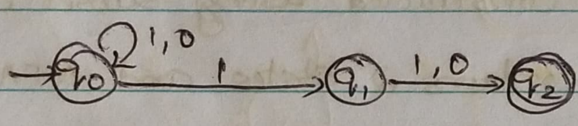
* For each set $S \subseteq Q_N$ and for i/p symbol a in Σ

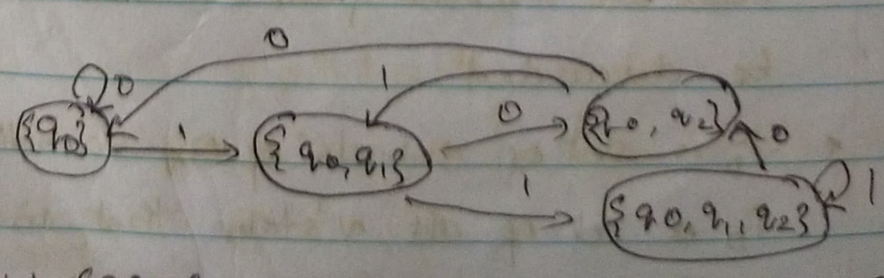
$$\delta_D(S, a) = \bigcup_{P \in S} \delta_N(P, a)$$

to compute $\delta_D(S,a)$, we take all the states P in S and on i/p a on p, take union of all these states

eg1:



δ	0	1
$\rightarrow q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_2\}$
$* q_2$	ϕ	ϕ
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$* \{q_0, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$
$* \{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$



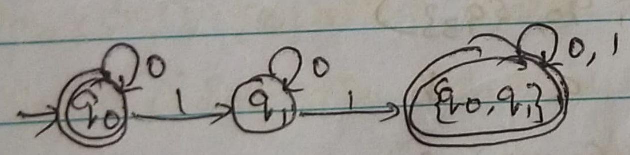
Note: $\{q_1\}$ & $\{q_2\}$ are not accessible from start state q_0

eg2: Construct a DFA equivalent to $m = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$

δ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_1	$\{q_0, q_1\}$

Ans: Subset Construction Method

δ	0	1
$* \rightarrow q_0$	q_0	q_1
q_1	q_1	$\{q_0, q_1\}$
$* \{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_0\}$

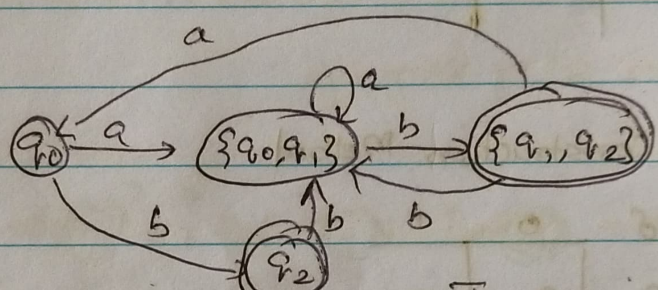


Eg3: Find a DFA equivalent to NFA $m = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ where δ is given by

State	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_2\}$
$\{q_1\}$	$\{q_0\}$	$\{q_1\}$
q_2	ϕ	$\{q_0, q_1\}$

Ans.

δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
$* q_2$	ϕ	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$* \{q_1, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$



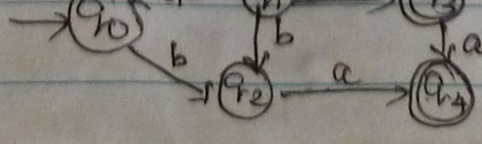
Transition diagram

T eg4: Construct a DFA equivalent to $M = (\{q_0, q_1, q_2, q_3\}$
$\{0, 1\}, \delta, q_0, \{q_3\})$

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_3	q_3
$* q_3$	ϕ	q_2

T eg5: Construct a DFA equivalent to NFA

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_0, \{q_3, q_4\})$



Equivalence of NFA & DFA

Theorem: For every NFA, there is an equivalent DFA that accept the same set of language

Proof: Let $m = (Q, \Sigma, \delta, q_0, F)$ be an NFA accept L, we construct a DFA $m' = (Q', \Sigma, \delta', q_0', F')$ where

$Q' = 2^Q$ (In DFA, state is subset of NFA)

$q_0' = \{q_0\}$ (subset $\{q_0\}$ in NFA)

F' = set containing all the subsets of F

δ = transition fn that m can reach any of the possible states $\delta(q, a) = \{q_0, \ldots q_k\}$, that is represented as a single state.

if $\delta'([q_1, q_2, \ldots q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \ldots$

