The Sunayev-Zel'dovich Effect

M.R. Tousif
Graduate Student
(tousif@unm.edu)

Department of Physics & Astronomy,
University of New Mexico, USA

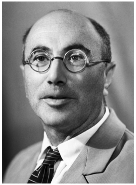
May 10, 2023

Outline

1. The Physics of Sunayev-Zel’dovich Effect (SZE)
2. Results and Discussions based on SZE

The Sunayev- Zel'dovich Effect(SZE)

- The theoretical foundation of the Sunyaev-Zel'dovich effect was laid in the early 1970s (Sunyaev & Zel'dovich 1970), but is based on earlier work on the interactions of photons and free electrons by (Kompaneets 1956; Dreicer 1964; Weymann 1965).



(Yakov Borisovich Zeldovich, 1914-1987)

(Rashid Alevich Sunayev, 1943-)

- The Sunyaev-Zel'dovich (S-Z) effect is the scattering of cosmic microwave background radiation (CMBR) photons off electrons in the gas that fills the gravitational potential well in clusters of galaxies.

Types of SZE

- Kinetic Zel'dovich Effect
  - first-order effect due to the bulk velocity of the cluster with respect to the CMBR.
- Thermal Zeldo'vich Effect
  - second-order effect due to the thermal velocities of the electrons in the gas.

Typical picture of SZE

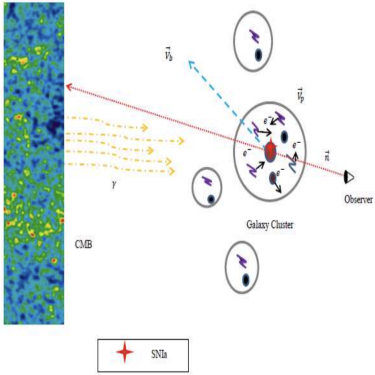


Figure: Galaxy cluster hosts a supernova type SN-Ia. Line of sight direction(Red dotted arrow) and the blue dashed arrow is the direction of the bulk flow. The solid black arrows shows the peculiar velocities of cluster members.(arXiv: 1703.02021v3)

Physical Process

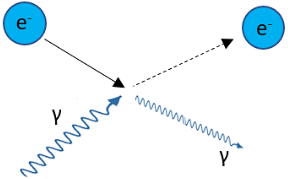
Inverse Compton Scattering;

$$e^{-}(q) + \gamma(p) = e^{-}(q') + \gamma(p')$$

$$\begin{aligned} &q + p = q' + p' \\ &E_e(q) + E_\gamma(p) = E_e(q') + E_\gamma(p') \end{aligned}$$

"We make assumptions:"

- Non-relativistic regime, T, $T_e \ll m_e$, T is the CMB temperature at the redshift where the scattering takes place, while T_e is the electron temperature.
- Isotropic scattering, taking $|M|^2$ as well as the distribution functions to be independent of $\hat{p}, \hat{q}$.



Theory Formalism

Boltzmann Equation for photon distribution in presence of collision with plasma in a galaxy cluster or any other medium,

$$\left[\frac{\partial}{\partial t} - Hp\frac{\partial}{dp}\right] f(p,t) = c\,[f(p)]_{SZ} \tag{1}$$

f(p,t)=localized

$$c\,[f(p)]_{SZ} = \frac{1}{2E(p)} \int \frac{d^3q}{(2\pi)^3 2E_e(q)} \int \frac{d^3q'}{(2\pi)^3 2E_e(q')} \int \frac{d^3p'}{(2\pi)^3 2E_e(p')} \sum_{spins} |M|^2 \cdot (2\pi)^4 \delta_D^{(4)}[p+q-p'-q'] \cdot [f_e(q')f(p') - f_e(q)f(p)] \tag{2}$$

Tools to deal with eq.2:

$$f_e(q') = f_e(q+p-p') \approx f_e(q), \int \delta_D^{(3)}(p+q-p'-q')d^3(q') = 1 \tag{3}$$

Theory Formalism (Thermal Zel'dovich Effect)

Collision term gets simplified under appropriate assumptions we have,

$$c\,[f(p)]_{SZ} = \frac{n_e \sigma_T}{4\pi p} \int p' dp' \int d\Omega' \cdot \Bigg[\delta_D^{(1)}(p-p') + \frac{(p'-p)\cdot q}{m_e} \frac{\partial}{\partial p'} \delta_D^{(1)}(p-p') + \frac{1}{2}\left(\frac{(p'-p)\cdot q}{m_e}\right)^2 \frac{\partial^2}{\partial p'^2} \delta_D^{(1)}(p-p') \Bigg] [f(p') - f(p)] \tag{4}$$

$$x = \frac{p}{T}, T = \frac{T_0}{a}, \ a = \frac{1}{1+z} \ , y = \int \frac{n_e T_e \sigma_T}{m_e} dt, dt = a\, d\chi \tag{5}$$

