3-D mass map reconstruction

Method and Plane in S19A

Xiangchong Li
Colaborators:
Naoki Yoshida, Masamune Oguri
Shiro Ikeda, Wentao Luo

Goal

$$\gamma = \mathbf{T}\delta + \text{noise},$$

where **T** includes both physical signal and systematics

Physical Signal

$$\kappa(\vec{\theta}, z_s) = \frac{3H_0^2\Omega_M}{2c^2}\int_0^{\chi_s} d\chi_l \frac{\chi_l\chi_{sl}}{\chi_s}\frac{\delta(\vec{\theta}, z_l)}{a(\chi_l)},$$

$$\gamma_L(\vec{\theta}, z_s) = \int d^2\theta' D(\vec{\theta} - \vec{\theta'})\kappa(\vec{\theta'}, z_s),$$

where

$$D(\vec{\theta}) = -\frac{1}{\pi}(\theta_1 - i\theta_2)^{-2}.$$

Systematics

- photo-z uncertainty;
- Masking;
- Smoothing in transverse plane;
- Pixelization.

The Goal is to inverse the matrix **T** and estimate 3-D density contrast field: $\delta(\vec{\theta}, z_l)$.

Prior Information

Since the 3-D inversion problem is an ill-posed problem, we have to add prior information.

Model Dictionary

$$\delta(\vec{\theta}, z) = \Sigma_i \Phi_i(\vec{\theta}, z) x_i,$$

where Φ_i is the model basis, and x_i is the projected modes. e.g.,

1. Point Mass;
2. Fourier Space (sine, cosine);
3. Starlets.

Regularization

$$\hat{x} = \arg\min_{x} \left\{ \frac{1}{2} \left\| \Sigma^{-\frac{1}{2}} (\gamma - \mathbf{T}\Phi x) \right\|_2^2 + \lambda \|x\|_p^p \right\},$$

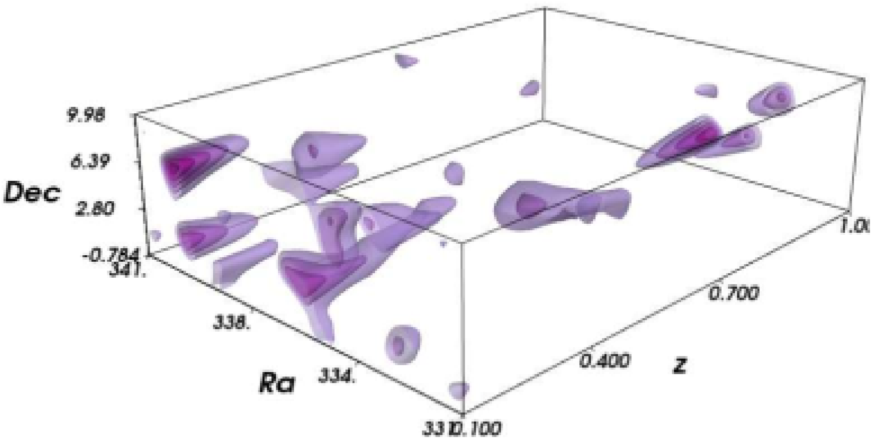
$$\|x\|_p^p = (\sum_i |x|_i^p).$$

e.g.,

1. $p = 2$, Ridge regression (Wiener Filter);
2. $p = 1$, Lasso (Sparse);
3. $p = 0$, Best subset (sparsest but unsolvable).

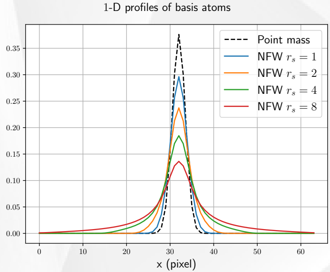
Choose a model dictionary and a regularization (Prior distribution of x).

Wiener Filter in S16A



Masamune Oguri, 2018 (HSC), Wiener Filter

NFW Model dictionary



- pixel size: 1 arcmin
- Gaussian Smoothing: 1.5 arcmin

NFW Atoms

$$\phi_\alpha(\vec{r}) = \frac{f}{2\pi\theta_\alpha^2} F(|\vec{\theta}|/\theta_\alpha)\delta_D(z),$$
$$(\alpha = 1..N)$$

$$F(x) = \begin{cases} -\frac{\sqrt{c^2-x^2}}{(1-x^2)(1+c)} + \frac{\operatorname{arccosh}\left(\frac{x^2+c}{x(1+c)}\right)}{(1-x^2)^{3/2}} & (x < 1), \\ \frac{\sqrt{c^2-1}}{3(1+c)}\left(1 + \frac{1}{c+1}\right) & (x = 1), \\ -\frac{\sqrt{c^2-x^2}}{(1-x^2)(1+c)} + \frac{\arccos\left(\frac{x^2+c}{x(1+c)}\right)}{(x^2-1)^{3/2}} & (1 < x \leq c), \\ 0 & (x > c). \end{cases}$$

Takada & Jain (2003)

Adaptive Lasso (regularization/Prior on x)

Approximation to l^0 regularization with two steps of l^1 lasso estimation

Zou (2007)

first step: lasso

$$\hat{x} = \arg\min_{x} \left\{ \frac{1}{2} \left\| \Sigma^{-\frac{1}{2}} (\gamma - \mathbf{A}x) \right\|_2^2 + \lambda \|x\|_1^1 \right\}.$$

$$\hat{w} = \frac{1}{\left| \hat{x}'^{\,\mathrm{lasso}} \right|^{\tau}}, \tag{1}$$

