Neutrino Oscillation, an Aperture to Look Through BSM Physics

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NuPAC Seminar
03.28.2023

Outline

- ☑ Section A: Overview of Neutrino Physics
- ☑ Section B: Neutrino Oscillations in Vacuum and Matter, MSW Effect and Neutrino Mass ordering.
- ☑ Section C: Neutrino Mass and Double Beta Decay in the context BSM
- ☑ Summary

Section A

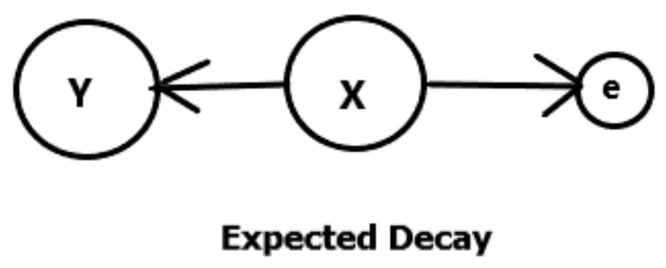
Overview of Neutrino Physics

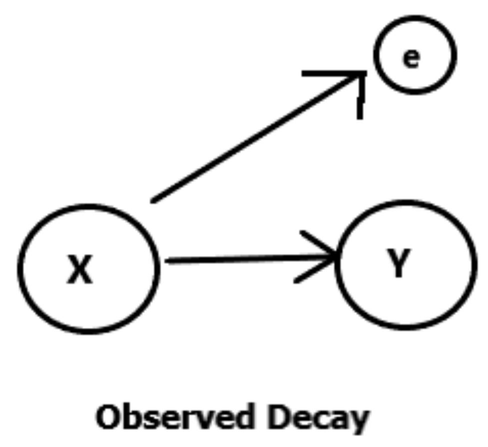
Radioactive Beta Decay

In the year of 1914

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-}$$

Energy Conservation





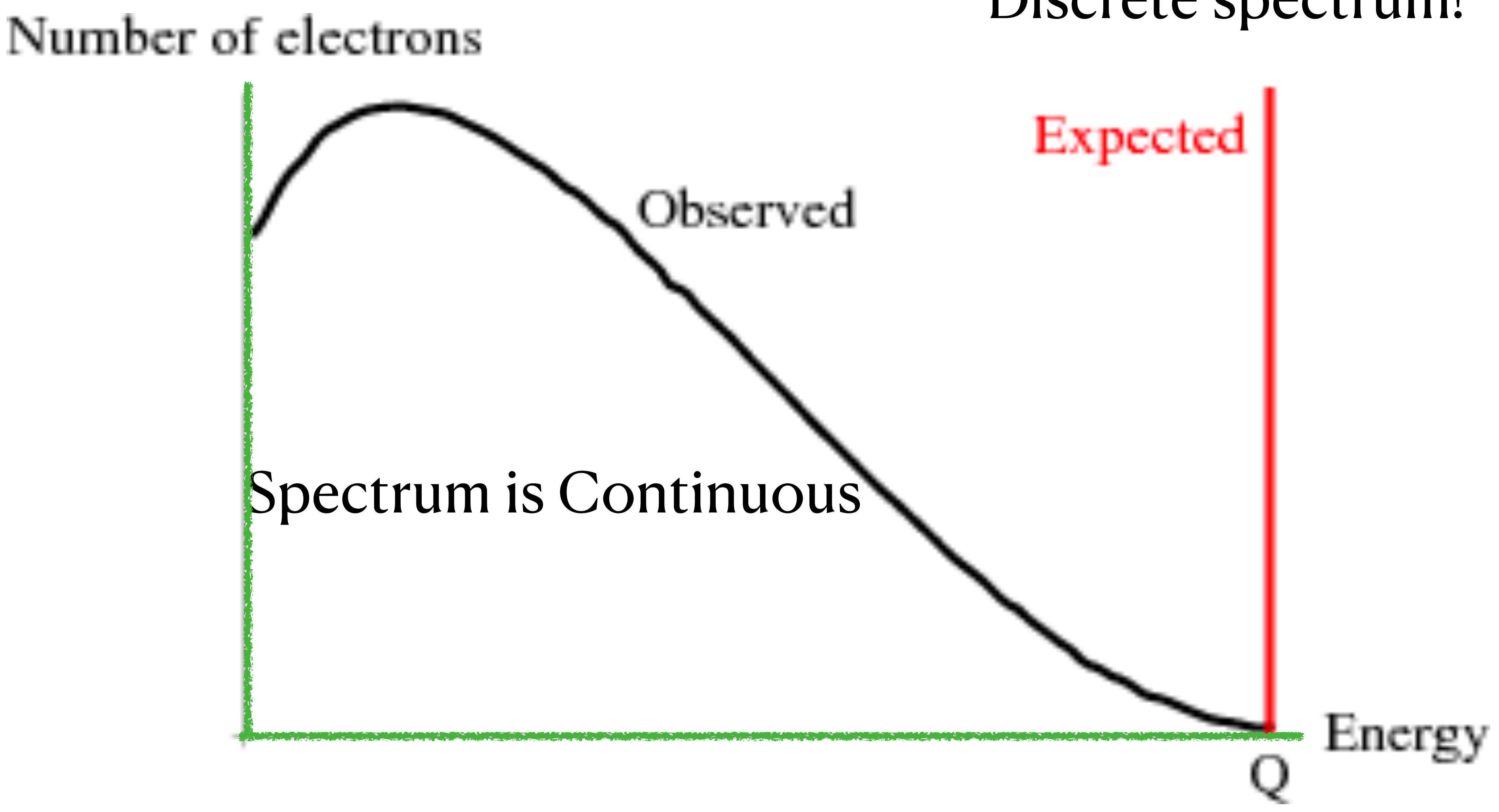
$$E_X = E_Y + E_{e^-}$$

Momentum Conservation $\overrightarrow{P_X} = \overrightarrow{P_Y} + \overrightarrow{P_{e^-}}$

$$E_X = M_X \; and \; \overrightarrow{P_Y} = -\overrightarrow{P_e} \qquad M_X^2 = \sqrt{|P|^2 + M_Y^2} + \sqrt{|P|^2 + m_e^2}$$

$$E_e = \frac{M_X^2 + m_e^2 - M_Y^2}{2M_X} \qquad Q_\beta = M_X(A, Z) - M_Y(A, Z+1) - m_e$$

Discrete spectrum!

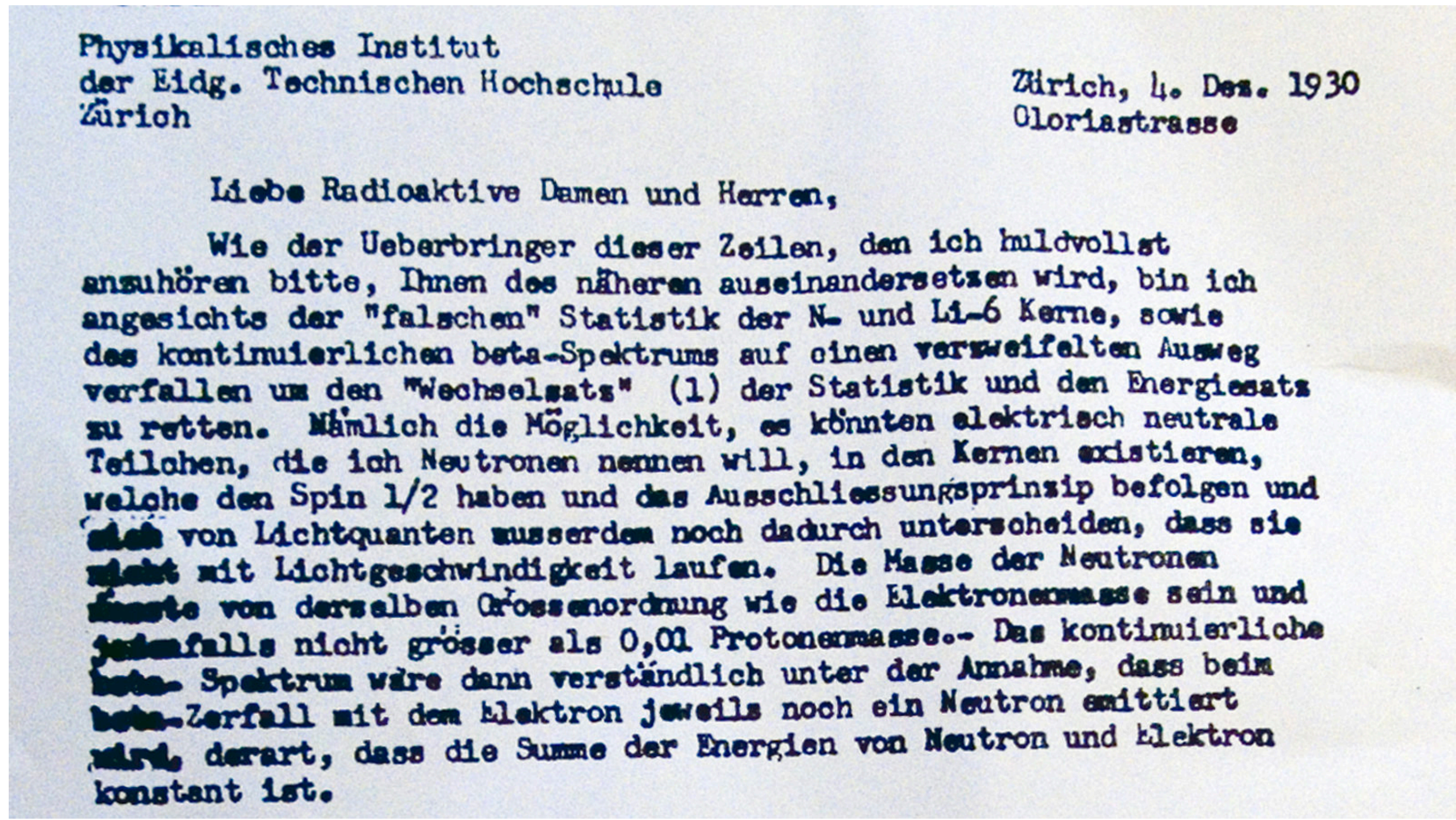


Conservation of Energy, Momentum as well as Spin statistics theorem was in Danger!

${}^{14}N = 14p + 7e^{-} = 21$ Odd spin (based on Nuclear Model)

${}^{14}N$: Even spin observed, the spin-statistics theorem seemed to be violated

Pauli Hypothesis: Desperate Remedy



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

https://www.math.utah.edu/~beebe/talks/2015/qtm/pdf/pauli-1930-ltc.pdf

There is an additional neutral particle with a $s = \dfrac{1}{2}$ carry 1 % of mass across the detector without leaving any trace such that the sum of neutron and electron energy is constant.

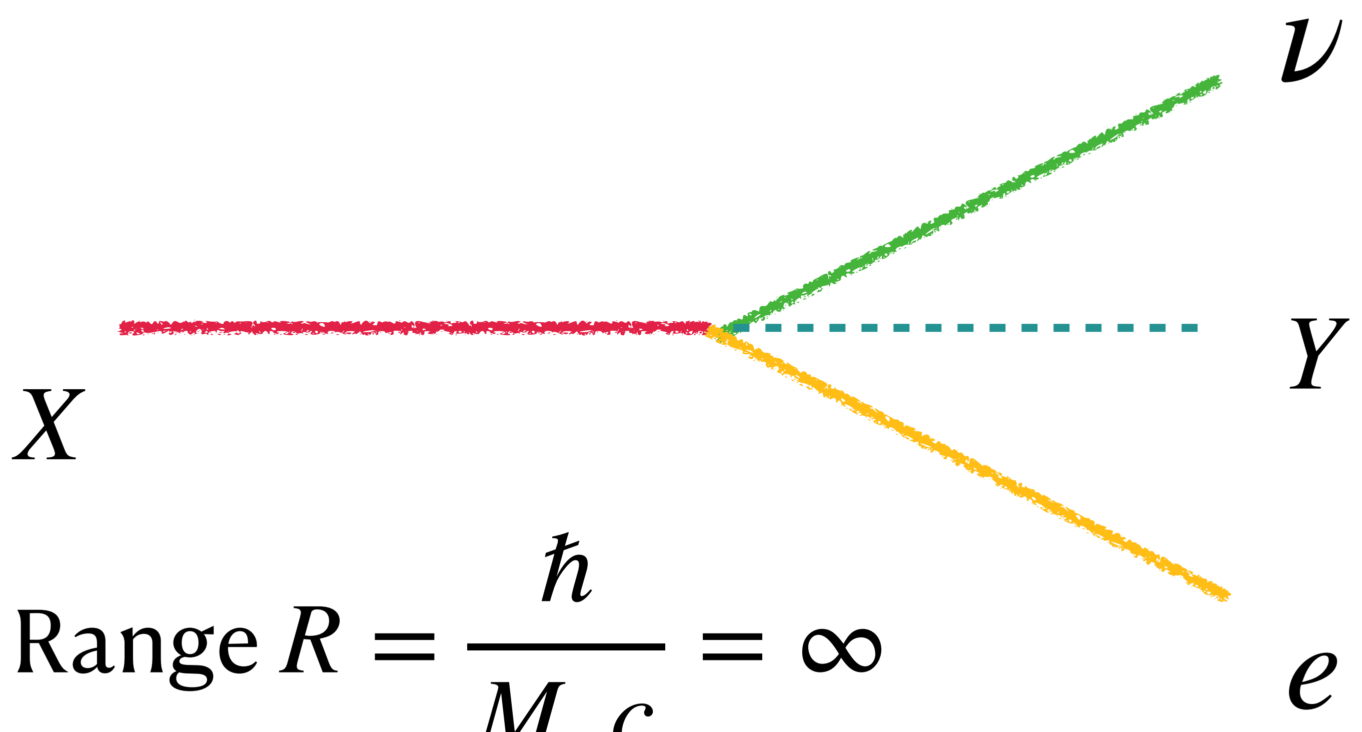
$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + Neutron $$

$$ Q = (M_X - M_Y - m_e)\, c^2 $$

1930 : Wolfgang Pauli

$^{14}N = 14p + 7e^{-} + 7n = 28$, Even Spin (Nuclear Model rescued)

1934: **Fermi Theory of Beta Decay**



Range $R = \dfrac{\hbar}{M_m c} = \infty$

Energy, Momentum Conservation , spin-statistics satisfied in the three-body beta decay process

"I have done a terrible thing. I have postulated a particle that can not be detected"

Unfortunately, Pauli was wrong and later neutrinos were detected!

Discovery of Invisible Neutrinos

- **Electron neutrino ν_e : 1956**

  Reactor anti-neutrinos: $\bar{\nu}_e + p \rightarrow n + e^+$

  Nobel Prize to Frederick Reines in 1995

Savannah River plant near South Carolina

Clyde Cowan

Frederick Reines

- **Muon neutrino ν_μ : 1962** at BNL

  Neutrinos from pion decay:

  $\pi^- \rightarrow \mu^- + \nu_{(\mu)}$

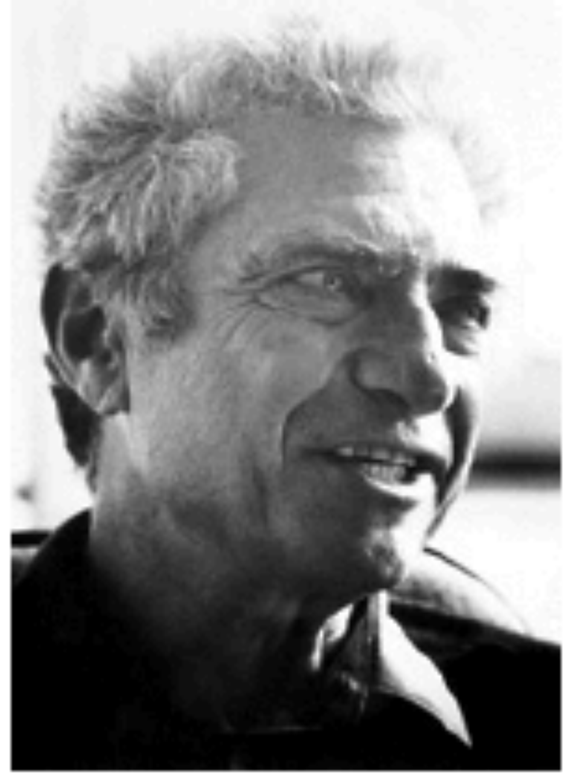
  $\nu_{(\mu)} + N \rightarrow N' + \mu^-$

  Always a muon, never an e^- / e^+

  Nobel Prize in 1988

Leon M. Lederman

Melvin Schwartz



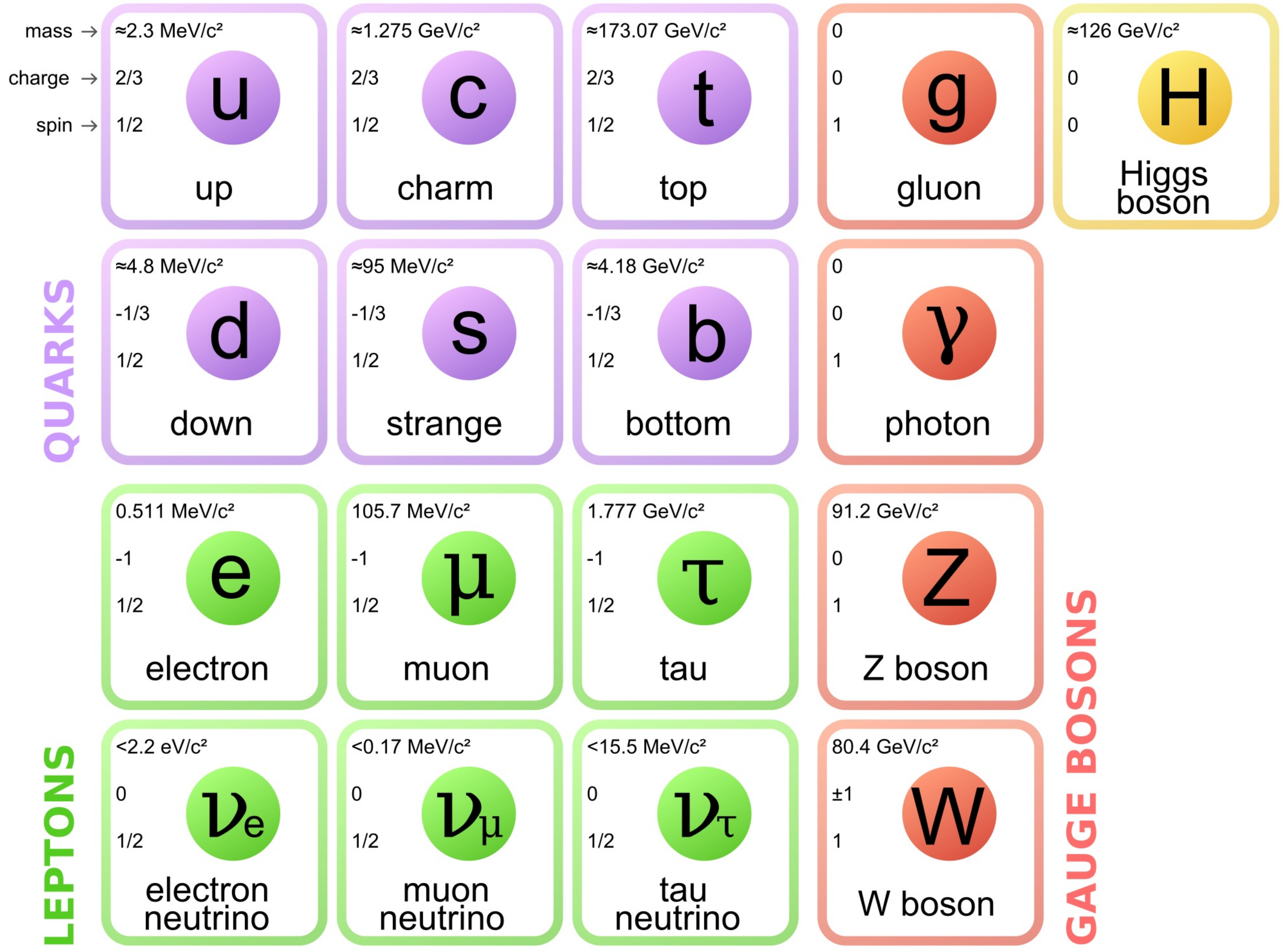
Jack Steinberger

- **Tau neutrino ν_τ : 2000**

  DONUT experiment at Fermilab: $\nu_\tau + N \rightarrow \tau + N'$

Neutrinos in Standard Model

- spin 1/2 particles
- colourless
- electrically neutral
  $\Rightarrow$ only weak interaction via massive vector bosons ($W^{\pm}$, Z^0)
- 3 neutrinos, associated with e, μ, τ
- lepton number of each family conserved
- neutrinos are left handed and antineutrinos right handed



Standard Model

Theory of Neutrinos in the SM

Dirac Lagrangian for fermion field $L(x) = \bar{\psi}(x)\left(i\overleftrightarrow{\partial\!\!\!/}_{\mu} - m\right)\psi(x) = 0$

$\bar{\psi} = \psi^{\dagger}\gamma^{0}$ and $\overleftrightarrow{\partial}_{\mu} = \dfrac{\overrightarrow{\partial}_{\mu} - \overleftarrow{\partial}_{\mu}}{2}$ and

$\partial\!\!\!/ = \gamma^{\mu}\partial_{\mu}$

Using Euler-Lagrange equation $\partial_{\mu}\dfrac{\partial L}{\partial\left(\partial_{\mu}\bar{\psi}\right)} - \dfrac{\partial L}{\partial\bar{\psi}} = 0$

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}, \quad \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} \qquad \gamma^{0}\gamma^{\mu\dagger}\gamma^{0} = \gamma^{\mu}$$

$$\psi = \psi_{L} + \psi_{R} \; and \; \bar{\psi} = \bar{\psi}_{L} + \bar{\psi}_{R}$$

$$\psi_{L} = \frac{1-\gamma^{5}}{2}\psi = P_{L}\psi$$

$$\psi_{R} = \frac{1+\gamma^{5}}{2}\psi = P_{R}\psi$$

Projection Operator

$$P_{R} + P_{L} = 1$$

$$P_{R}^{2} = P_{R}$$

$$P_{L}^{2} = P_{L}$$

$$\boxed{\bar{\psi}_{R}\, i\gamma_{\mu}\partial_{\mu}\psi_{R} + \bar{\psi}_{L} i\gamma_{\mu}\partial_{\mu}\psi_{L} - m\left(\bar{\psi}_{R}\psi_{L} + \bar{\psi}_{L}\psi_{R}\right) = 0}$$

$$\boxed{\begin{array}{c} i\;\partial\!\!\!/\psi_{R} = m\psi_{L} \\ i\;\partial\!\!\!/\psi_{L} = m\psi_{R} \end{array}}$$

$$\{\gamma^{5}, \gamma^{\mu}\} = 0, \quad \left(\gamma^{5}\right)^{2} = 1, \quad \left(\gamma^{5}\right)^{\dagger} = \gamma^{5}$$

$$m = m_{\nu} = 0 \quad \boxed{\begin{array}{c} i\;\partial\!\!\!/\psi_{R} = 0 \\ i\;\partial\!\!\!/\psi_{L} = 0 \end{array}} \longrightarrow \text{Weyl Equations}$$

$$\gamma_{c}^{5} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; \gamma_{c}^{0} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \; \vec{\gamma}_{c} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

Chiral fields ψ_{L} *and* ψ_{R} (i.e Weyl spinors)

Massive Neutrinos

Majorana Theory, 1937

$$i\gamma^\mu\partial_\mu\psi_L = m\psi_R \qquad (1) \qquad\qquad i\gamma^\mu\partial_\mu\psi_R = m\psi_L \qquad (2)$$

Taking hermitian conjugate of (2) and multiply right with γ^0 and using the Property of Dirac matrix we obtain

$$-i\partial_\mu\bar{\psi}_R\gamma^\mu = m\bar{\psi}_L \quad \rightarrow (3)$$

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu$$

$$C^\dagger = C^{-1}$$

$$C^T = -C$$

Taking transpose of (3) and multiply C on the left we obtain

$$i\gamma^\mu\partial_\mu C\bar{\psi}_R^T = mC\bar{\psi}_L^T \quad \rightarrow (4)$$

Comparing Eq.(4) & (1) $\Rightarrow \quad \psi_R = C\bar{\psi}_L^T$

ψ is a Majorana Field given by: $\quad \psi = \psi_L + \psi_R = \psi_L + C\bar{\psi}_L^T \quad \Rightarrow \quad \boxed{\psi = \psi_L + \psi_L^C = \psi^C}$

$$\downarrow$$

Majorana Condition , Neutrinos are massive
Beyond Standard Model

Dirac equation

$$\left(i\gamma^\mu\partial_\mu - m_\nu\right)\psi = 0$$

$$\left(i\gamma^\mu\partial_\mu - m_\nu\right)\psi^C = 0$$

Charged Fermion: $\pm q\gamma^\mu A_\mu$

Majorana condition not valid

Kinetic Effect: Same for both Dirac and Majorana $\Longrightarrow 0\nu\beta\beta$

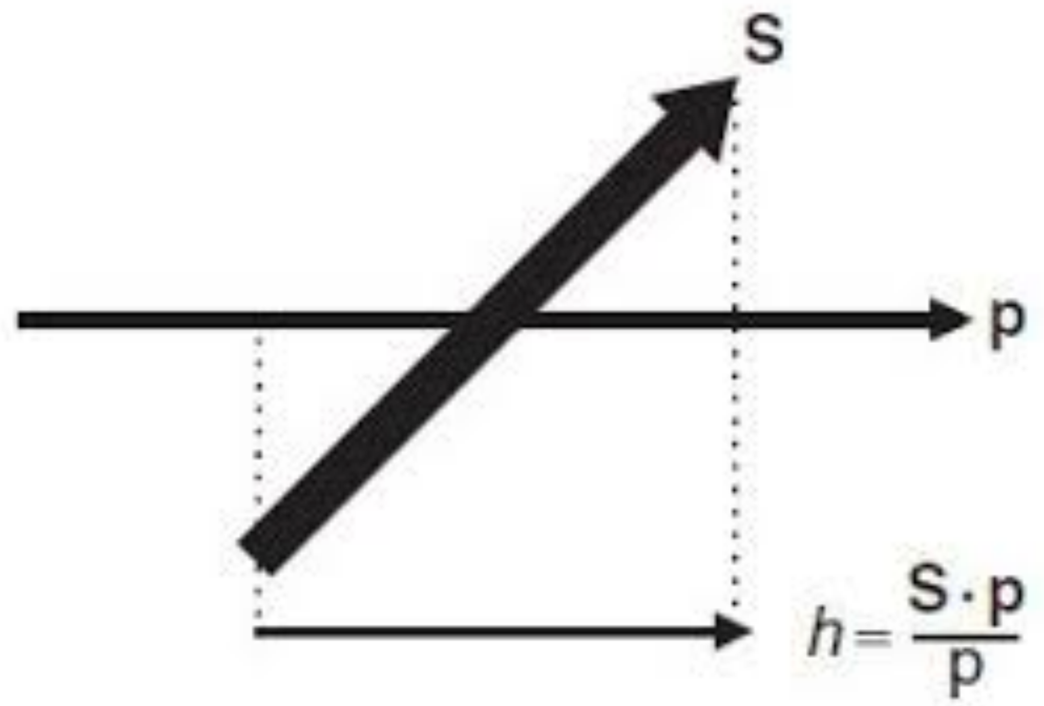
Helicity and Handedness of Neutrinos

$$\hat{h} = \frac{\vec{S}\cdot\vec{P}}{s|\vec{P}|} = \frac{\vec{\Sigma}\cdot\vec{P}}{|\vec{P}|}$$

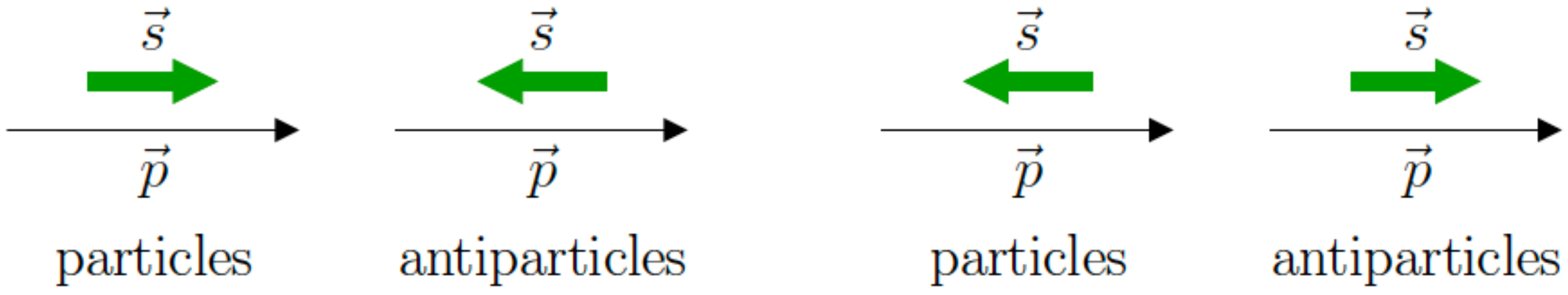
$\hat{h}$ has eigenvalues ± 1

$$\frac{\vec{\Sigma}\cdot\vec{P}}{|\vec{P}|}\psi_L = -\psi_L$$

$$\frac{\vec{\Sigma}\cdot\vec{P}}{|\vec{P}|}\psi_R = +\psi_R$$



Massless right-handed chiral fields ψ_R has definite positive helicity and Massless left-handed chiral fields ψ_L has definite negative helicity



Helicity States for ψ_R

Helicity States for ψ_L $SU(2)_L$

Neutrinos: Left Handed

Antineutrinos: Right Handed

What about C, P, CP, and CPT ?

