۹۸۴۱۱۴۳۲

محمد عرفان زارع زردینی

سری دوم تمرین تئوری درس سیگنال و سیستم

€ $ طلا نفت

سؤال 1

A) $h[n] = \left(\frac{1}{5}\right)^n u[n]$

بررسی علّی بودن: شرط علی بودن ($h[n]=0$ for $n<0$)

$h[n]=0$ for $n<0 \Rightarrow$ ✓ علّی

بررسی پایداری: شرط پایداری ($\sum_{k=-\infty}^{\infty}|h[k]|<\infty$)

$\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n = \frac{1\left(1-\left(\frac{1}{5}\right)^n\right)}{\frac{4}{5}} = \frac{5}{4} < \infty \Rightarrow$ ✓ پایداری

B) $h[n] = (0.8)^n u[n+2]$

بررسی علی بودن:

$h[n] \neq 0$ for some $n<0 \Rightarrow$ ✗ علّی

بررسی پایداری:

$\sum_{n=0}^{\infty}(0.8)^n = \frac{1\left(1-\left(\frac{4}{5}\right)^n\right)}{\frac{1}{5}} = 5 < \infty \Rightarrow$ ✓ پایداری

روز جمهوری اسلامی ایران (تعطیل)

C) $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$

بررسی علی بودن:

$h[n]=0$ for $n<0 \Rightarrow$ ✓ علّی

بررسی پایداری

$\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n + (1.01)^n = \frac{1\left(1-\left(-\frac{1}{2}\right)^n\right)}{\frac{3}{2} = 1-\left(-\frac{1}{2}\right)} + \frac{1\left(1-(1.01)^n\right)}{-0.01}$

$= \frac{2}{3} + 100\times 1.01 - 100 < \infty \Rightarrow$ پایداره

روز طبیعت (تعطیل)

۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹ ۱۰ ۱۱ ۱۲ ۱۳ ۱۴ ۱۵ ۱۶ ۱۷ ۱۸ ۱۹ ۲۰ ۲۱ ۲۲ ۲۳ ۲۴ ۲۵ ۲۶ ۲۷ ۲۸ ۲۹ ۳۰ ۳۱

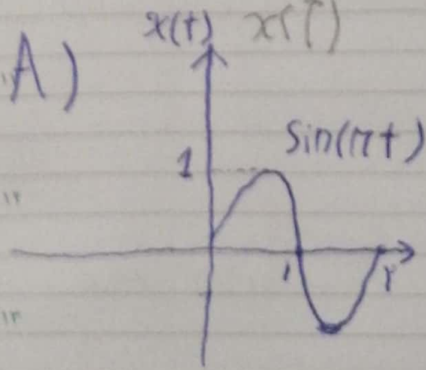
D) $h(t)=e^{-4|t|}$

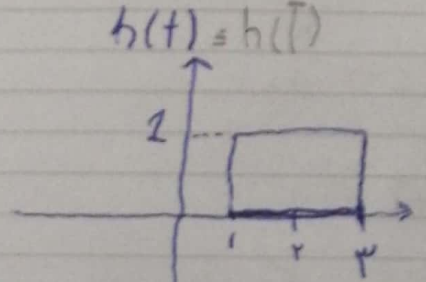
$h(t)\neq 0$ for $t<0$

علّی نیست چون

$\int_{-\infty}^{+\infty}|e^{-4|t|}|\,dt = 2\int_{0}^{\infty}e^{-4t}\,dt = \frac{-1}{4}(e^{-\infty}-e^{0}) = \frac{2}{4}<\infty$ پایدار است چون

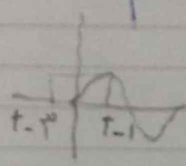
A)

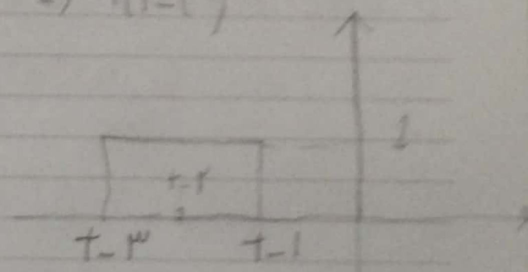




سوال ۲

$\Rightarrow h(t-\tau)$





$t<0 \Rightarrow y(t)=0$

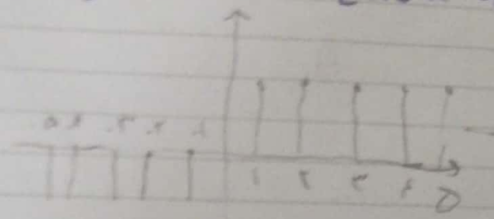
$t<1 \Rightarrow y(t)=0$

$1\leq t\leq 2 \Rightarrow y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)\,d\tau = \int_{0}^{t-1}\sin(\pi\tau)\times 1 = (-\cos\pi(t-1)+1)\times\frac{1}{\pi}$