$\delta([q_1, q_2 \ldots q_i], a) = [p_1, p_2 \ldots p_j]$ iff

$\delta(\{q_1, q_2 \ldots q_i\}, a) = \{p_1, p_2 \ldots p_j\}$

Basis: $|x| = 0$, $\delta(q_0, \epsilon) = \{q_0\}$, by definition

$\delta(q_0', \epsilon) = [q_0] = q_0'$ is true for length $= 0$

Induction: Assume this is true for the strings y with $|y| \leq m$. Let x be string of length $m+1$

$x = ya$

$\delta(q_0, y) = \{p_1, p_2 \ldots p_j\}$ & $\delta(q_0, ya) = \{r_1, r_2 \ldots r_k\}$

As $|y| \leq m$, by induction hypothesis

$$\delta'(q_0', y) = [P_1, P_2 \ldots P_j]$$

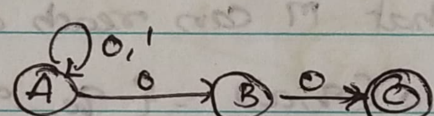
Also, $\{r_1, r_2 \ldots r_k\} = \delta(q_0, ya) = \delta(\{P_1, P_2 \ldots P_j\}, a)$

By def δ'

$$\delta'([P_1, P_2 \ldots P_j], a) = [r_1, r_2 \ldots r_k]$$

$$\delta'(q_0', ya) = \delta'(\delta'(q_0', y), a) = \delta'([P_1, P_2 \ldots P_j], a)$$

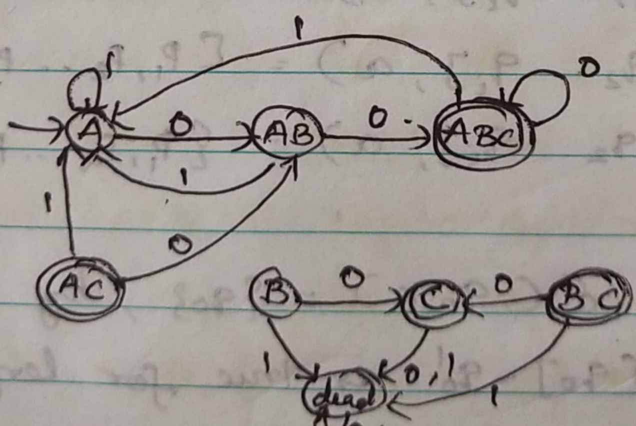
$$= [r_1, r_2 \ldots r_k]$$



NFA

DFA

Subsets



At A, if 0, can go to either A or B.

" if 1, ", to A

Proof by construction

Given NFA, Let's show how to build an equiv DFA

Let $m = (Q, \Sigma, \delta, q_0, F)$ → NFA given

Construct

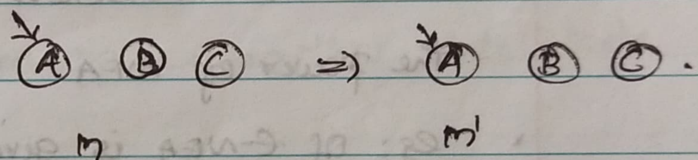
$m' = (Q', \Sigma, \delta', q_0', F')$ → This DFA building

Q' - Diff. set of states

where

$Q' = P(Q)$. Assume ∀ state, there is possible subset

$q_0' := \{q_0\}$

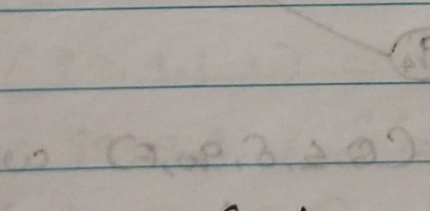


Subset containing $\{q_0\}$

$F' =$

$\delta'(R, a) = \{ q \mid q \in Q$ and $q \in \delta(r, a)$ for some $r \in R$

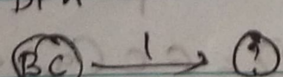
NFA



$\delta(B, 1) = \{B, C\}$

$\delta(C, 1) = \{C, A\}$

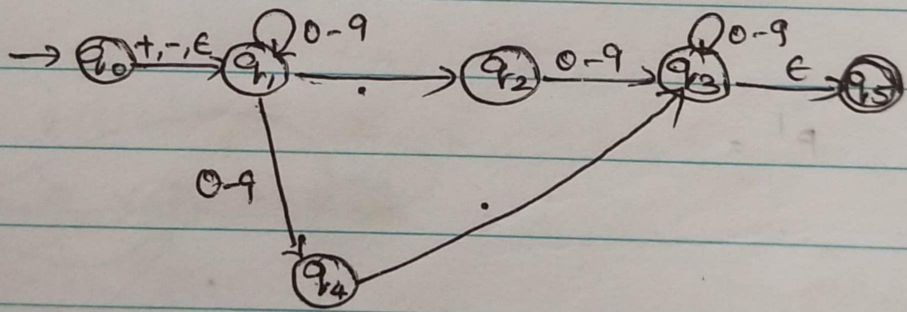
DFA



(BCE) → DFA state is the set of states in NFA

NFA with ε-transitions (εNFA)