C, P, CP, CPT symmetry of SM

C: Charge conjugation $\hat{C}|\nu\rangle_L = \eta|\bar{\nu}\rangle_L \quad \Rightarrow \times \quad Violated$

P: Parity $\hat{P}|\nu\rangle_{LH} = \zeta|\nu\rangle_{RH} \quad \Rightarrow \times \quad Violated$

Right handed neutrinos are Singlet

LSND Experiment

$P(\nu_\mu \rightarrow \nu_e)$

Sterile Neutrinos

CP: Charge Parity $\hat{C}\hat{P}|\nu\rangle_{LH} = \chi|\bar{\nu}\rangle_{RH} \quad \Rightarrow \quad ✓ \quad Allowed$

$P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} \quad \Rightarrow CP \ Conserved$ $\qquad P_{\nu_\alpha \rightarrow \nu_\beta} \neq P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} \quad \Rightarrow CP \ Violated$

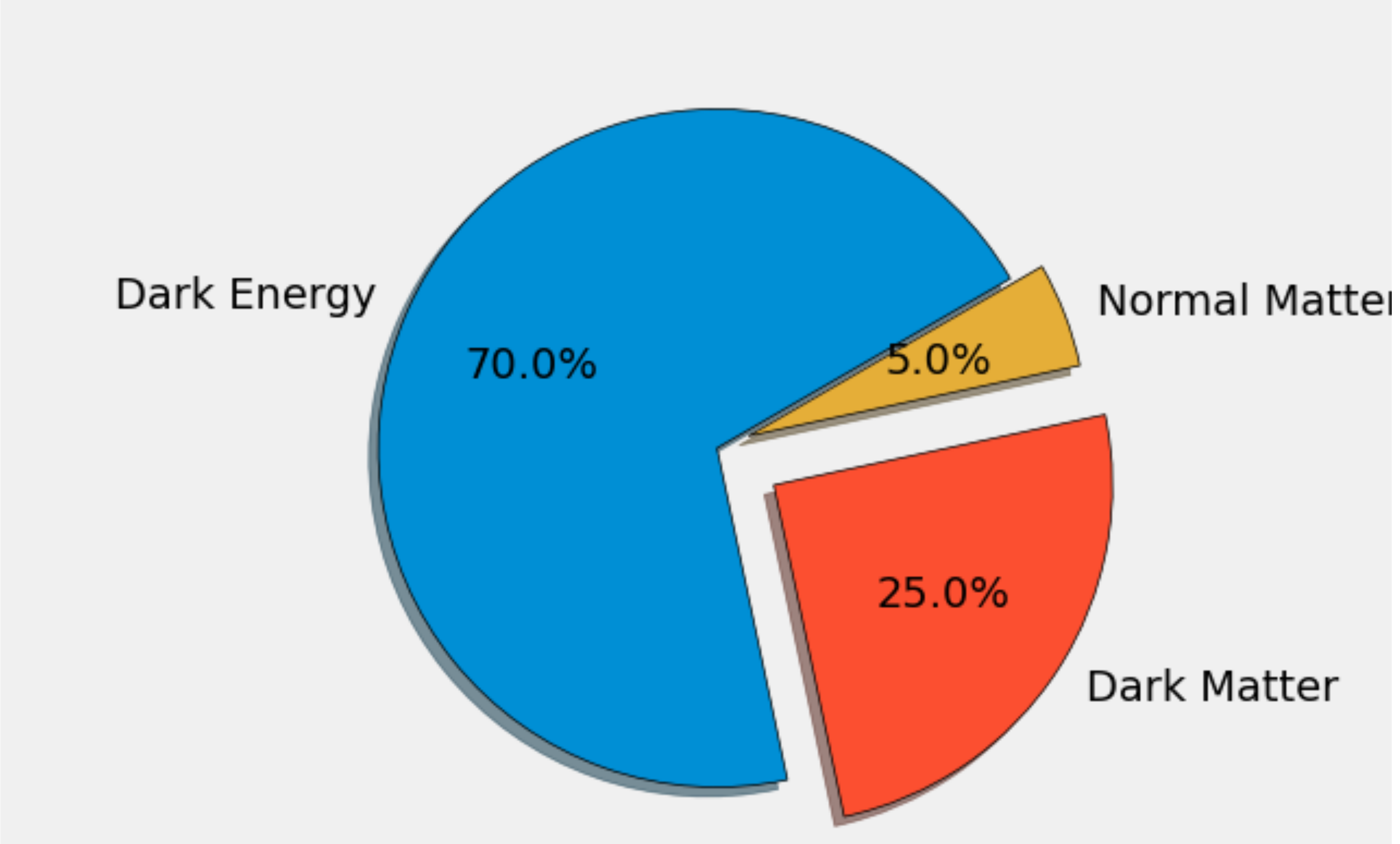
CPT: Charge Parity Time reversal

$P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \quad \Rightarrow CPT \ Conserved$ $\qquad P_{\nu_\alpha \rightarrow \nu_\beta} \neq P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \quad \Rightarrow CPT \ violated$

We can test the CP and CPT symmetry of the Standard Model in the Neutrino Oscillation Experiment.

Neutrinos and Dark Matter connection

- ν_{RH} , are singlet standard model groups $SU(3)_C \times SU(2)_L \times U(1)_Y$ and can be a possible dark matter candidate depending on their mass (Extra species, $N_{eff} = 3.04 + 1 + ..$, in Cosmology)



Energy budget of Universe

https://arxiv.org/pdf/1610.02743.pdf

$H_0 = 100\,h\ [km/sec/Mpc]$

$h = 0.68$

- Dark matter density: $\Omega_{DM}h^2 = 0.119$,
- Neutrinos density : $\Omega_\nu h^2 = \dfrac{\sum m_{\nu,i}}{94eV}$

Neutrinos to be dark matter, It has to be $\Omega_{DM}h^2 = \Omega_\nu h^2$: $\dfrac{\sum m_{\nu,i}}{94\,eV} = 0.119$

The individual mass has to be $1/3\ of \sum m_{\nu,i} \approx 3.73\,eV$

- Sterile neutrinos with a mass greater than $3.73\ eV$could be dark matter, and KeV mass are warm dark matter candidates in literature.

Future experiment: CMB-S4

https://arxiv.org/pdf/1807.06209.pdf

Sources of Neutrinos

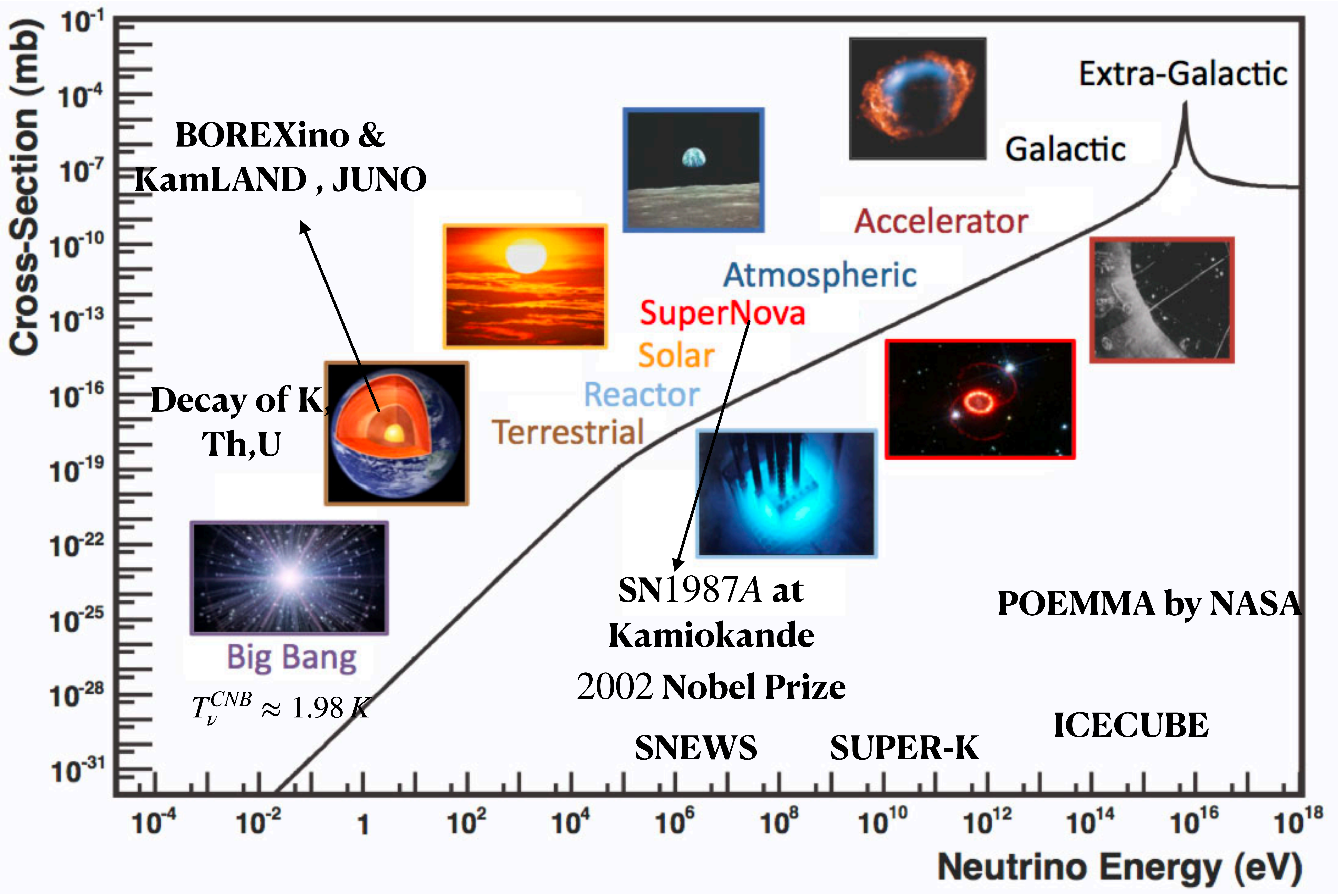
- Some of the lowest-energy ones come from the Big Bang, while the most energetic seen thus far have come from an extragalactic source.

- The neutrino cross-section is a measure of how likely the neutrino is to be stopped by regular matter. The higher energy a neutrino has, the more likely it is to interact.

Neutrinos are here, there & everywhere !!!

1 barn $= 10^{-24}\,cm^2$

Rev.Mod.Phys. 84 (2012) 1307-1341



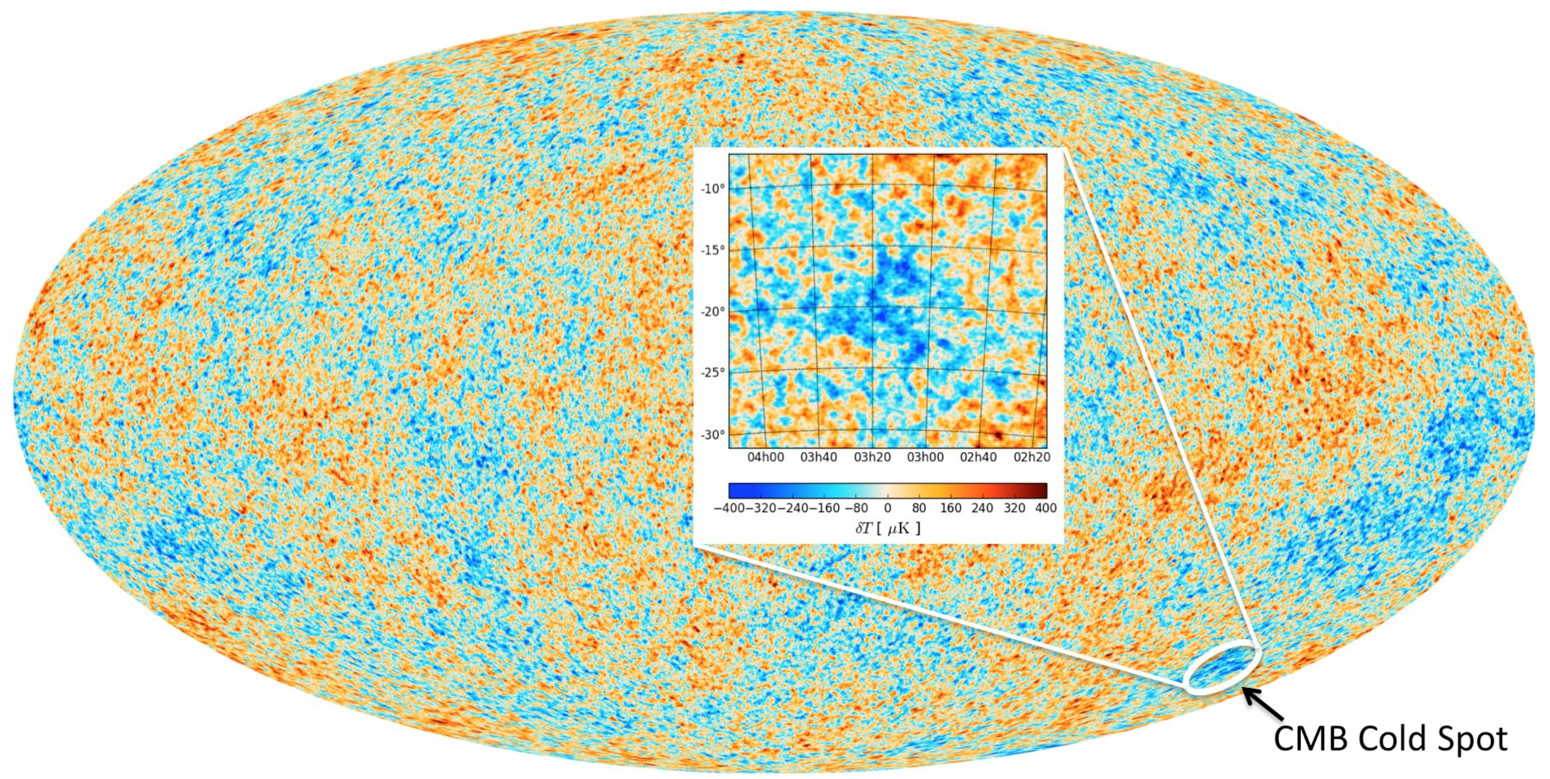
Neutrinos in Early Universe

$\mu_\nu = -\mu_{\bar{\nu}}$ means no matter-antimatter asymmetry in the early universe and under equilibrium condition.

Let us first consider the relativistic limit $p >> m$, $E \approx p$. Here, we set the chemical potential to zero. The number density is then given by

$$n = g\int_0^\infty \frac{d^3p}{(2\pi)^3}\frac{1}{e^{\frac{p}{T}} \pm 1}$$

$$= \frac{gT^3}{2\pi^3}\int_0^\infty \frac{x^2dx}{e^x \pm 1}$$



CMB Map

where we have set $x = p/T$ and used Isotropic distribution $d^3p = 4\pi^2 dp$

$$n_\gamma = \frac{gT_\gamma^3}{2\pi^2}\int_0^\infty \frac{x^2dx}{e^x - 1}$$

$$n_\gamma = \frac{2T_\gamma^3}{\pi^2}\zeta(3)$$

$$n_\gamma = 410\ photons/cm^3$$

$$n_\nu = \frac{gT_\nu^3}{2\pi^2}\int_0^\infty \frac{x^2dx}{e^x + 1}$$

$$n_\nu = \frac{6T_\nu^3}{2\pi^2}2\zeta(3)\frac{3}{4}$$

$$n_\nu = 336\ neutrinos/cm^3$$

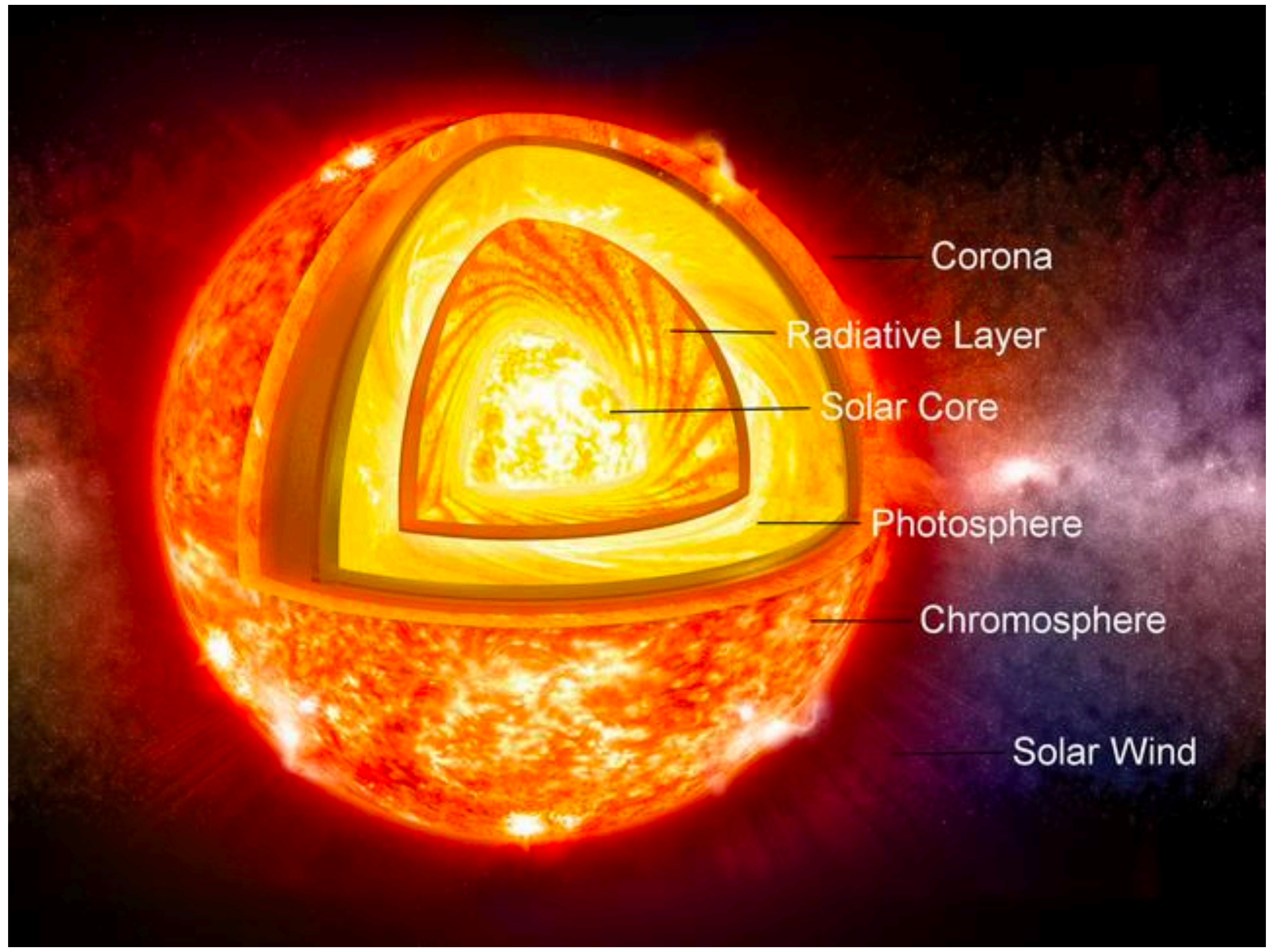
$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

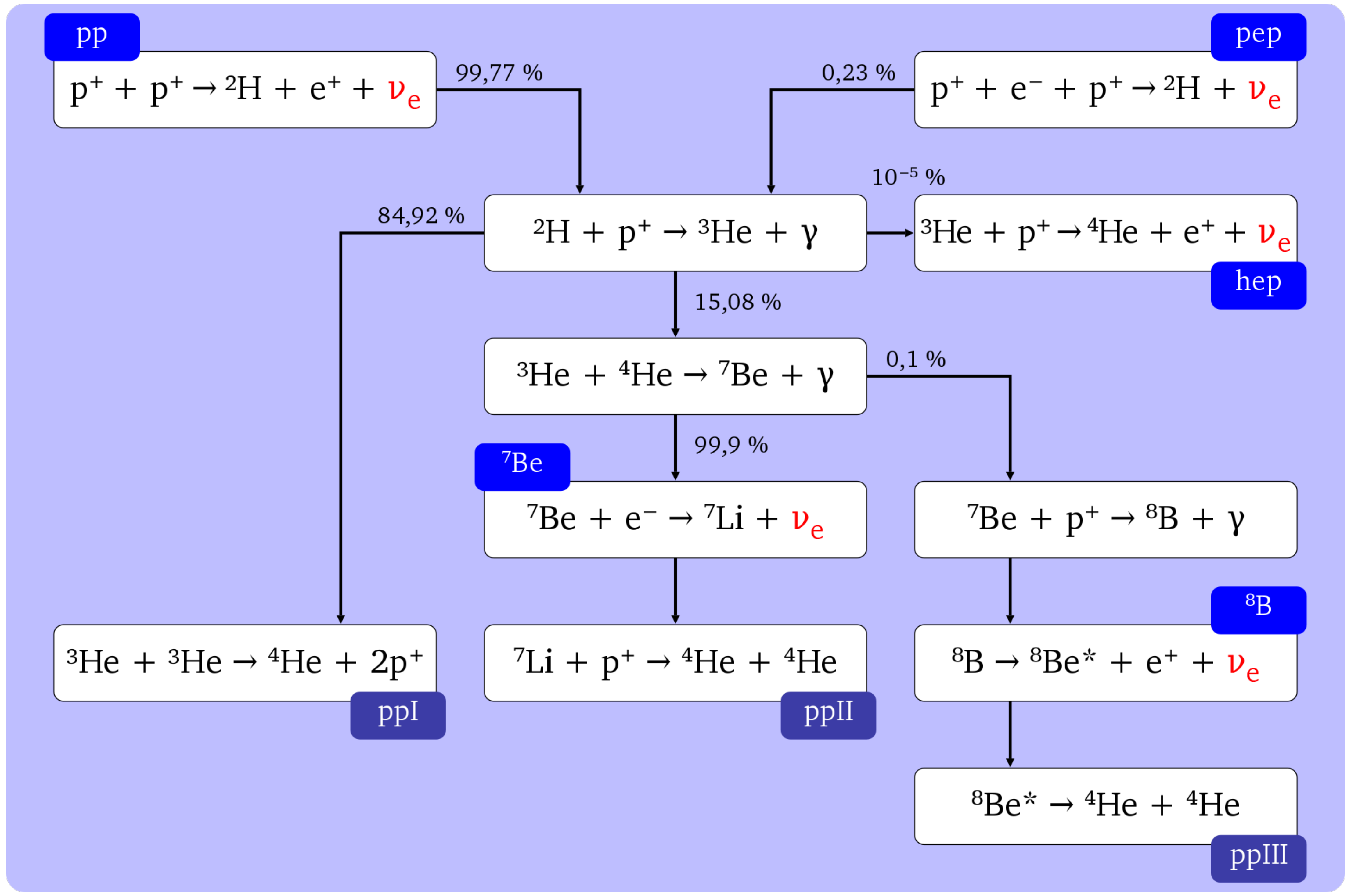
$$T_{CMB} \approx 2.73K$$

$$T_{CNB} \approx 1.98K$$

Neutrinos second most abundant particle after photon !

Sun cannot shine without Neutrinos





One can do the Same calculations for photons

Net Process in the PP chain $4^1H \to^4 He + 2e^+ + 2\nu_e + 26.7\,MeV$

$$E_2 = \frac{ch}{\lambda_{Peak}}, \lambda_{Peak} = 500\,nm$$

$$L_\odot = 3.828 \times 10^{26}\,W\,m^{-2} sec^{-1}$$

$$n_\gamma = 10^{17}$$

$$n_\nu = \frac{E_1}{E_2}\frac{Area_{finger}}{Area}$$

$$= 2 \times \frac{3.916 \times 10^{26} W\,sec}{26.7\,MeV}\frac{1\,cm^2}{4\pi\,1.5 \times 10^{13}\,cm^2}$$

$$= 6.5 \times 10^{10}$$

65 Billions of neutrinos pass through per centimeter square of finger area $\longrightarrow$ Huge flux

Neutrino From Supernovae

In a supernova, SN1987A typical energy is released $E_{SN} \approx 2.6 \times 10^{45}$ Joule and an explosion event happen in the LMC the closest galaxy to our Milky Way.

$$\text{Distance} = 50\,\text{kilo parsec}, \quad 1\text{pc} = 3.08 \times 10^{16}\,\text{m}$$

Taking the average energy 4.2 MeV per neutrino of supernovae neutrino one can calculate the flux on earth

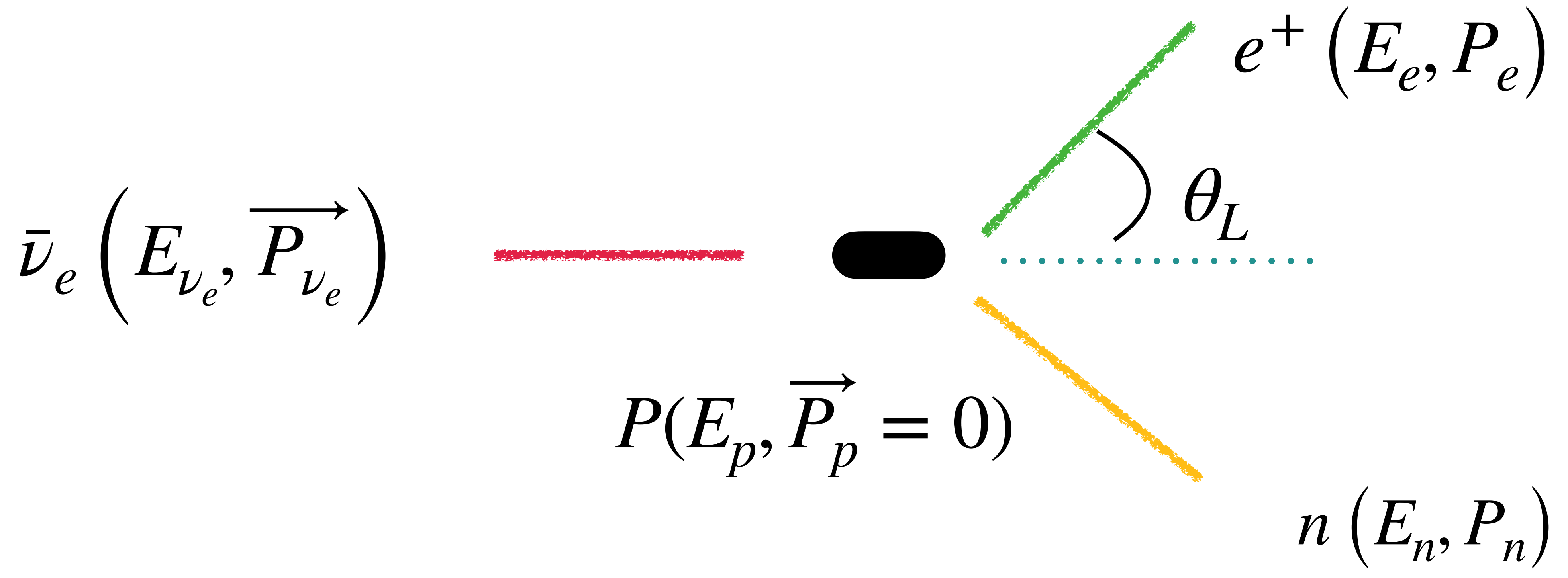
$$n_\nu = \frac{E_{SN}}{A_{LMC}} \times A_{Earth} \approx 10^{14}$$

However, only $(10 - 14)$ neutrinos were detected in Kamiokande and IMB detector in the year of 1987.

Catching neutrinos is an unusual process: Cross section study is important.