Re-writing Botzmann equation with variables x and y we get,

$$\boxed{\frac{\partial f(x,y)}{\partial y} = x^2 \frac{\partial^2}{\partial x^2} f(x,y) + 4x \frac{\partial}{\partial x} f(x,y)} \tag{6}$$

Photon distribution function in absense of collision: $f(x, y = 0) = \frac{1}{e^x - 1}$

Thermal S-Z Effect (cont'd)

Solution of (eq.6) for the photon distribution function $f(x, y)$ in presence of compton parameter y:

- Approx. solution with $y(<< 1)$;

$$f(x,y) = \frac{1}{e^x - 1} + \frac{y}{x^2}\frac{\partial}{\partial x}\left[x^4 \frac{\partial}{\partial x}\right]\left(\frac{1}{e^x - 1}\right) \tag{7}$$

$$\frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4 \frac{\partial}{\partial x}\right]\left(\frac{1}{e^x - 1}\right) = \frac{xe^x}{(e^x - 1)^2}\left(\frac{x}{tanh(x/2)} - 4\right)$$

- General solution :

$$f(x,y) = \frac{1}{\sqrt{4\pi y}T_{cmb}}\int_0^\infty exp\left(4y - \frac{1}{4y}\left[ln(T/T_{cmb}) + 5y\right]^2\right)dT \tag{8}$$

$$x = \frac{h\nu}{kT_{CMB}}, y \propto n_e \sigma_T T_e$$

- Spectral Radiance from a galaxy cluster :

$$B_{\nu,T} = \frac{2h\nu^3}{c^2} f(x, y = 0) \rightarrow \textit{Planck function} \quad (9)$$

$$B_{\nu,T}|_C = \frac{2h\nu^3}{c^2} f(x, y) \rightarrow \textit{Planck function with collisions} \quad (10)$$

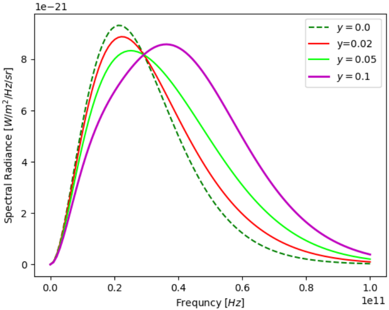


Figure: Blackbody spectrum with Thermal S-Z effcet. y is compton parameter determines the shape of the CMB spectrum.

Photon temperature

Temperature of Blackbody in presence of compton parameter y can be expressed as (compatible with observation),

$$\frac{T(x)_y}{T_{CMB}} = \frac{1}{ln\left[1 + f(x,y)^{-1}\right]} \tag{11}$$

When compton parameter y=0(i.e no collison b/w electron and photon in the galaxy cluster or any other medium along the l.o.s);

$$\frac{T(x)_0}{T_{cmb}} = 1, \;\; f(x,0) = \frac{1}{e^x - 1} \tag{12}$$

- Photon temperature in the approximation limit ($y << 1$):

$$\frac{T(x)_y}{T_{cmb}} = 1 + y\left(\frac{x}{tanh(x/2)} - 4\right)$$

- Photon temperature in general is given by (eq.11) with the substitution of f(x,y) from (eq.8)

Temperature Profile

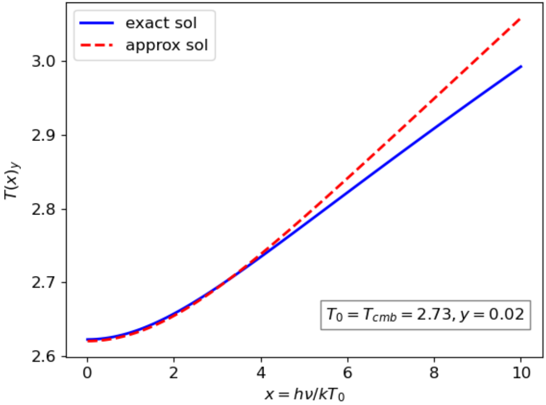


Figure: Effective photon temperature arising from the Zel'dovich-Sunyaev mechanism with compton parameter y.

