Constraining the dense matter equation of state using neutron star observations

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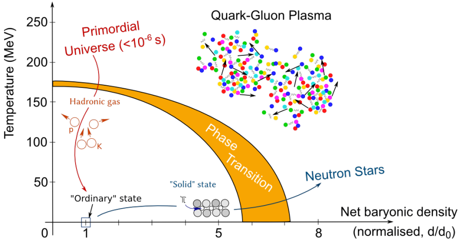
Motivation

The strong force is still poorly understood at high densities and low temperatures.



- Understanding the composition of neutron stars can thus give important information on the behavior of the strong interaction.

Compact Object as an Astrophysical Laboratory

- Neutron stars are the dead remnants of massive stars.
- They can be up to twice as massive as our sun.
- A teaspoon of neutron star matter would weigh 4 billion tons.

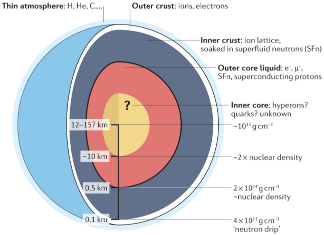


Figure: Typical Neutron Star structure, P.C: Google

The macroscopic properties of nuclear matter are described by equation of state .

Equation of State(EoS)

Pressure and Energy density relation of nuclear matter.

- EoS 1:
  Meta Model+Speed of sound approach
- EoS 2: MIT Bag Model describes quark matter, strange matter on overall density scale.

EoS 1: Meta Model up to (1-2)n_{sat}

- The general properties of nuclear interactions are often characterized in terms of the nuclear empirical parameters:

$$e_{sat} = E_{sat} + \frac{1}{2!}K_{sat}x^2 + \frac{1}{3!}Q_{sat}x^3 + \frac{1}{4!}Z_{sat}x^4 + \cdots \quad (1)$$

$$e_{sym} = E_{sym} + L_{sym}x + \frac{1}{2!}K_{sym}x^2 + \frac{1}{3!}Q_{sym}x^3 + \frac{1}{4!}Z_{sym}x^4 + \cdots \quad (2)$$

$$x = \frac{n - n_{sat}}{3n_{sat}} \quad (3)$$

Energy per nucleon in nuclear matter, defined as

$$\frac{E}{A} \approx e_{sat} + e_{sym} \quad (4)$$

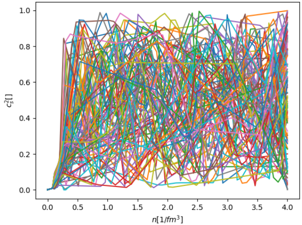
$$\epsilon = n\frac{E}{A}; \quad p = -\frac{dE}{dV} = n^2\frac{dE/A}{dn}$$

J. Margueron et al. , Phys.Rev.C 97 (2018) 2, 025805

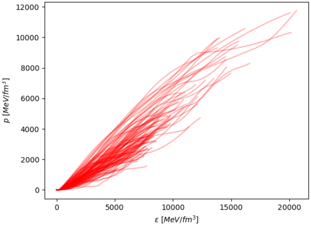
Speed of Sound approach

$n_{break} < n < 25n_{sat} \qquad p(n) = p(n_i) + \int_{n_i}^{n} c_s^2(n')\mu(n')dn'$

$$\epsilon(n) = \epsilon(n_i) + \int_{n_i}^{n} \mu(n')dn'$$



(a) Speed of sound



(b) pressure-energy density plot

R. Somasundaram et al, Phys. Rev. C 107, 025801

MIT Bag Model

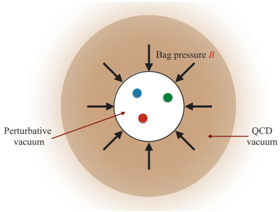


Figure: Hadron bag

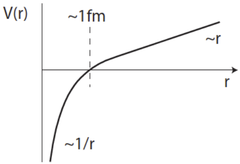


Figure: QCD Potential

Quarks are confined inside the bag!
Non-interacting.