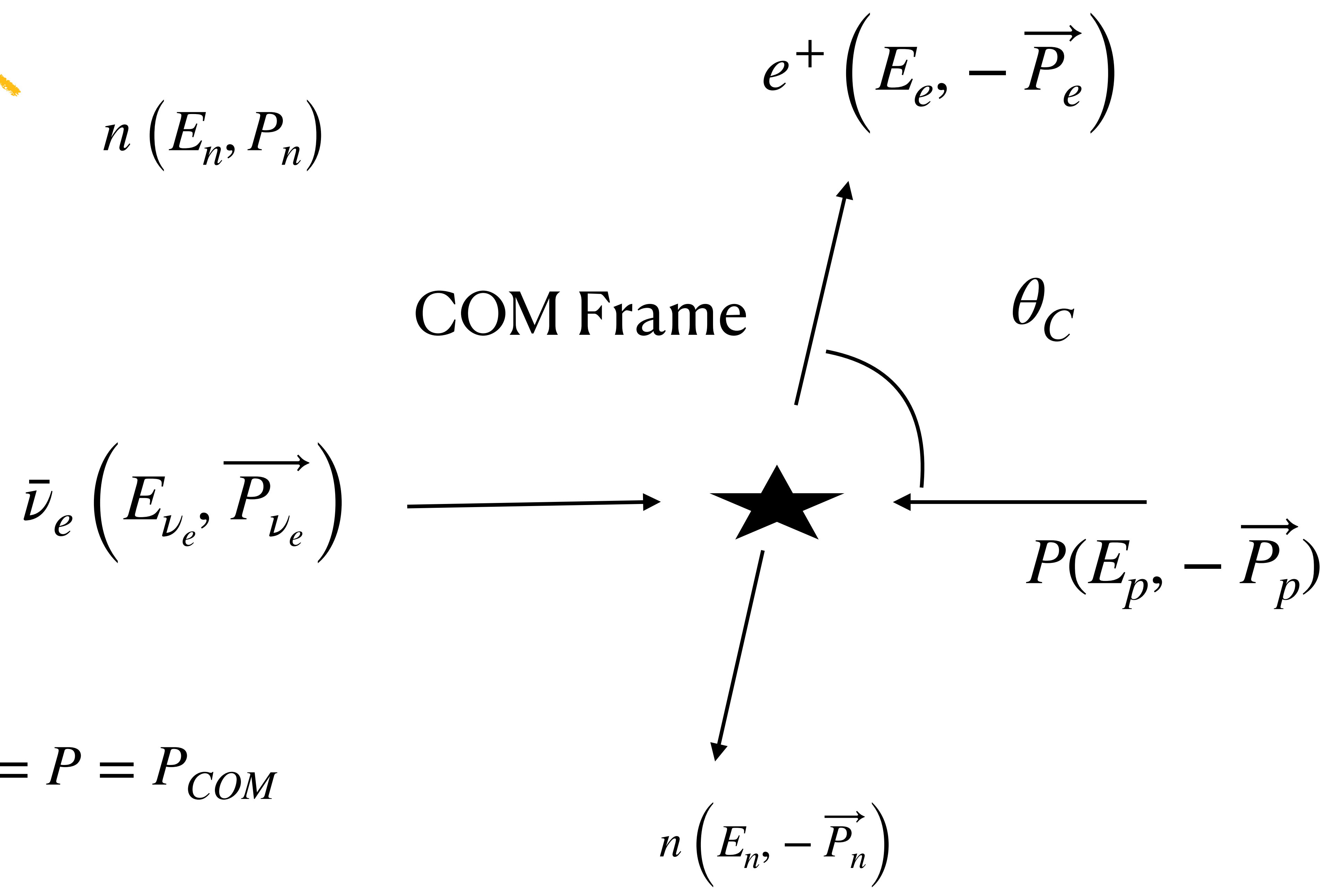
Relativistic Kinematics of Neutrino Scattering with a point like Object

$$\bar{\nu}_e + p \rightarrow e^+ + n$$
$$\nu_e + n \rightarrow e^- + p$$



$$s = \left(P_e + P_p\right)^2$$

$$S_{Lab} = \left(E_{\nu_e} + E_p\right)^2 - \left(\overrightarrow{P_{\nu_e}} + \overrightarrow{P_p}\right)^2 = m_p^2 + 2E_{\nu_e} m_p$$



$$S_{COM} = \left(2P^2 + m_p^2 + 2P\sqrt{P^2 + m_p^2}\right)$$

$$|\overrightarrow{P_{\nu_e}}| = |-\overrightarrow{P_p}| = P = P_{COM}$$

Total Cross Section $\sigma \approx G_F^2 P_{COM}^2$

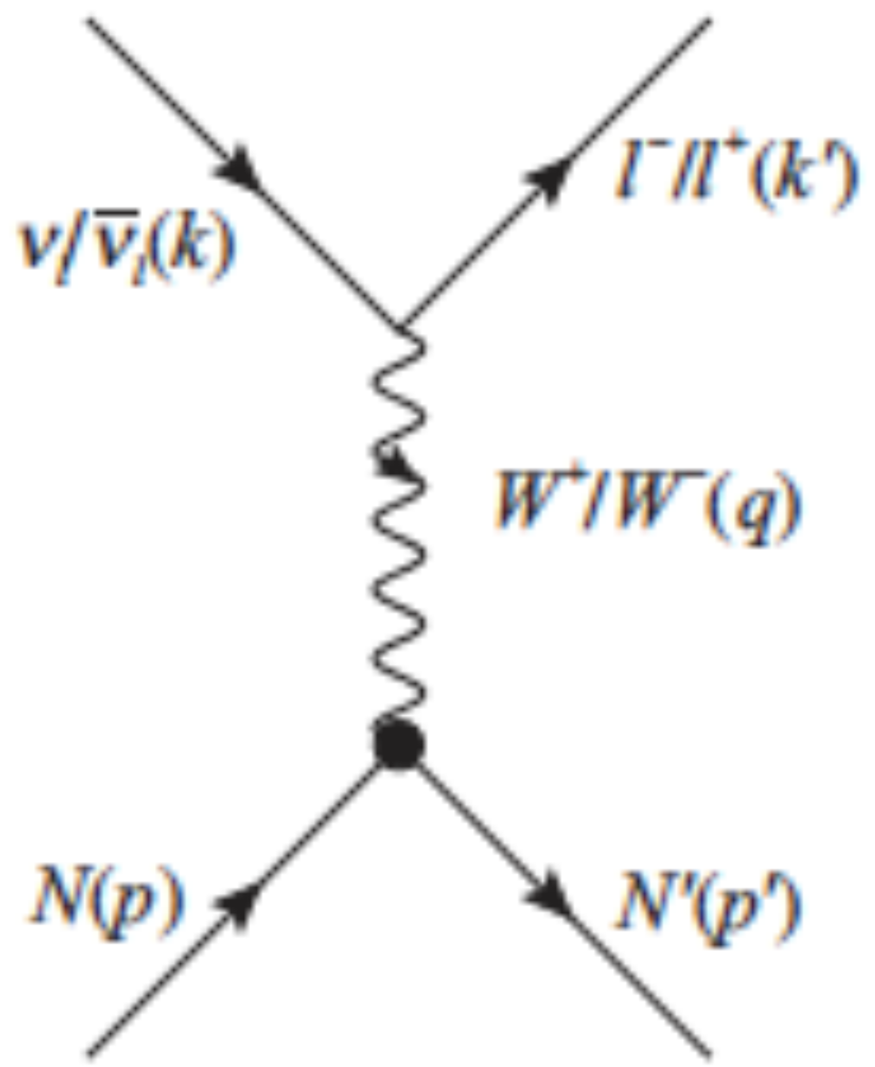
Neutrino Nucleon Scattering $\longrightarrow$

$$\sigma \approx G_F^2 E_\nu^2 \;\; for \; E_\nu \ll m_p$$
$$\sigma \approx G_F^2 E_\nu m_p \;\; for \; E_\nu \gg m_p$$

Neutrino electron Scattering $\longrightarrow$

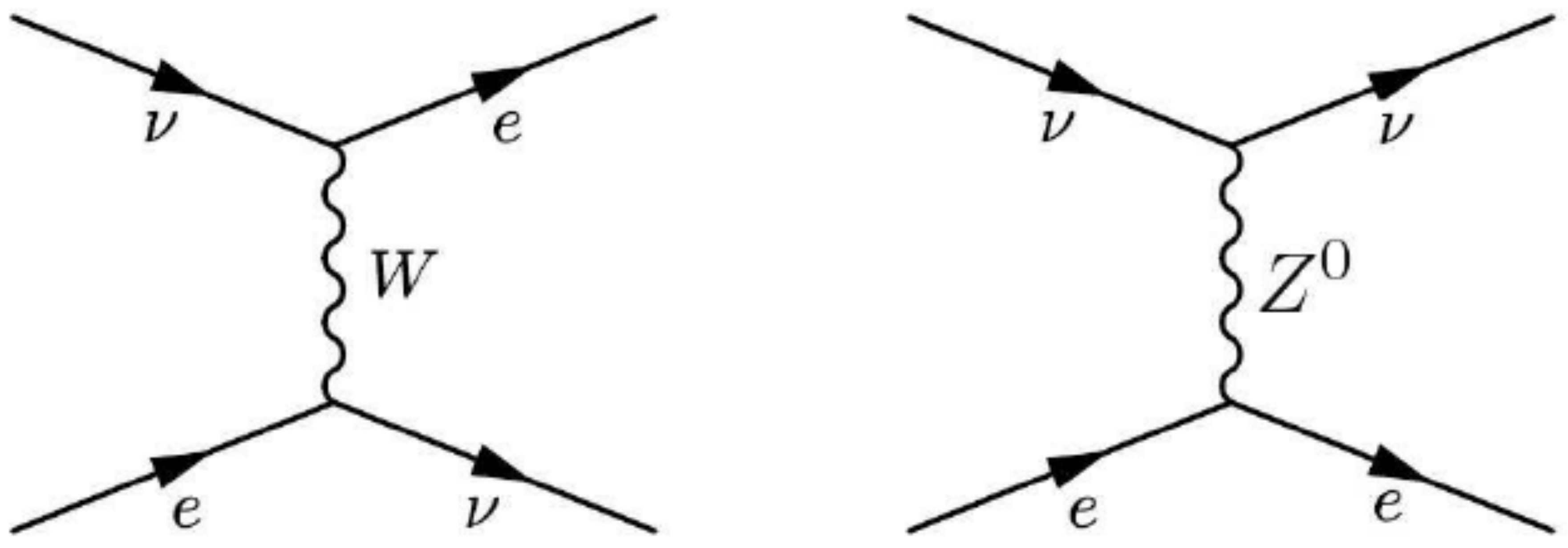
$$\sigma \approx G_F^2 E_\nu^2 \;\; for \; E_\nu \ll m_e$$
$$\sigma \approx G_F^2 E_\nu m_e \;\; for \; E_\nu \gg m_e$$

A typical cross section in weak interaction



$\bar{\nu}_e + p \rightarrow e^+ + n$ (QES)

$N = n, p$
$N' = p, n$

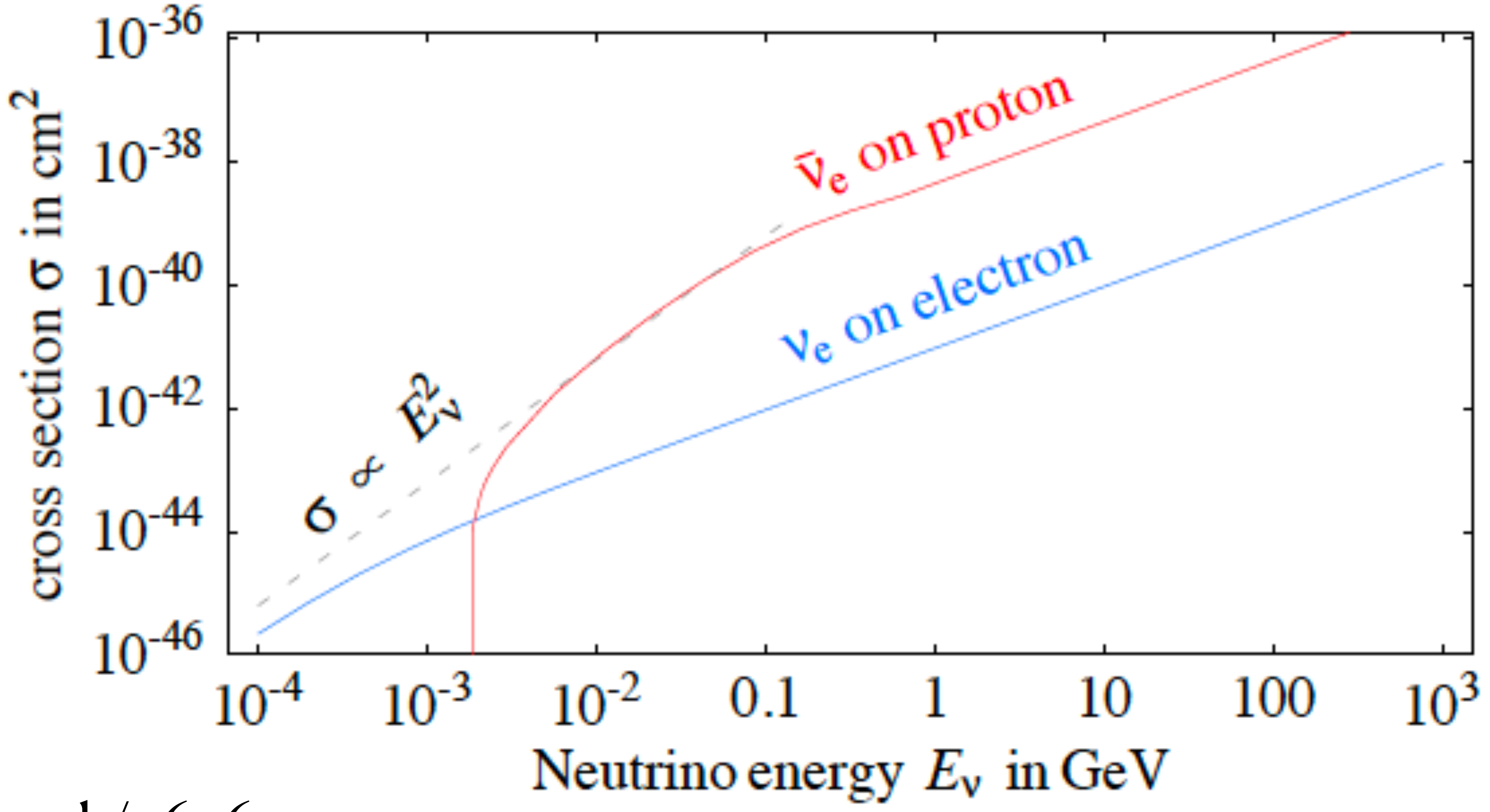


$\nu_e + e^- \rightarrow \nu_e + e^-$ (ES)

- At $E_\nu \ll m_p$ the conservation of energy gives

$E_\nu = E_{e^+} + m_n - m_p = E_{e^+} + 1.29\,MeV$

- Therefore neutrino energy can be deduced by measuring electron energy alone. Since $E_{e^-} \geq m_{e^-}$ the reaction is only possible if $E_\nu \geq 1.80\,MeV$.



https://arxiv.org/abs/hep-ph/0606054

Neutrino Mean Free Path

Typically a neutrino with Energy E_ν interacts with nucleon and has a cross-section in the low energy limit

$$\sigma \approx 10^{-44}\left(\frac{E_\nu}{m_e c^2}\right)^2 cm^2 \qquad (E_\nu \ll GeV)$$

Neutrinos produced in Boron decay in the sun with an Average solar density $\rho_\odot = 1gm/cm^3$

$${}^{8}_{5}B \rightarrow 2\,{}^{4}_{2}H_e + e^+ + \nu_e \quad \rightarrow \text{PP III Chain} \qquad E_{\nu_e} = 10\text{ MeV}$$

Number density is given by $n_e = N_A \times \rho_\odot$ with $N_A = 6.023 \times 10^{23}$

Mean free path, $l = \dfrac{1}{n_e \sigma} = 4.18 \times 10^{17}$ cm. $\longrightarrow$ In general, neutrinos can leave many astrophysical systems (i.e supernovae, galaxies etc) with no interactions

That makes Neutrino an amazing messenger far from the cosmos!

Neutrino Events Kinematics

For a $2-2$ scattering process $AB \to CD$

The counting rates can be written as. $n_s = (n_b v_b) \times n_t \times \sigma_s$

Total Flux $[Area \times time]^{-1}$

Contains the physics $[Area]$

- $n_b v_b =$ beam flux
- $v_b =$ The relative velocity between the beam and target(i.e if the target not stationary)
- $n_t =$ No of particles in the target
- $\sigma_s =$ scattering cross-section

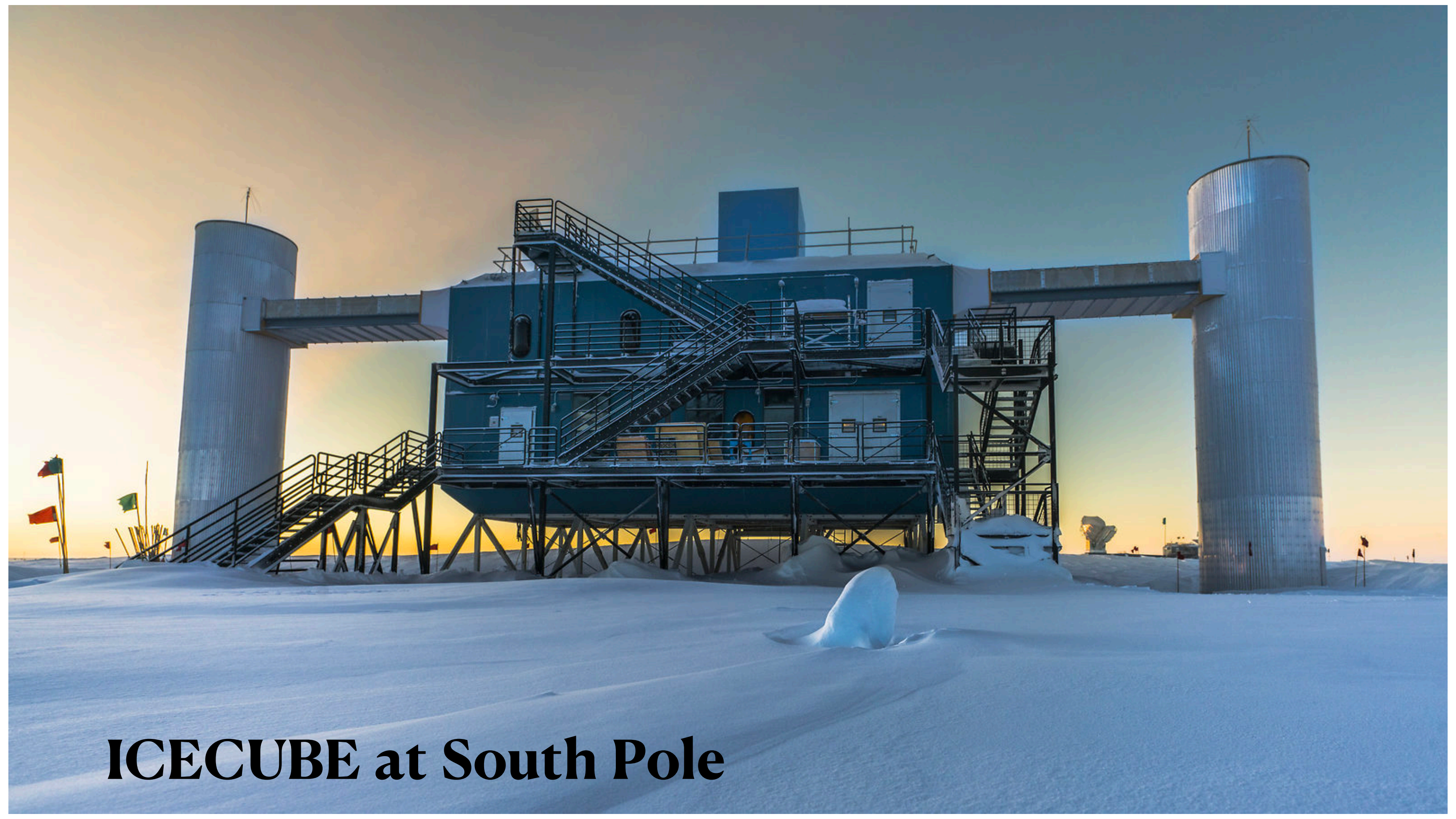
Total flux 10^{10} cm^{-2} s^{-1} and $\sigma_s \approx 10^{-45}$ cm^2, about 10^{30} target atoms are required to produce one event per day.

$1\,gm = 1\,mol \equiv 6.023 \times 10^{23}$

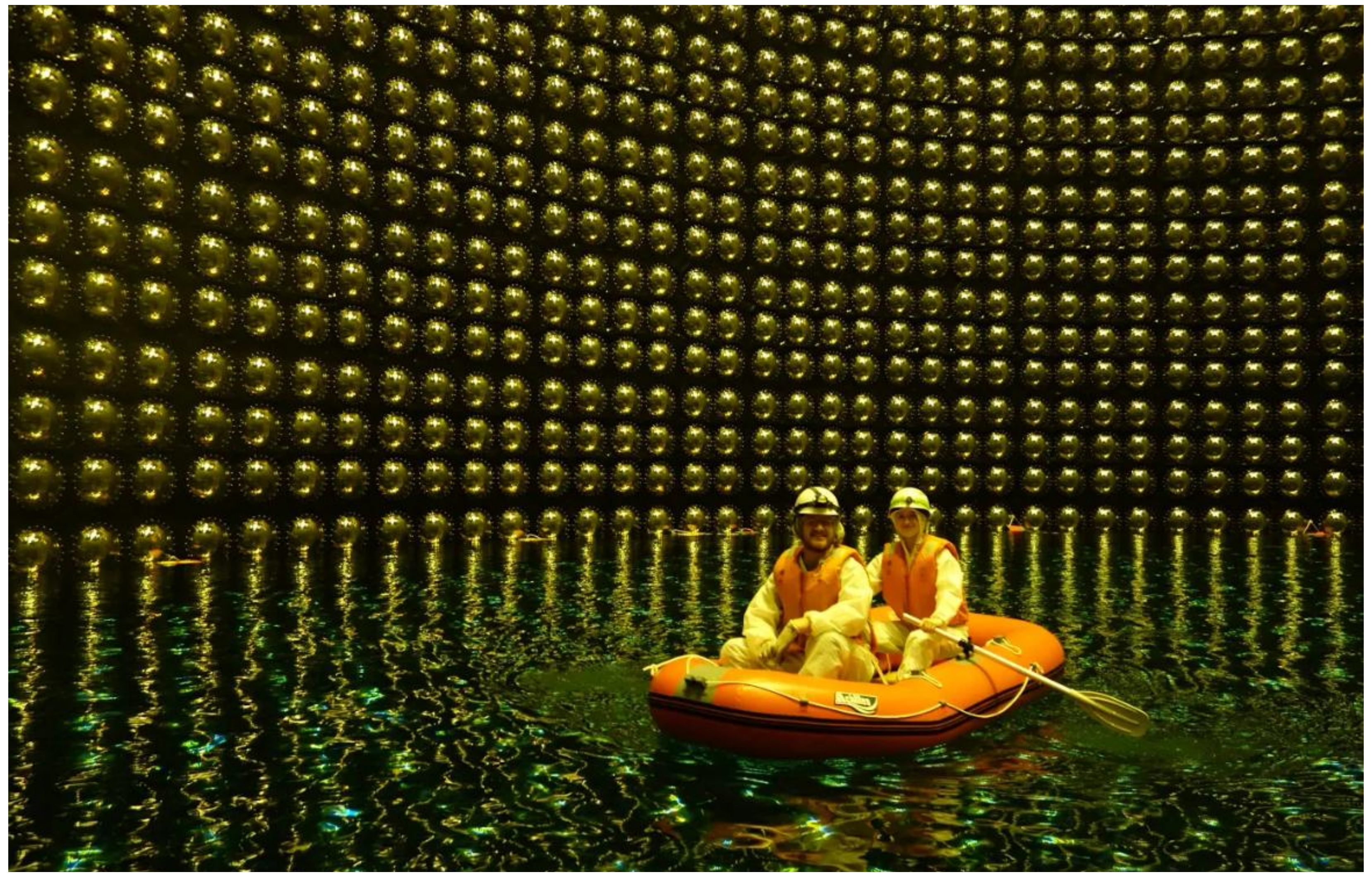
$1\,ton \approx 10^{30}\ target\ atoms$

Tons size target, a big gigantic detector is necessary for the neutrino detection experiment.

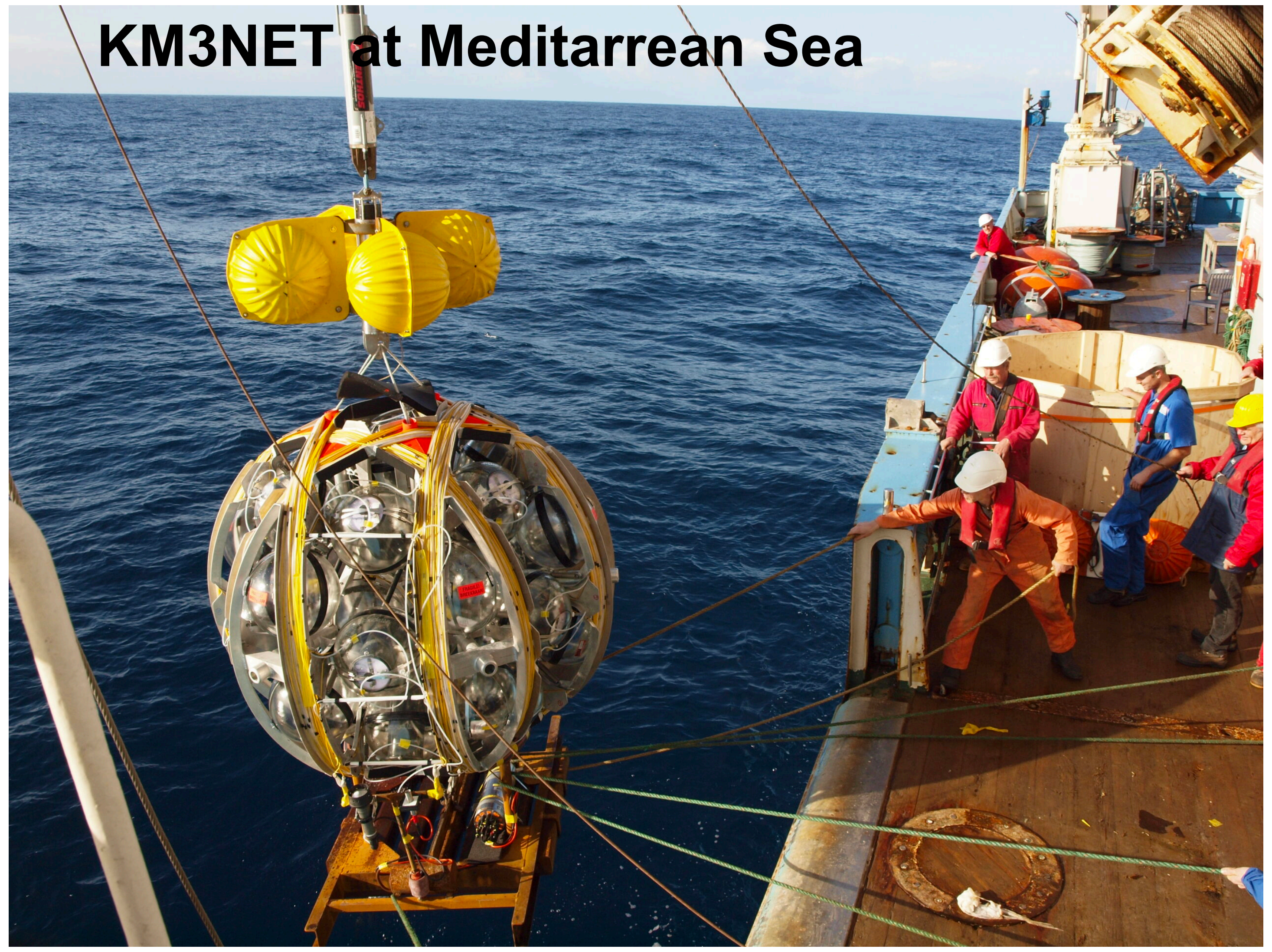
Modern-day Neutrino Detectors



Neutrino Detection involves several methods on the surface, underground, inside the sea, or in the ice or space.