$2\leq t\leq 3 \Rightarrow$ $1\leq t\leq 2$ مشابه $\Rightarrow = (1-\cos\pi(t-1))\times\frac{1}{\pi}$

$3\leq t\leq 5$ $y(t)=\int_{t-3}^{2}\sin\pi t\,d\tau = (\cos(\pi(t-1))-1)\frac{1}{\pi}$

B) $x[n] = u[n] - u[-n]$ $\qquad h[n] = \begin{cases} (\frac{1}{۲})^n & n \geq 0 \\ ۴^n & n < 0 \end{cases}$



$y[n] = \sum_{k=-\infty}^{+\infty} (u[k] - u[-k])\, h[n-k] - \sum_{k=-\infty}^{-1} h[n-k] =$

$n-k \geq 0 \Rightarrow k \leq n \rightarrow h[n-k] = (\frac{1}{۲})^{n-k}$

$n-k < 0 \Rightarrow k > n \rightarrow h[n-k] = ۴^{n-k}$

$\sum_{k=0}^{n} (\frac{1}{۲})^{n-k} + \sum_{k=n+1}^{+\infty} ۴^{n-k} - \sum_{k=n}^{-1} ۴^{n-k} - \sum_{k=-\infty}^{n-1} (\frac{1}{۲})^{n-k}$

$\sum_{k=0}^{n} (\frac{1}{۲})^{n-k} = (\frac{1}{۲})^n \sum_{k=0}^{n} (۲)^k = (\frac{1}{۲})^n \times \frac{1-(۲)^{n+1}}{1-۲} = (\frac{1}{۲})^n (۲^{n+1} - 1) = ۲ - (\frac{1}{۲})^n$

$\sum_{k=n+1}^{+\infty} ۴^{n-k} = \sum_{k=0}^{\infty} ۴^{n-k} - \sum_{k=0}^{n} ۴^{n-k} = ۴^n \times \frac{1}{1-\frac{1}{۴}} - ۴^n \sum_{k=0}^{n} (\frac{1}{۴})^k = \frac{۴^{n+1}}{۳} - ۴^n \times \frac{1-\frac{1}{۴}^{n+1}}{1-\frac{1}{۴}}$

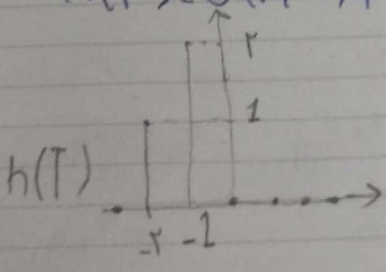
$= \frac{۴^{n+1}}{۳} (1 - 1 + (\frac{1}{۴})^{n+1}) = \frac{1}{۳}$

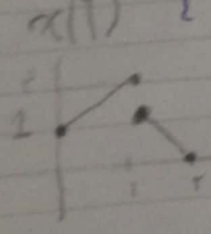
ادامه
عقب تر

C)

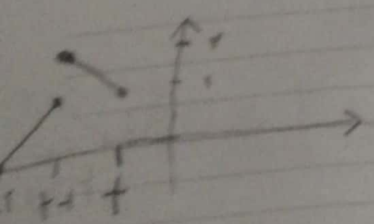
$x(t) = \begin{cases} t+1 & 0 \leq t \leq 1 \\ ۲-t & 1 \leq t \leq ۲ \\ 0 & O.W \end{cases}$

$h(t) = \delta(t+۲) + ۲\delta(t+1)$





$y(t) = \int x(T) h(t-T)$



$t < -۲ \Rightarrow y(t) = 0$

$-۲ \leq t \leq -1 \rightarrow y(t) = \int_{-۲}^{-1} ۲(T+1)\, dT = -\frac{1}{۲}$

$-1 < t \leq 0 \Rightarrow \int_{-1}^{0} ۲(T+1)\, dT + \int_{-1}^{0} ۲(1-T) dT = 1 + \frac{۵}{۲}$

