۹۸۲۱۱۴۳۳۲

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تمرین سری چهارم

۱)

A) $x(t)=[e^{-at}\cos(\omega_0 t)]u(t) \quad a>0$

$$e^{-at}\cos(\omega_0 t)u(t)=\frac{1}{2}e^{-at}e^{j\omega_0 t}u(t)+\frac{1}{2}e^{-at}e^{-j\omega_0 t}u(t)$$

$$\Rightarrow X(j\omega)=\frac{1}{2[a-j\omega_0+j\omega]}+\frac{1}{2[a+j\omega_0+j\omega]}$$

*B) $x(t)=\sum_{n=-\infty}^{+\infty} e^{-2|n|t}\,u(t)$

برای این بخش که بیشتر به حالت گسسته مربوط می‌شود ایده خاصی ندارم.

$$e^{-2|n|t}u(t)=\sum_{-\infty}^{0}\frac{1}{2}e^{+2nt}u(t)+\sum_{0}^{\infty}\frac{1}{2}e^{-2nt}u(t)$$

C) $x(t)=\begin{cases}1+\cos(\pi t) & |t|\le 1\\ 0 & |t|>1\end{cases}$

$$=\left(1+\frac{1}{2}e^{j\pi t}+\frac{1}{2}e^{-j\pi t}\right)y(t)\Rightarrow x(t)=y(t)+\frac{1}{2}e^{j\pi t}y(t)+\frac{1}{2}e^{-j\pi t}y(t)$$

$$\Rightarrow X(\omega)=Y(\omega)+\frac{1}{2}Y(\omega-\pi)+\frac{1}{2}Y(\omega+\pi)$$

D) $x(t)=\left[\frac{\sin(\pi t)}{\pi t}\right]\left[\frac{\sin(2\pi(t-1))}{\pi(t-1)}\right]$

① $x_1(t)=\frac{\sin\pi t}{\pi t}\longleftrightarrow X(j\omega)=\begin{cases}1 & -\pi<\omega<\pi\\ 0 & \text{در غیر این صورت}\end{cases}$

② $x_2(t)=\frac{\sin(2\pi(t-1))}{\pi(t-1)}\longleftrightarrow X(j\omega)=\begin{cases}e^{-j\omega} & |\omega|<2\pi\\ 0 & \text{در غیر این صورت}\end{cases}$

$x(t) = ① \cdot ② = \frac{1}{2\pi} X_1(jw) \cdot X_2(jw)$

$$X(jw) = \begin{cases} e^{-jw} & |w|<\pi \\ (\frac{1}{2\pi})(3\pi+w)e^{-jw} & -3\pi<w<-\pi \\ (\frac{1}{2\pi})(3\pi-w)e^{-jw} & \pi<w<3\pi \\ 0 & \text{در غیر اینصورت} \end{cases}$$

۲)

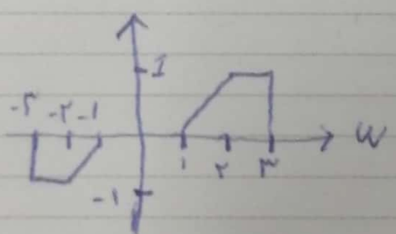
A) $X(jw) = \frac{2\sin(3(w-2\pi))}{w-2\pi}$

$$x(t) = \begin{cases} e^{j2\pi t} & -3<t<3 \\ 0 & \text{در غیر اینصورت} \end{cases}$$

B) $X(jw) = \cos(4w + \frac{\pi}{3})$ تبدیل فوریه عکس

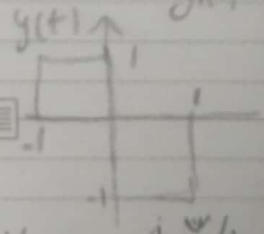
$\Rightarrow x(t) = \frac{1}{2} e^{-j\frac{\pi}{3}} \delta(t-4) + \frac{1}{2} e^{j\frac{\pi}{3}} \delta(t+4)$

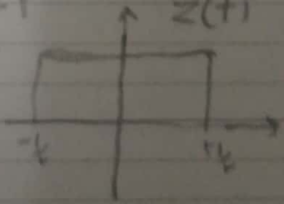
C)



$x(t) = \frac{1}{2\pi} \int X(jw) e^{jwt} dw$

$\Rightarrow x(t) = \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}$





۳) الف)