Super- Kamiokande at Japan



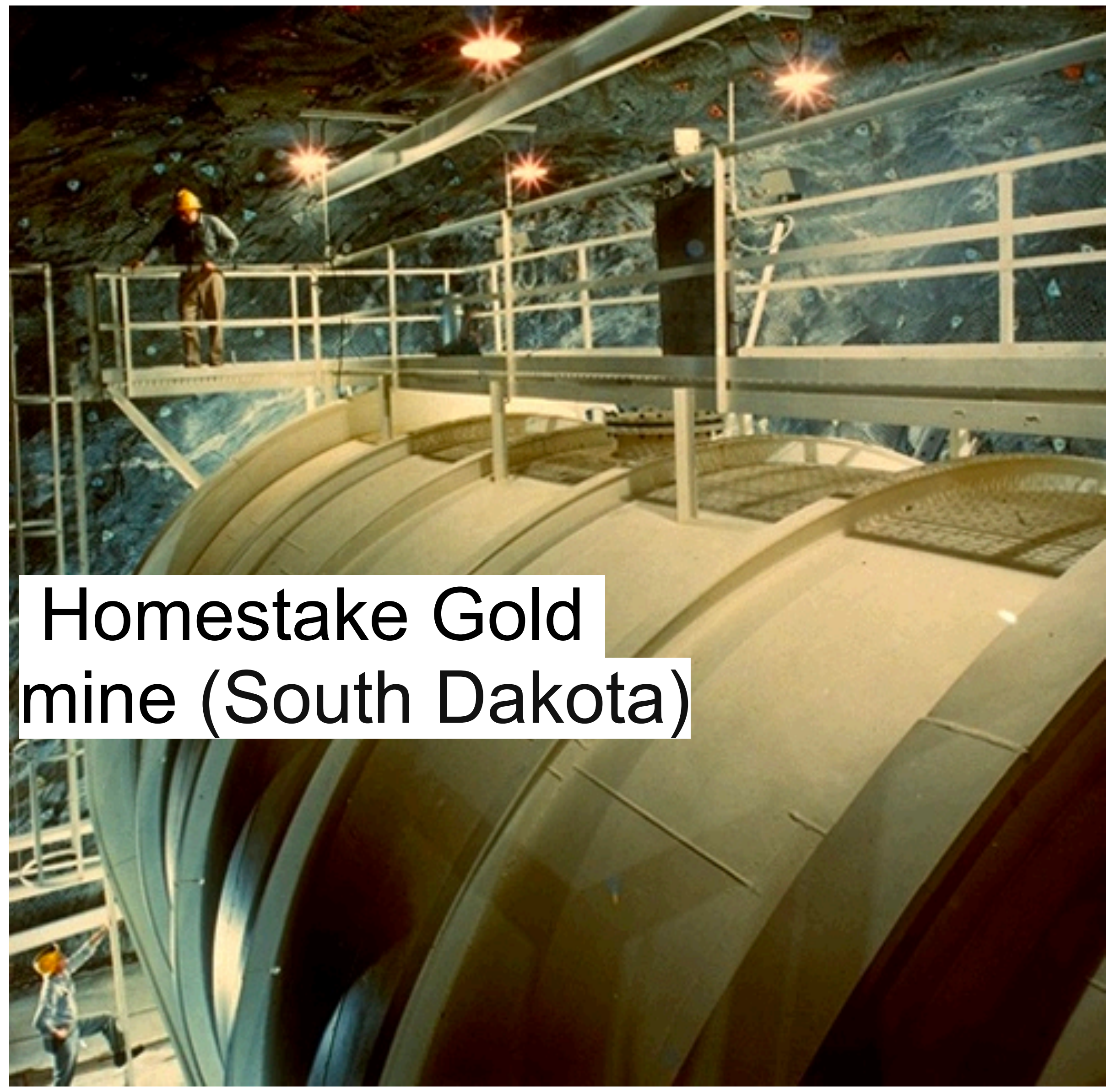
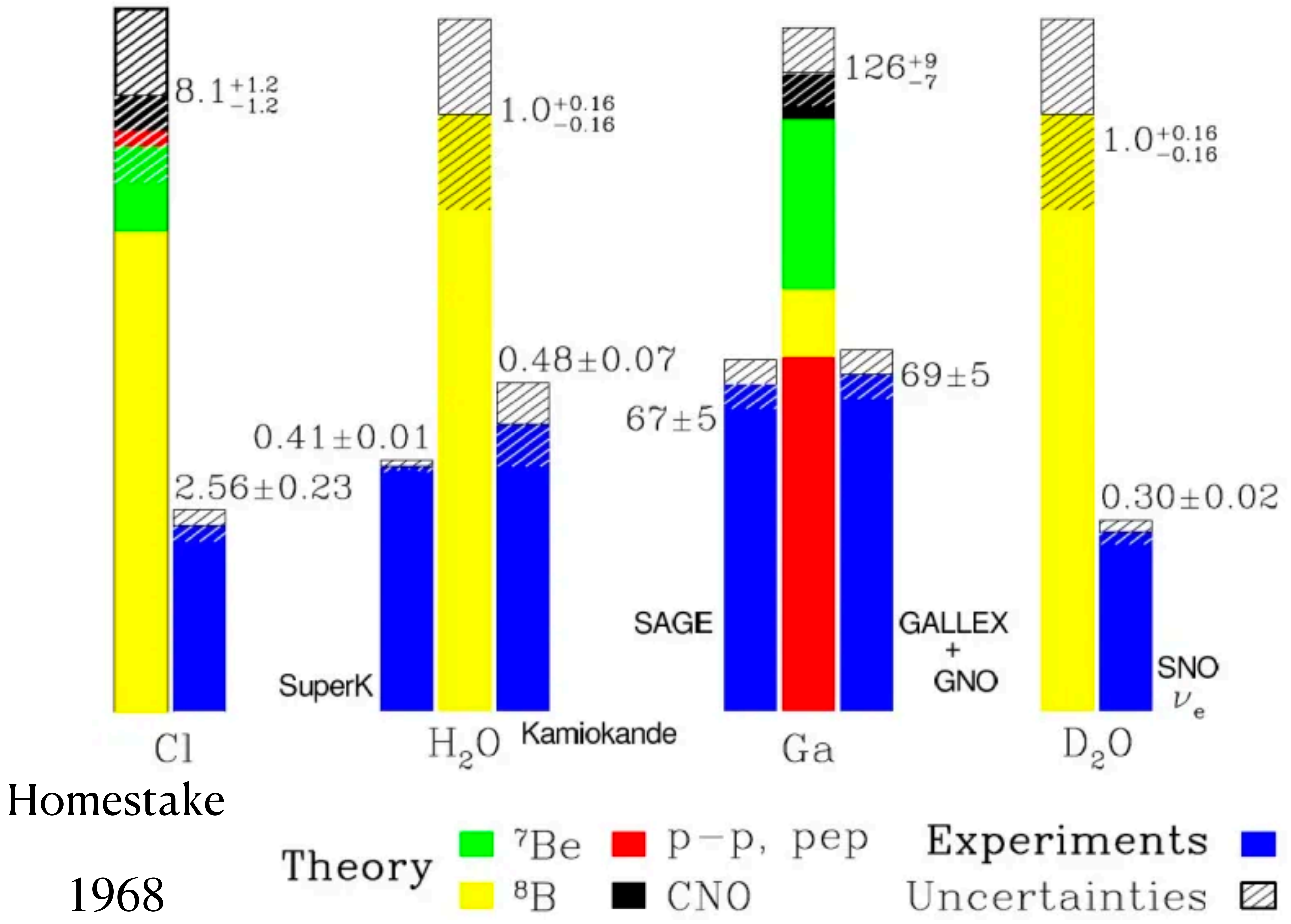


Image Credit: Collaboration Site

Solar Neutrino Puzzle

Solar Neutrino Unit (SNU): One capture per second and per 10^{36} target atoms.

Homestake	$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$	(CC)
SAGE+GALLEX	$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$	(CC)
SuperK+Kamiokande	$\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$	(ES)
SNO	$\nu_e + {}^2H \rightarrow e^- + p + p$	(CC)



https://arxiv.org/pdf/hep-ex/0312045.pdf

hep-ph/0412068

Solar Neutrino Puzzle and Initial Thoughts

- ☑ Perhaps we do not understand the Sun well enough. Maybe a better theory of the internal structure of the Sun would predict fewer neutrinos, in agreement with the measurements.

- ☑ Perhaps we don't understand neutrinos well enough; maybe they have some features beyond the standard theory of neutrinos that account for the anomaly.

$$\downarrow\downarrow$$

Around 1970, most speculation including Pontecarvo centers on some variation of a theory that there are three kinds (called "flavors") of neutrinos, and that their passage through matter may cause one neutrino flavor to "oscillate" into another.

https://arxiv.org/abs/2202.12421

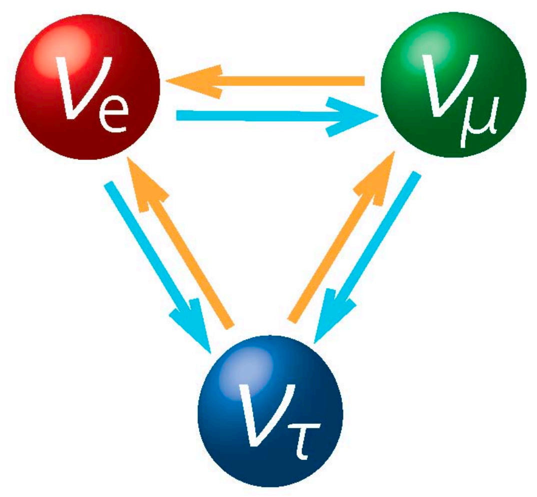
SNO Observations and Solar Neutrino Problem Solved (2002)

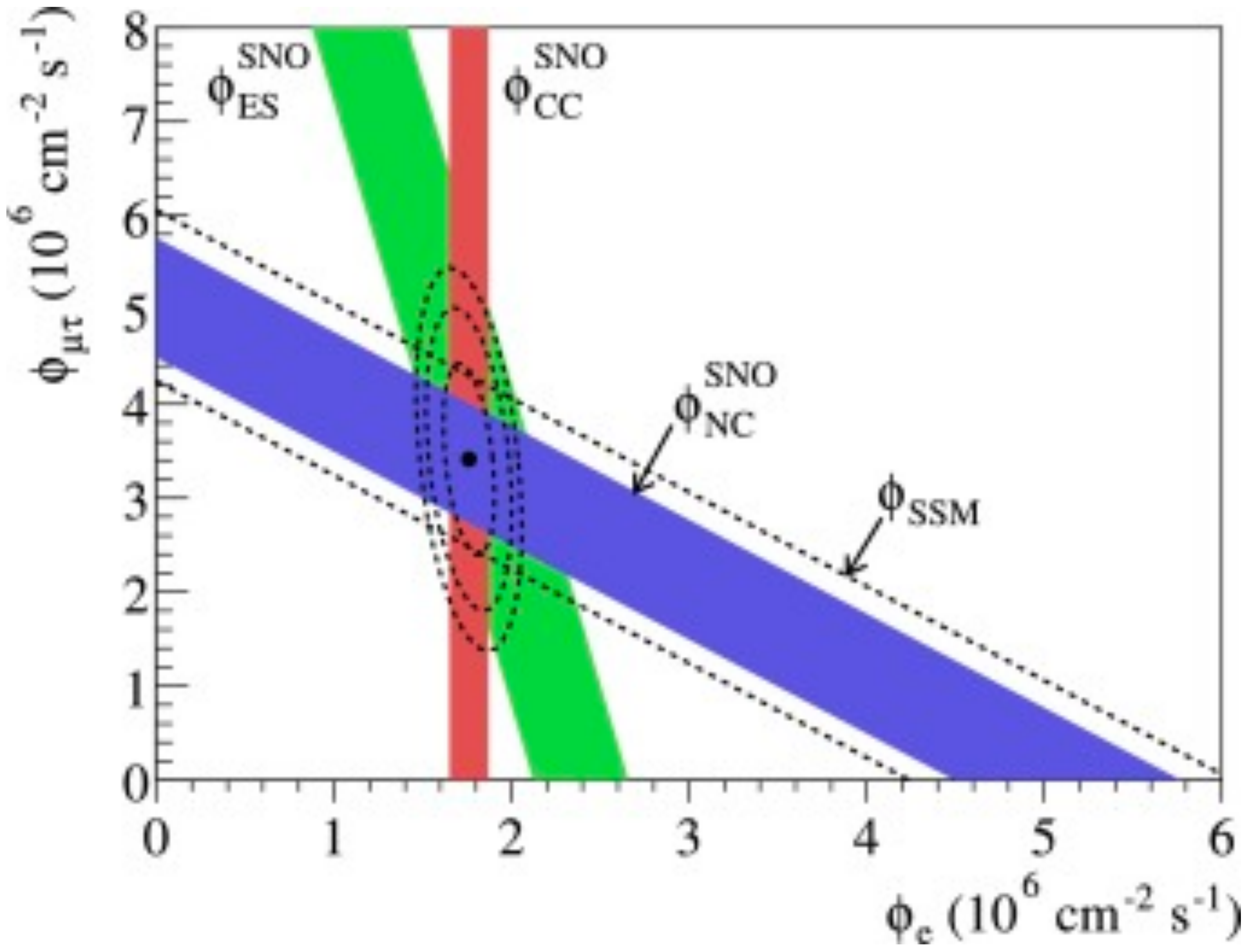
$$\nu_e + {}^2H \rightarrow e^- + p + p \quad \text{(CC)}$$

$$\nu_\alpha + {}^2H \rightarrow \nu_\alpha + p + n \quad \text{(NC)}$$

$$\nu_\alpha + e^- \rightarrow \nu_\alpha + e^- \quad \text{(ES)}$$

The idea of Neutrino Oscillation got validation with SNO observation ←



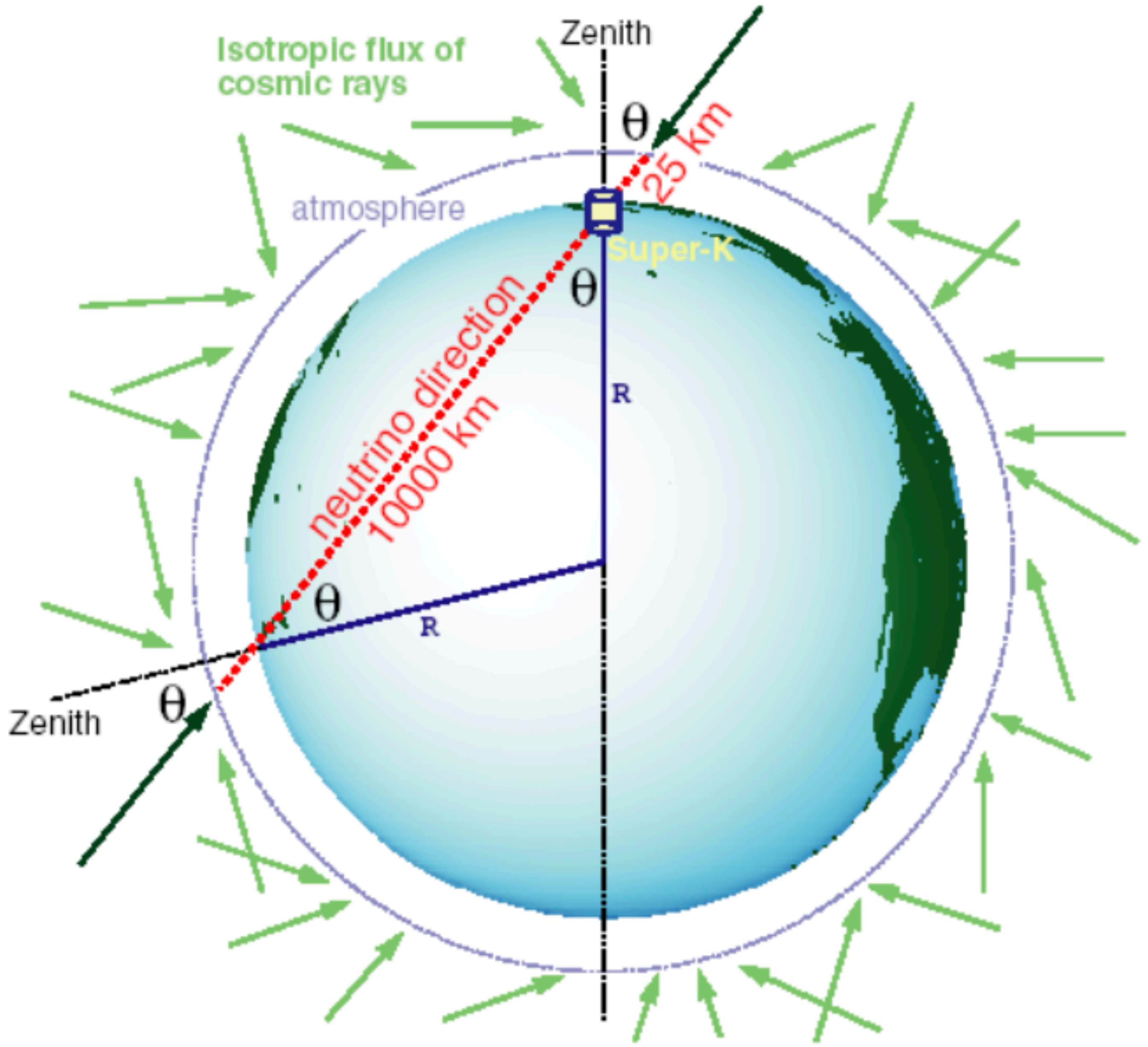


2002 Nobel Prize: Ray Davis's the first evidence in 1968 that the electron neutrino flux at the Earth, is substantially less than would be predicted by the SSM.

Atmospheric Neutrino Anomaly

$$\pi^+ \to \mu^+ + \nu_\mu \;,\quad \pi^- \to \mu^- + \bar{\nu}_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \;,\quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$



Muons travel a distance before decay: $\quad d \approx c\tau_\mu \gamma_\mu = 0.3\,\dfrac{E_\mu}{0.3\,GeV}$

τ_μ is the muon life-time and $\gamma_\mu = \dfrac{E_\mu}{m_\mu}$ is the relativistic dilation factor

Without Oscillations flux of atmospheric neutrinos would be up/down symmetric

In the energy, $E \leq 1GeV$ most of the muons decay before hitting the ground, the ratio of neutrino fluxes should satisfy the following ratios

$$\boxed{\frac{\phi_{\nu_\mu} + \phi_{\bar{\nu}_\mu}}{\phi_{\nu_e} + \phi_{\bar{\nu}_e}} \approx 2 \qquad \frac{\phi_{\nu_\mu}}{\phi_{\nu_{\bar{\mu}}}} \approx 1 \qquad \frac{\phi_{\nu_e}}{\phi_{\nu_{\bar{e}}}} \approx \frac{\phi_\mu^+}{\phi_\mu^-}}$$

At energy higher than $E \gg 1$ GeV the fraction of muon which hit the ground before decaying increases leading to an increase of flavor ratios

$$\frac{\phi_{\nu_\mu} + \phi_{\bar{\nu}_\mu}}{\phi_{\nu_e} + \phi_{\bar{\nu}_e}} > 2$$

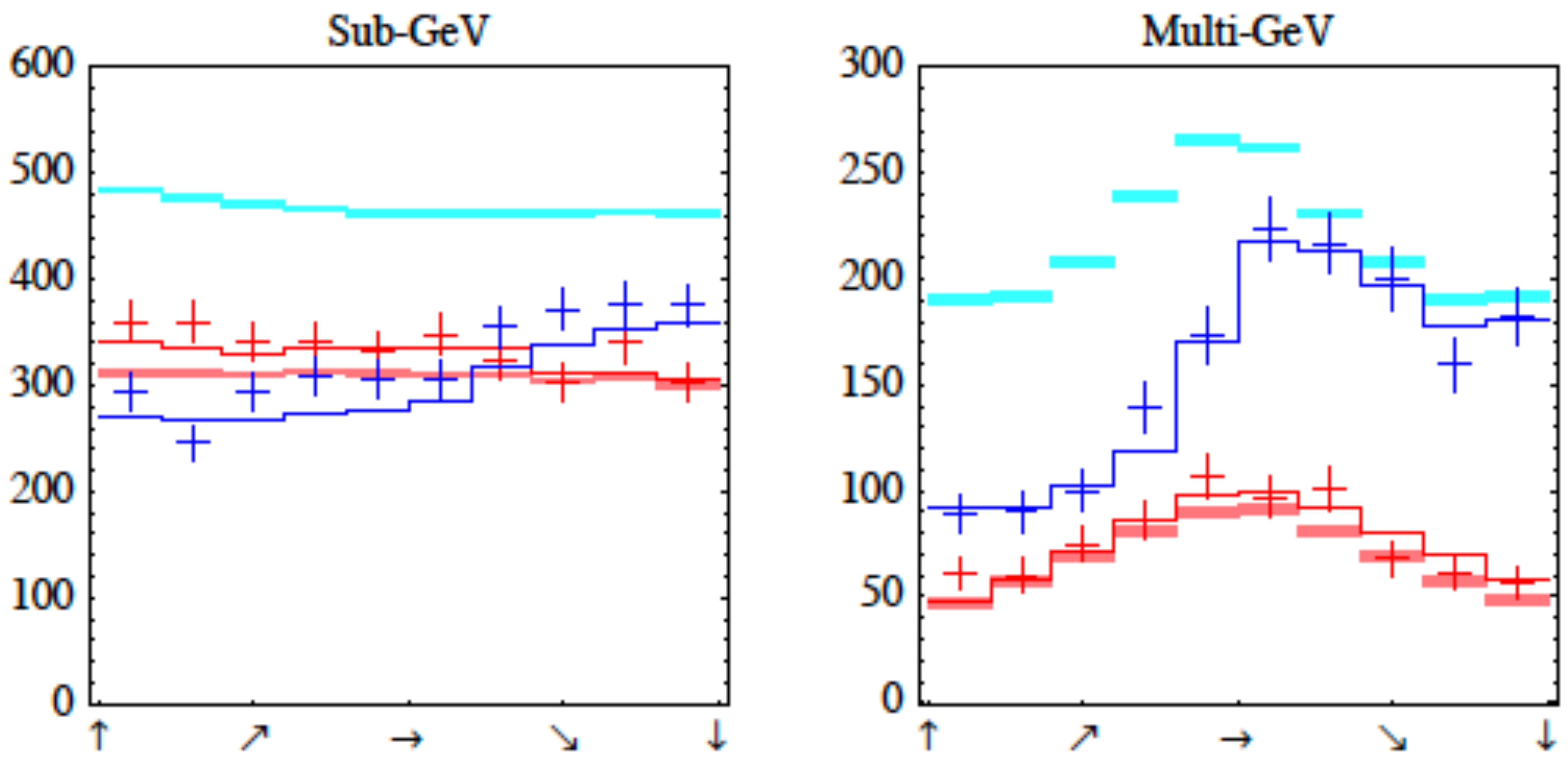
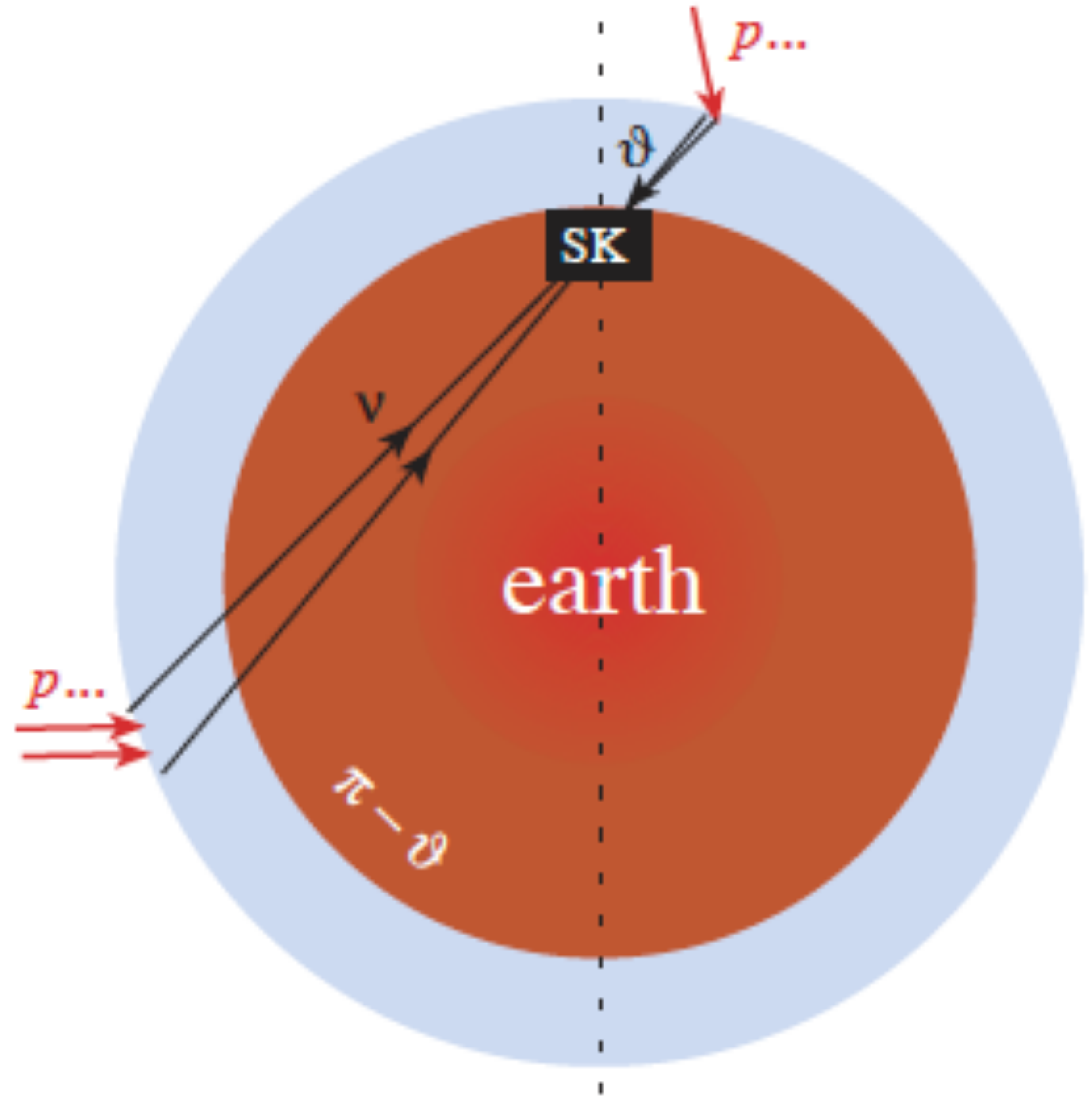


Fig: SK data: The number of $e^{\pm}$ (*red*) and $\mu^{\pm}$ (*blue*) as a function of the direction of scattered lepton. The horizontal axis is $\cos\nu$ ranging -1 (vertically up-going events) and $+1$ (vertically down-going events). The crosses are the data with their errors and the thin lines are best fit with oscillation and thick lines are no oscillation expectation.



$$R = \frac{(N_\mu/N_e)_{exp}}{(N_\mu/N_e)_{theory}}$$

$R = 1 \rightarrow$ No Oscillation

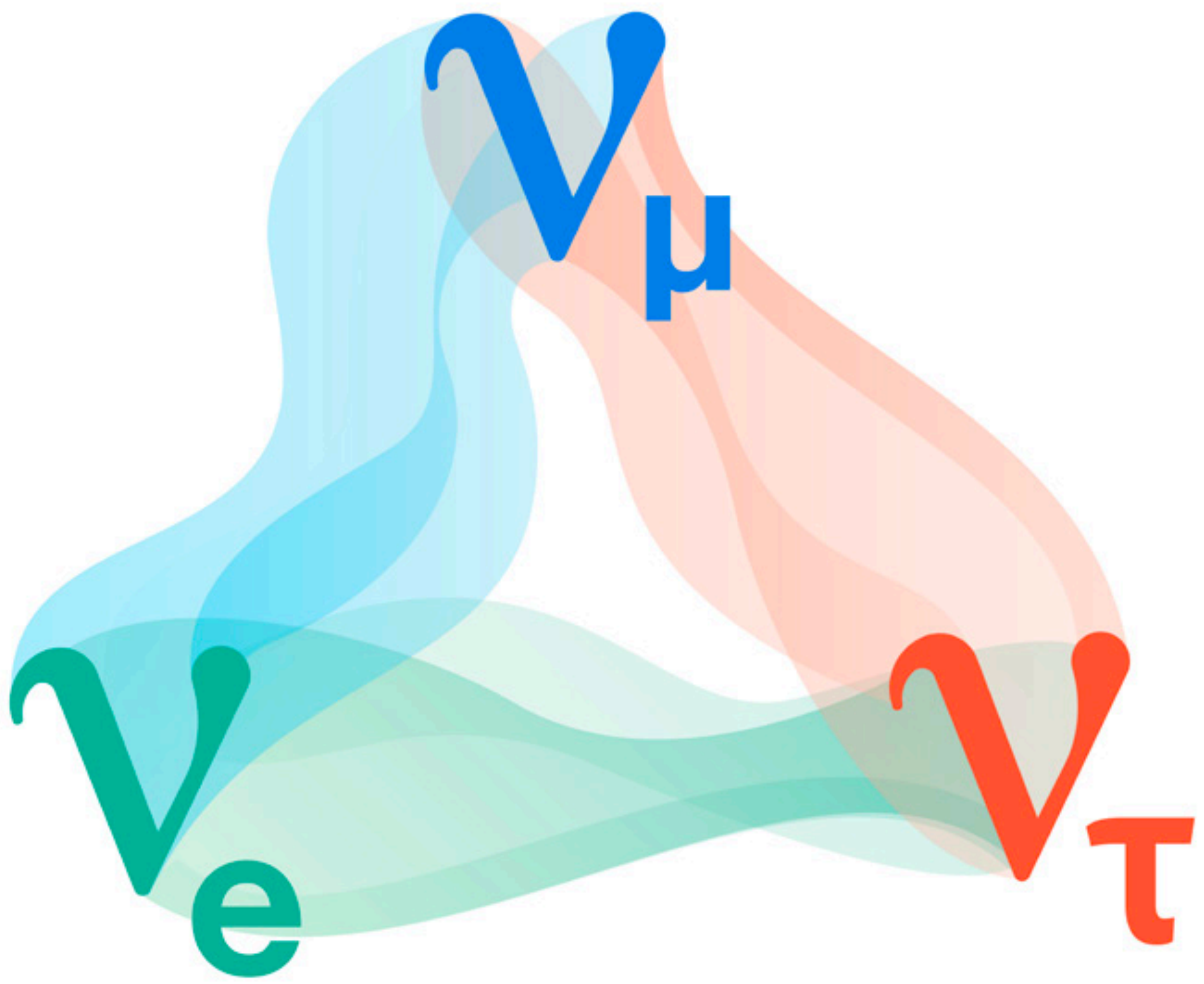
$R < 1 \rightarrow$ Oscillation

→ Strengthen the Theory of Neutrino Oscillation

For SK: $R = 0.63 - 0.65$ (Sub GeV-Multi GeV)