- number density: $n = \sum_{u,d,s} \int_0^\infty \frac{g}{2\pi^3} f(p) d^3p$
- Energy density: $\epsilon = \sum_{u,d,s} \int_0^\infty \frac{g}{2\pi^3} \epsilon(p) f(p) d^3p + B$
- Pressure: $P = \sum_{u,d,s} \int_0^\infty \frac{g}{2\pi^3} \frac{p^2}{3\epsilon} f(p) d^3p - B$
- g is the degeneracy factor= 2×3(spin$\times$ color)

$$f(p) = \frac{1}{e^{\frac{\epsilon-\mu}{T}} + 1} \tag{5}$$

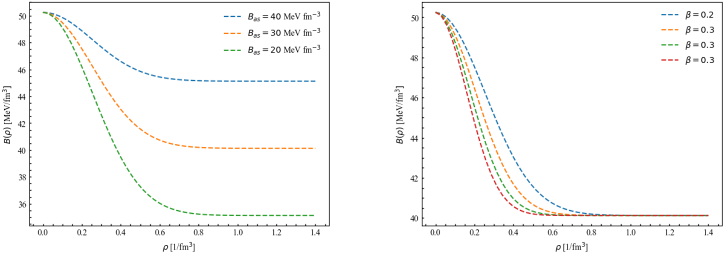
Quarks are appear to be quasi-free at short distance: K. Johnson, Acta Phys.Polon.B6

Density dependent bag pressure

Density dependent Bag pressure

$$B(\rho) = B_{as} + \frac{\Delta B}{2}\left[1 + e^{-\beta(\rho/\rho_0)^2}\right] \quad (6)$$

B_{as} is the asymtotic density, $\Delta B = B - B_{as}$



Zero Temperature Solution

We construct EOS making various approximation: at zero temperature quarks are filled up to Fermi level with Fermi energy $E_f < \mu_f$, distribution function approaches to unity, means all the Fermi level below Fermi energy are completely filled and abobe are empty.

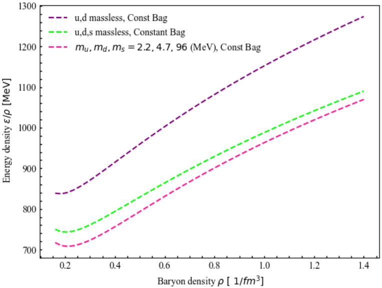
$$P = -B(\rho) + \sum_f \frac{1}{4\pi^2}\left[\mu_f k_f\left(u_f^2 - \frac{5}{2}m_f^2\right) + \frac{3}{2}m_f^4 ln\left(\frac{\mu_f + k_f}{m_f}\right)\right] \tag{7}$$

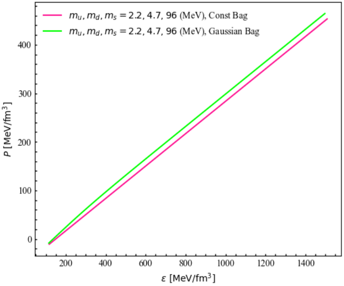
$$\epsilon = B(\rho) + \sum_f \frac{3}{4\pi^2}\left[\mu_f k_f\left(u_f^2 - \frac{5}{2}m_f^2\right) - \frac{1}{2}m_f^4 ln\left(\frac{\mu_f + k_f}{m_f}\right)\right] \tag{8}$$

$$\rho = \sum_f \frac{k_f^3}{3\pi^2} \tag{9}$$

k_f=Fermi momentum of quark flavor f given by $k_f = \sqrt{\mu_f^2 - m_f^2}$, μ_f is the chemical potential of flavor f.

Quark Matter EOS





- Minimum energy per baryon occurs correspond to the zero pressure across the baryon density ensuring thermodynamic self-consistency.
- Massive quark flavors appreciably satisfies the stability window of $^{56}Fe = 928$ MeV(E/A) up to several times of nuclear saturation density.

Stellar Structure Equations

Tolman-Oppenheimer-Volkoff(TOV) Equation for compact object generally written as,

$$\frac{dm(r)}{dr} = \frac{4\pi r^2 \epsilon(r)}{c^2} \tag{10}$$

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2 r^2}\left[1 + \frac{P(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1} \tag{11}$$

The boundary conditions: $P(0) = P_c$ and P(R=r)=0 with m(r)=M to ensure hydrostatic equilibrium.