Relative Change in Photon Temperature

We have,

$$\frac{T(x)_y}{T_{cmb}} = 1 + y\left(\frac{x}{tanh(x/2)} - 4\right) \tag{13}$$

In the Rayleigh-Jeans limit $(x \to 0)$

$$\frac{T(x)_y}{T_{cmb}} = 1 - 2y \tag{14}$$

that means,

$$\frac{\Delta T(x)_y}{T_{cmb}} \approx -2y$$

ΔT is independent of redshift!

We normalize the relative change in intensity due tSZ process over a pure black-body intensity then we arrived at following relation,

$$\boxed{\frac{\Delta I_{tSZ}}{I_\nu|_{cmb}} = \left[y \frac{xe^x}{(e^x - 1)} \left(\frac{x}{tanh(x/2)} - 4 \right) \right]} \tag{15}$$

Kinetic S-Z effect

$$\boxed{\frac{\Delta I_{kSZ}}{I_\nu|_{cmb}} = -\frac{u_b}{c}\frac{xe^x}{(e^x-1)}\int n_e\sigma_T dl = -\frac{xe^x}{(e^x-1)}\int \frac{u_b}{c}\, n_e\sigma_T dl} \quad (16)$$

In the above equation we have used the fact that $\frac{q}{m_e} = u_b$(i.e bulk velocity of electrons in a galaxy cluster) and $dt = \frac{dl}{c}$ where dl is the line of sight distance to the galxay cluster.

Order of magnitude comparison: tSZ and kSZ effect

tSZ dominates over the kSZ, typically by an order of magnitude for galaxy clusters, because the thermal velocity of electrons

$$\sqrt{\frac{k_B T_e}{m_e}} = 4 \times 10^4 \left(\frac{T_e}{10^8 K}\right) kms^{-1} \tag{17}$$

Bulk velocities $u_b \leq 10^3 kms^{-1}$

Spectrum: tSZ and kSZ effect

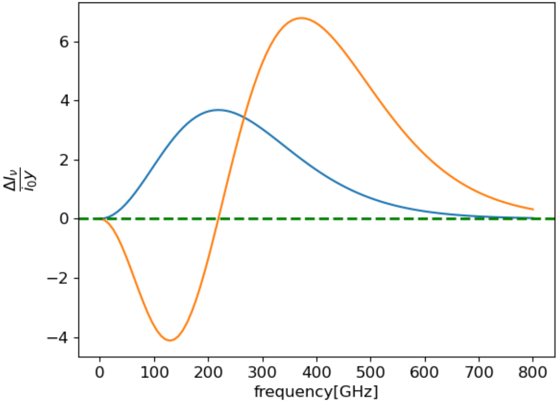


Figure: The frequency dependences of the thermal and kinematic S-Z effects, expressed as a function of intensity.

Conclusion

- There seems to be a possibility that distortions in CMB spectrum are caused by hot electrons inverse Compton scattering off the background photons, we feel it is worthwhile to re-examine this mechanism.
- An expression for the photon temperature heated by the electrons are studied.
- Relative photon temperature is computed here and we see it is independent of red-shift, allows us to detect clusters via the S-Z effect back to their epoch of formation.

Implications of S-Z Effect

- The S-Z effect gives complementary information to X-ray observations of the same gas.
- The kinematic effect provides a direct measurement of cluster peculiar velocities.
- Hubble constant: combine with X-ray observations, S-Z effect can be useful to estimate Hubble rate of the universe,

$$H_0 = \frac{8(T_0 K_B \sigma_T)^2}{m_e^2 c^3 K(T_e)} \cdot \left(\frac{T_e}{\Delta T_{RJ}}\right)^2 \theta X_{SB} \left((1+z)^3 - (1+z)^{3/2}\right)$$

X-ray surface brightness : $X_{SB} = \frac{K(T_e)\, l\, n_e^2}{(1+z)^4}$ of cube of gas of side l at a red-shift z, with an electron number density n_e and temperature T_e and $K(T_e)$ is an emissivity constant.

Acknowledgements

- Thank you Prof. Philstrom for teaching "Advanced Astrophysics II ".
- Papers:
  - "Aspects of the Zel'dovich-Sunayev mechanism" PRD Vol 21 No 2, 15 Jan 1990.
  - The Establishment of Thermal Equilibrium between Quanta and Electrons*, SOVIET PHYSICS JETP Vol 4, No 5, JUNE, 1957
  - The Sunayev-Zel'dovich effect in clusters by Michael Jones.

Thank You and Questions?