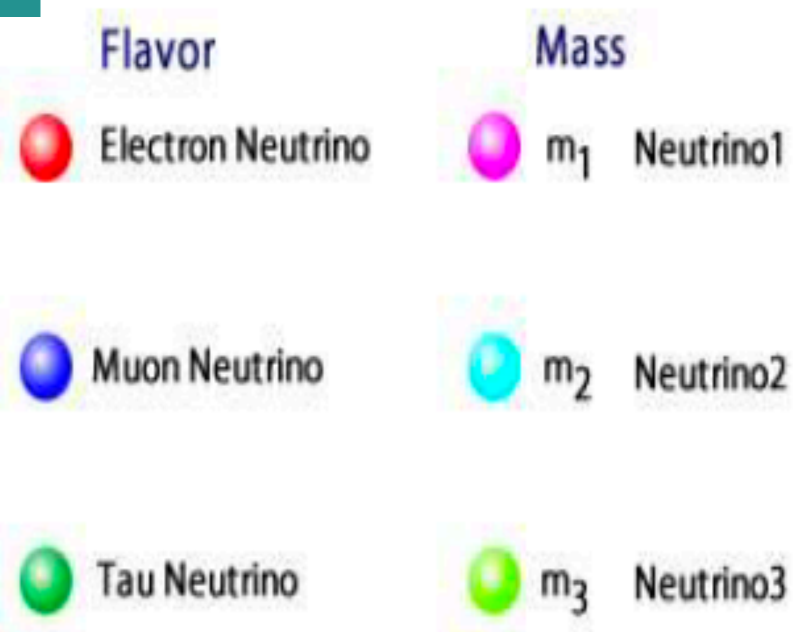
Section B

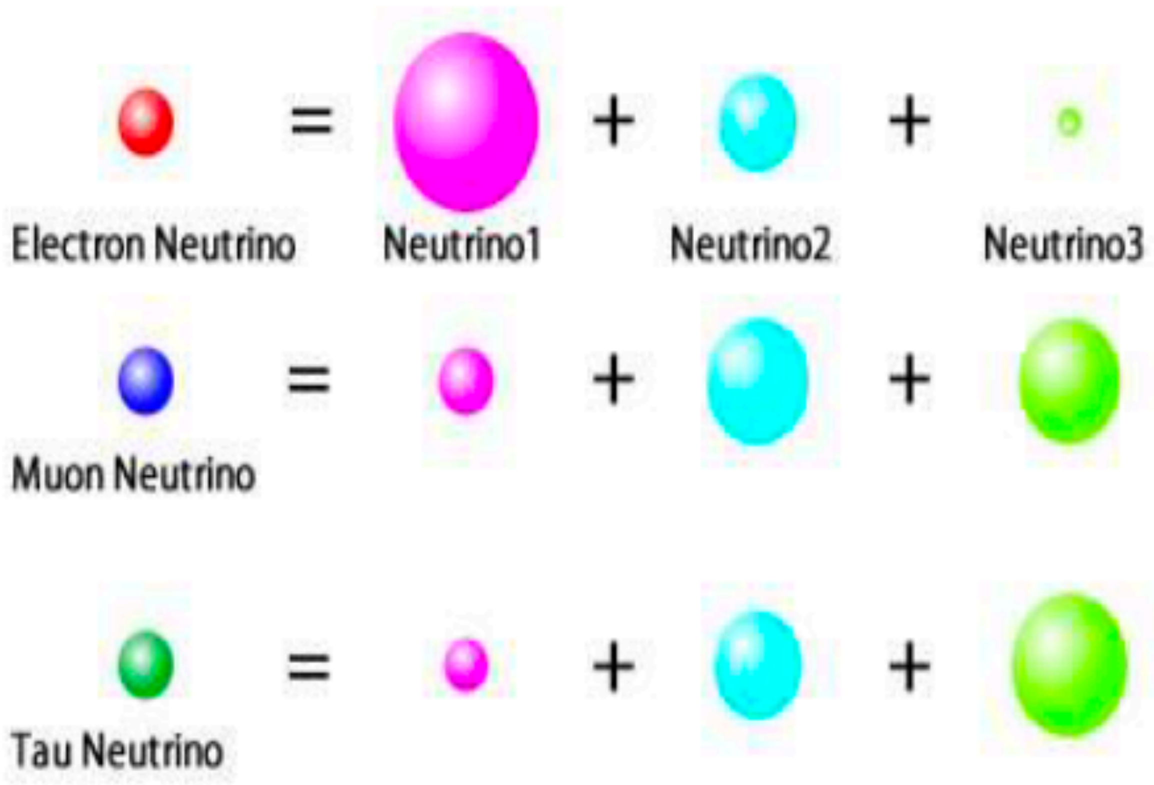
Neutrino Oscillation



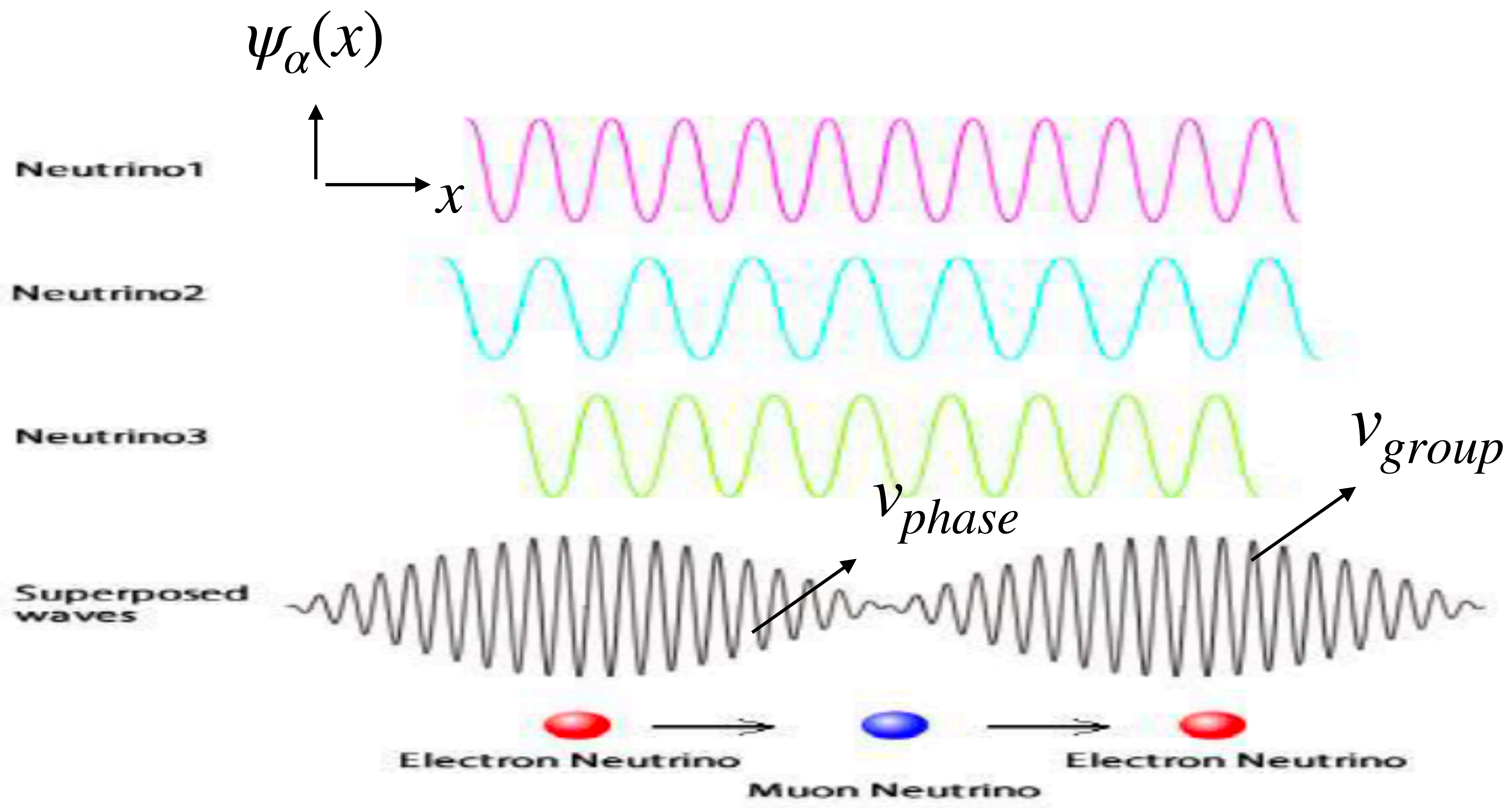
Interference in Neutrinos

Each neutrinos have different mass eigen states.





The flavor of a neutrino is determined as a superpositio the mass eigen states.



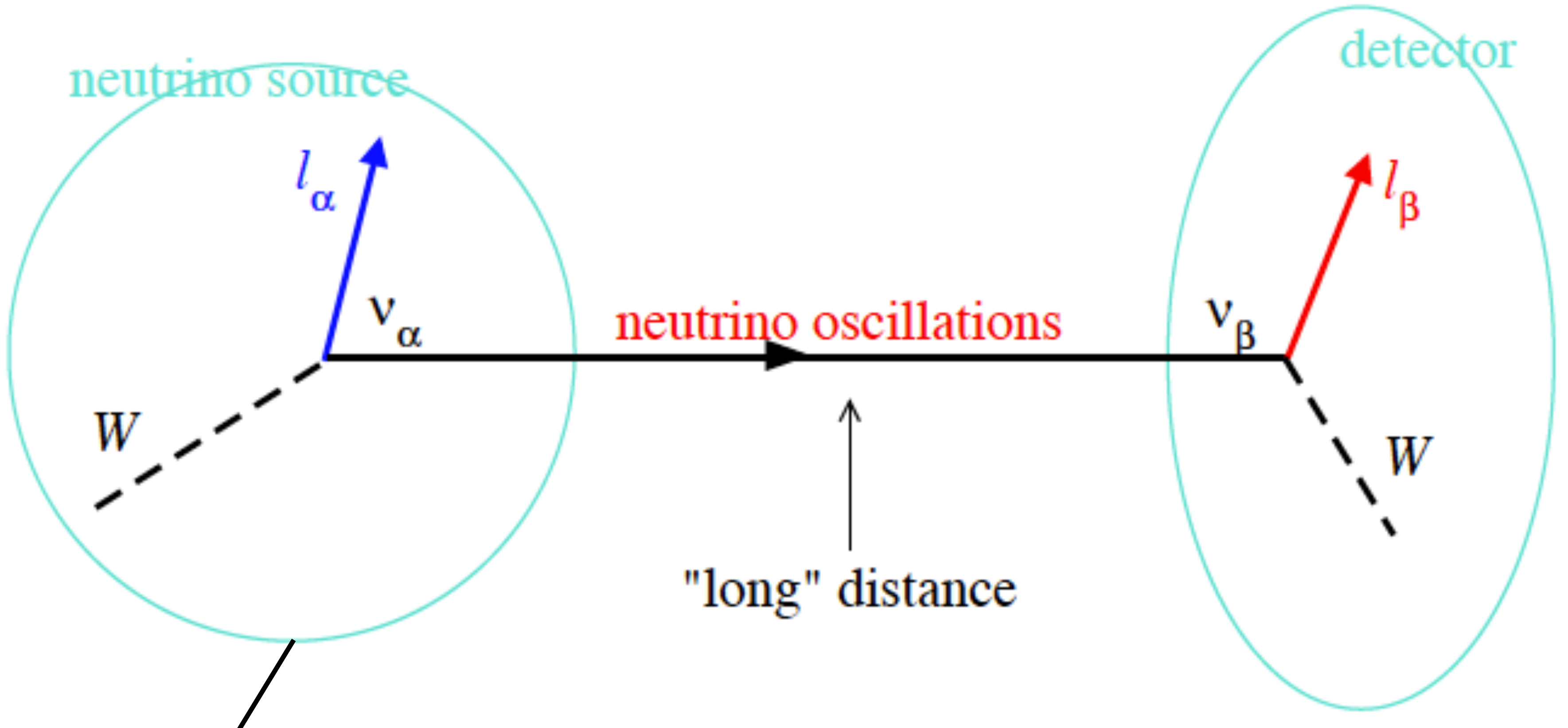
Phase velocities $\Delta v_{phase} \approx \frac{\Delta m^2}{2E}, \quad \Delta m^2 = m_k^2 - m_j^2$

☑This occurs when neutrinos have non-zero mass and non-zero mixing angles.

Phase Difference $\Delta\phi = \Delta v_{phase} t$

PC credit: Superkamiokande site

Neutrino Oscillation Picture



Flavors conversion happens

Production

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

Coherent superposition of mass eigenstates

Propagation

$$|\nu_j\rangle : e^{-iE_j t}$$

Schrodinger Picture $\quad i\frac{d|\nu_j\rangle}{dt} = H|\nu_j\rangle$

$H(t_0) = H(t) = H \qquad H|\nu_j\rangle = E_j|\nu_j\rangle$

$$|\nu_j(t)\rangle = e^{-iE_j t}|\nu_j(0)\rangle \qquad V(t,t_0)^\dagger V(t,t_0) = I = V(t,t_0)V(t,t_0)^\dagger$$

Detection

$$\langle\nu_\beta| = \sum_j \langle\nu_j| U_{\beta j}$$

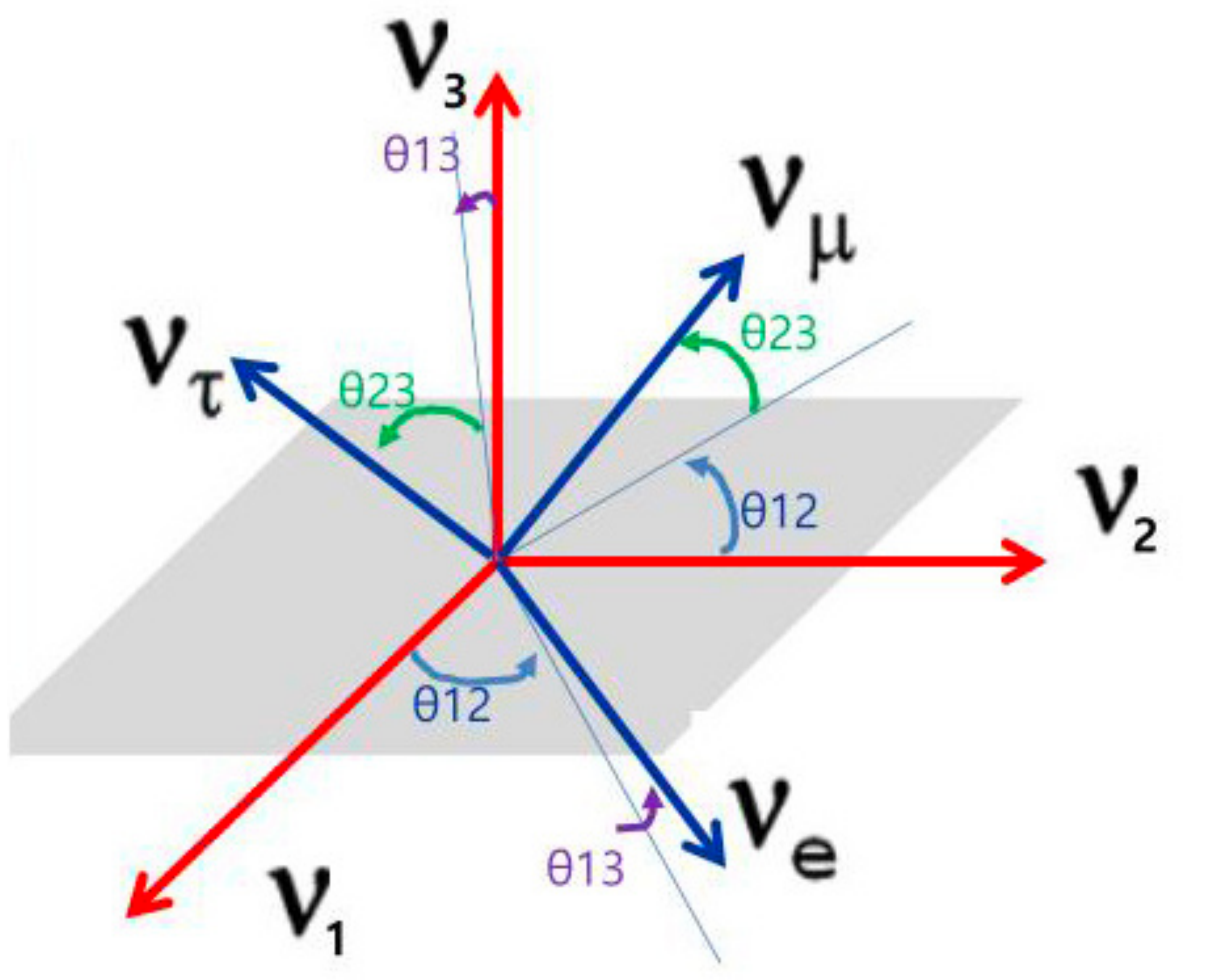
U = Mixing matrix

$$U^\dagger U = I = UU^\dagger$$

Three Neutrino Mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$c_{ij} = \cos\theta_{ij} \; , s = \sin\theta_{ij}$$



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}}_{\text{Atmospheric Experiments}} \underbrace{\begin{bmatrix} c_{13} & 1 & s_{13}e^{i\delta} \\ 0 & 0 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}}_{\text{Reactor and Long Baseline Experiments}} \underbrace{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Solar Experiments}} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$J = \hat{J}\sin\delta \qquad \hat{J} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}$$

$\delta =$ CP violating phase $\quad 0 \le \delta < 2\pi$

$\delta = 0, \pi \qquad$ CP conserved.

Three Euler Rotation can generate U has three mixing angles and one CP violating phase

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = C\sin\delta \;, C = f\left(\hat{J}, \Delta m_{ij}\right)$$

$\longrightarrow$ Long Baseline Experiment

We will concentrate here on two-flavor mixing which is much easier to visualize.

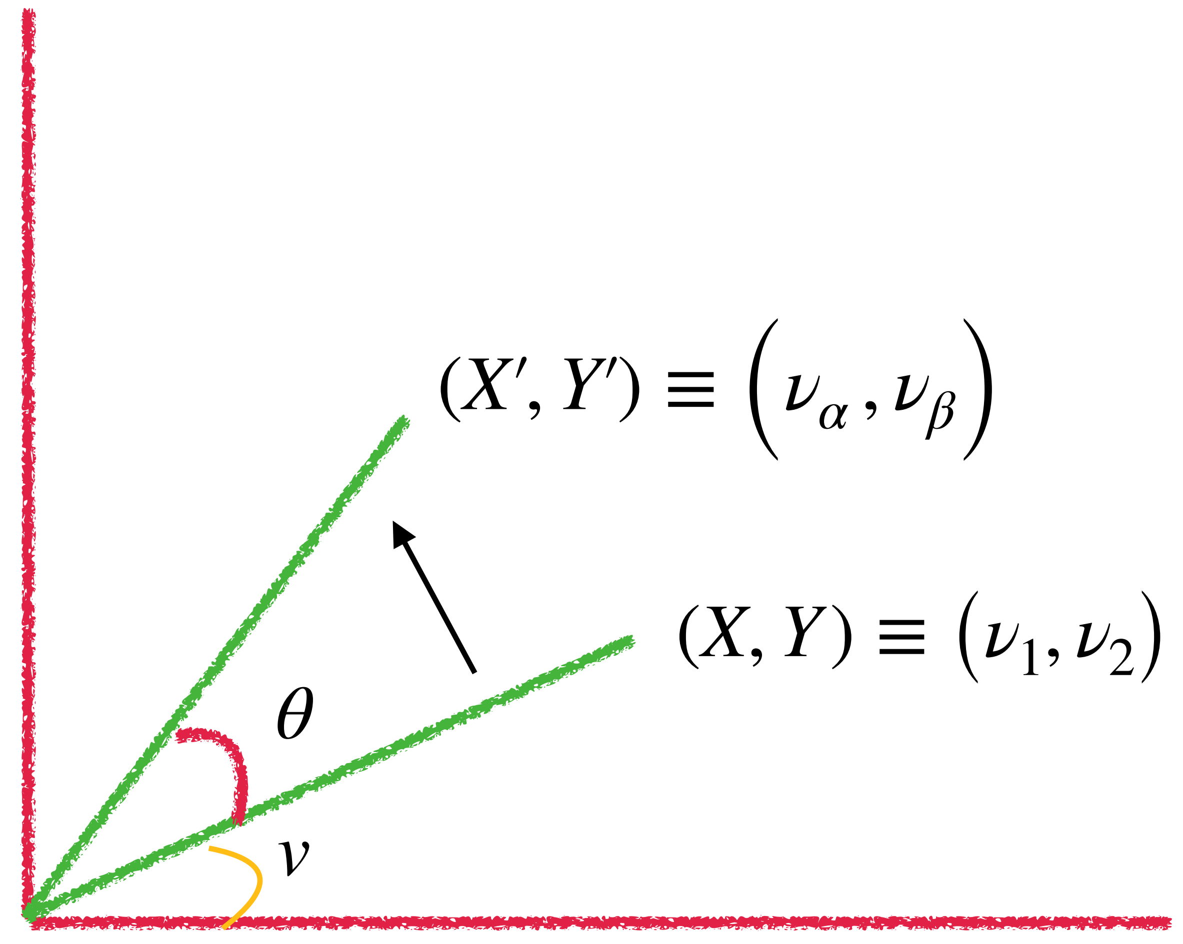
Mixing Matrix in Two flavor System

$X = r\cos v \qquad Y = r\sin v \qquad X' = r\cos(v+\theta) \quad Y' = r\sin(v+\theta)$

$$X' = \cos\theta\, x - \sin\theta\, y$$

$$Y' = \sin\theta\, x + \cos\theta\, y$$

$$\longrightarrow \begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$



Active Rotation in 2D

Counterclockwise direction

$$\nu_\alpha = \cos\theta\,\nu_1 - \sin\theta\,\nu_2$$

$$\nu_\beta = \sin\theta\,\nu_1 + \cos\theta\,\nu_2$$

$$\longrightarrow \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \boxed{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$U = U_{PMNS} \qquad U^\dagger U = I = UU^\dagger$ Unitary in nature

Parametrization of neutrino mixing matrix is important for Neutrino oscillation study.

Parametrization of Mixing Matrix

Usually, $N \times N$ matrix has N^2 real independent parameters : $\frac{N(N-1)}{2}$ Mixing angles $\frac{N(N+1)}{2}$ Phases

PMNS Matrix $N=3$ three mixing angles and six phases, not all phases are physical.

$$\nu_{kL} = e^{\phi_k}\nu_{kL} \ , \quad (k=1,2,3) \qquad l_{\alpha L} = e^{\phi_\alpha} l_{\alpha L} \ , \quad (\alpha = 1,2,3)$$

$2N-1$ phases can be eliminated with the re-definition of neutrino and leptons field

$$\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2} \quad \text{Physical phase left}$$

$$N=3 \Rightarrow \begin{cases} \frac{N(N-1)}{2} = 3 & \text{Mixing angles} \\ \frac{(N-1)(N-2)}{2} = 1 & \text{Physical phase} \end{cases}$$

$$N=2 \Rightarrow \begin{cases} \frac{N(N-1)}{2} = 1 & \text{Mixing angle} \\ \frac{(N-1)(N-2)}{2} = 0 & \text{Physical phase} \end{cases}$$

Total six parameters: $\theta_{12}, \ \theta_{13}, \ \theta_{23} \ , \delta_{CP}, \Delta m^2_{21}, |\Delta m_{31}|^2$

Total two parameters: $\theta_{ij}, \Delta m^2_{ij}$

Vacuum Oscillation in Two Neutrinos

Two generation mixing, have one mixing angle and no CP violating phase

Assumption:

Neutrino Produced $x = 0$ with energy E

$$|\nu(x=0)\rangle = |\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

At some distance x $\quad |\nu(x)\rangle = e^{-iE_1x}\cos\theta|\nu_1\rangle + e^{-iE_2x}\sin\theta|\nu_2\rangle$

The probability of new flavors' ν_β appearance at a distance $x \equiv L$ is

$$\boxed{P_{\nu_\alpha\to\nu_\beta}(x=L) = |\langle\nu_\beta|\nu(x=L)\rangle|^2 = \sin^2 2\theta\,\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right)}$$

The effective Hamiltonian for a neutrino mass eigenstate with mass m_j is given below,

$$H|\nu_j\rangle = E_j|\nu_j\rangle$$

$$E_j = \sqrt{P^2+m_j^2} \quad\rightarrow\quad E_j = \left(P + \frac{m_j^2}{P^2}\right)^{\frac{1}{2}}$$

$$E_j = P + \frac{m_j^2}{2P}\ ; \qquad E_j \approx \frac{m_j^2}{2E}\ ;$$

neutrinos with MeV energy limit $P \approx E$

m_2, m_1 are non-degenerate at least on of them is non zero.

A proper unit was introduced to make dimensionless

$$\frac{\Delta m_{21}^2 L}{4E} \equiv \frac{\left(\Delta m_{21} c^2\right)^2 L}{4E\hbar c} = 1.27 \frac{\Delta m_{21}^2}{[eV^2]} \frac{L}{[Km]} \frac{[GeV]}{E}$$

$\hbar = c = 1$

$\hbar c \approx 0.197\, GeV - fm$

$$\boxed{P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\theta_v \, \sin^2 \left(1.27 \frac{\Delta m_{21}^2}{[eV^2]} \frac{L}{[Km]} \frac{[GeV]}{E} \right)}$$

Transition Probability

Oscillation Amplitude

Frequency of Oscillation

$$P_{\nu_\alpha \to \nu_\alpha}(L, E) = 1 - P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\theta_v \, \sin^2 \left(1.27 \frac{\Delta m_{21}^2}{[eV^2]} \frac{L}{[Km]} \frac{[GeV]}{E} \right)$$

Survival Probability

$$P_{\nu_\alpha \to \nu_\alpha}(L, E) + P_{\nu_\alpha \to \nu_\beta}(L, E) = 1$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}(L, E) + P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = 1$$

Closed Quantum Evolution Total probability conserved

$\Delta m_{21}^2 \to -\Delta m_{21}^2$ can not distinguish whether $m_2 > m_1$ or vice versa.

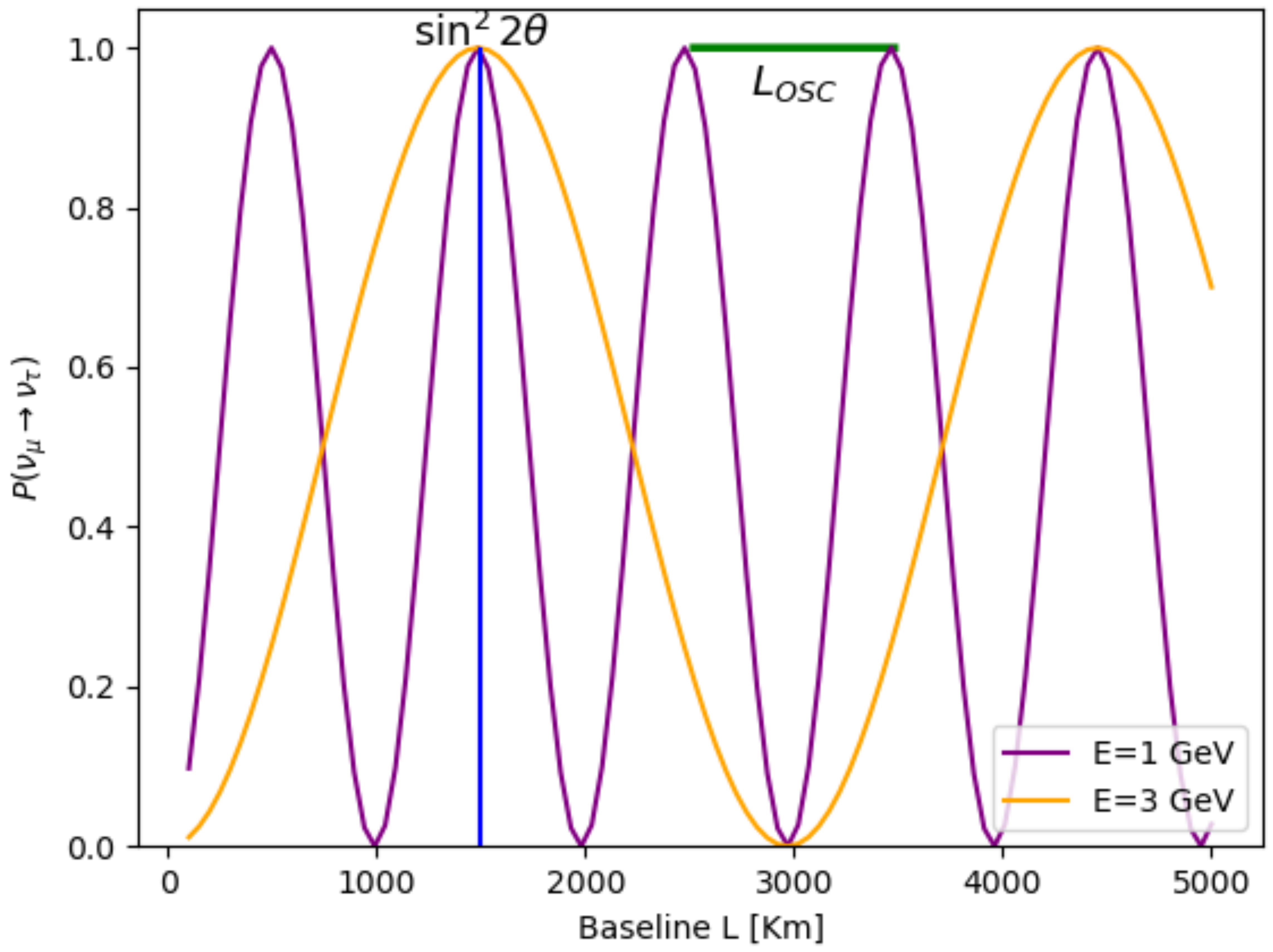
Oscillation Length and Test of Accuracy

$$L_{OSC} = \frac{4\pi E}{\Delta m^2}$$

$$L_{OSC} = 2.48\,Km \times \frac{E}{\Delta m^2}\frac{GeV}{eV^2}$$

Neutrino energy E= 1 GeV and 3 GeV, and for Δm^2_{32}, Oscillation length should be L_{osc} = 992 km and 2976 km respectively.

From we get L_{osc}= 990.45 km and 2975.05 km, respectively.



$$\sin^2 2\theta = 1;\ \text{Oscillation is maximal at } \theta = \pi/4$$

Neutrino Oscillations is a L/E phenomenon.

DUNE, T2K, NoVA

No Oscillation $\theta = 0$, flavor, and mass eigenstates are identical.

