

The Sunayev-Zel'dovich Effect

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Outline

- 1 The Physics of Sunayev-Zel'dovich Effect (SZE)
- 2 Results and Discussions based on SZE



The Sunayev- Zel'dovich Effect(SZE)

- The theoretical foundation of the Sunyaev-Zel'dovich effect was laid in the early 1970s (Sunyaev & Zel'dovich 1970), but is based on earlier work on the interactions of photons and free electrons by (Kompaneets 1956; Dreicer 1964; Weymann 1965).



(Yakov Borisovich
Zeldovich, 1914-1987)



(Rashid Alevich Sunayev,
1943-)

- The Sunyaev-Zel'dovich (S-Z) effect is the scattering of cosmic microwave background radiation (CMBR) photons off electrons in the gas that fills the gravitational potential well in clusters of galaxies.

Types of SZE

- Kinetic Zel'dovich Effect
 - ▶ first-order effect due to the bulk velocity of the cluster with respect to the CMBR.
- Thermal Zeldo'vich Effect
 - ▶ second-order effect due to the thermal velocities of the electrons in the gas.

Typical picture of SZE

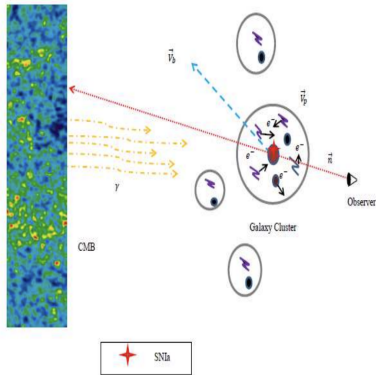


Figure: Galaxy cluster hosts a supernova type SN-Ia. Line of sight direction (Red dotted arrow) and the blue dashed arrow is the direction of the bulk flow. The solid black arrows show the peculiar velocities of cluster members. (arXiv: 1703.02021v3)

Physical Process

Inverse Compton Scattering;

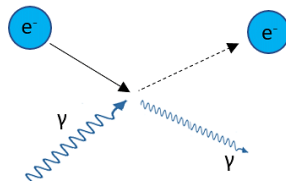
$$e^-(q) + \gamma(p) = e^-(q') + \gamma(p')$$

$$q + p = q' + p'$$

$$E_e(q) + E_\gamma(p) = E_e(q') + E_\gamma(p')$$

“We make assumptions:”

- Non-relativistic regime, $T, T_e \ll m_e$, T is the CMB temperature at the redshift where the scattering takes place, while T_e is the electron temperature.
- Isotropic scattering, taking $|M|^2$ as well as the distribution functions to be independent of \hat{p}, \hat{q} .



Theory Formalism

Boltzmann Equation for photon distribution in presence of collision with plasma in a galaxy cluster or any other medium,

$$\left[\frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right] f(p, t) = c [f(p)]_{SZ} \quad (1)$$

$f(p, t)$ = localized

$$\begin{aligned} c [f(p)]_{SZ} &= \frac{1}{2E(p)} \int \frac{d^3 q}{(2\pi)^3 2E_e(q)} \int \frac{d^3 q'}{(2\pi)^3 2E_e(q')} \int \frac{d^3 p'}{(2\pi)^3 2E_e(p')} \sum_{spins} |M|^2 \cdot \\ &\quad (2\pi)^4 \delta_D^{(4)}[p + q - p' - q'] \cdot [f_e(q')f(p') - f_e(q)f(p)] \end{aligned} \quad (2)$$

Tools to deal with eq.2:

$$f_e(q') = f_e(q + p - p') \approx f_e(q), \quad \int \delta_D^{(3)}(p + q - p' - q') d^3(q') = 1 \quad (3)$$



Theory Formalism (Thermal Zel'dovich Effect)

Collision term gets simplified under appropriate assumptions we have,

$$c[f(p)]_{SZ} = \frac{n_e \sigma_T}{4\pi p} \int p' dp' \int d\Omega' \cdot \left[\delta_D^{(1)}(p - p') + \frac{(p' - p) \cdot q}{m_e} \frac{\partial}{\partial p'} \delta_D^{(1)}(p - p') + \frac{1}{2} \left(\frac{(p' - p) \cdot q}{m_e} \right)^2 \frac{\partial^2}{\partial p'^2} \delta_D^{(1)}(p - p') \right] [f(p') - f(p)] \quad (4)$$

$$x = \frac{p}{T}, T = \frac{T_0}{a}, a = \frac{1}{1+z}, y = \int \frac{n_e T_e \sigma_T}{m_e} dt, dt = a d\chi \quad (5)$$

