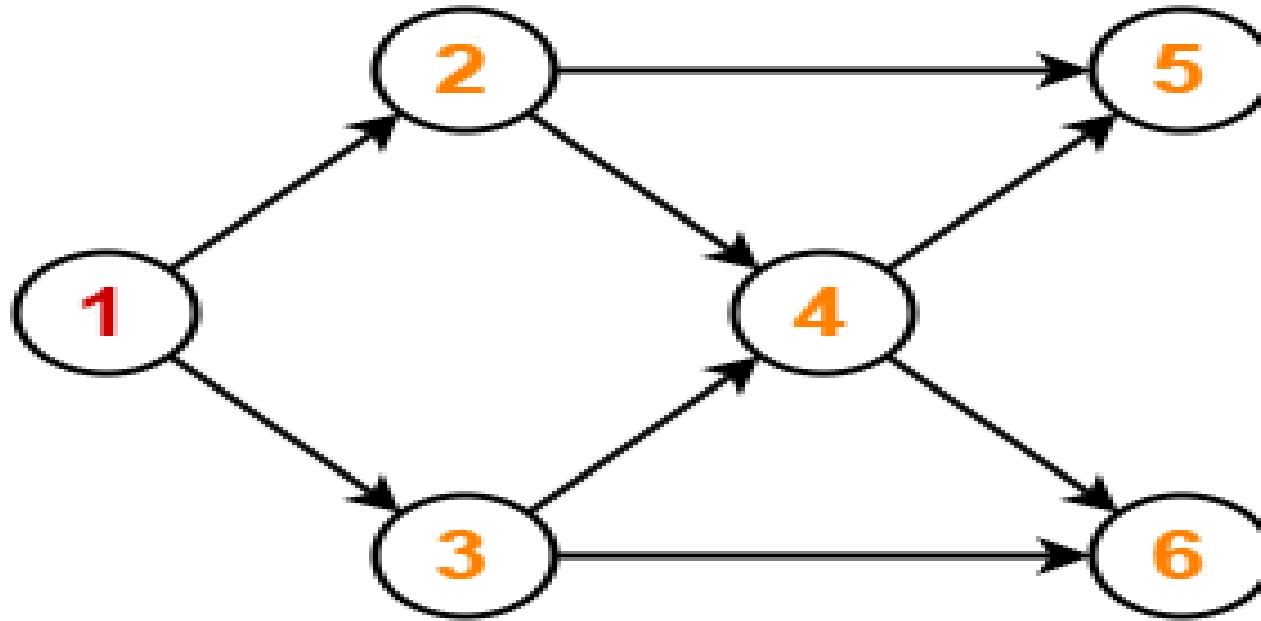


# Topological Sorting

- Topological sorting for Directed Acyclic Graph (DAG) is a linear ordering of vertices such that for every directed edge  $(u,v)$ , vertex  $u$  comes before  $v$  in the ordering.
- A topological sort of a graph is an ordering of its vertices along a horizontal line so that all directed edges go from left to right.
- If the graph contains a cycle, then no linear ordering is possible.
- Topological Sorting for a graph is not possible if the graph is not a DAG.

# Topological Sorting



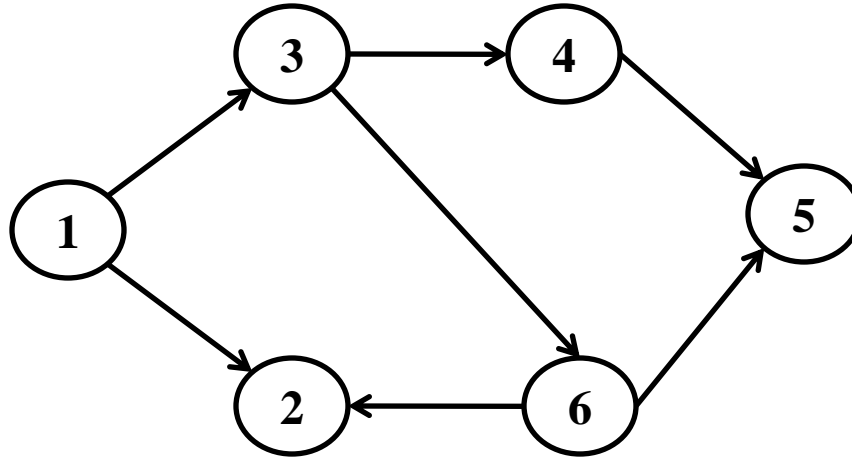
**Topological Sort Example**

One possible Topological Sort=[1,2,3,4,5,6]

# Topological Sorting Algorithm

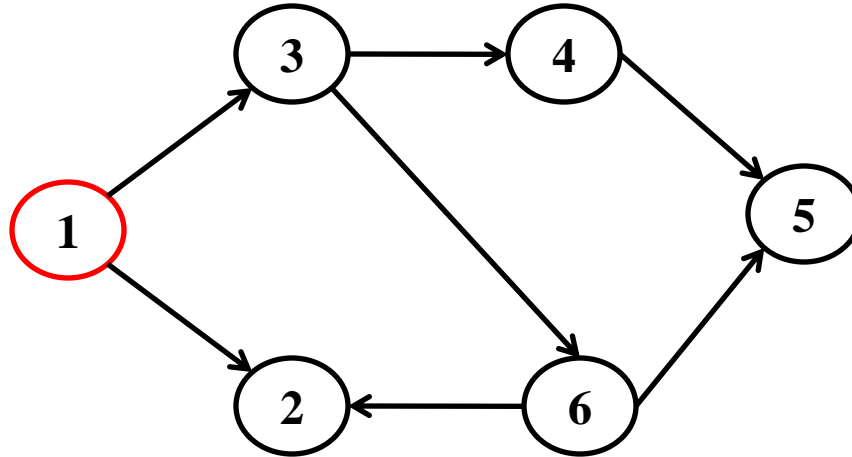
1. Identify a node with no incoming edges( $\text{indegree}=0$ )
2. Add this node to the ordering.
3. Remove this node and all its outgoing edges from the graph.
4. Repeat step 1 to 3 until the graph becomes empty