Comparative Study of Oscillation Probability in Two Neutrinos

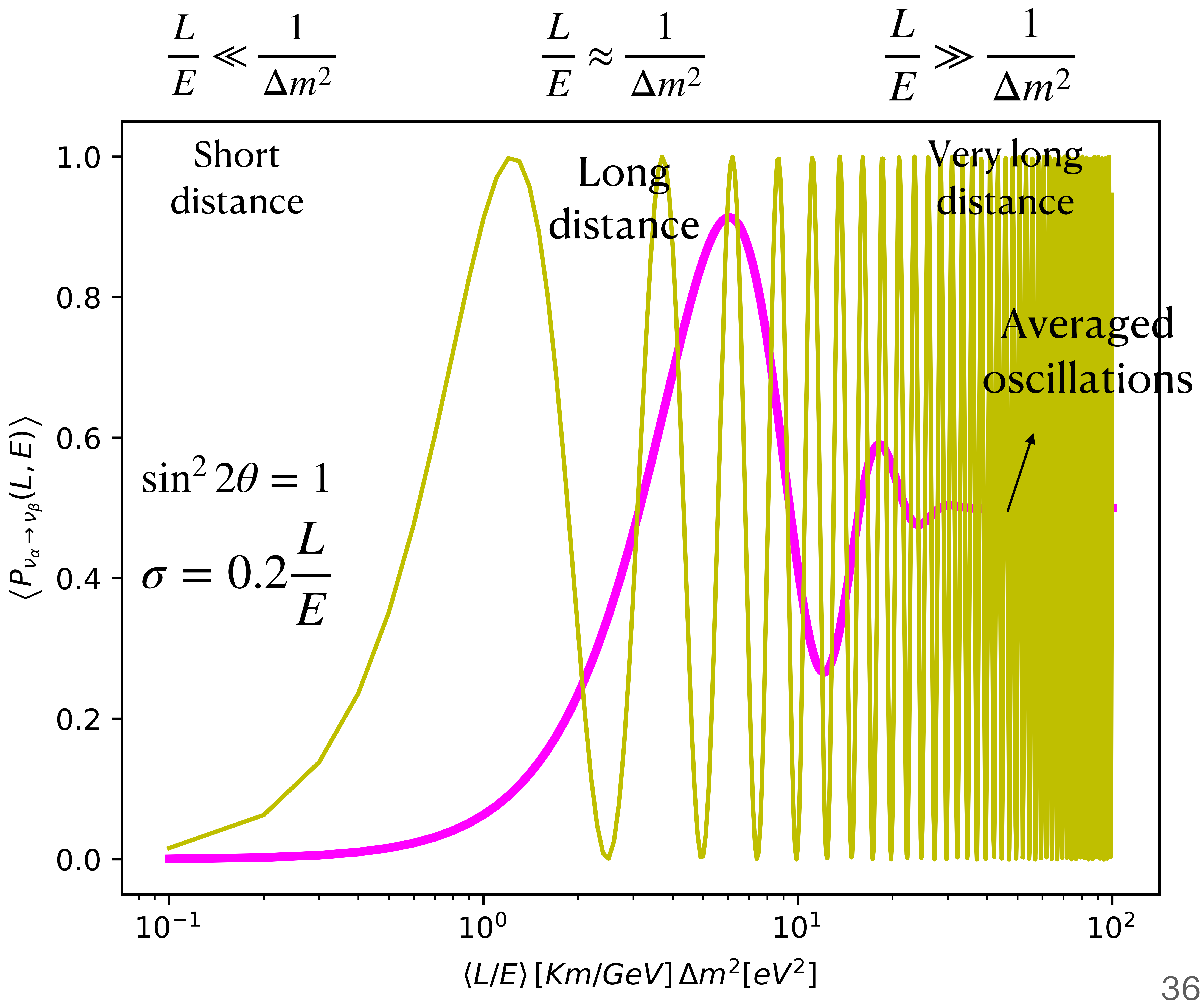
Impossible to measure oscillation probability precisely over a distance L and neutrino energy E, source and detector always has spatial uncertainty

$$\left\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \right\rangle = \frac{1}{2}\sin^2(2\theta)\left(1 - \left\langle \left(\cos\frac{\Delta m^2 L}{2E} \right) \right\rangle \right)$$

$$\left\langle \cos\frac{\Delta m^2 L}{2E} \right\rangle = \int cos\left(\frac{\Delta m^2 L}{2E}\right) \phi\left(\frac{L}{E}\right) d\left(\frac{L}{E}\right)$$

$$\phi\left(\frac{L}{E}\right) = \frac{1}{\sqrt{2\pi\sigma_{L/E}^2}} \exp\left(\frac{\left(L/E - \langle L/E \rangle\right)^2}{2\sigma_{L/E}^2}\right)$$

$$\left\langle \cos\left(\frac{\Delta m^2 L}{2E}\right) \right\rangle = cos\left(\frac{\Delta m^2}{2}\left\langle \frac{L}{E} \right\rangle\right) \exp\left[-\frac{1}{2}\left(\frac{\Delta m^2}{2}\sigma_{L/E}\right)^2\right]$$

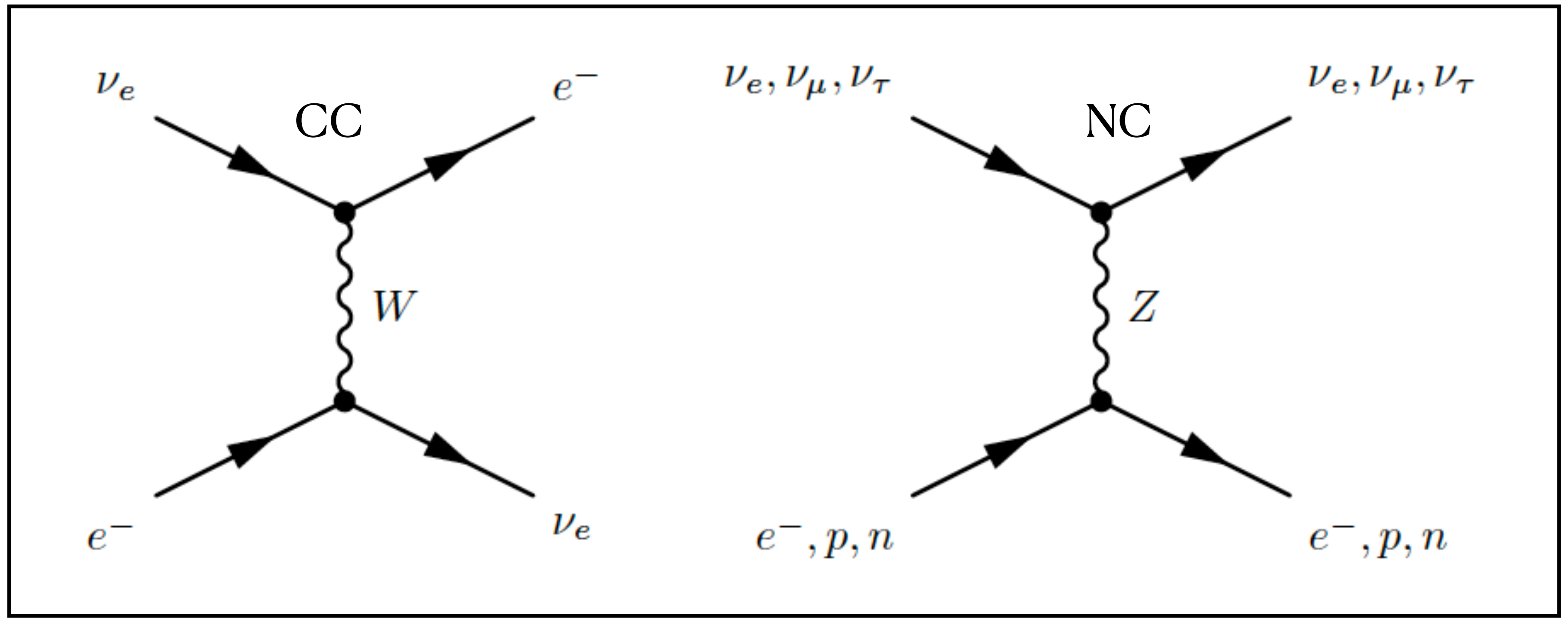


These are all interesting phenomena in a vacuum what about the matter?

Neutrino Oscillation in Matter

- ☑ 1978 L. Wolfenstein discovered the matter effect in Neutrinos, which faces a potential due to **coherent** elastic scattering in the medium particles: electrons and nucleons.

- ☑ The potential is analogous to the index of refraction of the medium modifies the mixing of neutrinos.



Remark:

$$\sigma_{Lab} \approx \sigma_{incoherent} \approx G_F E_\nu M_N = 10^{-38} cm^2 \frac{E_\nu M_N}{GeV^2}$$

$$l_{inchorent} = \frac{1}{N \sigma_{inchorent}} \qquad N \approx N_A \approx 10^{24}/cm^3 \; , M_N \approx 1 GeV$$

$$l_{incoherent} = \frac{10^{17} cm}{E/MeV}$$

Mean free path high, potential due to incoherent scattering is neglected!

Neutron Star or Supernovae: extreme dense environment

$$N_{core} \approx 10^{12} N_A/cm^3 \qquad l_{incoherent} \approx 1 Km$$

Significant Incoherent scatterings contributes to matter potential!

Matter Potential

$$H_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e(x)\gamma^{\rho}\left(1-\gamma^5\right)e(x)\right]\left[\bar{e}(x)\gamma_{\rho}\left(1-\gamma^5\right)\nu(x)\right]$$

$$V_{CC} = \sqrt{2}G_F N_e$$

$$H_{eff}^{NC}(x) = \frac{G_F}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left[\bar{\nu}_\alpha(x)\gamma^{\rho}\left(1-\gamma^5\right)\nu_\alpha(x)\right]\left[\bar{f}(x)\gamma_{\rho}\left(1-\gamma^5\right)f(x)\right]$$

$$V_{NC} = -\frac{1}{2}\sqrt{2}G_F N_n$$

The effective matter potential is $V = V_{CC} + V_{NC} = \sqrt{2}G_F\left(N_e - \frac{N_n}{2}\right)$,

* V is positive for neutrinos, for antineutrinos sign of V gets reversed.

That potential V is very small, $\sqrt{2}G_F \approx 7.63\times 10^{-14}\frac{eV}{N_A/cm^3}$

$$V_{CC} \approx 7.63\times 10^{-14}\frac{\rho}{gm/cm^3}\times Y_e\ eV\ ;\ Y_e = N_e/N_N \approx 0.5$$

Due to this matter potential neutrino mass eigenstate ν_k follow,

$$\begin{aligned} m^2_{k(matter)} &= \left(E+V_{CC}\right)^2 - |\vec{P}|^2 \\ &= E^2 + V_{CC}^2 + 2EV_{CC} - |\vec{P}|^2 \\ &= m_k^2 + 2EV_{CC};\ \ E \gg V_{CC} \end{aligned}$$

Table: Matter potential in the various medium

Medium	Matter density (g/cm^3)	$V_{CC}(eV)$
Solar core	~ 100	$\sim 10^{-12}$
Earth core	~ 10	$\sim 10^{-13}$
Supernova	$\sim 10^{14}$	~ 1

Neutrino Flavor evolution in matter

Total Hamiltonian in matter $H = H_0 + H_I$ with $H_I|\nu_\alpha\rangle = V|\nu_\alpha\rangle$; $V = V_{CC} + V_{NC}$ $\quad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k}|\nu_k\rangle \quad \rightarrow \quad |\nu_k\rangle = \sum_\alpha U_{k\alpha}|\nu_\alpha\rangle$

In the Schrödinger picture a neutrino state $|\nu_\alpha\rangle$ as ; $i\dfrac{d|\nu_\alpha(t)\rangle}{dx} = H|\nu_\alpha(t)\rangle$ with $|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle$

The amplitude of $\nu_\alpha \rightarrow \nu_\beta$ transition is given by $\psi_{\alpha\beta}(t) = \langle\nu_\beta|\nu_\alpha(t)\rangle, \quad P = |\psi_{\alpha\beta}(t)|^2$

The Schrödinger equation with effective hamiltonian H_f in the flavor basis $\quad i\dfrac{d\psi_\alpha}{dx} = H_f\psi_\alpha \qquad \psi_\alpha = \begin{pmatrix}\psi_{\alpha e}\\ \psi_{\alpha\mu}\end{pmatrix}$

$$H_f = \left(UM^2U^\dagger\right) + A \qquad A = 2EV_{CC}$$

$$H_f = \begin{pmatrix}-\Delta m^2\cos 2\theta + A & \sin^2 2\theta\\ \sin^2 2\theta & \Delta m^2\cos 2\theta - A\end{pmatrix} \qquad A = \begin{pmatrix}A & 0\\ 0 & 0\end{pmatrix},\ U = \begin{pmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{pmatrix},\ M^2 = \begin{pmatrix}0 & 0\\ 0 & \Delta m^2_{21}\end{pmatrix}$$

H_f can be diagonalized $\qquad U_M^T H_f U_M = H_M$ where $H_M = \dfrac{1}{4E}diag\left(-\Delta m^2_M, \Delta m^2_M\right)$ in Mass basis in matter

Mikhiyev-Smirnov-Wolfstein (MSW) Effect

$$\nu_e = \cos\theta_M \nu_{1M} + \sin\theta_M \nu_{2M}$$

$$\nu_\mu = -\sin\theta_M \nu_{1M} + \cos\theta_M \nu_{2M}$$

The unitary mixing matrix in matter $U_M = \begin{pmatrix} cos\theta_M & -sin\theta_M \\ \sin\theta_M & cos\theta_M \end{pmatrix}$

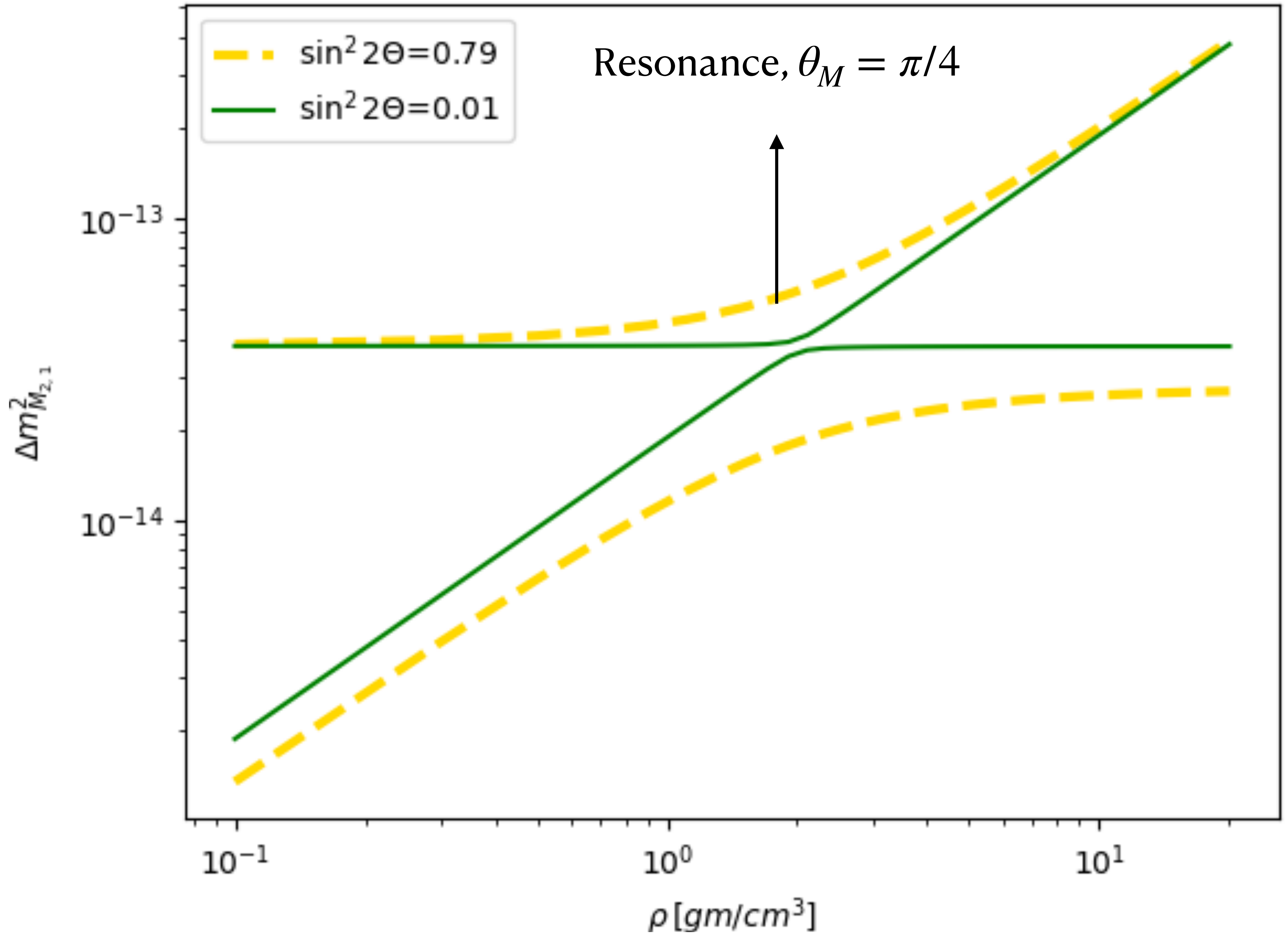
Effective mixing angle in matter : $\tan 2\theta_M = \dfrac{\tan 2\theta}{1 - \dfrac{A}{\Delta m^2 \cos 2\theta}}$

Resonance : $A^R = \Delta m^2 \cos 2\theta \rightarrow N_e^R = \dfrac{\Delta m^2 cos2\theta}{2\sqrt{2} G_F N_e}$

The effective mass square difference in the matter:

$$\Delta m_M^2 = \sqrt{\left(\Delta m^2 \sin^2 2\theta + (\Delta m^2 \cos 2\theta - A)^2\right)}$$

$\Delta m_M^2|^R = \sqrt{\left(\sin^2 2\theta\right)}$ has minimum value, mixing is maximal leading towards complete transition i.e MSW effect



Level Crossing Phenomenon

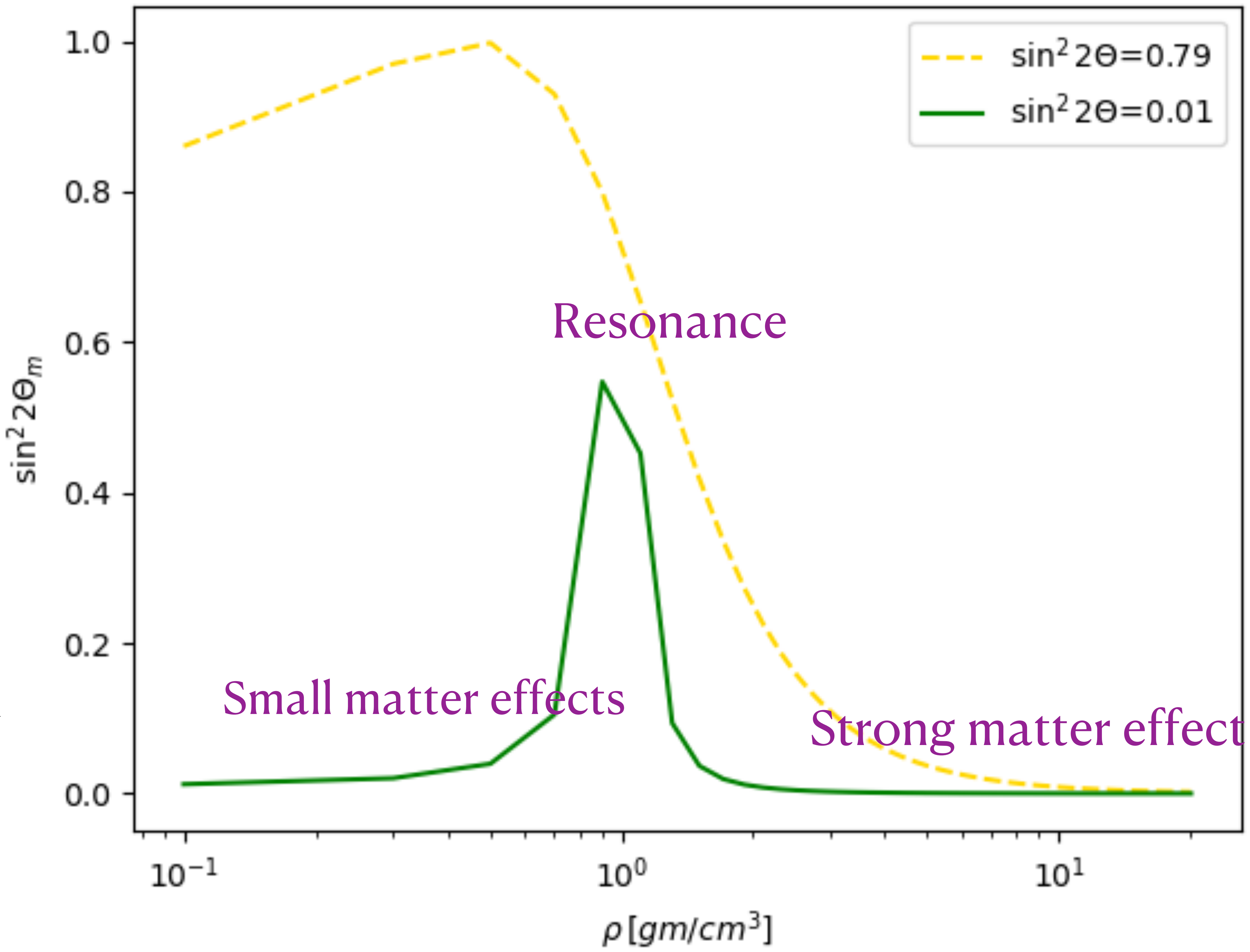
Oscillation amplitude in Matter

$$sin2\theta_M = \frac{\Delta m^2\, sin2\theta}{\sqrt{\left(\Delta m^2 \sin^2 2\theta + (\Delta m^2 \cos 2\theta - A)^2\right)}}$$

$A \ll cos\, 2\theta$: Small matter effects, vacuum oscillations

$A \gg cos\, 2\theta$: Matter effect dominates, oscillations suppressed

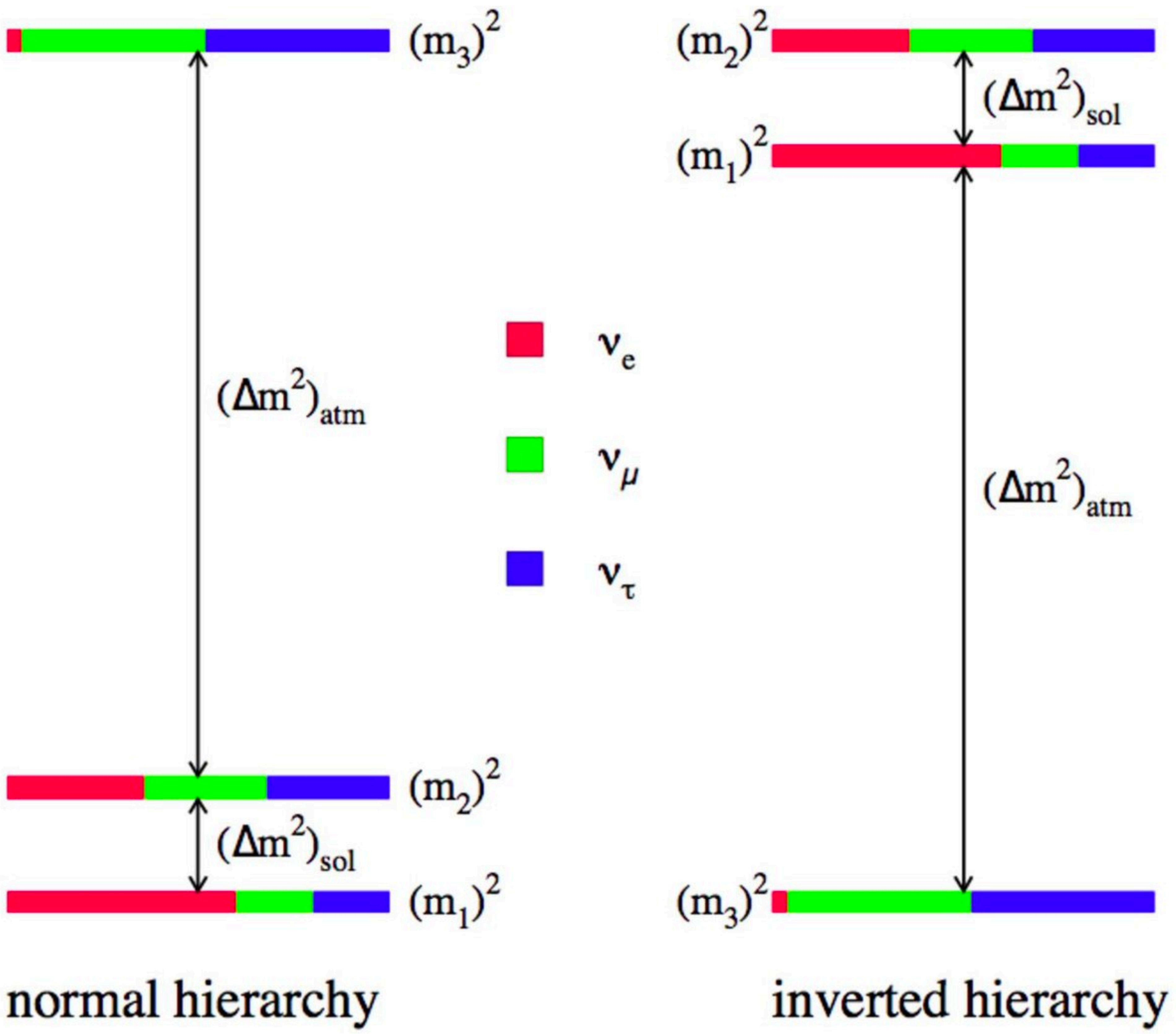
$A = cos\, 2\theta$: Resonance takes place , oscillations maximum



Matter effect in Oscillation

A is positive for neutrinos, resonance can only exist for $\theta < \pi/4 \rightarrow \cos 2\theta > 0$, because for $\theta > \pi/4, \cos 2\theta < 0$. However for antineutrinos things get reversed.

Neutrino Mass Scheme in Three Neutrinos



Solar: $\Delta m_{21}^2 > 0; \quad m_2 > m_1$

Atmosphere: $|\Delta m_{31}^2|$

What is the absolute scale of neutrino mass of a neutrino?

Absolute scale of neutrino mass in the Oscillation Picture

Normal Hierarchy: $m_2^2 = m_1^2 + \Delta m_{sol}^2 \quad ; \; m_3^2 = m_1^2 + \Delta m_{atm}^2$

Inverted Hierarchy

$$m_1^2 = m_3^2 + \Delta m_{atm}^2 \quad ; \; m_2^2 = m_1^2 + \Delta m_{sol}^2 = m_3^2 + \Delta m_{atm}^2 + \Delta m_{sol}^2$$

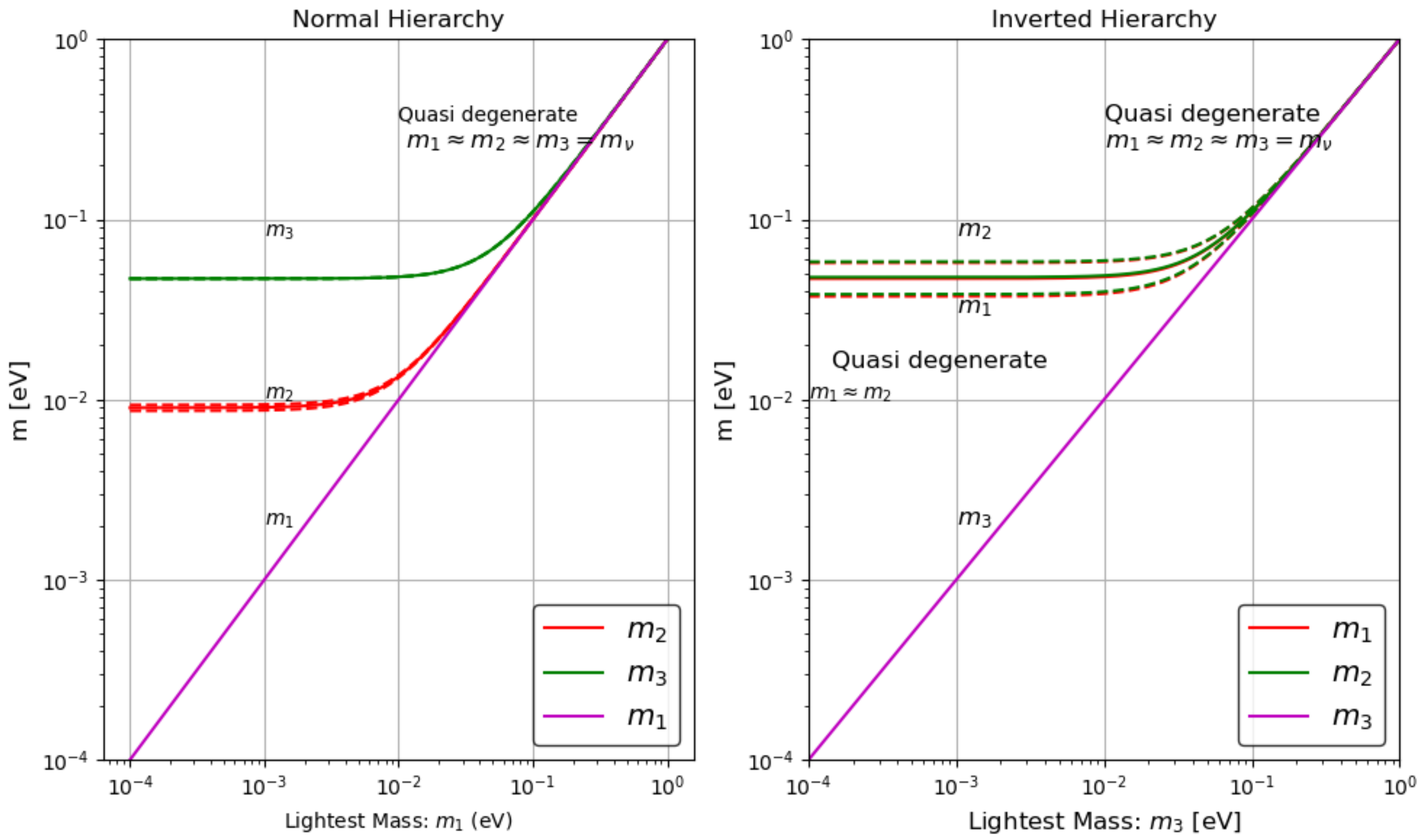
Quasi Degenerate Region: $m_1 \approx m_2 \approx m_3 = m_\nu$

$$m_\nu \gg \sqrt{\Delta m_{atm}^2} \approx 5 \times 10^{-2} eV$$

Lowest Mass Region: $\sqrt{\Delta m_{atm}^2}$ is significant

$m_1 \approx m_2$ (always quasi-degenerate only differ by Δm_{sol}^2)

At least two neutrinos are massive with $m_\nu > 8 \times 10^{-3}$ eV.