Re-writing Boltzmann equation with variables x and y we get,

$$\boxed{\frac{\partial f(x, y)}{\partial y} = x^2 \frac{\partial^2}{\partial x^2} f(x, y) + 4x \frac{\partial}{\partial x} f(x, y)} \quad (6)$$

Photon distribution function in absence of collision: $f(x, y = 0) = \frac{1}{e^x - 1}$

Thermal S-Z Effect (cont'd)

Solution of (eq.6) for the photon distribution function $f(x, y)$ in presence of compton parameter y :

- Approx. solution with $y(<< 1)$;

$$f(x, y) = \frac{1}{e^x - 1} + \frac{y}{x^2} \frac{\partial}{\partial x} \left[x^4 \frac{\partial}{\partial x} \right] \left(\frac{1}{e^x - 1} \right) \quad (7)$$

$$\frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \frac{\partial}{\partial x} \right] \left(\frac{1}{e^x - 1} \right) = \frac{xe^x}{(e^x - 1)^2} \left(\frac{x}{\tanh(x/2)} - 4 \right)$$

- General solution :

$$f(x, y) = \frac{1}{\sqrt{4\pi y} T_{cmb}} \int_0^\infty \exp \left(4y - \frac{1}{4y} [\ln(T/T_{cmb}) + 5y]^2 \right) dT \quad (8)$$

$$x = \frac{h\nu}{kT_{CMB}}, y \propto n_e \sigma_T T_e$$

- Spectral Radiance from a galaxy cluster :

$$B_{\nu,T} = \frac{2h\nu^3}{c^2} f(x, y = 0) \rightarrow \text{Planck function} \quad (9)$$

$$B_{\nu,T|C} = \frac{2h\nu^3}{c^2} f(x, y) \rightarrow \text{Planck function with collisions} \quad (10)$$

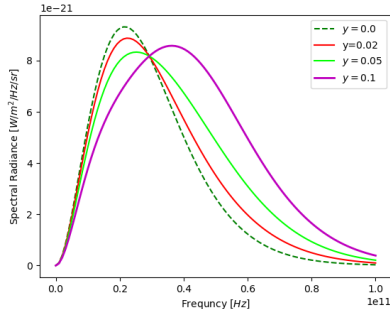


Figure: Blackbody spectrum with Thermal S-Z effect. y is compton parameter determines the shape of the CMB spectrum.

Photon temperature

Temperature of Blackbody in presence of compton parameter y can be expressed as (compatible with observation),

$$\frac{T(x)_y}{T_{CMB}} = \frac{1}{\ln[1 + f(x, y)^{-1}]} \quad (11)$$

When compton parameter $y=0$ (i.e. no collision b/w electron and photon in the galaxy cluster or any other medium along the l.o.s);

$$\frac{T(x)_0}{T_{cmb}} = 1, \quad f(x, 0) = \frac{1}{e^x - 1} \quad (12)$$

- Photon temperature in the approximation limit ($y \ll 1$):

$$\frac{T(x)_y}{T_{cmb}} = 1 + y \left(\frac{x}{\tanh(x/2)} - 4 \right)$$

- Photon temperature in general is given by (eq.11) with the substitution of $f(x, y)$ from (eq.8)

Temperature Profile

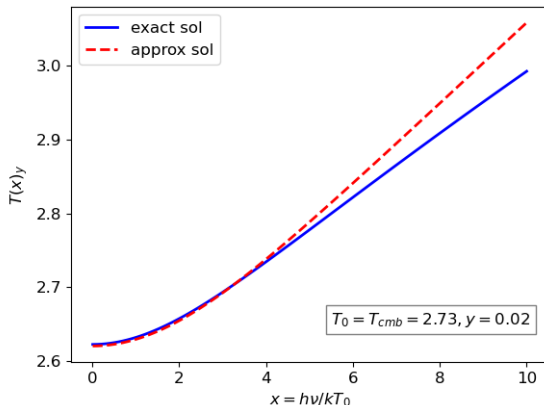


Figure: Effective photon temperature arising from the Zel'dovich-Sunyaev mechanism with compton parameter y .