**Write the topological sorting for the DAG given below**



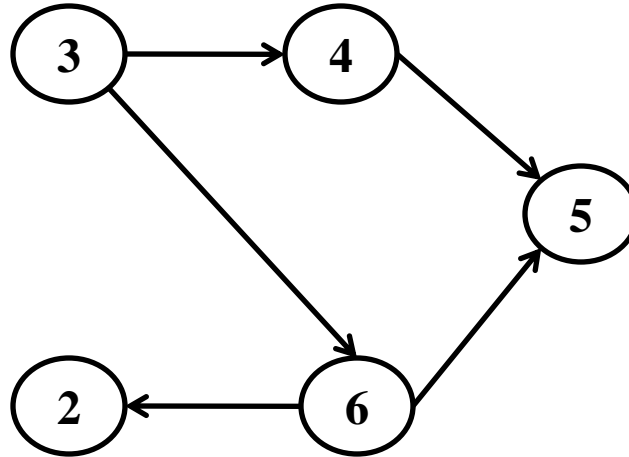
Topological Ordering:

**Write the topological sorting for the DAG given below**



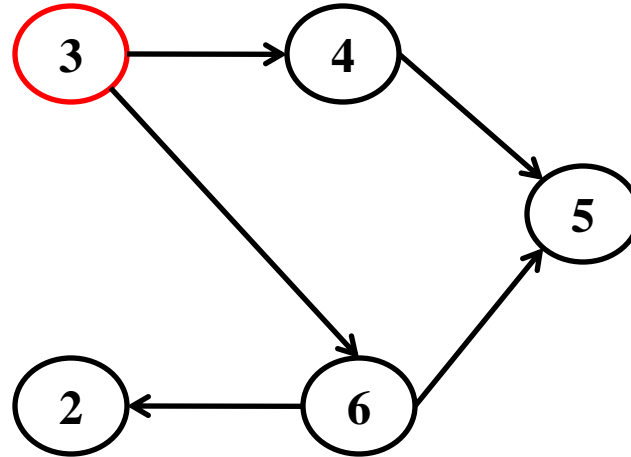
Topological Ordering:1

**Write the topological sorting for the DAG given below**



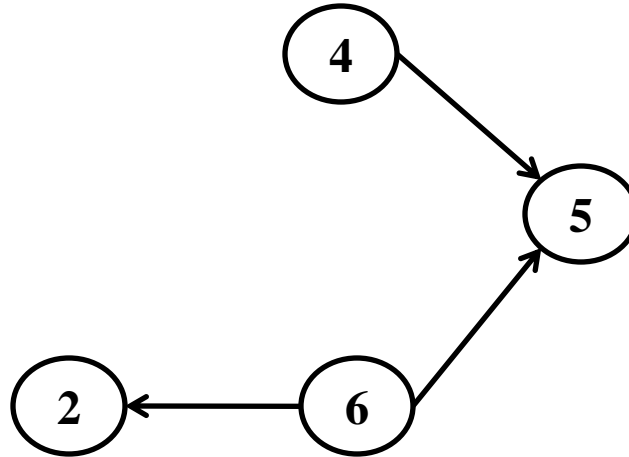
Topological Ordering:1

**Write the topological sorting for the DAG given below**



Topological Ordering: 1, 3

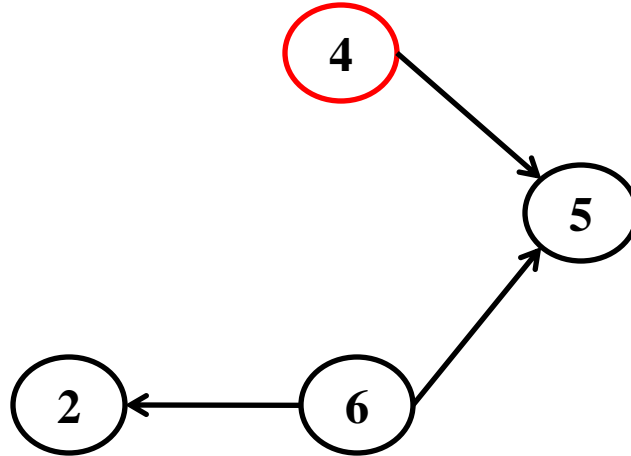
**Write the topological sorting for the DAG given below**