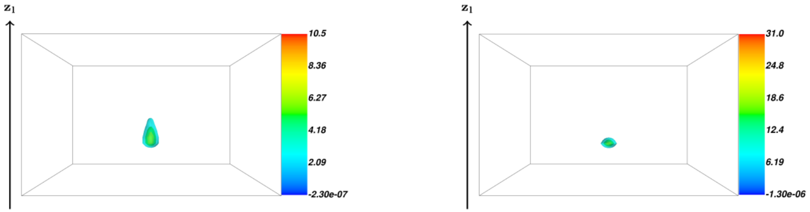
hyper-parameter: $\tau = 2$.

second step: weighted lasso

$$\hat{x} = \arg\min_{x} \left\{ \frac{1}{2} \left\| \Sigma^{-\frac{1}{2}} (\gamma - \mathbf{A}x) \right\|_2^2 + \hat{w} \lambda_{\mathrm{ada}} \|x\|_1^1 \right\}.$$

Results for Noiseless Simulation

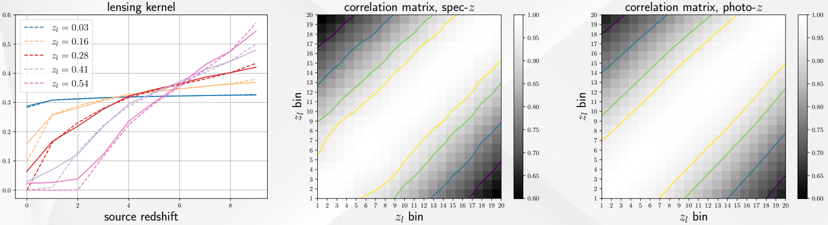
- HSC (s16) number densitty;
- HSC (s16) redshifts(best estimation);
- No noise; no photo-z uncertainty.



(i) lasso

(ii) adaptive lasso

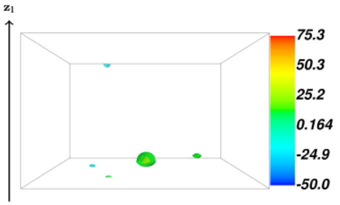
Correlated Lensing Kernels



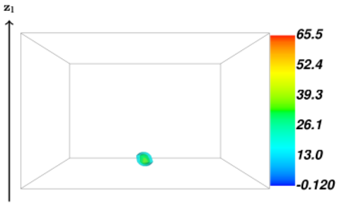
The lensing kernels for two neighbouring lens redshift bins are too correlated (The shapes of them are similar). Therefore, it is difficult to distinguish in which bins the mass is actually located.

Results for HSC mock catalogs

- HSC (s16) number densitty;
- HSC (s16) redshifts(best estimation);
- HSC (s16) shape noise; HSC photo-z uncertainty.



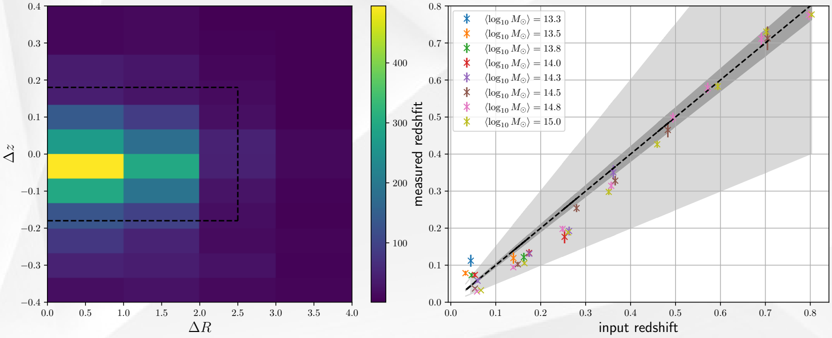
(i) NFW: $\lambda = 3.5$



(ii) NFW: $\lambda = 5.0$

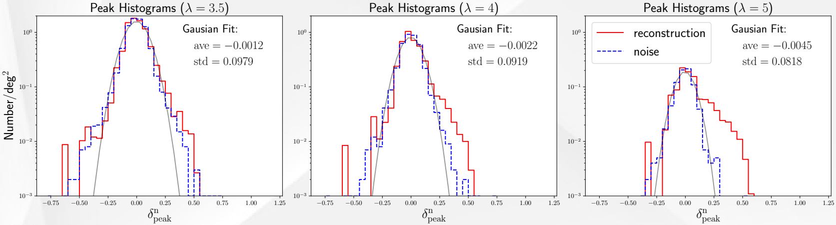
Redshift estimation

We simulation halos with different mass at different redshift; each halo with 100 noise realizations. The following figures show the offsets of detected peaks from the input positions.



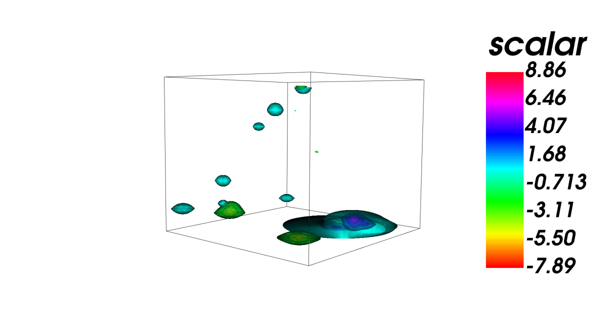
Peak Histogram

1000 pure noise realizations with HSC shape noise and photoz uncertainty.



Preliminary run on 1 deg^2 region of XMM

- S19A data
- corrected for additive bias
- set multiplicative bias to zero



Plans

- Use Planck CMB mass map to improve performance on high-z?
- Cluster detection on S19A?
- X-ray clusters?
- Welcome to join the project!