* NFA can go from one state to another without consuming any input symbol.
* Makes the transition easier, but don't increase the power of NFA
* eg: of ε-NFA is given below



* ε-NFA by $A = (Q, \Sigma, \delta, q_0, F)$ where
  1) δ: defines a state in Q and member in $\Sigma \cup \{\epsilon\}$ ie either i/p symbol or ε, ε can't be a member of alphabet $Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^{Q}$

* δ and $\hat{\delta}$ is different for ε-NFA
$\hat{\delta}(q_0, +-) = \delta(q_0, \epsilon) = \epsilon.C(q_0)$ then $\delta((q_0, \epsilon), +-) = \delta(q_0, +-) = q_1$

	ϵ	+-	.	0,1-9
q_0	q_1	q_1	ϕ	ϕ
q_1	ϕ	ϕ	q_2	q_1, q_4
q_2	ϕ	ϕ	ϕ	q_3
q_3	q_5	ϕ	ϕ	q_3
q_4	ϕ	ϕ	q_3	ϕ
q_5	ϕ	ϕ	ϕ	ϕ

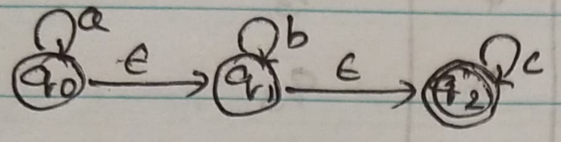
ϵ-closure - ϵ-closed path from that state itself

eg: Compute $\hat{\delta}(q_0, 5.6)$

1) $\delta(q_0, \epsilon) = \epsilon.CLOSE(q_0) = \{q_0, q_1\}$
2) $\delta(\{q_0, q_1\}, 5) = \delta(q_0, 5) \cup \delta(q_1, 5) = \{q_1, q_4\}$
3) $\epsilon.C(\{q_1, q_4\}) = \{q_1, q_4\}$
4) $\delta(\{q_1, q_4\}, \cdot) = \{q_2, q_3\}$
5) $\epsilon.C\{q_2, q_3\} = \{q_2, q_3, q_5\}$
6) $\delta(\{q_2, q_3, q_5\}, 6) = \{q_3\}$
7) $\epsilon.C\{q_3\} = \{q_3, q_5\}$ which includes the final state q_5, so that the string is accepted.

NFA with Epsilon and NFA without ϵ-moves

Q₁: Convert ϵ-NFA to NFA



Step 1: Find ϵ-closure for all states

2: Find δ for all possible i/p

3: Find ϵ-closure of step 2.

I $\epsilon.c\,(q_0) = \{q_0, q_1, q_2\}$

II $\delta(\{q_0, q_1, q_2\}, a) = \{q_0\}$

$\delta(\{q_0, q_1, q_2\}, b) = \{q_2\}$

$\delta(\{q_0, q_1, q_2\}, c) = \{q_2\}$

III $\epsilon.c\,\{q_0\} = \{q_0, q_1, q_2\}$

$\epsilon.c\,\{q_2\} = \{q_2\}$

$\epsilon.c\,\{q_2\} = \{q_2\}$

Repeat the steps until all the states in Q are completed

$F = q_0 \cup F$ If F contains $\epsilon.c\,(q_0)$, otherwise F

I $\epsilon.c\,(q_1) = \{q_1, q_2\}$.

