GREEDY STRATERGY

- Greedy strategy is an approach to solve optimisation problem.
- Greedy algorithm consists of number of stages, considering one input at a time.
- At each stage, a decision is made by using some criteria, If the inclusion of next input into partially constructed optimal solution will result in an infeasible solution, the this input is not added to the optimal solution, otherwise it is added.
- Once the decision is made, it cannot be undone.

Control abstraction:

```
Algorithm Greedy(a,n)
// a[1:n] contains the n inputs
{
   solution=φ//Initialize solution
   for i=1 to n do
   {
       x:=Select(a);
       if Feasible(solution,x) then
             solution=Union(solution,x)
   }
return solution;
}
```

Fractional Knapsack Problem

- Fractional knapsack problem can be solved by using greedy strategy.
- We are given n objects and a bag or knapsack.
- Object i has a weight w_i and the knapsack has a capacity M. If a fraction x_i, $0 \leqslant x_i \leqslant 1$,of object i is placed into the knapsack, then a profit of $p_i x_i$ is earned
- The knapsack problem is to fill the knapsack such that it maximizes total profit

- The fractional knapsack problem can be sated as

$$maximize \sum_{n=1}^{n} (x_i . pi)$$

subject to constraint,

$$\sum_{n=1}^{n} (x_i . wi) \leqslant M \text{ and } 0 \leqslant x_i \leqslant 1$$

- **Example:**

Given n=3, W={18,15,10} and P={25,24,20}, M=20.

Selection criteria 1: items on the decreasing order of profit.

The object 1 has the largest profit, and it is placed into knapsack first. Only two units of knapsack capacity left. Next we consider object 2 and only 2/15 can be placed.

X1=1

X2=2/15;

X3=0; x={1,2/15,0}

Weight=w1x1+w2x2+w3x3=1*18+2/15*15+0*10=20.

Profit=p1x1+p2x2+p3x3=1*25+2/15*24+0*20=28.2

This is not optimal.

Selection criteria 2: items on the increasing order of weight.

The object 3 has the least weight, and it is placed into knapsack first. Only ten units of knapsack capacity left. Next we consider object 2 and only 10/15 can be placed.

X1=0

X2=10/15;

X3=1;

Weight=w1x1+w2x2+w3x3=0*18+10/15*15+1*10=20

Profit=p1x1+p2x2+p3x3=0*25+10/15*24+1*20=36

This is also not optimal.

Selection criteria 3: items on the decreasing order of P_i/W_i.

OBJECT	O1	O2	O3
PROFIT(P)	25	24	20
WEIGHT(W)	18	15	10
P/W	1.38	1.6	2

Consider objects in the order x3,x2,x1

X3=1

X2=10/15;

X1=0;

Weight=w1x1+w2x2+w3x3=0*18+10/15*15+1*10=20

Profit=p1x1+p2x2+p3x3=0*25+10/15*24+1*20=36

This is optimal.

The knapsack problem gives optimal solution, when we consider in the decreasing order of P_i/W_i

Algorithm knapsack(m,n)

{

//P[1..n],W[1..n] be the profit and weights of n objects and objects are ordered in increasing order of Profit/Weight. X[1..n] be the solution vector.

U=m;

Initialize x[1..n]=0;

For i=1 to n do.

if (W_i>U) then break;//not able to place Oi in knapsack

X_i=1.0, U=U-W_i

If(i<=n) then X_i=U/$W_{i.}$

Return x[1..n].}

- ✓ If we do not consider the time to sort objects, the time complexity=O(n)

- Find the optimal solution for the following fractional Knapsack problem.

n=4, m = 60, W={40, 10, 20, 24} and P={280, 100, 120, 120}

OBJECT	O1	O2	O3	O4
P	280	100	120	120
W	40	10	20	24
P/W	7	10	6	5

Consider objects in decreasing order of Pi/Xi , ie in the order O2,O1,O3,O4

$x = \{1.0, 1.0, 10/20, 0\}$

$W = \sum W_i x_i = 60$ //

$profit, P = \sum P_i x_i = 280*1+100*1+120*\frac{10}{20}+120*0 = \underline{\underline{440}}$