Topological Ordering: 1,3

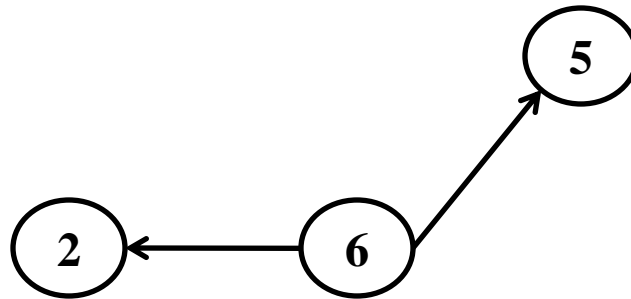


**Write the topological sorting for the DAG given below**



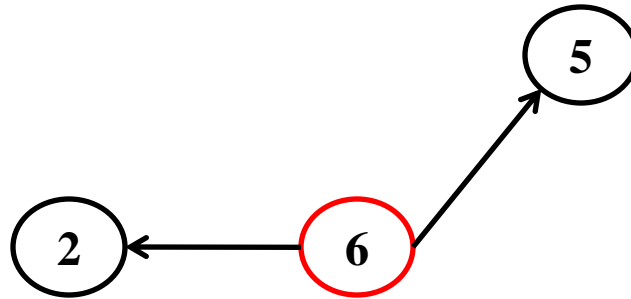
Topological Ordering: 1, 3, 4

**Write the topological sorting for the DAG given below**



Topological Ordering: 1,3,4

**Write the topological sorting for the DAG given below**



Topological Ordering: 1, 3, 4, 6

**Write the topological sorting for the DAG given below**



Topological Ordering: 1, 3, 4, 6

**Write the topological sorting for the DAG given below**



Topological Ordering: 1, 3, 4, 6, 2

**Write the topological sorting for the DAG given below**



Topological Ordering: 1, 3, 4, 6, 2

**Write the topological sorting for the DAG given below**



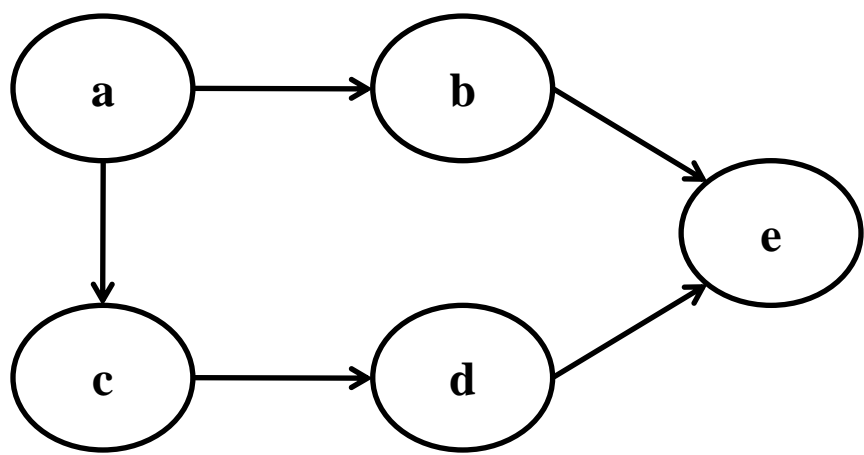
Topological Ordering: 1, 3, 4, 6, 2, 5

**Write the topological sorting for the DAG given below**

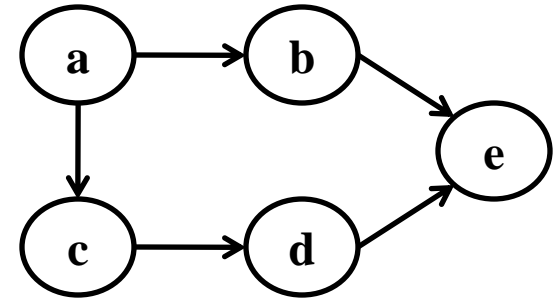
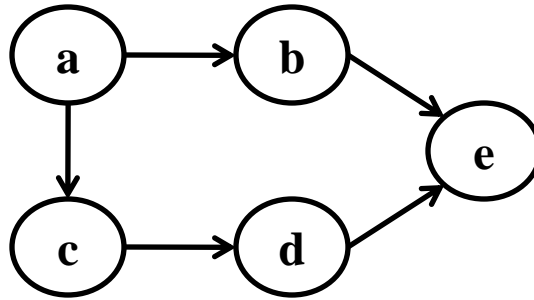
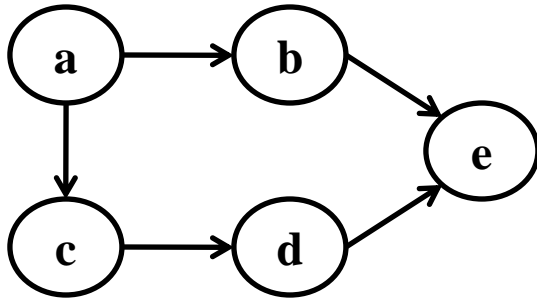
Topological Ordering: 1, 3, 4, 6, 2, 5



**Find the possible topological orderings for the following graph**

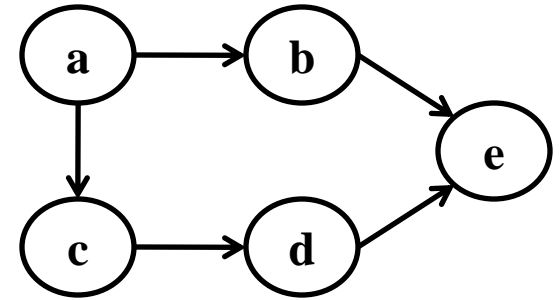
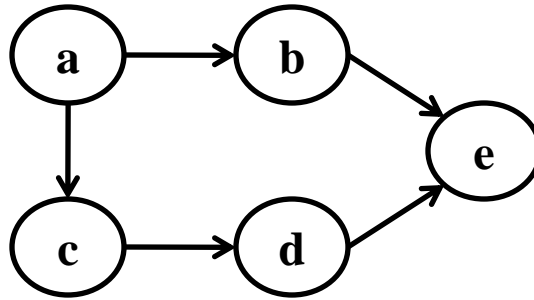
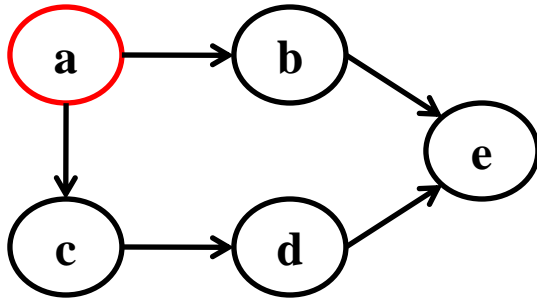