$y(t) = z(t+\frac{1}{2}) - z(t-\frac{1}{2})$

$Y(w) = e^{jw/2} Z(w) - e^{-jw/2} Z(w) = \text{sinc}(\frac{w}{2\pi})[e^{jw/2} - e^{-jw/2}] = 2j\sin(\frac{w}{2})\,\text{sinc}(\frac{w}{2\pi})$

$Z = \text{sinc}(\frac{w}{2\pi})$ $\quad y(t) = x'(t)$

$Y(w) = jwX(w) \Rightarrow X(w) = \frac{Y(w)}{jw}, \; w \neq 0$

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سه شنبه

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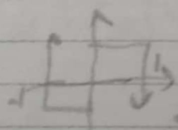
$x(t)=\int_{-\infty}^{t} y(\tau)\,d\tau \Rightarrow X(\omega)=\frac{Y(\omega)}{j\omega}+\pi Y(0)\delta(\omega)$ $\quad Y(\omega)=\frac{2\sin\frac{\omega}{2}}{\omega}$ $\quad \text{sinc}\left(\frac{\omega}{2\pi}\right)$ ۸

$\frac{2\sin\pi\frac{\omega}{2\pi}}{\pi\frac{\omega}{2\pi}}=\text{sinc}\left(\frac{\omega}{2\pi}\right)\Rightarrow X(\omega)=\text{sinc}^{2}\left(\frac{\omega}{2\pi}\right)$

۹ ب)

$Y(\omega)=|X(\omega)|e^{j\angle X(\omega)}=Y(\omega)e^{j3\omega}\Rightarrow x(t)=y(t+3)$



$z(\omega)=Y'(\omega)$

معکوس فوریه: $z(\omega)$ ، $z(\omega)$ ، $f(t)\longleftrightarrow g(\omega)$

خاصیت دوگانی $g(t)\longleftrightarrow 2\pi f(-\omega)$

۱۱

$h(t)=\frac{1}{2\pi}\,\text{sinc}\left(\frac{t}{2\pi}\right)$

$Z(\omega)=H\left(\omega-\frac{1}{2}\right)-H\left(\omega+\frac{1}{2}\right)+\delta(\omega+1)-\delta(\omega-1)$

$z(t)=e^{j\frac{t}{2}}h(t)-e^{-j\frac{t}{2}}h(t)+\frac{1}{2\pi}e^{-jt}-\frac{1}{2\pi}e^{jt}$

$+\infty$ $\quad +\infty$

۱۵ ادامه س ۳ - ب

$$z(t) = \frac{1}{\pi}\,\text{sinc}\left(\frac{t}{2\pi}\right) \times 2j \sin\frac{t}{2} - \frac{1}{2\pi} \times 2j\sin t = \frac{j}{\pi}\,\text{sinc}\left(\frac{t}{2\pi}\right)\sin\frac{t}{2} - \frac{j}{\pi}\sin t$$

شهادت حضرت فاطمه زهرا سلام الله علیها (۱۱ هـ ق) (تعطیل)

y(t) را اکنون می‌یابیم.

$$Y(w) = \int_{-\infty}^{w} z(\eta)\,d\eta$$

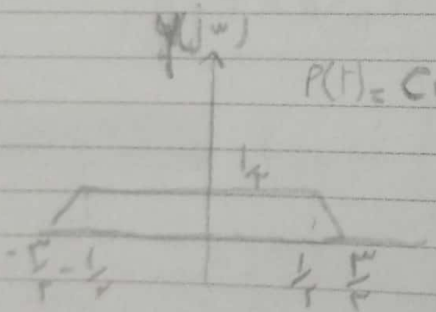
$$\Rightarrow y(t) = \frac{-1}{jt}\,x(t) + \pi x(0)\,\delta(t) = \frac{-1}{jt}\,x(t)$$

۴) الف)

$$p(t) = \sum_{-\infty}^{+\infty} a_k e^{jk\omega_0 t} \longleftrightarrow P(j\omega) = 2\pi \sum_{-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$$

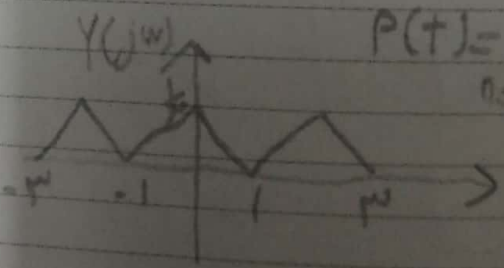
$$Y(j\omega) = \frac{1}{2\pi}\{X(j\omega) * P(j\omega)\} = \sum_{k=-\infty}^{\infty} a_k X(j(\omega - k\omega_0))$$