II $\delta(\{q_1, q_2\}, a) = \phi$

$\delta(\{q_1, q_2\}, b) = \{q_1\}$

$\delta(\{q_1, q_2\}, c) = \{q_2\}$

III $\epsilon.c\,\{q_1\} = \{q_1, q_2\}$

$\epsilon.c\,\{q_2\} = \{q_2\}$

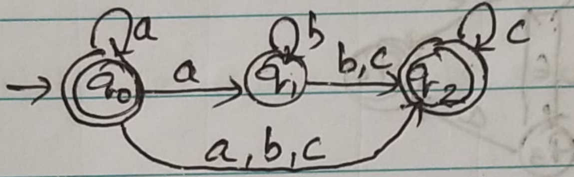
III $\epsilon.C(q_2, \epsilon) = \{q_2\}$

II $\delta(q_2, a) = \cancel{q_2}\ \phi$

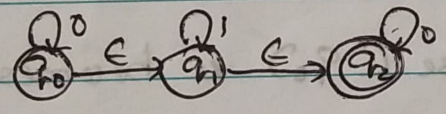
$\delta(q_2, b) = \phi$

$\delta(q_2, c) = \{q_2\}$

III $\epsilon.c\ \{q_2\} = \{q_2\}$



Q2 Convert ε-NFA given below to NFA



I $\epsilon.c(q_0) = \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, 0) = \{q_0, q_2\} \rightarrow \epsilon.c\ \{q_0, q_2\} = \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, 1) = \{q_1\} \Rightarrow \epsilon.c\ \{q_1\} = \{q_1, q_2\}$

II $\epsilon.c(q_1) = \{q_1, q_2\}$

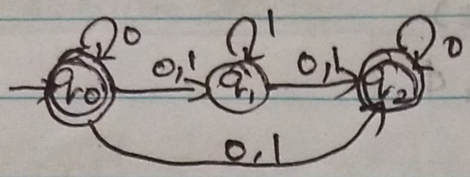
$\delta(\{q_1, q_2\}, 0) = \{q_2\} \Rightarrow \epsilon.c\ \{q_2\} = \{q_2\}$

$\delta(\{q_1, q_2\}, 1) = \{q_1\} \Rightarrow \epsilon.c\ \{q_1\} = \{q_1, q_2\}$

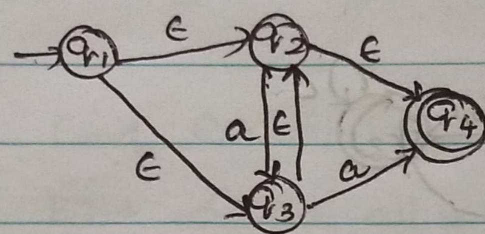
III $\epsilon.c(q_2) = \{q_2\}$

$\delta(\{q_2\}, 0) = \{q_2\} \Rightarrow \epsilon.c\ \{q_2\} = \{q_2\}$

$\delta(\{q_2\}, 1) = \phi$

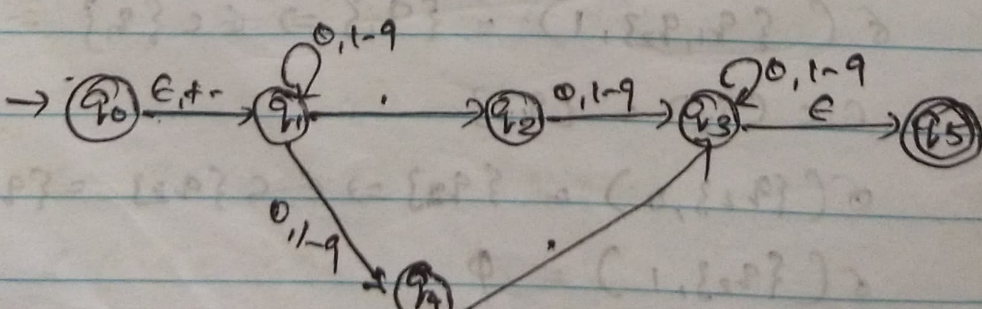


Q3: Convert ε-NFA to NFA



Conversion of ENFA to DFA

1. Compute ε-close (q_0) ie becomes the start state of DFA
2. Compute δ for all i/p alphabet
3. Then compute ε closure of step 2
4. Repeat the steps for newly created subsets
4. Final states are those which include the final state in ε-NFA



1. $\epsilon.c\ \{q_0\} = \{q_0, q_1\}$ is the start state of DFA

2. $\delta(\{q_0, q_1\}, +-) = \{q_1\} \Rightarrow \epsilon.c\ \{q_1\} = \{q_1\}$

$\delta(\{q_0, q_1\}, 0\text{-}9) = \{q_1, q_4\} \Rightarrow \epsilon.c\ \{q_1, q_4\} = \{q_1, q_4\}$

$\delta(\{q_0, q_1\}, \cdot) = \{q_2\} \Rightarrow \epsilon.c\ \{q_2\} = \{q_2\}$