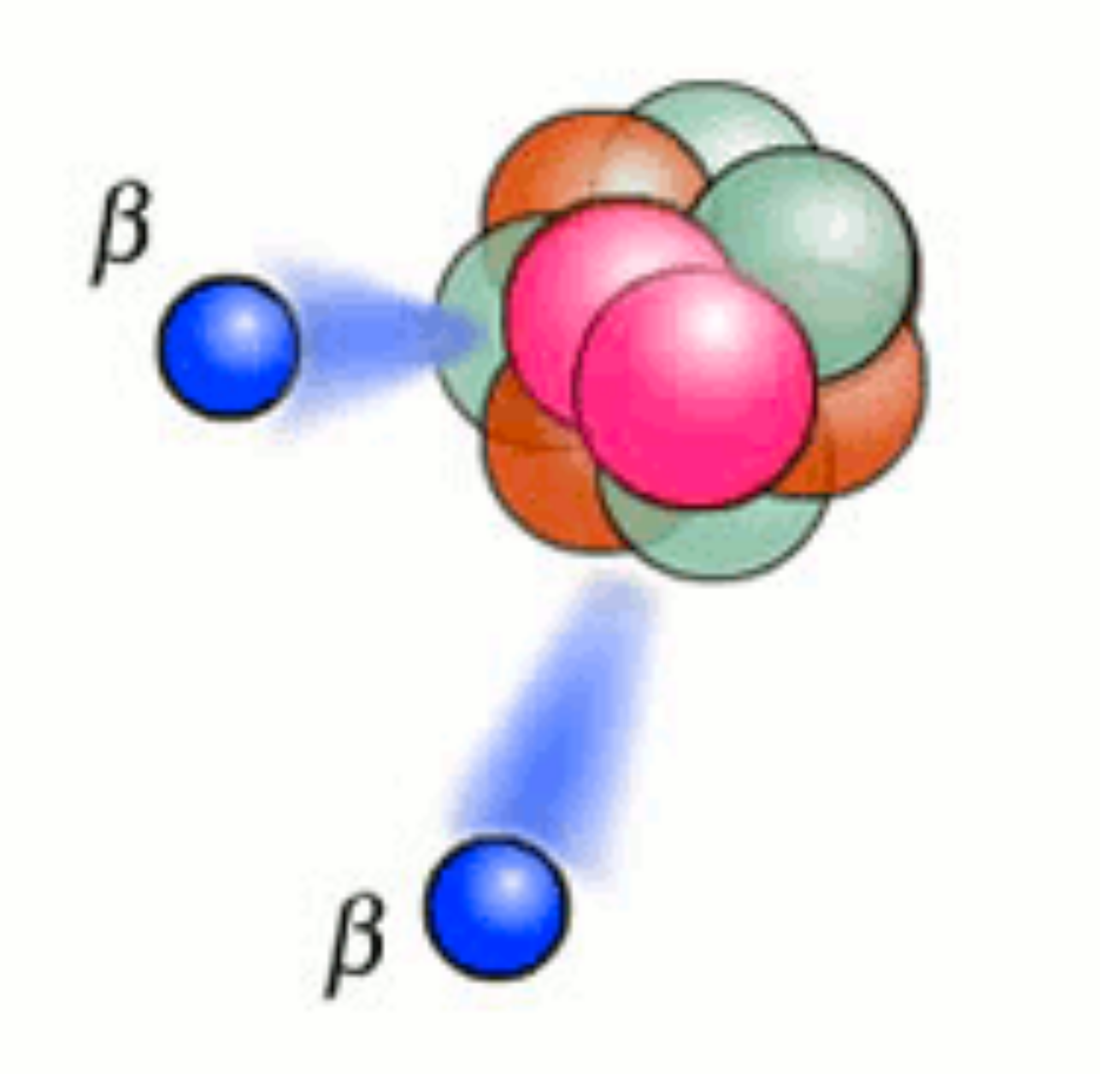
The solid line corresponds to best-fit values of mass square difference and dashed lines enclose 3σ ranges.

Can we look for a direct measurement of neutrino mass?

→ Possibly Yes if they are Majorana!

Section C

Double Beta Decay and Neutrino Mass in the context of BSM



Neutrino physics without Neutrinos

$$^{A}_{Z}X \rightarrow \, ^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e \quad \Rightarrow (1)$$

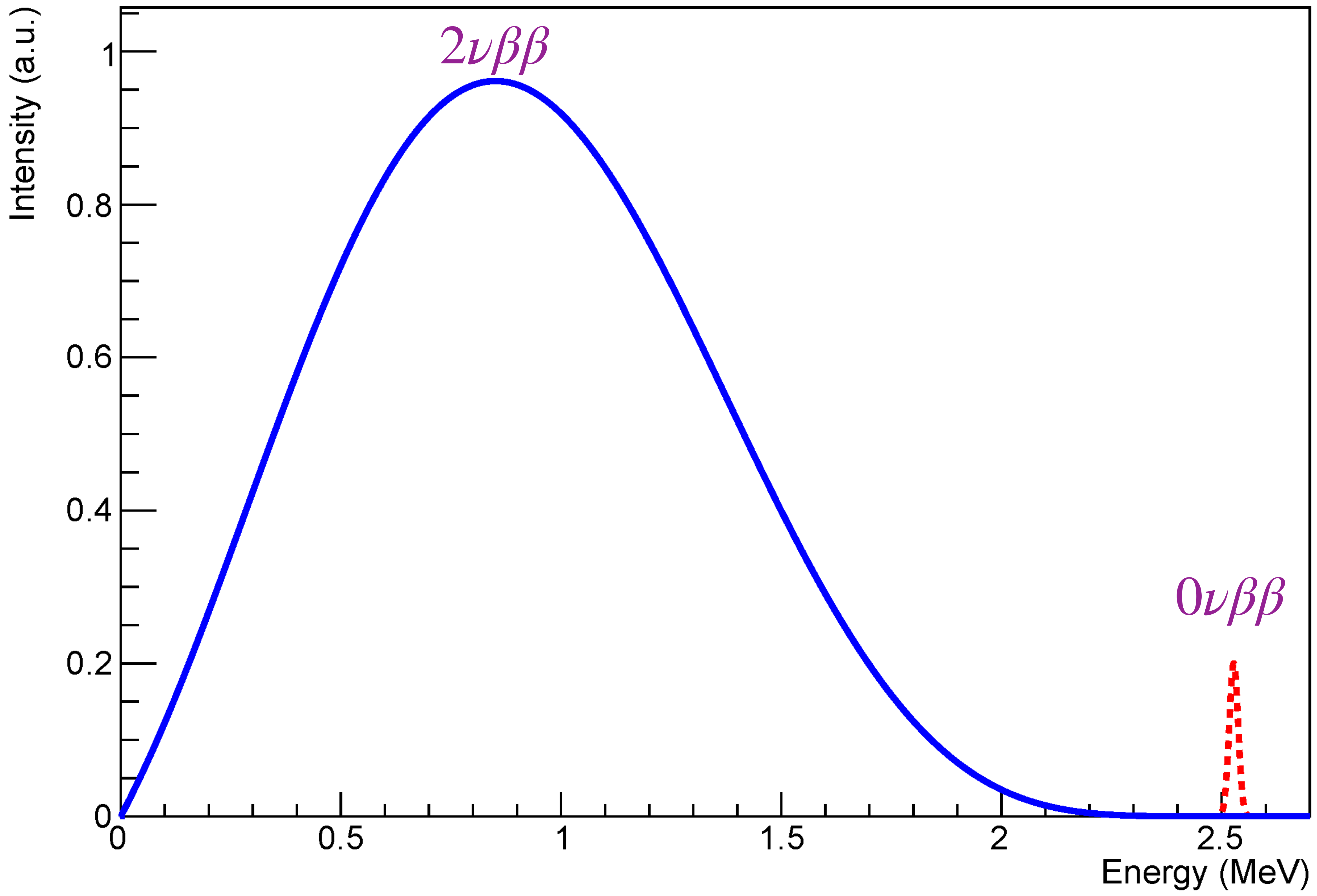
If $\bar{\nu}_e = \nu_e$

$$^{A}_{Z+1}Y + \nu_e \rightarrow \, ^{A}_{Z+2}Y + e^{-} \quad \Rightarrow (2)$$

Summing up (1) & (2) the net process is,

$$\boxed{^{A}_{Z}X \rightarrow \, ^{A}_{Z+2}Y + 2\,e^{-}}$$

$$M_X > M_Y + 2e^{-}$$



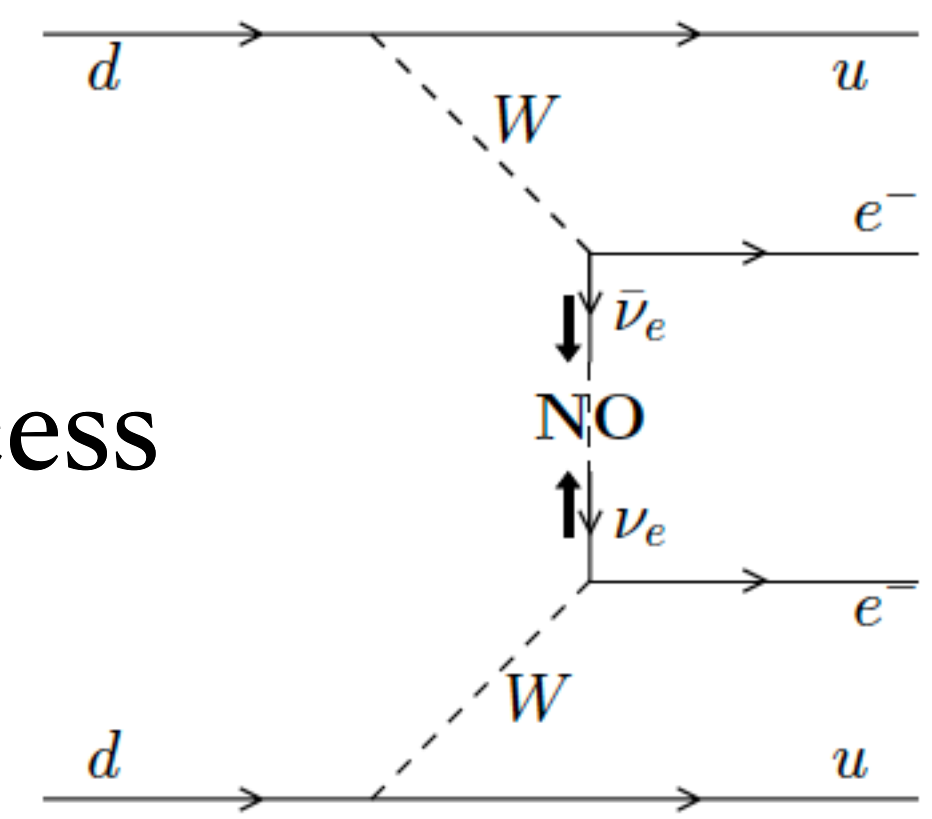
$$Q_{0\nu\beta\beta} = M_X(A, Z) - M_Y(A, Z+2) - 2m_e$$

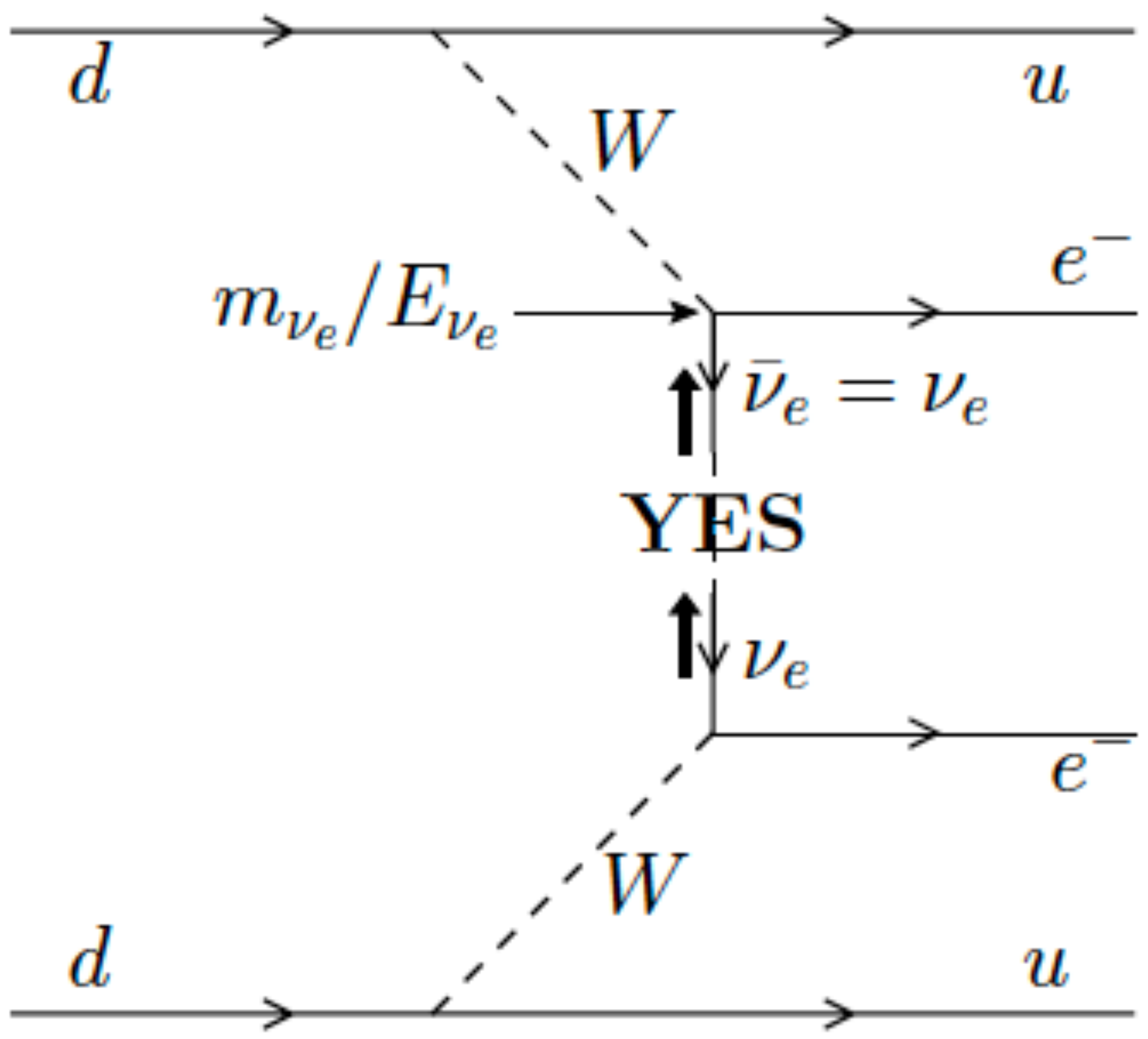
The sum kinetic energy of two electrons is equal to Q- the value of the reaction.

arxiv.org/pdf/2303.05127.pdf

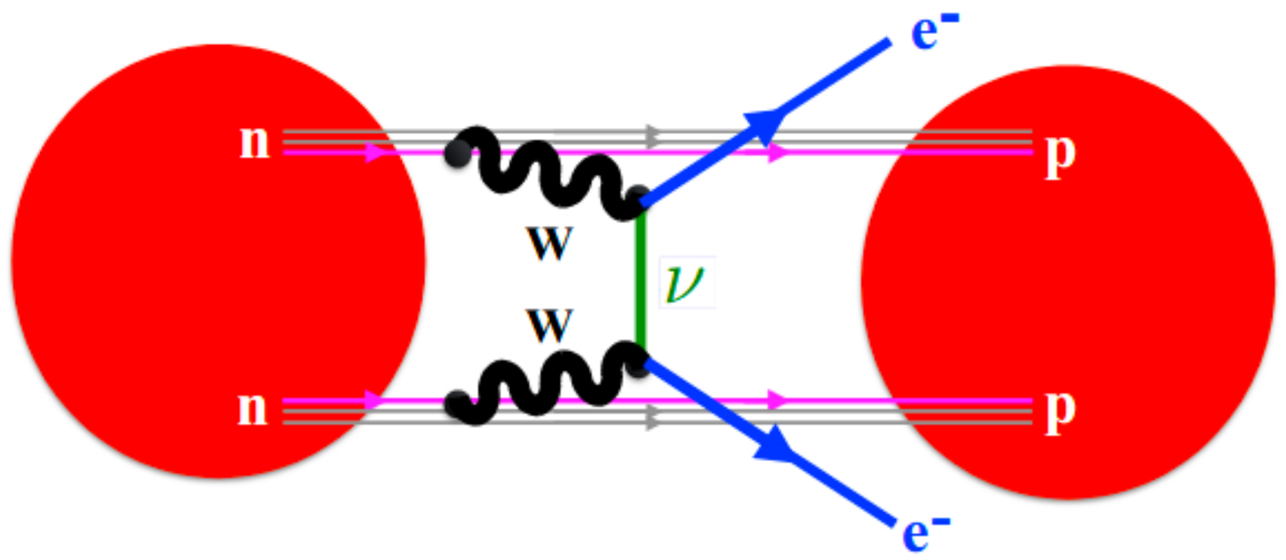
Feynman diagrams study

SM Process





Neutrinoless double beta decay



Forbidden $2\beta_{0\nu}$ process,
$\nu_e \neq \bar{\nu}_e$ and helicity mismatch

Rare $2\beta_{0\nu}$ Particle-antiparticle and helicity matching process

$$T_{1/2}^{0\nu} \approx 10^{21-30}\,years$$

The age of the universe is 13.7 billion years

https://downloads.hindawi.com/journals/ahep/2016/2162659.pdf

arxiv.org/pdf/2212.11099.pdf

Lepton Number violation and Matter-Antimatter Asymmetry

$$N(A,Z) \rightarrow N(A,Z+2) + 2e^- \quad (2\beta^-_{o\nu})$$

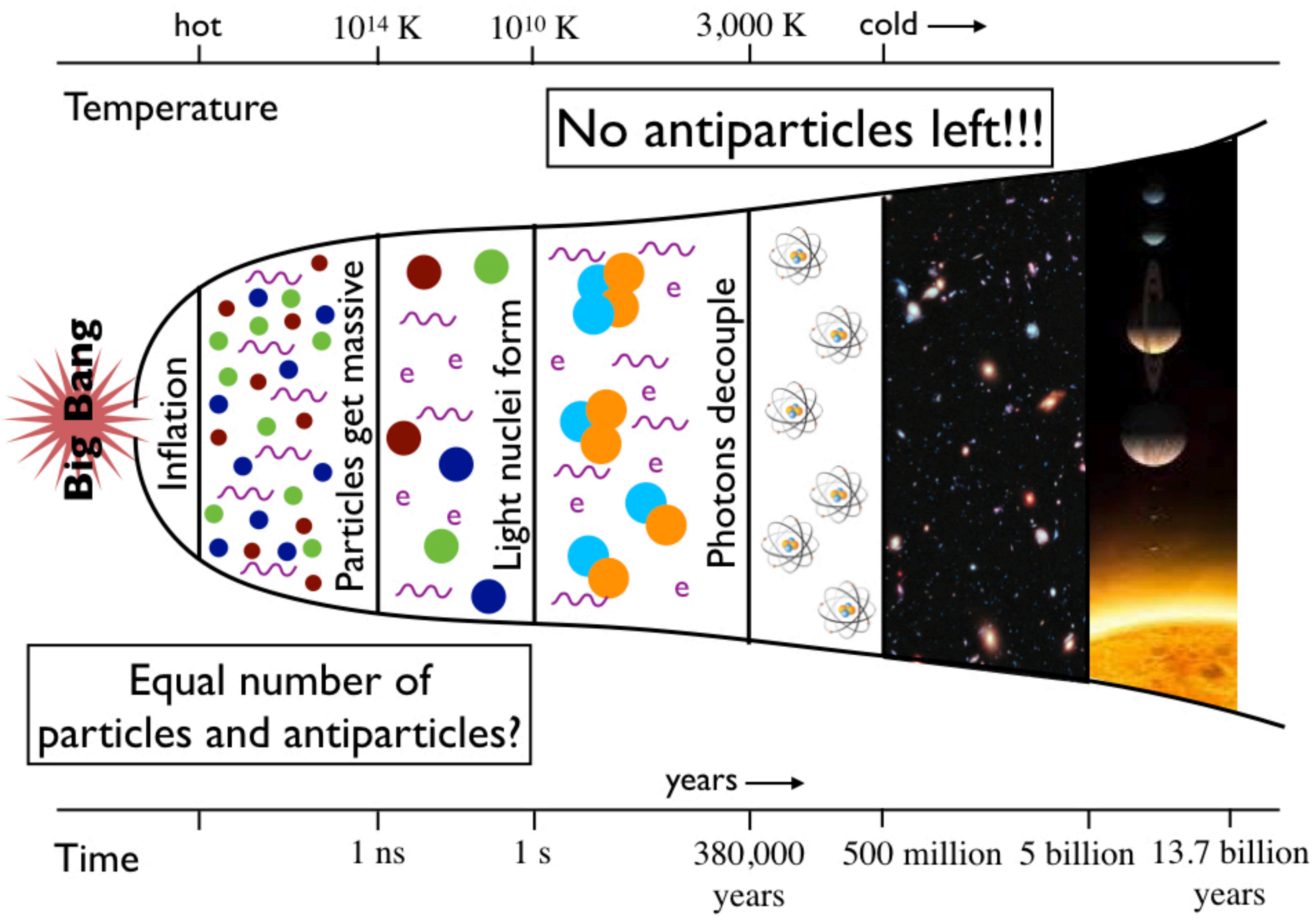
$$N(A,Z) \rightarrow N(A,Z-2) + 2e^- \quad (2\beta^+_{o\nu})$$

Lepton Number in the double beta decay process:

$$L_e : 0 = 0 \pm 2$$

$$\Delta L_e = \mp 2 \rightarrow \textit{Violated} \longrightarrow \textbf{BSM Physics}$$

- Leptogenesis proposes that the asymmetry is produced by a violation of lepton-number conservation.



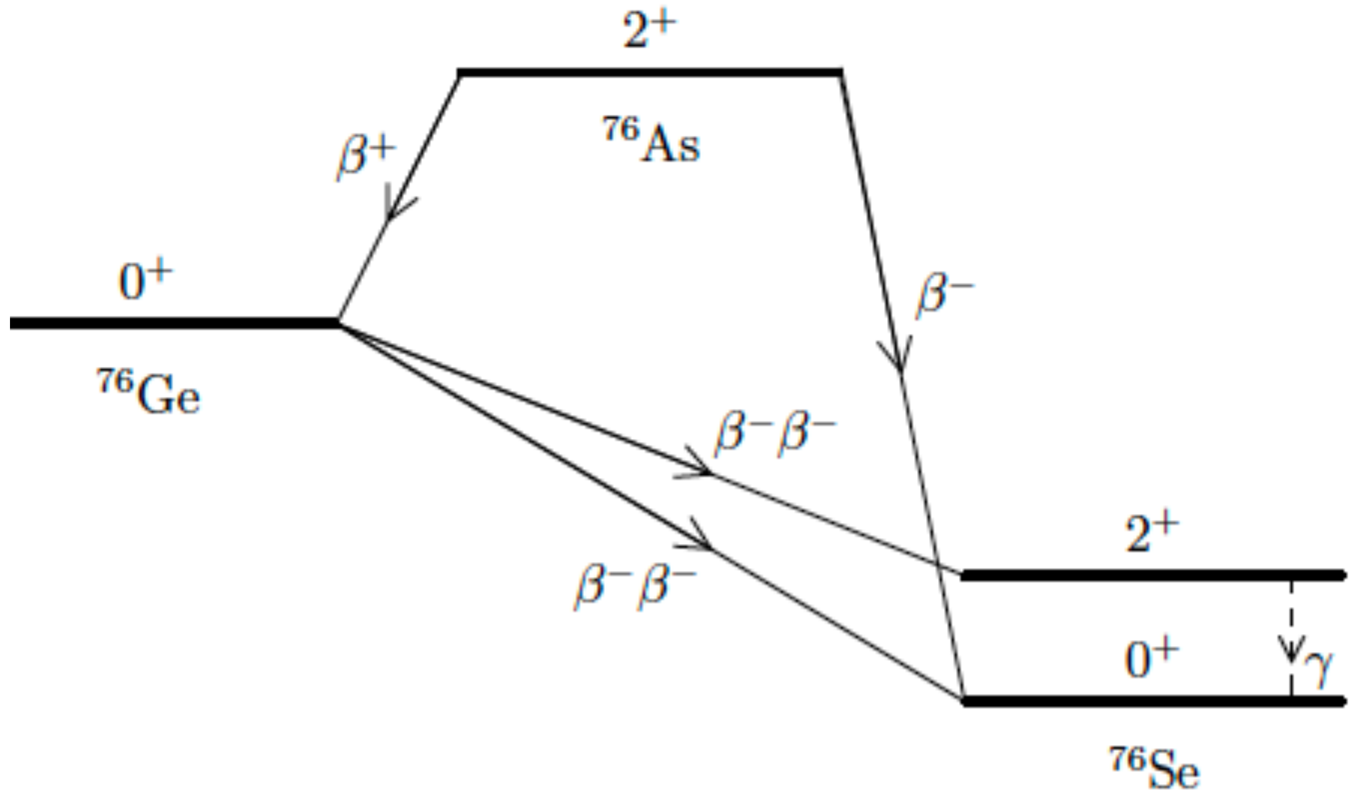
Proposed Experiment LEGEND at SNO, Canada

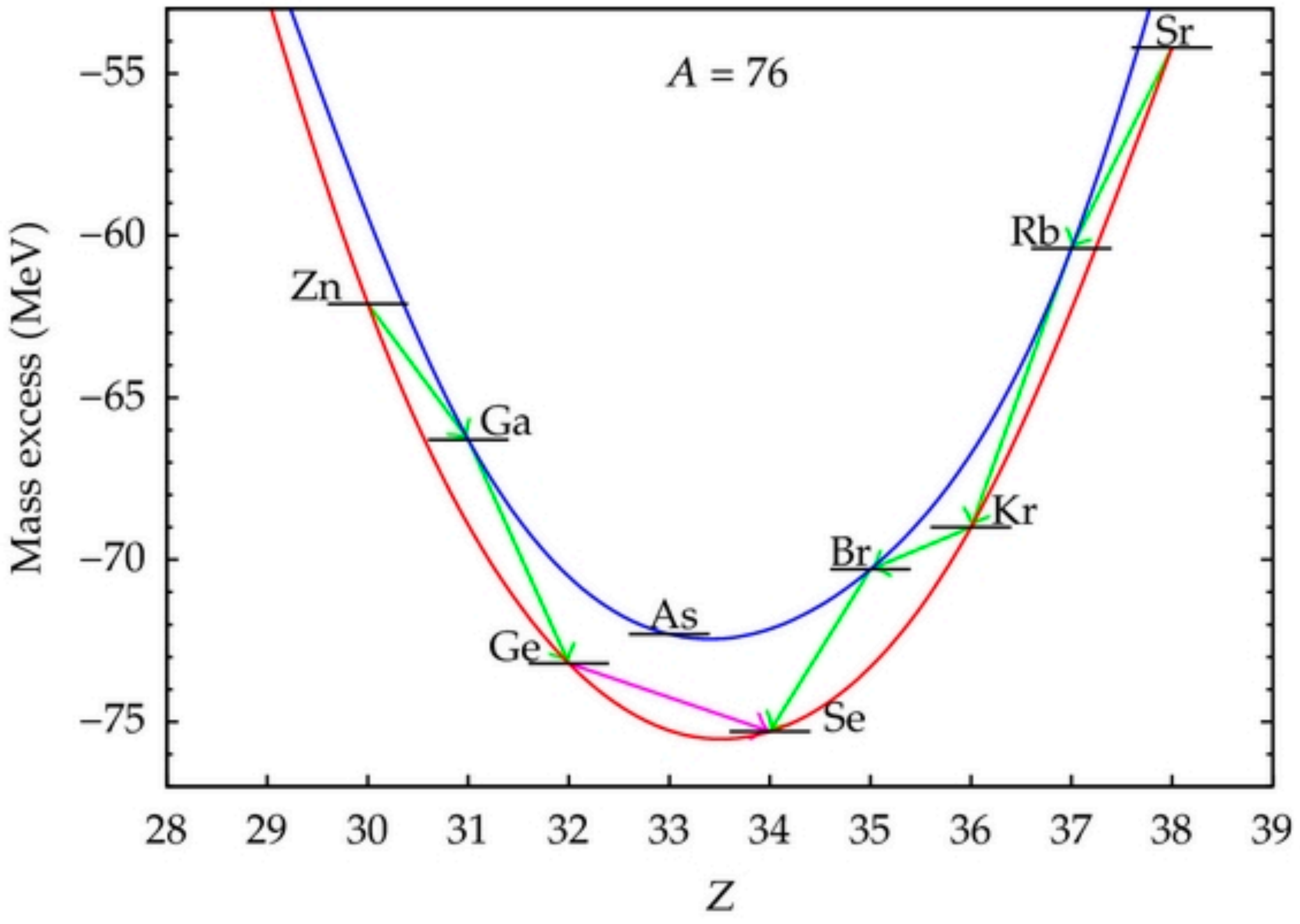
KAMLAND-Gen, Japan, nEXO in NM

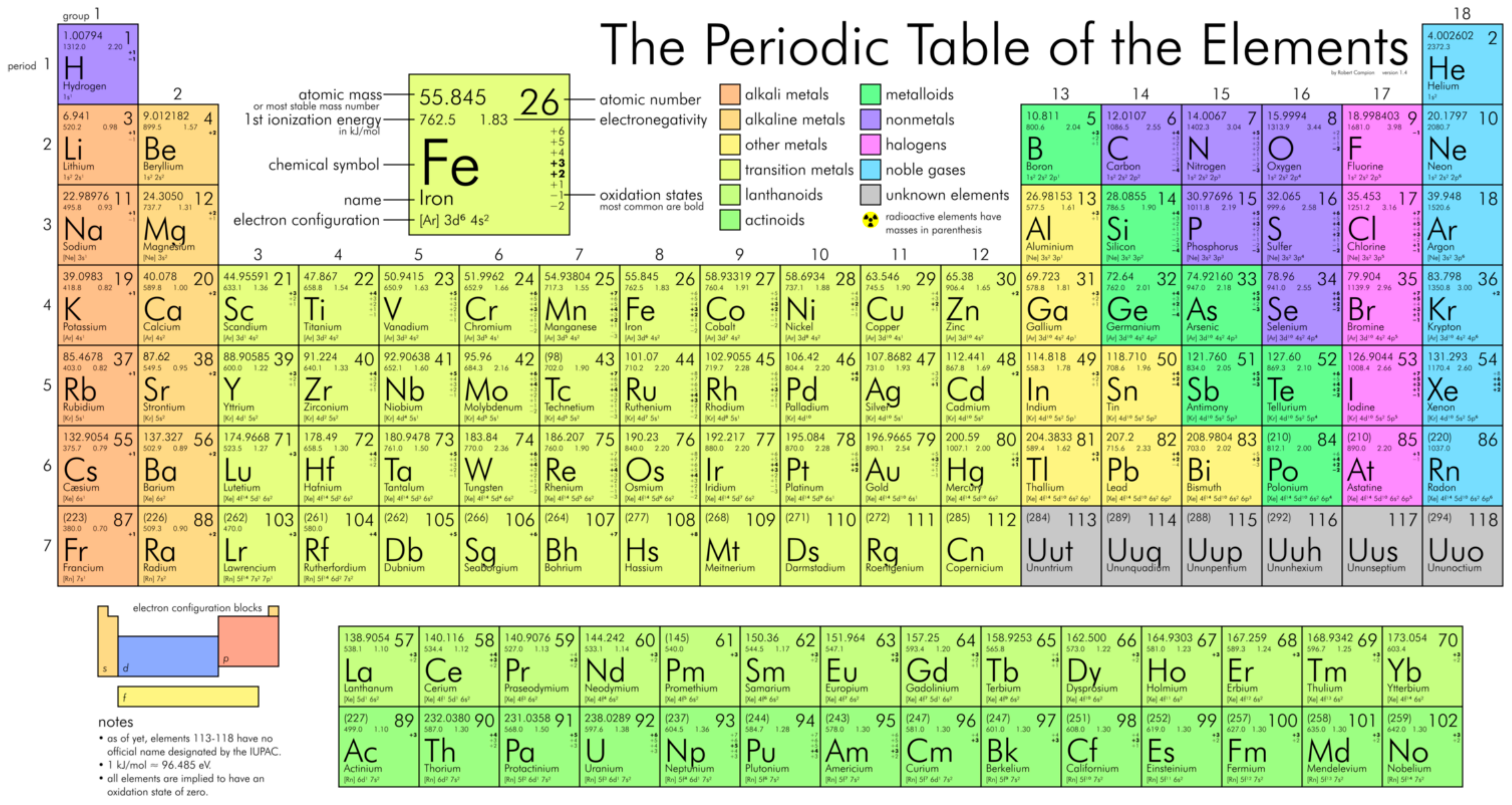
arxiv.org/pdf/2303.05127.pdf   Balentekin & Kayser: arxiv.org/pdf/1805.00922.pdf