**Find the possible topological orderings for the following graph**



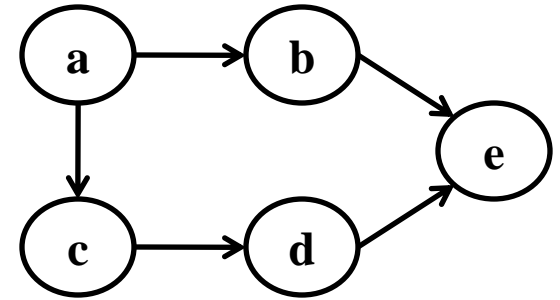
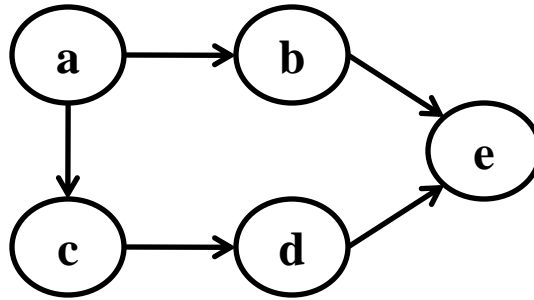
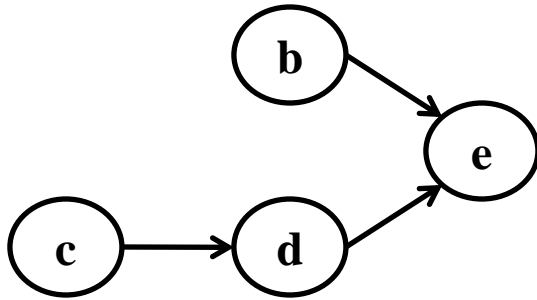
Topological Ordering-1:    Topological Ordering-2:    Topological Ordering-3:

**Find the possible topological orderings for the following graph**



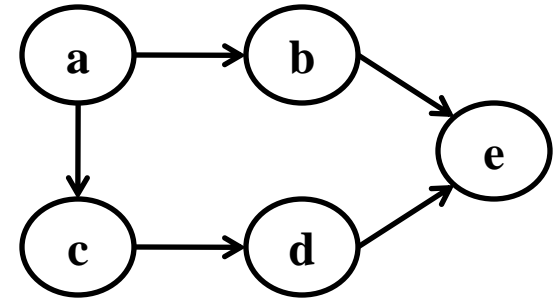
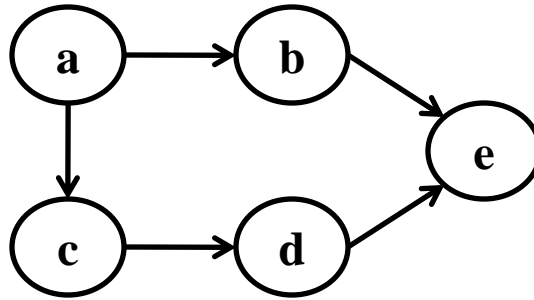
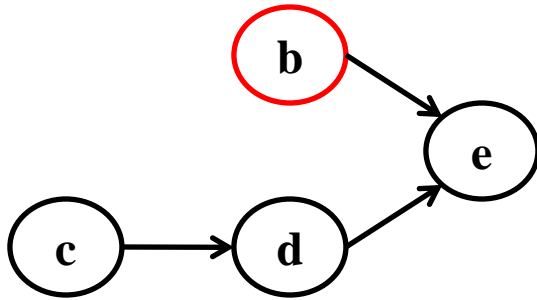
Topological Ordering-1:    Topological Ordering-2:    Topological Ordering-3:  
a

**Find the possible topological orderings for the following graph**



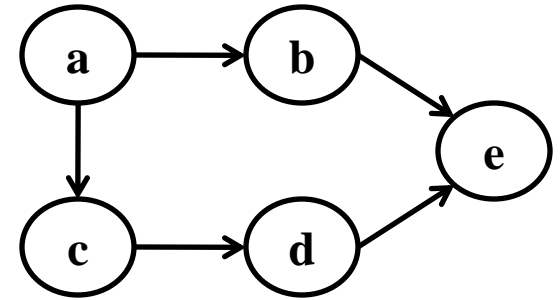
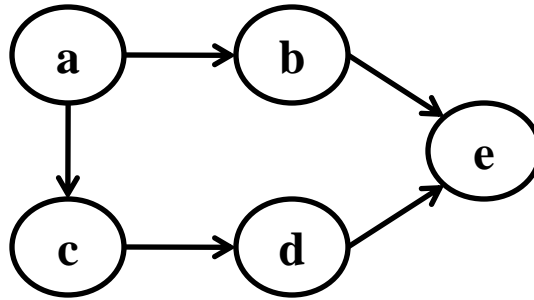
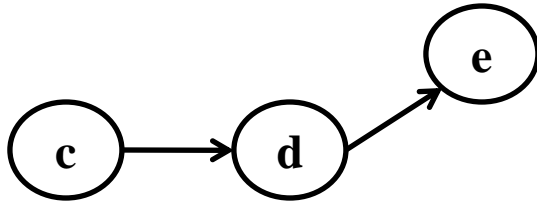
Topological Ordering-1:    Topological Ordering-2:    Topological Ordering-3:  
a