Newly created subsets are $\{q_1\}$, $\{q_2\}$, $\{q_1, q_4\}$

3. $\epsilon.c\ \{q_1\} = \{q_1\}$

$\epsilon.c\ (\delta\{q_1\}, +-) = \phi$

$\epsilon.c\ (\delta\{q_1\}, 0\text{-}9) = \{q_1, q_4\}$

$\epsilon.c\ (\delta\{q_1\}, \cdot) = \{q_2\}$

4. $\epsilon.c\ \{q_2\} = \{q_2\}$

$\epsilon.c\ (\delta\{q_2\}, +-) = \phi$

$\epsilon.c\ (\delta\{q_2\}, 0\text{-}9) = \{q_3, q_5\}$ → new subset

$\epsilon.c\ (\delta\{q_2\}, \cdot) = \phi$

5 $\epsilon.c\ \{q_1, q_4\} = \{q_1, q_4\}$

$\epsilon.c\ (\delta\{q_1, q_4\}, +-) = \phi$

$\epsilon.c\ (\delta\{q_1, q_4\}, 0\text{-}9) = \{q_1, q_4\}$

$\epsilon.c\ (\delta\{q_1, q_4\}, \cdot) = \{q_2, q_3, q_5\}$ → new subset

6. $\epsilon.c\ \{q_3, q_5\} = \{q_3, q_5\}$

$\epsilon.c\ \delta(\{q_3, q_5\}, +-) = \phi$

$\epsilon.c\ \delta(\{q_3, q_5\}, 0\text{-}9) = \{q_3, q_5\}$

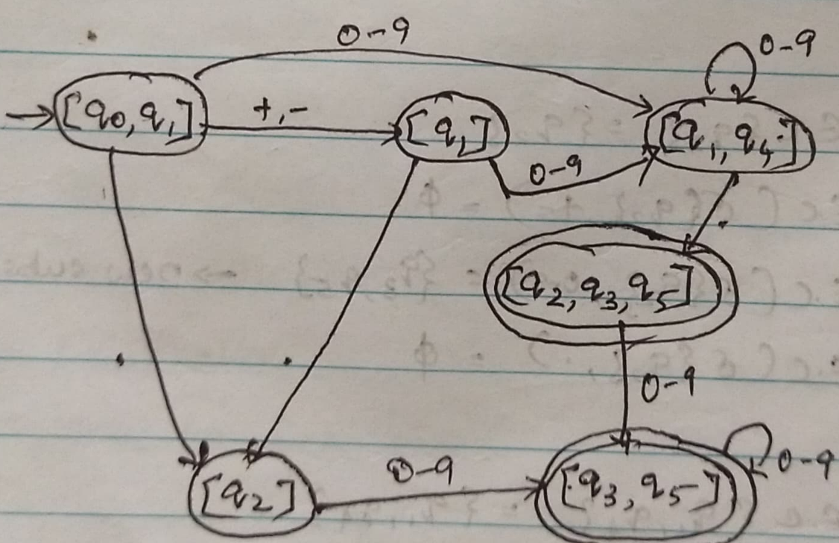
$\epsilon.c\ \delta(\{q_3, q_5\} \cdot) = \phi$

7 $\epsilon.c\ \{q_2, q_3, q_5\} = \{q_2, q_3, q_5\}$

$\epsilon.c\ \delta(\{q_2, q_3, q_5\}, +-) = \phi$

$\epsilon.c\ \delta(\{q_2, q_3, q_5\}, 0\text{-}9) = \{q_3, q_5\}$

$\epsilon.c\ \delta(\{q_2, q_3, q_5\}, \cdot) = \phi$



Regular Expression

- It's the algebraic representation of finite automata Language accepted by FA can be described by R.E R.E recognizes FA
- R.E is the i/p language for many s/m. eg: Lexical analyser generator - Lex

Operators of R.E

- operators used in R.E are union, concatenation, closure and positive closure
- **Union** $L_1 = \{001, 10\}$ and $L_2 = \{\epsilon, 00\}$, then $L_1 \cup L_2 = \{001, 10, 00\}$ is the set of strings that are either in L_1 or L_2 or both
- **Concatenation** of Language L_1 & L_2 is the set of strings in L_1 and concatenating with L_2.
  $L_1 = \{001, 10, 111\}$ & $L_2 = \{\epsilon, 011\}$, then $L_1 L_2 = \{001, 10, 111, 001011, 10011, 111011\}$.
- Closure - L^* (kleen closure) - 0 or more occurrence of L
  - L^+ (Positive closure) - One or more occurrence of a string

Regular set - Any set represented by regular expression

Definition of R.E

- $\forall$ letter of Σ is a R.E
- ϵ is R.E
- If r_1 & r_2 are R.E, then
  1) (r_1) is R.E
  2) $r_1 r_2$ is R.E
  3) $r_1 + r_2$ is R.E
  4) R_1^* is R.E
  5) R_1^+ is R.E
- ϕ is R.E
- If any symbol, $L(a) = \{a\}$.