$0 < t \leq 1 \Rightarrow \int_{0}^{1} ۲(1-T)\, dT = ۳$

D) $x[n] = 3^n u[-n-1] + (\frac{1}{3})^n u[n]$ $\qquad h[n] = (\frac{1}{4})^n u[n+3]$

$$z[n] = \sum_{-\infty} \left(3^k u[-k-1] + (\tfrac{1}{3})^k u[k]\right) (\tfrac{1}{4})^{n-k} u[n+3-k]$$

$$= (\tfrac{1}{4})^n \sum_{-\infty}^{\infty} (12)^k \underbrace{u[-k-1]}_{-k-1 \geq 0 \Rightarrow k \leq -1} \cdot \underbrace{u[n+3-k]}_{k \leq n+3} + \sum_{k=-\infty}^{+\infty} (\tfrac{4}{3})^k \underbrace{u[k]}_{k \geq 0} \cdot \underbrace{u[n+3-k]}_{k \leq n+3}$$

$\Rightarrow 0 \leq k \leq n+3 \rightarrow n \geq -3$

$\rightarrow$ if $n+3 \geq -1 \Rightarrow n \geq -4 \Rightarrow \sum_{-\infty}^{-1} (12)^k = \sum_{k=0}^{\infty} (\frac{1}{12})^k - 1 = \frac{12}{11} - 1 = \frac{1}{11}$

$\rightarrow$ if $n+3 \leq -1 \rightarrow n \leq -4 \rightarrow \sum_{-\infty}^{n+3} (12)^k = \sum_{k=0}^{+\infty} (\frac{1}{12})^k + \sum_{k=0}^{n+3} (12)^k - 1$

$$= \frac{1}{11} - \frac{1-(12)^{n+4}}{11} = \frac{(12)^{n+4}}{11}$$

$$\sum_{k=0}^{n+3} (\tfrac{4}{3})^k = \frac{1-(\frac{4}{3})^{n+4}}{-\frac{1}{3}} = 3\left((\tfrac{4}{3})^{n+4} - 1\right) \rightarrow z[n] = \frac{1}{11} + 3\left((\tfrac{4}{3})^{n+4} - 1\right)$$

$n \geq -3$

۳) از روی عبارت (۱) با $y(t)$ به این نتیجه می‌رسیم که برای قسمت الف، از طرفین معادله دیفرانسیل انتگرال می‌گیریم

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t) + b_1 \frac{dx(t)}{dt} + b_2 \frac{d^2 x(t)}{dt^2}$$

$$\xrightarrow{\int} a_2 \int_{-\infty}^{t} \frac{d^2 y(\tau)}{d\tau^2} d\tau + a_1 \int_{-\infty}^{t} \frac{dy(\tau)}{d\tau} d\tau + a_0 \int y(\tau) d\tau$$

$$= b_0 \int x(\tau) d\tau + b_1 \int \frac{dx(\tau)}{d\tau} d\tau + b_2 \int \frac{d^2 x(\tau)}{d\tau^2} d\tau$$

$$\longrightarrow a_2 \left(\frac{dy(t)}{dt} - \frac{dy}{dt}(-\infty)\right) + a_1 \left(y(t) - y(-\infty)\right) + a_0 \int_{-\infty}^{t} y(\tau) d\tau$$

$$= b_0 \int_{-\infty}^{t} x(\tau) d\tau + b_1 \left(x(\tau) - x(-\infty)\right) + b_2 \left(\frac{dx(t)}{dt} - \frac{dx}{dt}(-\infty)\right)$$

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Causality ≡ intial at rest $\rightarrow \begin{cases} \dfrac{d^n x(-\infty)}{dt^n} = 0 \\ \qquad n = 0, 1, \dots \\ \dfrac{d^n y(-\infty)}{dt^n} = 0 \end{cases}$