**Find the possible topological orderings for the following graph**



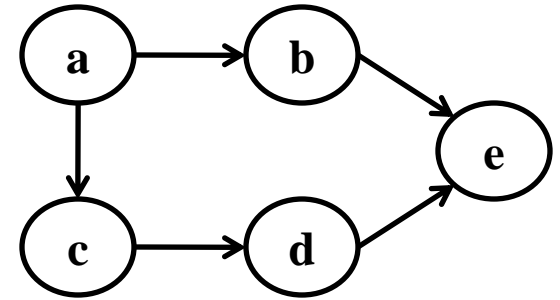
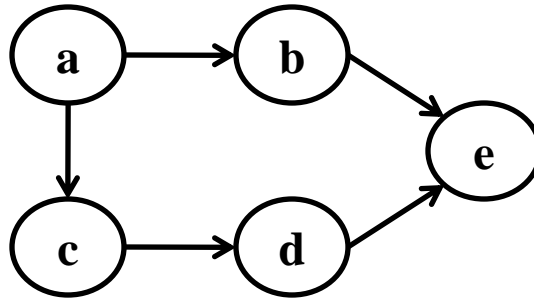
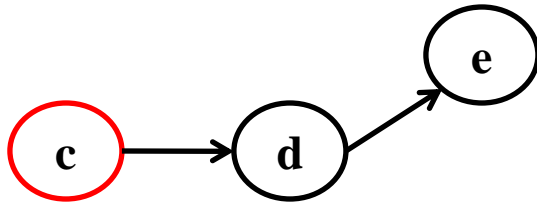
Topological Ordering-1: a,b  
Topological Ordering-2:  
Topological Ordering-3:

**Find the possible topological orderings for the following graph**



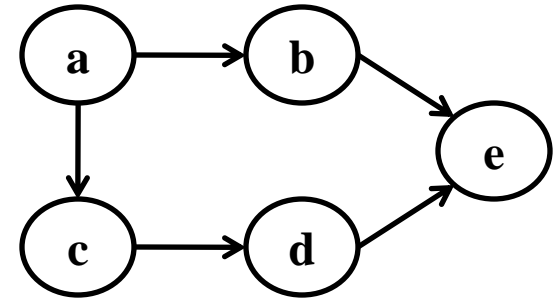
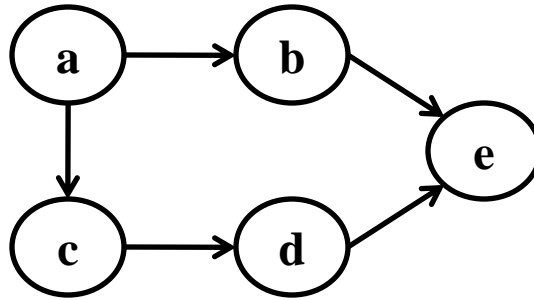
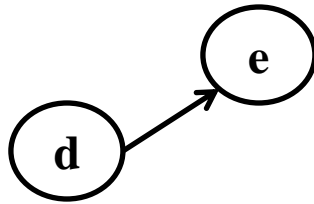
Topological Ordering-1: a,b  
Topological Ordering-2:  
Topological Ordering-3:

**Find the possible topological orderings for the following graph**



Topological Ordering-1: a,b,c    Topological Ordering-2:    Topological Ordering-3:

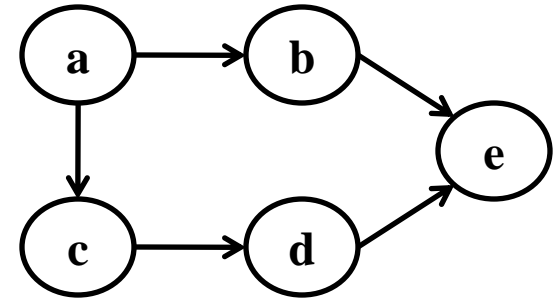
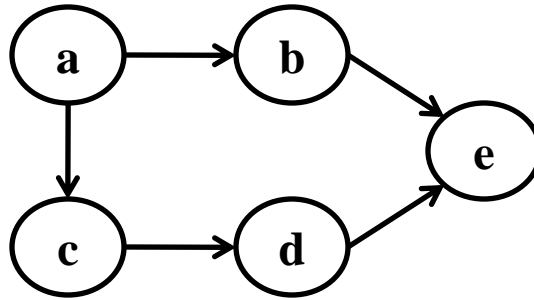
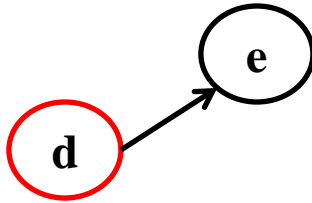
**Find the possible topological orderings for the following graph**



Topological Ordering-1: a,b,c  
Topological Ordering-2:  
Topological Ordering-3:

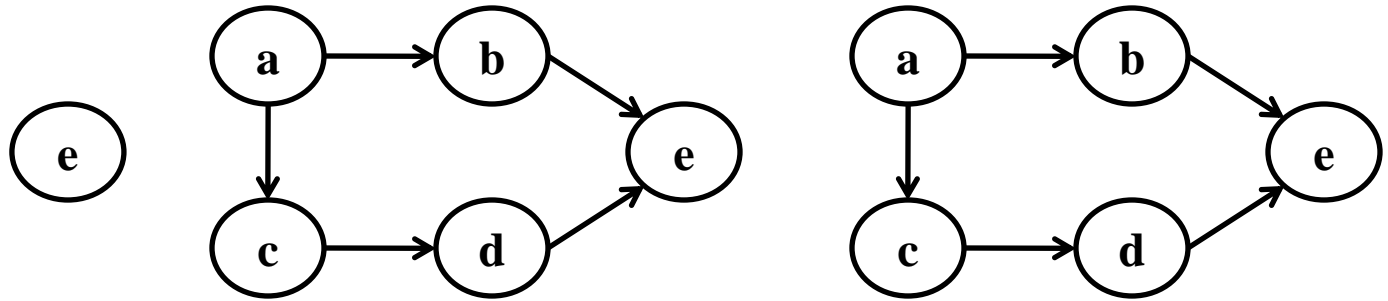


**Find the possible topological orderings for the following graph**



Topological Ordering-1: a,b,c,d    Topological Ordering-2:    Topological Ordering-3:

**Find the possible topological orderings for the following graph**

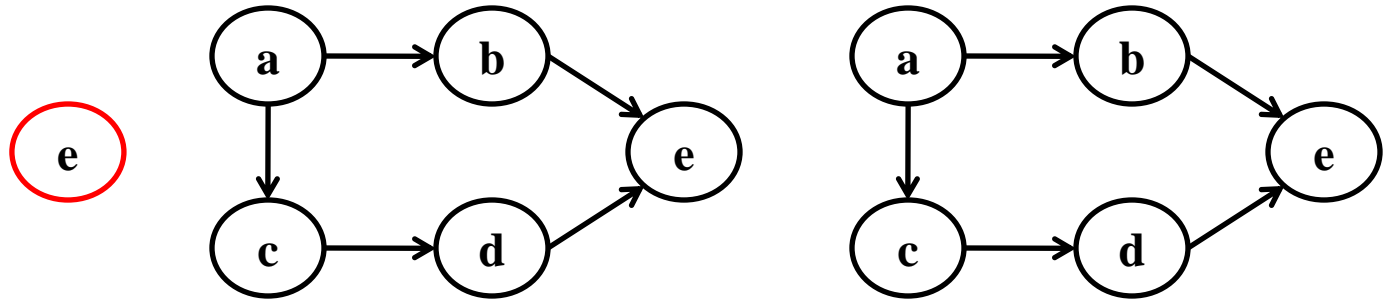


Topological Ordering-1:  
a,b,c,d

Topological Ordering-2:

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

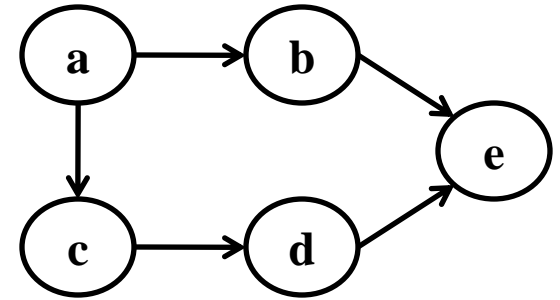
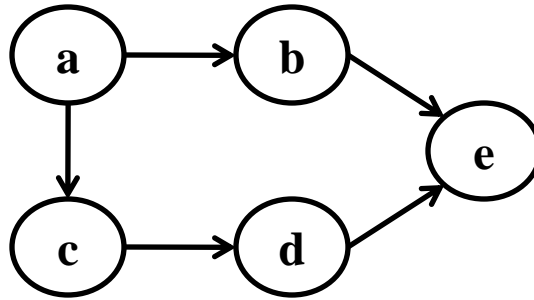


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:

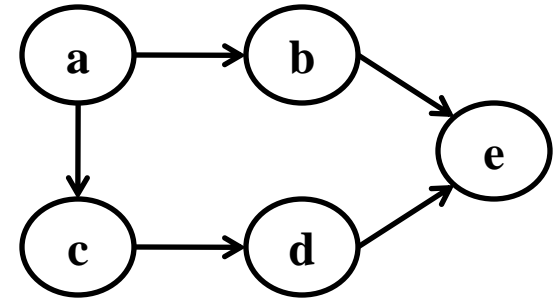
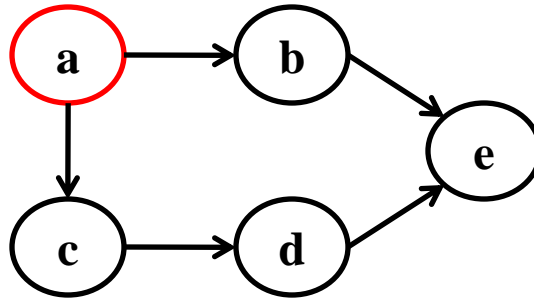
Topological Ordering-3:

**Find the possible topological orderings for the following graph**



Topological Ordering-1: a,b,c,d,e    Topological Ordering-2:    Topological Ordering-3:

**Find the possible topological orderings for the following graph**

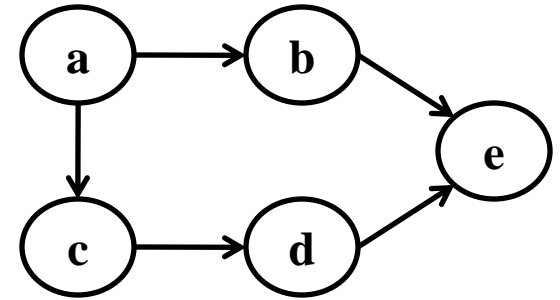
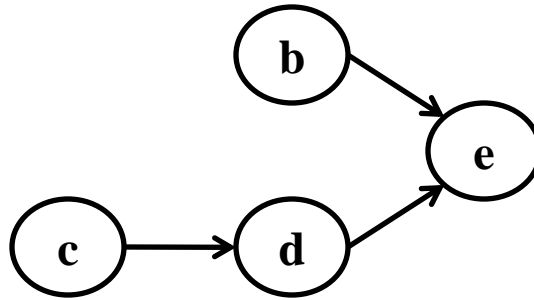