Designing of Regular Expression

Eg: Describe the foll. set by R.E

a) $\{101\}$ b) $\{abba\}$ c) $\{01, 10\}$ d) $\{\epsilon, ab\}$
e) $\{abb, a, b, bba\}$ f) $\{\epsilon, 0, 00, 000\}$
g) $\{1, 11, 111\}$

a) Any symbol is R.E 101 is concatenated with 1, 0 and 1

b) abba

c) 01+10

d) ϵ + ab

e) abb + a + b + bba

f) 0^*

g) 1^+

Eg2: Write R.E for the foll. set

~~Ans.~~ 1) L_1 = Set of strings $\Sigma = \{0,1\}$ ending in 00

Ans: $(0+1)^* 00$

2) L_2 = Set of strings $\Sigma = \{0,1\}$ beginning with 0

Ans: $0(0+1)^*$

3) L_2 = begin with 0 & ends with 1

Ans: $0(0+1)^* 1$

4) $L_3 = \{\epsilon, 11, 1111, \ldots\}$

$(11)^*$

Eg3: Construct R.E over $\Sigma = \{a,b,c\}$ containing atleast one a and atleast one b

A: $(a+b+c)^* a (a+b+c)^* b (a+b+c)^* + (a+b+c)^* b (a+b+c)^* a (a+b+c)^*$

4. Write R.E for $\Sigma = \{0,1\}$ whose 10^{th} symbol from right end is 1

A $(0+1)^* 1 (0+1)^9$

5 Write R.E for the set of strings of equal no. of 0's & 1's such that in every prefix, the no. of 0's differs from no. of 1's by atmost 1

A $(01+10)^*$

6. Write R.E for $\Sigma = \{0,1\}$ not containing 101 as substring

A: $(0^* 1^* 00)^* 0^* 1^*$

7. Write R.E over $\Sigma = \{a,b\}$ with even no: of a's followed by odd no. of b's

A. $(aa)^*(bb)^* b$

8. $\Sigma = \{0,1\}$, has no pair of consecutive 0's

$(1+01)^* (0+\epsilon)$

9. Write R.E for $L = \{a,b\}^*$, $n_a(w) \bmod 3 = 0$

A: No. of a's divided by 3 will be 0

$(b^* a b^* a b^* a b^*)^*$

Identities of R.E

1. $\phi + R = R$
2. $\phi R = R\phi = \phi$
3. $\epsilon R = R\epsilon = R$
4. $\epsilon^* = \epsilon$, $\phi^* = \epsilon$
5. $R + R = R$
6. $R^* R^* = R^*$
7. $R R^* = R^* R$
8. $(R^*)^* = R^*$
9. $\epsilon + R R^* = R^* = \epsilon + R^* R$

10) $(PQ)^* P = P(QP)^*$

11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

12) $(P+Q) R = PR + QR$

$R(P+Q) = RP + RQ$

Construction of R.E from DFA

* For ∀ R.L, there exist a DFA, for this we are using Arden's theorm

Arden's theorm

* Let P & Q be 2 R.E over Σ. If P doesn't contain null string ϵ, then

$$R = Q + RP \text{ has a sol}^n \text{ that } R = QP^*$$

* Put the value of R in RHS repeatedily

$$R = Q + (Q + RP)P = Q + QP + RP^2$$
$$= Q + QP + (Q + RP)P^2$$
$$= Q + QP + QP^2 + RP^3$$
$$= Q(1 + P + P^2 + P^3 + \dots)$$

$$R = Q + RP \iff R = QP^*$$

Use of Arden's theorm to find R.E from DFA

1) Transition diagram shouldn't have ϵ-transition
2) It must have only a single initial state
3) It vertices are $q_1, q_2 \dots q_n$
4) q_i the final state
5) w_{ij} denote R.E representing the set of Labels of edges from q_i to q_j. We can get the foll. set of equation in $q_1 \dots q_n$