We aslo worked on enthaply formulation of solving TOV: Fasten enough

Tidal Deformation

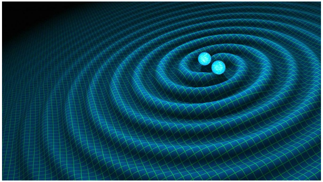
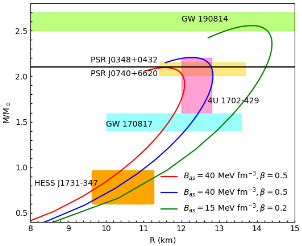


Figure: Binary neutron star inspiral.

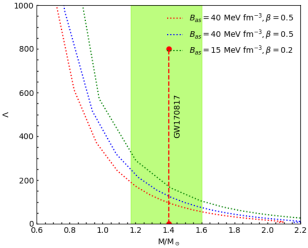
$$Q_{ij} = -\Lambda \epsilon_{ij}$$

ϵ_{ij} : External tidal field,
Q_{ij} : Quadrupole moment.
Tidal deformability: $\Lambda = \frac{2}{3} k_2 (R/M)^5$,

Results: 1



(a) Mass-Radius relation

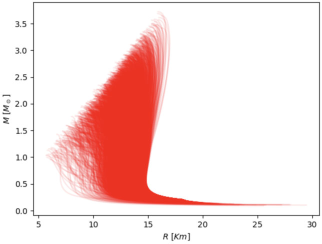


(b) Tidal deformability-Mass relation

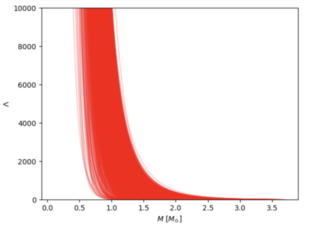
Figure: Zero Temperature Bag Model , $B^{1/4}$=(130-140) MeV

Results: 2

We have constructed 100000 EOS based on Meta Model and speed of sound approach and estimated the Mass-radius and Tidal deformabilities for each individual EOS.



(a) Mass-Radius relation

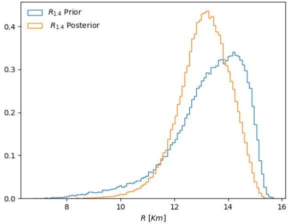


(b) Tidal deformability-Mass relation

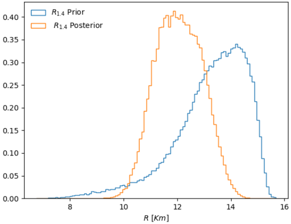
NICER Observations

The likelihood of a certain EOS given an M-R posterior $p(M, R|NICER)$ from a NICER measurement can be written as,

$$\mathcal{L}(EOS|NICER) = \int_0^{M_{TOV}} dM\, p\,(M, R(EOS)|NICER) \qquad (12)$$



(c) J0030+451 2019 M-R dataset



(d) PSR J0437–4715 2024 M-R dataset

Figure: EOS inference based on the NICER measurement of Pulsars

Next Step

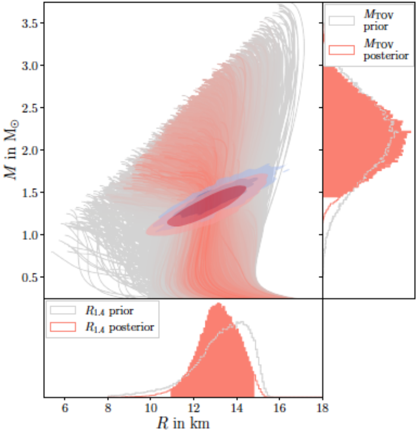


Figure: M-R posterior for PSR J0030+451

Arxiv: 2402.04172

Learning Outcome and Conclusion

- Studied Neutron star EOS formulation and hand on experience of solving stellar structure equations for structural studies of NS: i.e nuclear matter equation of state.
- With density dependent Bag model, we find equatorial radius of $11 - 13.7$ km and Tidal deformability of 200 in dimensionless unit, which consistent with GW observations.
- We also able to incorporate compact object mass of mass 0.77 $M\odot$ within radius $9.7 - 11.2$ km
- Using the PSR datasets we find the the likelihood of MM+CSM model EOS and canonical posterior distribution on an equatorial radius at mass 1.4 $M_{\odot}$.

I would like express my sincere gratitude to my mentors: Oleg, Soumi and Rahul. Thank you all!

Questions ?