$p(t) = \cos(\frac{t}{2})$

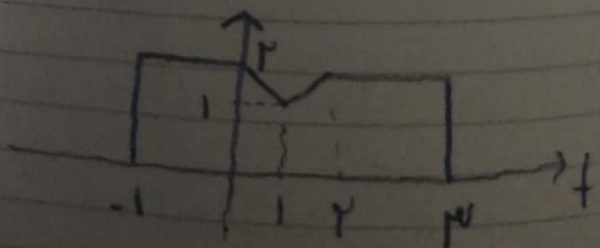


ب)

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - n\pi)$$



۵)



الف) $y(t) = x(t+1)$ حقیقی و زوج

$\measuredangle Y(j\omega) = 0$

$\measuredangle X(j\omega) = -\omega$

$Y(j\omega) = e^{j\omega} X(j\omega)$

ب) $X(j0) = \int_{-\infty}^{+\infty} x(t)\, dt = 7$

c) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega$

$\int_{-\infty}^{\infty} X(j\omega)\, d\omega = ۲\pi x(0) = ۴\pi$

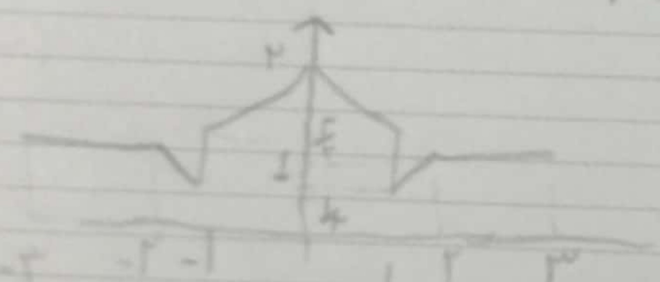
d) $\int_{-\infty}^{+\infty} X(j\omega)\, \frac{۲\sin(\omega)}{\omega}\, e^{j۲\omega}\, d\omega$

$Y(j\omega) = \frac{۲\sin\omega}{\omega} e^{۲j\omega}$

$\Rightarrow y(t) = \begin{cases} 1 & -۳ < t < -1 \\ 0 & \text{در غیر اینصورت} \end{cases}$

e) $\int_{-\infty}^{+\infty} |X(j\omega)|^{۲}\, d\omega$

$Re\{X(j\omega)\} \to Ev\{x(t)\} = \{\frac{[x(t) + x(-t)]}{۲}\}$



الف) $X(j\omega_1, j\omega_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(t_1, t_2)\, e^{-j(\omega_1 t_1 + \omega_2 t_2)}\, dt_1\, dt_2$ (۴

$= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} x(t_1, t_2) e^{-j\omega_1 t_1} dt_1 \right] e^{-j\omega_2 t_2} dt_2$

$= \int_{-\infty}^{+\infty} X(j\omega_1, t_2)\, e^{-j\omega_2 t_2}\, dt_2 = F_{\omega_2}\{F_{\omega_1}\{x\{t_1, t_2\}\}\}$

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ب) $\mathcal{F}^{-1}_{\omega_1}(\mathcal{F}^{-1}_{\omega_2}(X(j\omega_1, j\omega_2))) = \mathcal{F}^{-1}_{\omega_1}(\mathcal{F}^{-1}_{\omega_2}(\mathcal{F}_{\omega_2}(\mathcal{F}_{\omega_1}(x(t_1, t_2)))))$

$$= \mathcal{F}^{-1}_{\omega_1}(\mathcal{F}_{\omega_1}(x(t_1, t_2))) = x(t_1, t_2) \Rightarrow x(t_1, t_2) = \mathcal{F}^{-1}_{\omega_1}(\mathcal{F}^{-1}_{\omega_2}(X(j\omega_1, j\omega_2)))$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega_1, j\omega_2)e^{j\omega_2 t_2}\, d\omega_2\right]e^{j\omega_1 t_1}\, d\omega_1$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} X(j\omega_1, j\omega_2)e^{j(\omega_1 t_1 + \omega_2 t_2)}\, d\omega_1\, d\omega_2$$