$q_1 = q_1 \omega_{11} + q_2 \omega_{21} + \cdots + q_n \omega_{n1} + \epsilon$

$q_2 = q_1 \omega_{12} + q_2 \omega_{22} + \cdots + q_n \omega_{n2}$

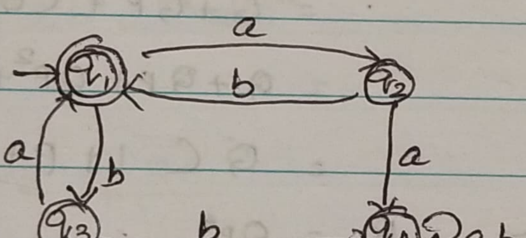
$q_3 = q_1 \omega_{13} + q_2 \omega_{23} + \cdots + q_n \omega_{n3}$

$\vdots$

$q_n = q_1 \omega_{1n} + q_2 \omega_{2n} + \cdots + q_n \omega_{nn}$

Solve this for the final state q_i. we add ϵ in the equation starts with starting state q_1 and solve it to find out the final state

eg: Find R.E for the transition diagram.



$q_1 = q_2 b + q_3 b + \epsilon$

$q_2 = q_1 a$

$q_3 = q_1 b$

$q_4 = q_2 a + q_3 b + q_4 a + q_4 b$

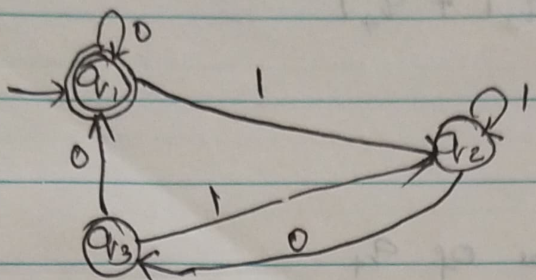
Put q_2 & q_3 in q_1

$q_1 = q_1 ab + q_1 ba + \epsilon$

$q_1 = \epsilon + q_1 (ab + ba)$ $(R = P + RQ = PQ^*)$

$q_1 = \epsilon (ab + ba)^*$

Q: Find R.E for the transition diagram



$$q_1 = q_1 0 + q_3 0 + \epsilon$$

$$q_2 = q_2 1 + q_1 1 + q_3 1$$

$$q_3 = q_2 0$$

put q_3 in q_2

$$q_2 = q_2 1 + q_1 1 + q_2 01$$

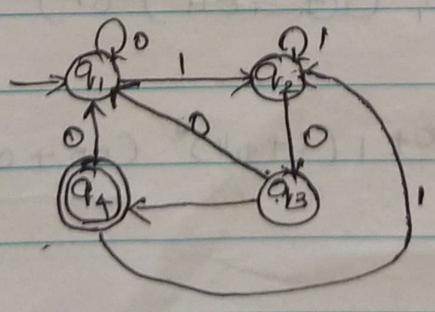
$$q_2 = q_2 (1+01) + q_1 1$$

$$q_1 = q_1 0 + (q_1 1 (1+01)^* 00) + \epsilon$$

$$q_1 = q_1 (0 + 1(1+01)^* 00) + \epsilon$$

$$= (0+1(1+01)^* 00)^*$$

Q: Find R.E for the given DFA



$q_1 = q_1 0 + q_3 0 + q_4 0 + \epsilon$

$q_2 = q_1 1 + q_2 1 + q_4 1$

$q_3 = q_2 0$

$q_4 = q_3 1$

Solve interms of q_4

$q_4 = q_3 1 = q_2 01$

Using q_2

$q_2 = q_1 1 + q_2 1 + q_2 011$

$q_2 = q_1 1 + q_2 (1 + 011)$

$q_2 = q_1 (1 (1 + 011)^*)$

Using q_1

$q_1 = q_1 0 + q_2 00 + q_2 010 + \epsilon$

$q_1 = q_1 0 + q_2 (00 + 010) + \epsilon$

$q_1 = q_1 0 + q_1 (1 (1 + 011)^*) (00 + 010) + \epsilon$

$= q_1 (0 + 1 (1 + 011)^* (00 + 010)) + \epsilon$

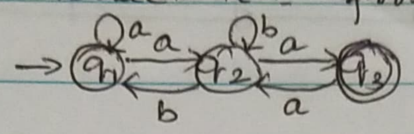
$q_1 = \epsilon (0 + 1 (1 + 011)^* (00 + 010))^*$

Put q_2 in q_4

$q_4 = q_1 (1 (1 + 011)^* 01)$

$= (0 + 1 (1 + 011)^* (00 + 010))^* (1 (1 + 011)^* 01)$

eg: Construct R.E from DFA



$q_1 = q_1 a + q_2 b + \epsilon$ — ①

$q_2 = q_2 b + q_1 a + q_3 a$ — ②

$q_3 = q_2 a$ — ③.