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

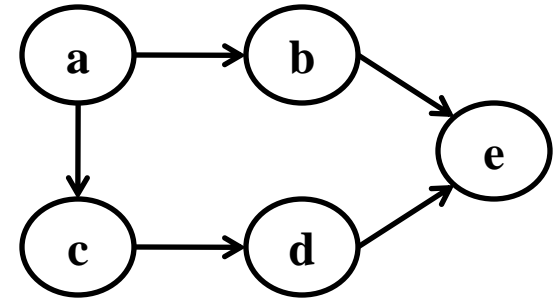
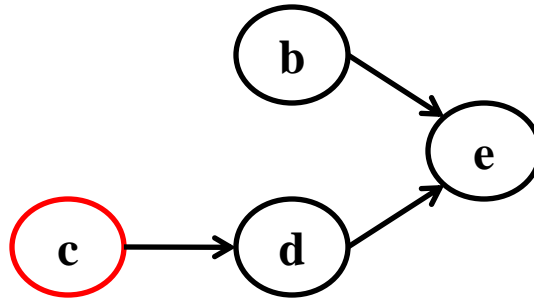


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

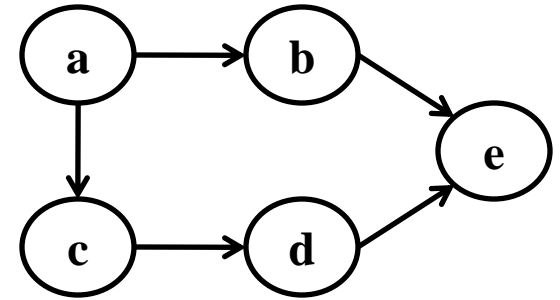
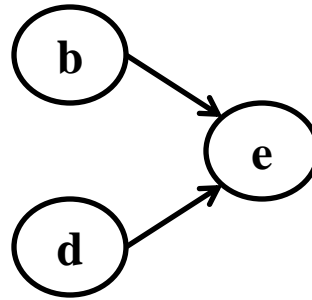


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c

Topological Ordering-3:

**Find the possible topological orderings for the following graph**



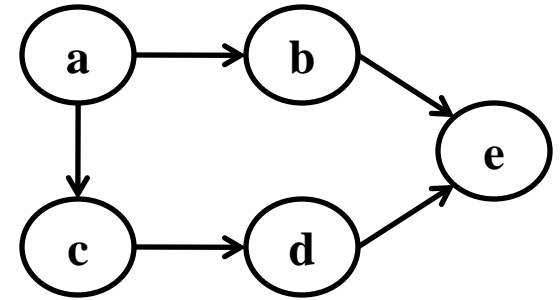
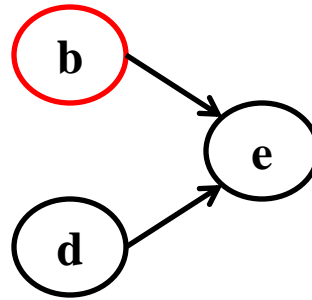
Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c

Topological Ordering-3:



**Find the possible topological orderings for the following graph**

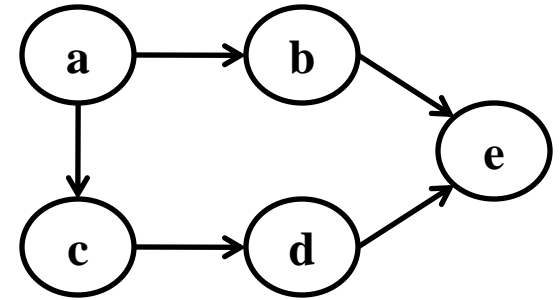
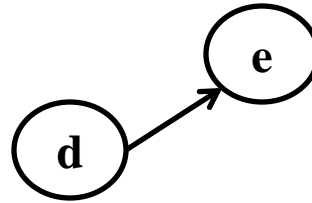


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

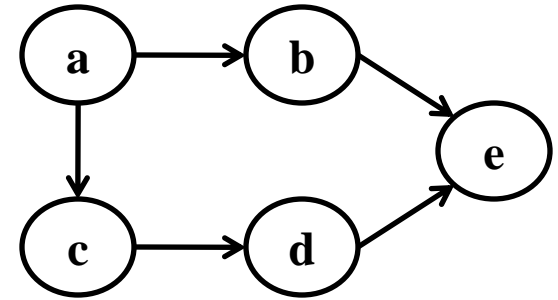
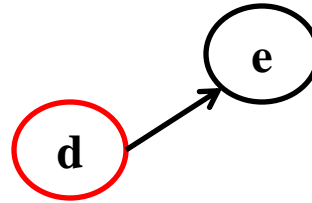