Relative Change in Photon Temperature

We have,

$$\frac{T(x)_y}{T_{cmb}} = 1 + y \left(\frac{x}{\tanh(x/2)} - 4 \right) \quad (13)$$

In the Rayleigh-Jeans limit ($x \rightarrow 0$)

$$\frac{T(x)_y}{T_{cmb}} = 1 - 2y \quad (14)$$

that means,

$$\frac{\Delta T(x)_y}{T_{cmb}} \approx -2y$$

ΔT is independent of redshift!

We normalize the relative change in intensity due tSZ process over a pure black-body intensity then we arrived at following relation,

$$\frac{\Delta I_{tSZ}}{I_{\nu}|_{cmb}} = \left[y \frac{x e^x}{(e^x - 1)} \left(\frac{x}{\tanh(x/2)} - 4 \right) \right] \quad (15)$$

Kinetic S-Z effect

$$\frac{\Delta I_{kSZ}}{I_{\nu}|_{cmb}} = -\frac{u_b}{c} \frac{x e^x}{(e^x - 1)} \int n_e \sigma_T dl = -\frac{x e^x}{(e^x - 1)} \int \frac{u_b}{c} n_e \sigma_T dl \quad (16)$$

In the above equation we have used the fact that $\frac{q}{m_e} = u_b$ (i.e bulk velocity of electrons in a galaxy cluster) and $dt = \frac{dl}{c}$ where dl is the line of sight distance to the galaxy cluster.

Order of magnitude comparison: tSZ and kSZ effect

tSZ dominates over the kSZ, typically by an order of magnitude for galaxy clusters, because the thermal velocity of electrons

$$\sqrt{\frac{k_B T_e}{m_e}} = 4 \times 10^4 \left(\frac{T_e}{10^8 K} \right) \text{ kms}^{-1} \quad (17)$$

Bulk velocities $u_b \leq 10^3 \text{ kms}^{-1}$

Spectrum: tSZ and kSZ effect

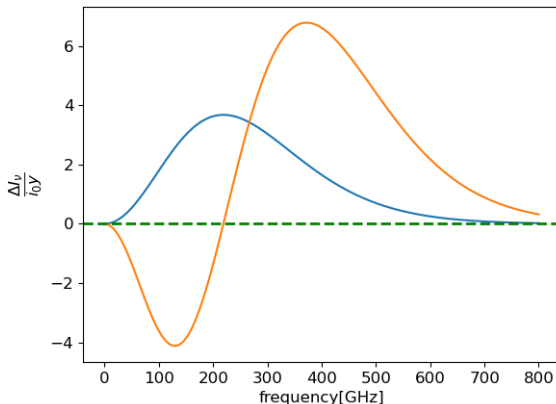


Figure: The frequency dependences of the thermal and kinematic S-Z effects, expressed as a function of intensity.

Conclusion

- There seems to be a possibility that distortions in CMB spectrum are caused by hot electrons inverse Compton scattering off the background photons, we feel it is worthwhile to re-examine this mechanism.
- An expression for the photon temperature heated by the electrons are studied.
- Relative photon temperature is computed here and we see it is independent of red-shift, allows us to detect clusters via the S-Z effect back to their epoch of formation.

Implications of S-Z Effect

- The S-Z effect gives complementary information to X-ray observations of the same gas.
- The kinematic effect provides a direct measurement of cluster peculiar velocities.
- Hubble constant: combine with X-ray observations, S-Z effect can be useful to estimate Hubble rate of the universe,

$$H_0 = \frac{8(T_0 K_B \sigma_T)^2}{m_e^2 c^3 K(T_e)} \cdot \left(\frac{T_e}{\Delta T_{RJ}} \right)^2 \theta X_{SB} \left((1+z)^3 - (1+z)^{3/2} \right)$$

X-ray surface brightness : $X_{SB} = \frac{K(T_e) I n_e^2}{(1+z)^4}$ of cube of gas of side l at a red-shift z , with an electron number density n_e and temperature T_e and $K(T_e)$ is an emissivity constant.

Acknowledgements

- Thank you Prof. Philstrom for teaching "Advanced Astrophysics II".
- Papers:
 - ▶ "Aspects of the Zel'dovich-Sunayev mechanism" PRD Vol 21 No 2, 15 Jan 1990.
 - ▶ The Establishment of Thermal Equilibrium between Quanta and Electrons*, SOVIET PHYSICS JETP Vol 4, No 5, JUNE, 1957
 - ▶ The Sunayev-Zel'dovich effect in clusters by Michael Jones.

Thank You and Questions?