Some Crucial Nuclear Physics for Double Beta Decay

35 isotopes i.e Ge, Te, Xe in nature for which beta decay is energetically forbidden, so they can go double beta decay







$$T_{1/2}^{0\nu} = G_{0\nu}\,|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2 \approx 10^{25} \text{ years}$$

Phase space factor

$\approx 10^{-15}\,yr^{-1}$

Nuclear Matrix elements

$|M_{0\nu}| \approx 1.5 - 4.6$

Effective Majorana mass

$\Delta I \left(0^+ \to 0^+\right) = 0$

Fermi Transition

J^P: Spin Parity

No change in parity

arxiv.org/pdf/2303.05127.pdf

https://downloads.hindawi.com/journals/ahep/2012/857016.pdf

Effective Majorana Neutrino Mass

$c = \cos\theta_{ab},\ s = \sin\theta_{ab}\quad 0 \le \theta_{ab} \le \pi/2$

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i$$

$$U^D = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

$$D^M = \begin{bmatrix} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{bmatrix}$$

$$U = U^D\, U^M = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$\lambda_1 = 0$ and λ_2, λ_3 are two unknown Majorana Phases

Majorana mass mass term: not invariant under U(1) gauge transformation

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i = \left|U_{e1}\right|^2 m_1 + e^{2i\lambda_2}|U_{e2}|^2 m_2 + e^{2i(\lambda_3-\delta_{13})}|U_{e3}|^2 m_3$$

$2i\lambda_2 \equiv \alpha_2\quad \&\ 2i(\lambda_3 - \delta_{13}) \equiv \alpha_3$

$$\boxed{m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i = \cos^2\theta_{13}\cos^2\theta_{12} m_1 + e^{i\alpha_2}\cos^2\theta_{13}\sin^2\theta_{12} m_2 + e^{i\alpha_3}\sin^2\theta_{13} m_3}$$

Phases have $0 \le \alpha_k \le 2\pi,\ \ k = 2{,}3$

Physically relevant cases: $\alpha_k = 0,\ \pi$ for which CP is conserved: $e^{i\alpha_k} = \pm 1$

$J = \hat{J}\sin\delta_{13}$

CP is conserved if $\delta_{13} = 0, \pi$ and $\lambda_2, \lambda_3 = 0, \pi/2, \pi, 3\pi/2$

Physically relevant cases: $\alpha_k = 0,\ \pi$, for which CP is conserved: $e^{i\alpha_k} = \pm 1$

There are four possibilities:

$$(++) \quad \alpha_2 = \alpha_3 = 0 \quad \rightarrow \quad m_{ee} = \left|U_{e1}\right|^2 m_1 + |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3 \longrightarrow \text{Maximum}$$

$$(--) \quad \alpha_2 = \alpha_3 = \pi \quad \rightarrow \quad m_{ee} = \left|U_{e1}\right|^2 m_1 - |U_{e2}|^2 m_2 - |U_{e3}|^2 m_3$$

$$(+-) \quad \alpha_2 = 0, \alpha_3 = \pi \quad \rightarrow \quad m_{ee} = \left|U_{e1}\right|^2 m_1 - |U_{e2}|^2 m_2 - |U_{e3}|^2 m_3$$

Minimum

$$(-+) \quad \alpha_2 = \pi, \alpha_3 = 0 \quad \rightarrow \quad m_{ee} = \left|U_{e1}\right|^2 m_1 - |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3$$

Effective Neutrino Mass in NH

$$m_{ee} = \left|U_{e1}\right|^2 m_1 + e^{i\alpha_2}|U_{e2}|^2\sqrt{m_1^2+\Delta m_{21}^2} + e^{i\alpha_3}|U_{e3}|^2\sqrt{m_1^2+\Delta m_{31}^2}$$

$(++), (+-) : m_{ee} \approx m_1$ Quasi degenerate

$$(--), (-+) : m_{ee} \approx m_1\left(|U_{e1}^2| - |U_{e2}^2|\right) \approx m_1 \cos 2\theta_{12}$$

In the region $m_1 \ll m_2 \ll m_3$:

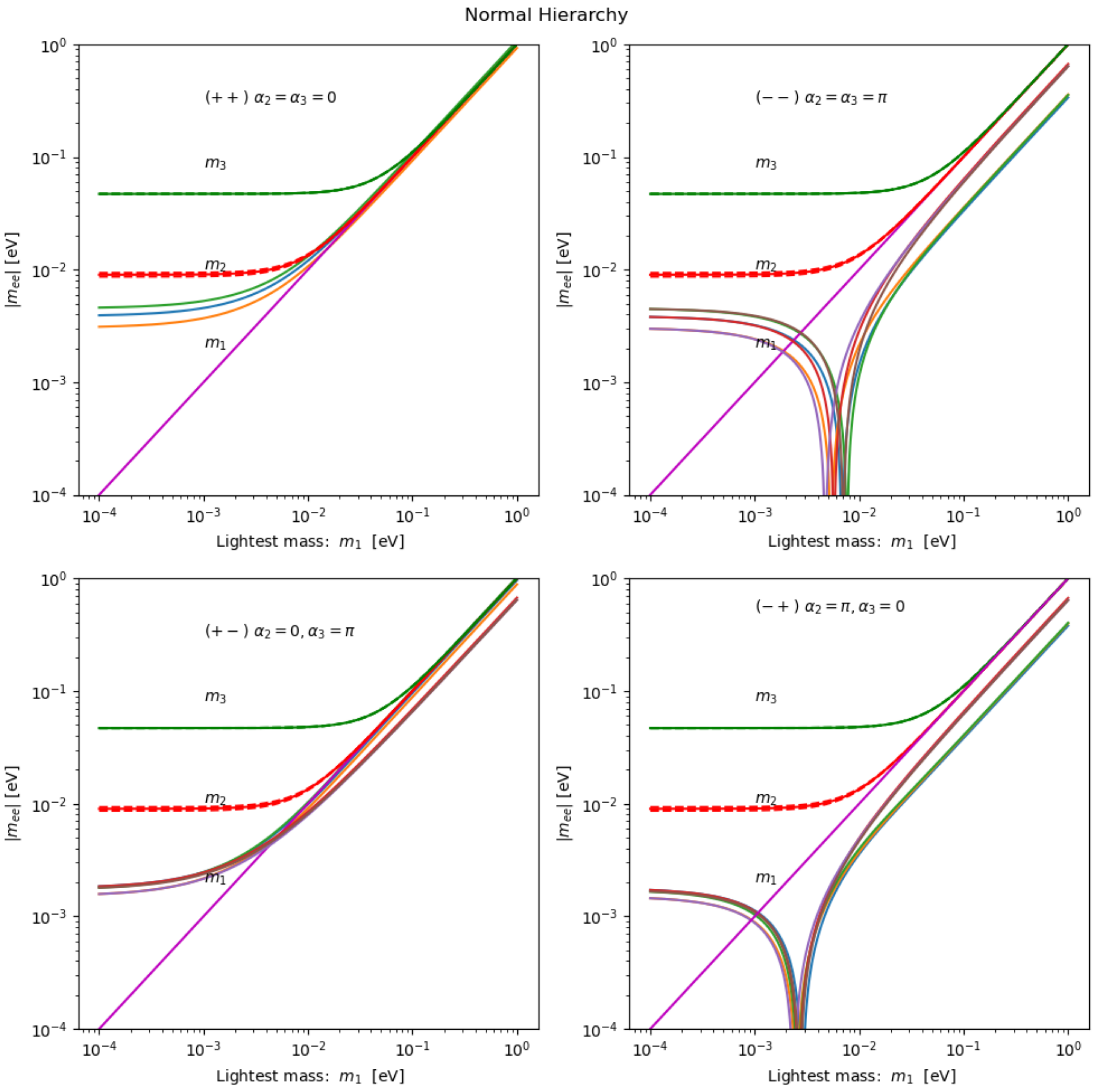
$$|m_{ee}|_{bf} \approx |\cos^2\theta_{12}^{bf} m_1 + e^{i\alpha_2}\sin^2\theta_{12}^{bf}\sqrt{\left(\Delta m_{21}^2\right)_{bf}}|$$

$$|m_{ee}|_{bf} = 0,\ m_1 \approx 4\times 10^{-3}\ \mathrm{eV}$$

For smaller m_1 :

$$|m_{ee}| \approx \left|\,|U_{e2}|^2\sqrt{\left(\Delta m_{21}^2\right)} + e^{i(\alpha_3-\alpha_2)}|U_{e2}|^2\sqrt{\left(\Delta m_{31}^2\right)}\right|$$

$$|m_{ee}|_{bf} = \sin^2\theta_{12}^{bf}\sqrt{\left(\Delta m_{21}^2\right)} \approx 2\times 10^{-3}\ \mathrm{eV}$$



Effective Neutrino mass in IH

$$m_{ee} = \left|U_{e1}\right|^2 \sqrt{m_3^2 + \Delta m_{31}^2} + e^{i\alpha_2}|U_{e2}|^2\sqrt{m_3^2 + \Delta m_{31}^2 + \Delta m_{21}^2} + e^{i\alpha_3}|U_{e3}|^2 m_3$$

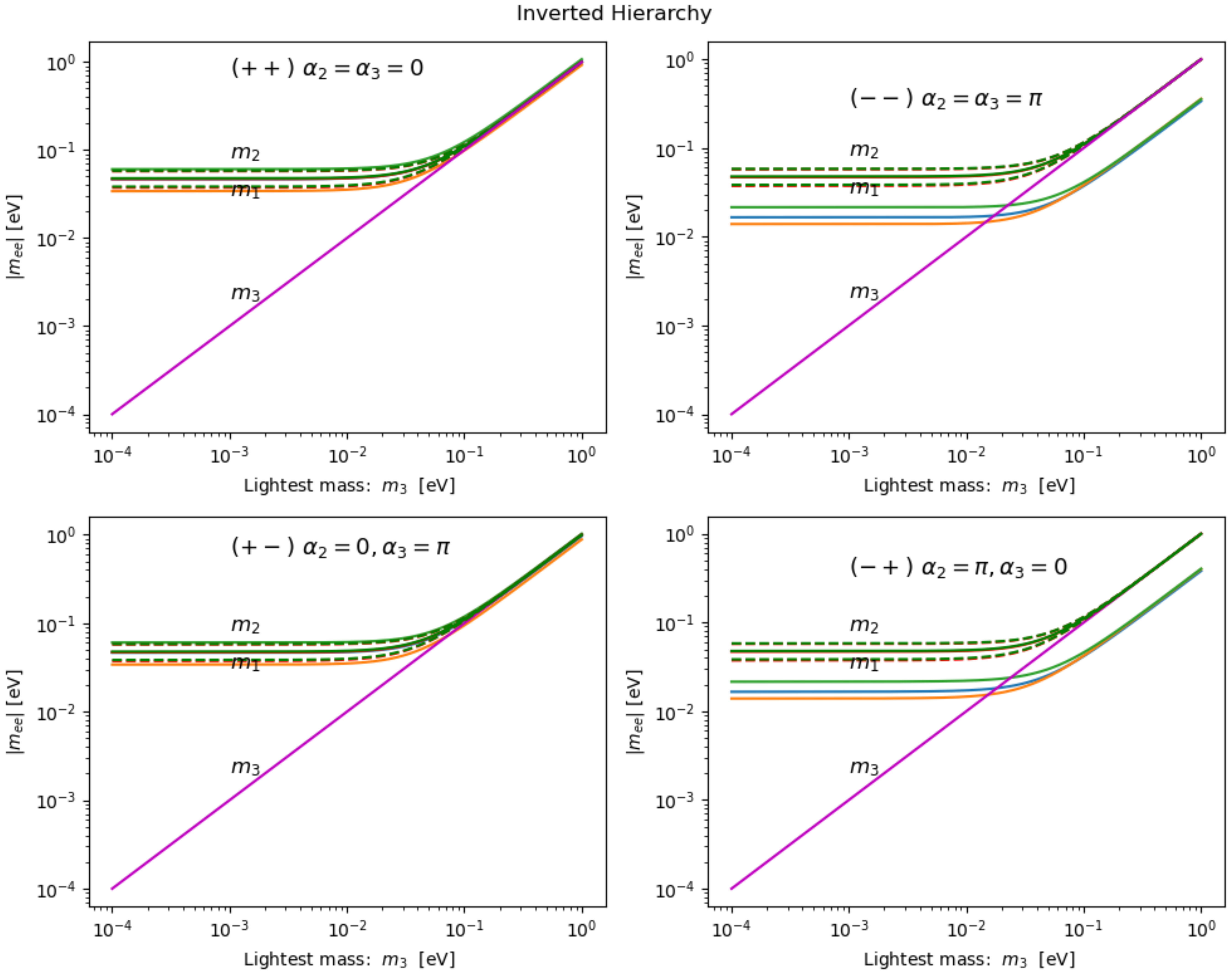
$(++), (+-) : m_{ee} \approx m_3$ Quasi degenerate

In the region $m_3 \ll m_1 \approx m_2$:

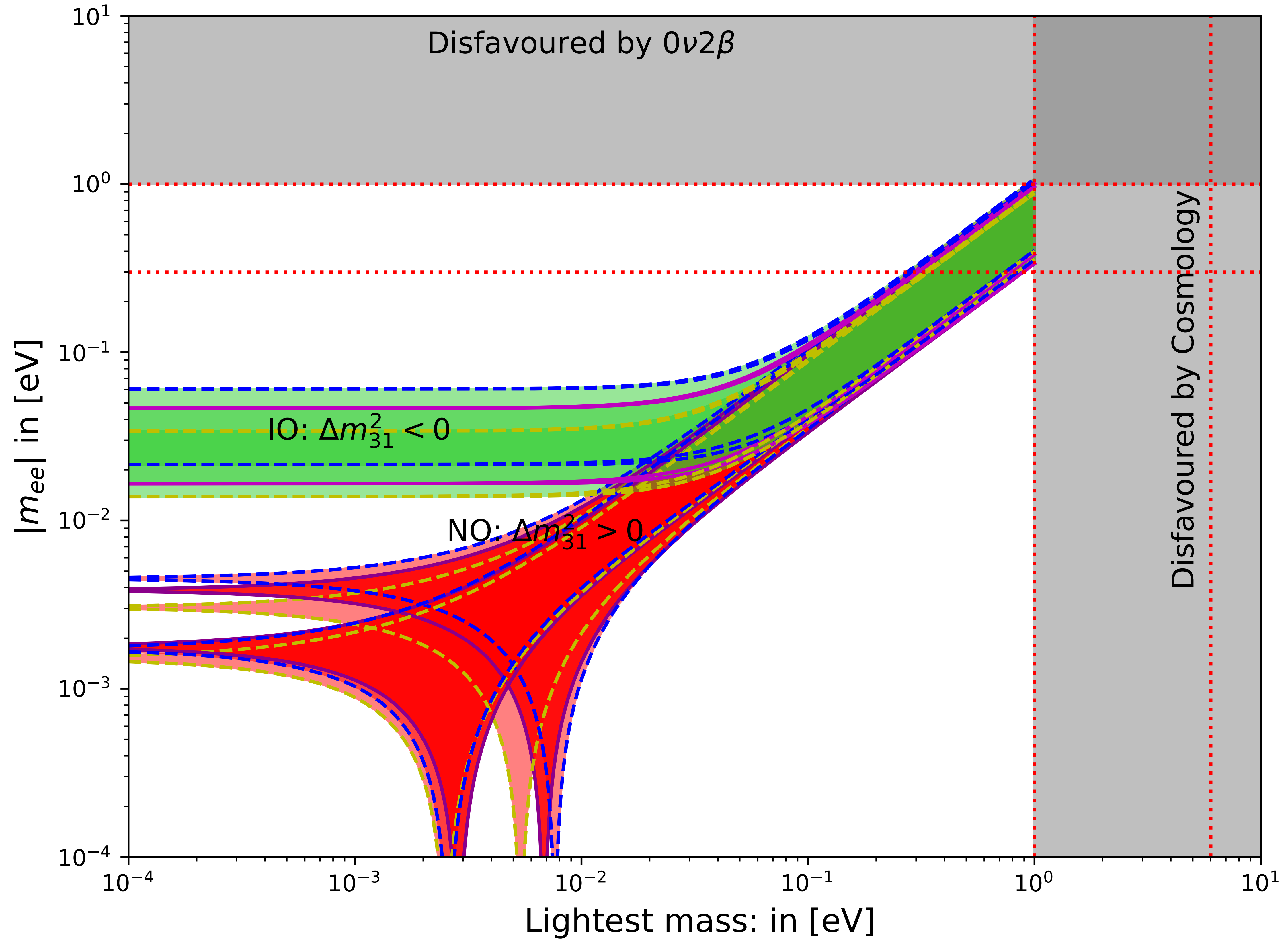
$$m_{ee} = \left(\left|U_{e1}\right|^2 + e^{i\alpha_2}|U_{e2}|^2\right)\sqrt{\Delta m_{31}^2}$$

$$(++), (+-) : m_{ee} \approx \sqrt{\Delta m_{31}^2}$$

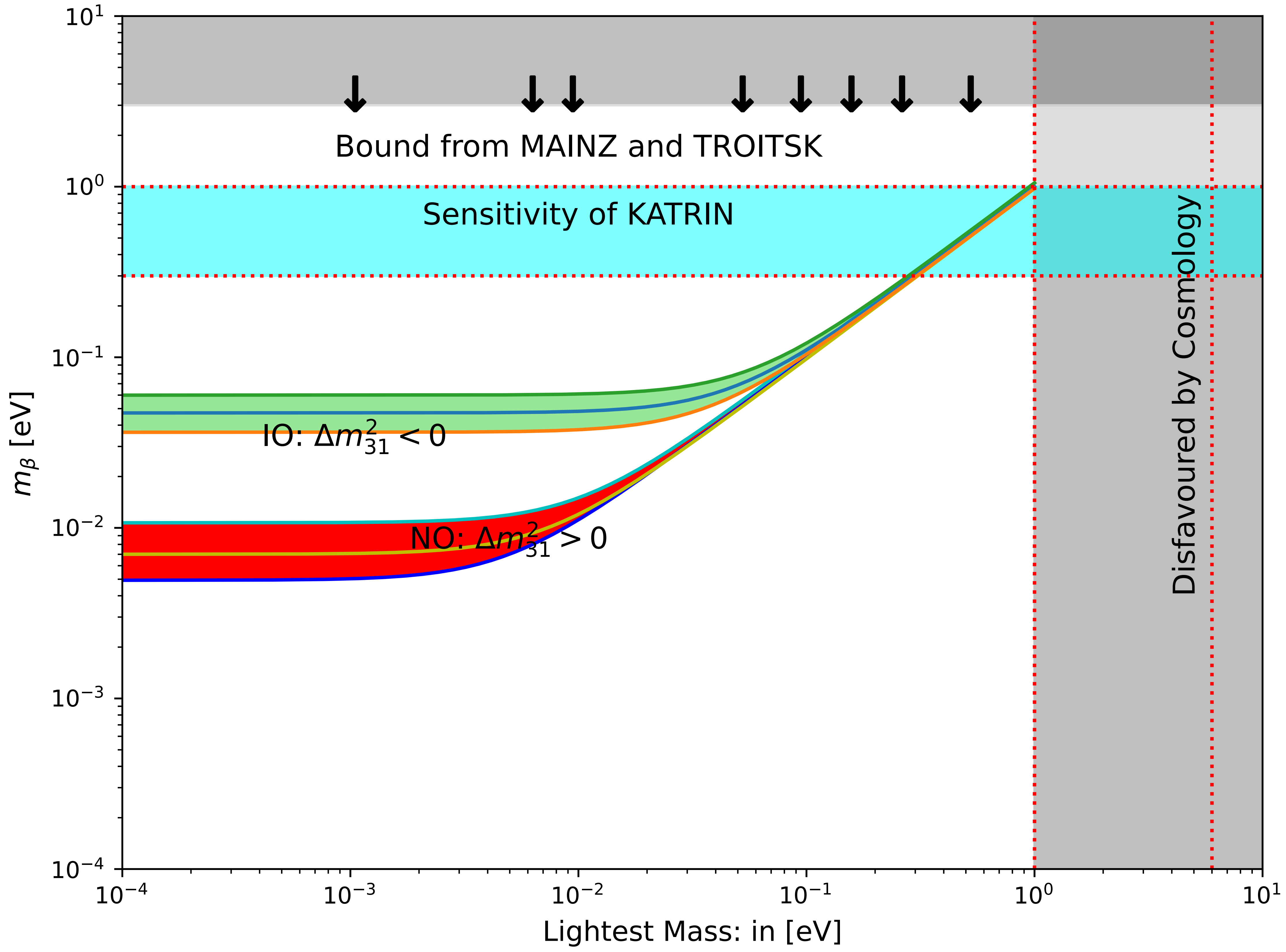
$$(--), (-+) : m_{ee} \approx \left(\left|U_{e1}\right|^2 - |U_{e2}|^2\right)\sqrt{\Delta m_{31}^2} \approx \cos 2\theta_{12}\sqrt{\Delta m_{31}^2}$$



IH ordering



Solid lines are best-fit values and dotted lines correspond to 3σ ranges.



Solid lines are best-fit values and dotted lines correspond to 3σ ranges.

$$9\times 10^{-3} \le |m_{ee}| \le 5\times 10^{-2}\,\mathrm{eV}\,;\; m_3 \le 10^{-2}\,\mathrm{eV}$$

$$7\times 10^{-3} \le |m_{ee}| \le 6\times 10^{-2}\,\mathrm{eV}$$

Neutrino mass in Cosmology

$$m_{cosmo} = \sum_{i=1}^{3} m_i = m_1 + m_2 + m_3$$

$N_{eff} = 3.04$ in Cosmology

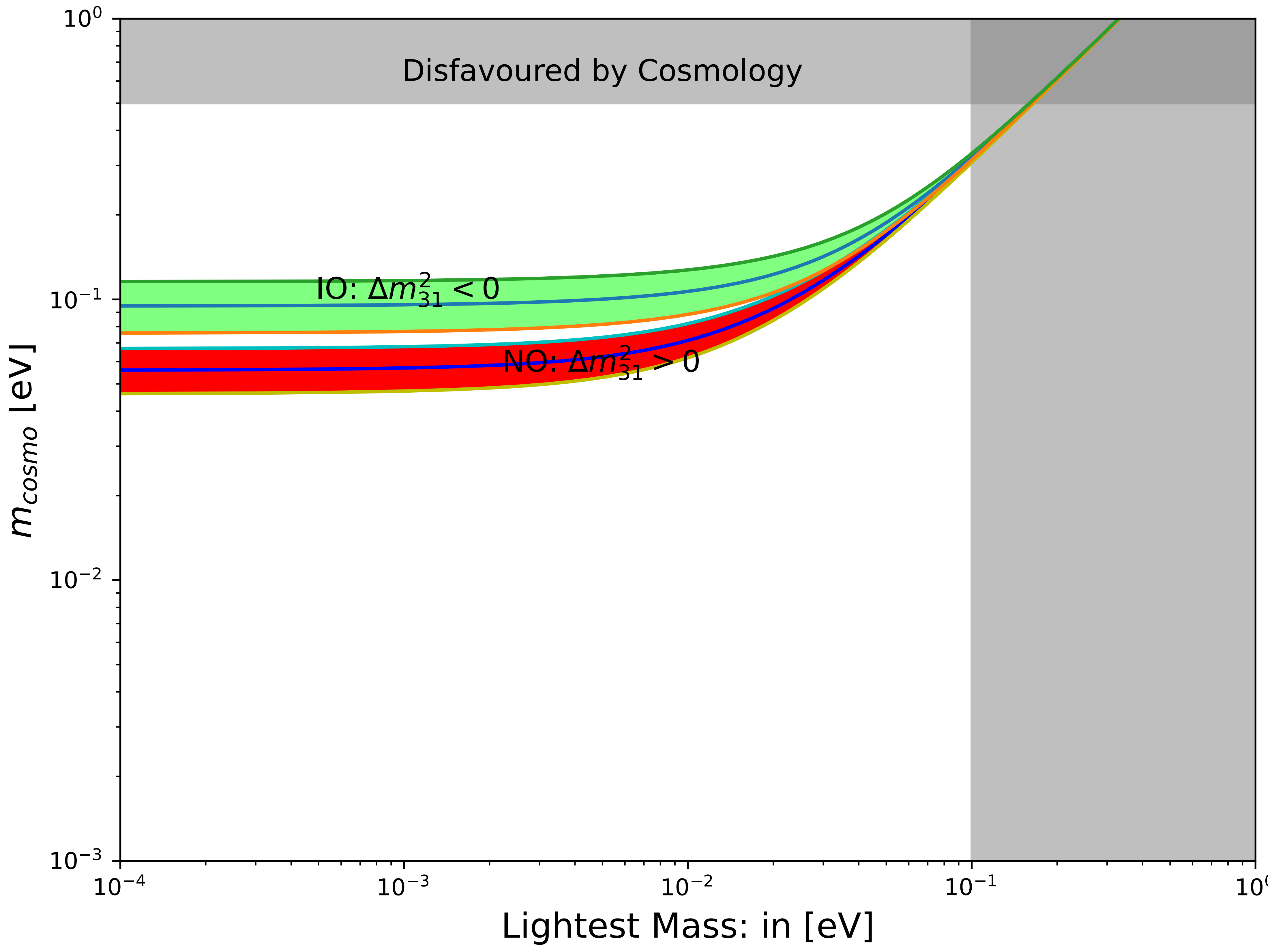
NO: m_1 is the lightest $m_1 < m_2 \ll m_3$

$$\boxed{m_{cosmo} = m_1 + \sqrt{m_1^2 + \Delta m_{21}^2} + \sqrt{m_1^2 + \Delta m_{31}^2}}$$

IO: m_3 is the lightest $m_3 < < m_1 < m_2$

$$\boxed{m_{cosmo} = \sqrt{m_3^2 + \Delta m_{31}^2} + \sqrt{m_3^2 + \Delta m_{31}^2 + \Delta m_{21}^2} + m_3}$$

Mass of neutrino $\sum_{i}^{3} m_\nu \leq 0.20\ \text{eV}$



Middle line are the best fit and above and below lines encloses 3σ range of values.

Summary

- At least two of the three neutrinos are heavy with a mass greater than 8 meV irrespective of normal or inverted mass ordering.
- In the theory of double beta-decay, the effective neutrino mass, we estimate between (9 – 50) meV. This limit can be better in near future, KATRIN next-generation ground-based experiments, CMB-S4 is sensitive to the sum of neutrinos up to $m_\nu \leq 0.23\, eV$.
- The Lepton number violation process in the laboratory would eventually establish the Majorana nature of neutrinos and would deepen our knowledge in understanding the matter-antimatter asymmetry of the universe to some extent.

Thank you