Solve it interms of q_3

substitute q_3 in equ ②

$q_2 = q_1 a + q_2 b + q_2 aa$

$q_2 = q_1 a + q_2 (b + aa)$ — ④

$q_2 = q_1 a (b+aa)^*$ — ⑤.

$q_1 = q_1 a + q_1 a (b+aa)^* b + \epsilon$

$q_1 = q_1 (a + a(b+aa)^* b) + \epsilon$

$q_1 = \epsilon (a + a(b+aa)^* b)^*$ — ⑥.

Substitute in ⑤

$q_2 = (a + a(b + aa^*)^* b)^* a (b+aa)^*$ — ⑦.

Substitute in ③

$q_3 = (a + a(b+aa^*)^* b)^* a (b+aa)^* a$

Construction of ϵ-NFA from Regular Expression (Kleen's Theorm)

Theorm: If r is a R.E representing the language $L(R) \subseteq \Sigma^*$, then there exist a NFA with ϵ-moves that accept $L(R)$

Proof

We prove theorm by induction on no. of operators in the R.E r. We prove that there is an NFA M with ϵ transitions having only one final state & no transition out of the final state such that $L(R) = L(M)$

Basis

Consider the case of zero operators. The elements in RE is Σ, ϵ, ϕ

1) $\rightarrow q_0 \xrightarrow{a} q_f$ R.E $= \{a\}$

2) $\rightarrow q_0 \xrightarrow{\epsilon} q_f$ R.E $= \{\epsilon\}$

3) $\rightarrow q_0 \quad q_f$ R.E $= \phi$

Hypothesis

Assume that the theorm is true for i operator where $i \geq 1$

Induction

Let R.E r have $i+1$ operator. Then there are 3 cases depending on the form of r.

Case1: $r = r_1 + r_2$

Both r_1 & r_2 must have i or $< i$ operators, thus there are 2NFA

$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{F_1\})$

$L(M_1) = L(r_1)$

$M_2 = (Q_2, \Sigma_2, \delta_2, q_2, \{F_2\})$

$L(M_2) = L(r_2)$

Assume that Q_1 & Q_2 are disjoint set. Now we construct a new NFA M with ϵ moves having q_0 as initial state & f_0 as final state.

Construct

$M = (Q_1 \cup Q_2 \cup \{q_0 f_0\}, \Sigma_1 \cup \Sigma_2, \delta, q_0, \{f_0\})$

where δ is defined by

1) $\delta(q_0, \epsilon) = \{q_1, q_2\}$

2) $\delta(q, a) = \delta_1(q, a) \ \forall \ q \in Q_1 - \{F_1\}$ &

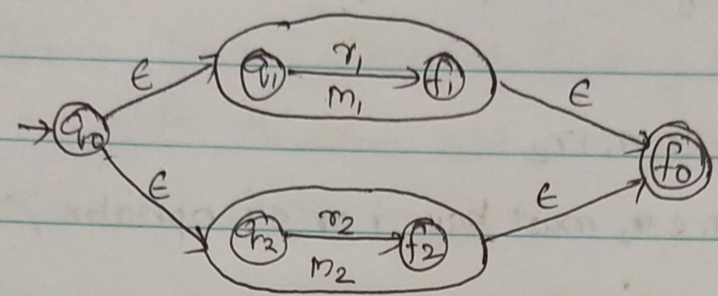
a in $\Sigma_1 \cup \{\epsilon\}$

3) $\delta(q, a) = \delta_2(q, a) \ \forall \ q \in Q_2 - \{F_2\}$ &

a in $\Sigma_2 \cup \{\epsilon\}$

4) $\delta_1(f_1, \epsilon) = \delta_2(f_2, \epsilon) = \{f_0\}$

Since there are no transition from f_1 or f_2 in M_1 or M_2, all transitions of M_1 & M_2 are present in M



$r_1 + r_2$

The construction of M is shown in fig. Any path in the transition dia. of M from q_0 to f_0 must begin by going to either q_1 or q_2 accepting ϵ. If path goes to q_1, it may follow any path in M_1 to f_1 & then go to f_0 by accepting ϵ.

Ml'ly path that begin by going to q_2 may follow any path in M_2 to f_2 & then go to f_0 by accepting ϵ. These are the only path from q_0 to f_0. This means that there is a path labelled x in M from q_0 to f_0 iff there is a path labelled x in M_1 from q_1 to F_1 or a path in M_2 from q_2 to f_2. Hence $L(M) = L(M_1) \cup L(M_2)$

Case 2

$r = r_1 r_2$

M_1 & M_2 are same as in Case 1