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

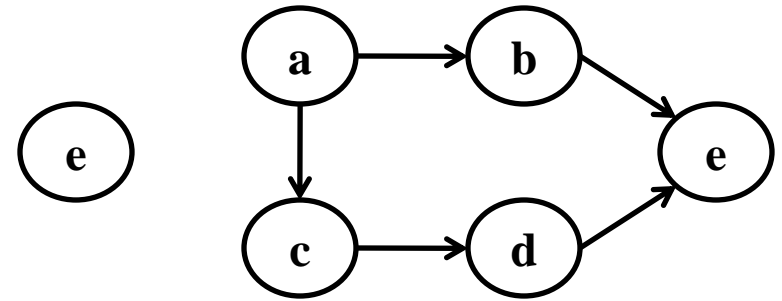


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

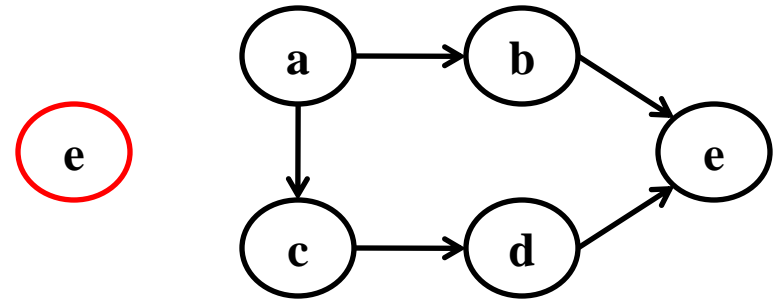


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

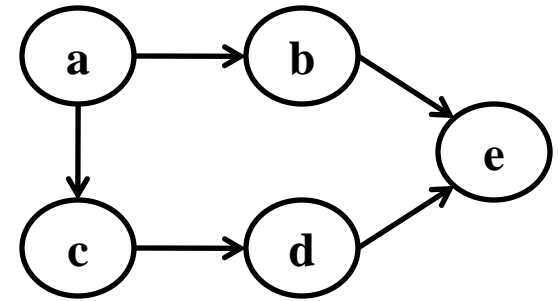


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

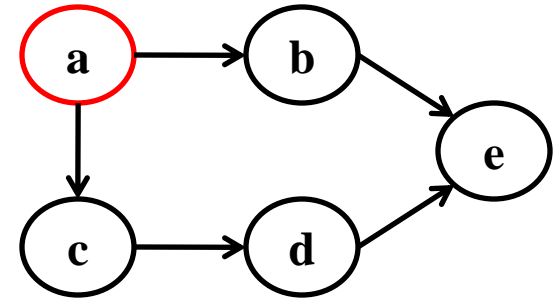


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:

**Find the possible topological orderings for the following graph**

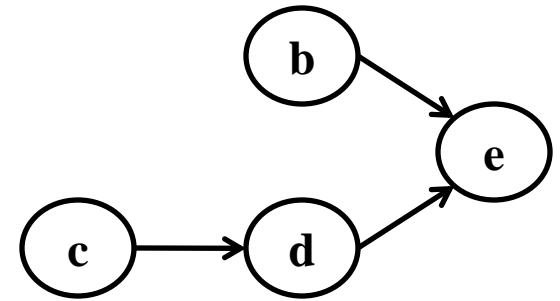


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:  
a,

**Find the possible topological orderings for the following graph**



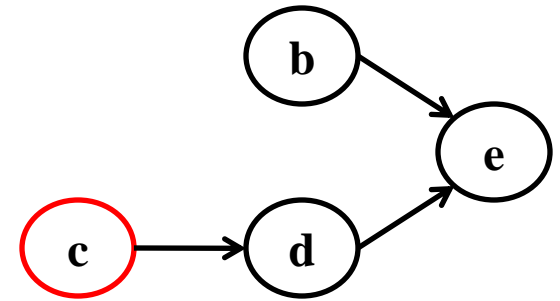
Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:  
a,



**Find the possible topological orderings for the following graph**

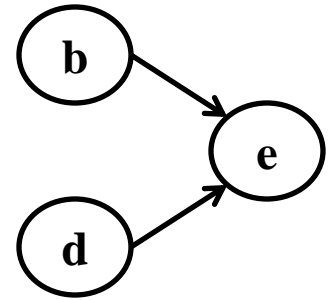


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:  
a,c

**Find the possible topological orderings for the following graph**



Topological Ordering-1:

a,b,c,d,e

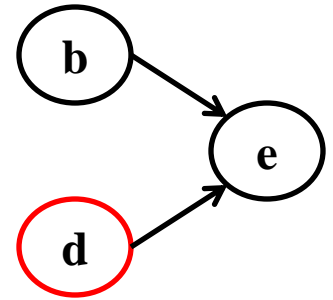
Topological Ordering-2:

a,c,b,d,e

Topological Ordering-3:

a,c

**Find the possible topological orderings for the following graph**

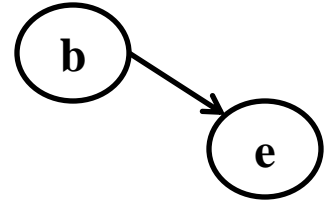


Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:  
a,c,d

**Find the possible topological orderings for the following graph**



Topological Ordering-1:

a,b,c,d,e

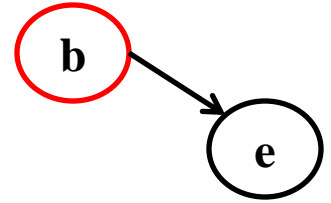
Topological Ordering-2:

a,c,b,d,e

Topological Ordering-3:

a,c,d

**Find the possible topological orderings for the following graph**



Topological Ordering-1:

a,b,c,d,e

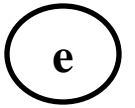
Topological Ordering-2:

a,c,b,d,e

Topological Ordering-3:

a,c,d,b

**Find the possible topological orderings for the following graph**



Topological Ordering-1:

a,b,c,d,e

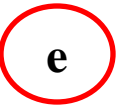
Topological Ordering-2:

a,c,b,d,e

Topological Ordering-3:

a,c,d,b

**Find the possible topological orderings for the following graph**



Topological Ordering-1:  
a,b,c,d,e

Topological Ordering-2:  
a,c,b,d,e

Topological Ordering-3:  
a,c,d,b,e

**Find the possible topological orderings for the following graph**

Topological Ordering-1:

a,b,c,d,e

Topological Ordering-2:

a,c,b,d,e

Topological Ordering-3:

a,c,d,b,e



# Topological Sorting Algorithm Complexity

- Time to determine the indegree for each node =  $O(E)$  time. This involves looking at each directed edge in the graph once.
- Time to determine the nodes with no incoming edges =  $O(V)$
- So Step 1 complexity =  $O(E + V)$
- Add nodes until we run out of nodes with no incoming edges. This loop could run once for every node— $O(V)$  times
- All together, the time complexity is  $O(V+E)$

# Topological Sorting Applications

- Scheduling jobs from the given dependencies among jobs
- Instruction Scheduling
- Determining the order of compilation tasks to perform in makefiles
- Data Serialization