pair of linear equations in two variables is presented by two lines.
- (ii) Algebraic methods. Following are the methods for finding the solutions(s) of a pair of linear equations:
-
- Substitution method
- Elimination method
- Cross-multiplication method.
- There are several situations which can be mathematically represented by two equations that are not linear to start with. But we allow them so that they are reduced to a pair of linear equations.
- Consistent system. A system of linear equations is said to be consistent if it has at least one solution.
- Inconsistent system. A system of linear equations is said to be inconsistent if it has no solution.

CONDITIONS FOR CONSISTENCY

Let the two equations be:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Then,

Relationship between coeff. or the pair of equations	Graph	Number of Solutions	Consistency of System
a1/a2 is not equal to b1/b2	Intersecting lines	Unique solution	Consistent
a1/a2 is equal to b1/b2 is not equal to c1/c2	Parallel lines	No solution	Inconsistent
a1/a2 is equal to b1/b2 is equal to c1/c2	Co-incident lines	Infinite solutions	Consistent

Problem Sums

- 10 students of class 10th took part in mathematics quiz. If the number of the girls is 4 more than the number of boys find the number of boys and the girls who took part in the quiz..
- 5 pencils and 7 pens together cost Rupees 50 whereas 7 pencils and 5 pens together cost rupees 46. Find the cost of one pencil and the that of one pen.

(i) Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

x + y = 10

x − y = 4

For x + y = 10,

x = 10 − y

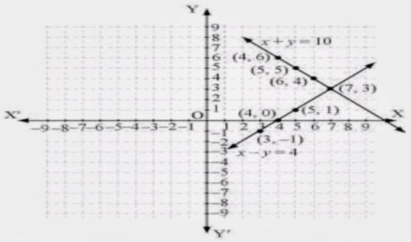
x	5	4	6
y	5	6	4

For x − y = 4,

x = 4 + y

x	5	4	3
y	1	0	−1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y.

According to the question, the algebraic representation is

5x + 7y = 50

7x + 5y = 46

For 5x + 7y = 50,

$$x = \frac{50 - 7y}{5}$$

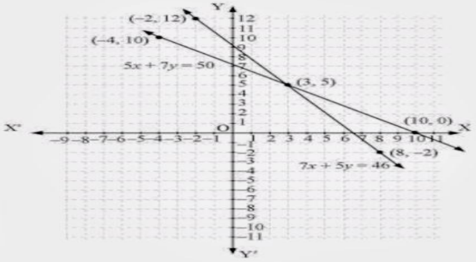
x	3	10	− 4
y	5	0	10

7x + 5y = 46

$$x = \frac{46 - 5y}{7}$$

x	8	3	− 2
y	− 2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Chapter :- 6

Topic :-
Converse Of BPT Theorem

If a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side.

Given :- ABC is a triangle.

AD:DB = AE:EC

To Prove :- DE||BC

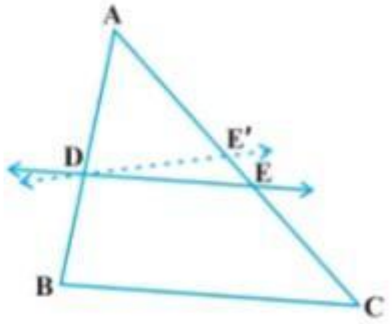
Construction :- Take a point E' on side AC.

Proof :- Let's assume DE not parallel to BC.

If it is not parallel to BC take DE' parallel to BC.

Now, AD:AB = AE':E'C (By BPT) - 1

But, AD:DB = AE:EC (Given) - 2



From 1 and 2 :-

AE:EC = AE':E'C

Now, Add +1 on both sides

AE/EC +1 = AE'/E'C +1

AE+EC/EC = AE'+E'C/E'C

=> AC/EC = AC/E'C

EC=E'C

It shows that E' & E must lie on same point and contradicts our assumption. Therefore DE||BC,

Hence proved..

Thank You

Have A Nice

Day !!!