= علیت

$$\rightarrow a_2 \frac{dy(t)}{dt} + a_1 y(t) + a_0 \int_{-\infty}^{t} y(\tau)\,d\tau = b_0 \int_{-\infty}^{t} x(\tau)\,d\tau + b_1 x(t) + b_2 \frac{dx(t)}{dt}$$

$$\xrightarrow{\int_{-\infty}^{t}} a_2 y(t) + a_1 \underbrace{\int_{-\infty}^{t} y(\tau)\,d\tau}_{I_1} + a_0 \underbrace{\int_{-\infty}^{t}\left(\int_{-\infty}^{\alpha} y(\tau)\,d\tau\right) d\alpha}_{I_2} = b_0 \underbrace{\int_{-\infty}^{t}\left(\int_{-\infty}^{\alpha} x(\tau)\,d\tau\right) d\alpha}_{I_3}$$

$$+ \underbrace{b_1 \int_{-\infty}^{t} x(\tau)\,d\tau + b_2}_{I_4} x(t)$$

$$\Rightarrow y(t) = -\frac{a_1}{a_2} I_1 - \frac{a_0}{a_2} I_2 + \frac{b_0}{a_2} I_3 + \frac{b_1}{a_2} I_4 + \frac{b_2}{a_2} x(t)$$

بخش ب)

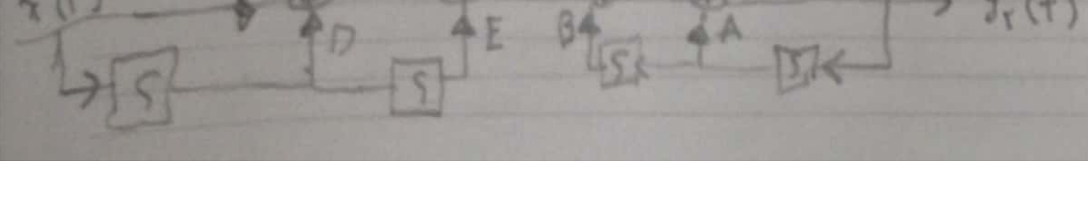
$$y(t) = A I_1 + B I_2 + C x(t) + D I_4 + E I_3$$

$$A = \frac{-a_1}{a_2}, \quad B = \frac{-a_0}{a_2}, \quad C = \frac{b_2}{a_2}, \quad D = \frac{b_1}{a_2}, \quad E = \frac{b_0}{a_2}$$

بخش ج) اتصال سری ← خروجی اول = ورودی دوم ← $x_2(t) = y_1(t)$

جایگذاری S_1 در S_2 → $y_2(t) = A \int_{-\infty}^{t} y_2(t)\,dt + B \int_{-\infty}^{t} \int_{-\infty}^{t} y_2(\alpha)\,d\alpha\,dt + C x_1(t)$

S_1: $+ D \int^{t} x_1(t)\,d\tau + E \int \left(\int_{-\infty}^{t} x_1(\alpha)\,d\alpha \right) d\tau$



$$y[n]-\frac{1}{2}y[n-1]=x[n],\quad x[n]=k\cos(\Omega_0 n)u[n] \quad (4$$

initial at rest $\rightarrow y[n]=0$ for $n<0$

معادله همگن: $y_h[n]-\frac{1}{2}y_h[n-1]=0 \Rightarrow y_h[n]=\frac{1}{2}y[n-1]$

$$K'(\tfrac{1}{2})^n=K'\tfrac{1}{2}\times(\tfrac{1}{2})^{n-1} \Rightarrow y_h[n]=K'(\tfrac{1}{2})^n u[n]$$

جواب خصوصی: $y_p[n]-\frac{1}{2}y_p[n-1]=k\cos(\Omega_0 n)u[n]$

$$y_p[n]=[A\cos\Omega_0 n+B\sin\Omega_0[n]]u[n]$$

$$\rightarrow A\cos\Omega_0 n\,u[n]+B\sin\Omega_0 n\,u[n]-\frac{1}{2}A\cos(\Omega_0 n-\Omega_0)u[n-1]$$

$$-\frac{1}{2}B\sin(\Omega_0 n-\Omega_0)u[n-1]=k\cos(\Omega_0 n)u[n]$$

$$\rightarrow A\cos(\Omega_0 n)+B\sin(\Omega_0 n)-\frac{1}{2}A\cos(\Omega_0 n)\cos(\Omega_0)-\frac{1}{2}A\sin(\Omega_0 n)\sin(\Omega_0)$$

$$-\frac{1}{2}B\sin(\Omega_0 n)\cos(\Omega_0)+\frac{1}{2}B\cos(\Omega_0 n)\sin\Omega_0=k\cos(\Omega_0 n)$$

$$\rightarrow \cos(\Omega_0 n)\left[A-\frac{1}{2}A\cos\Omega_0+\frac{1}{2}B\sin\Omega_0\right]+\underbrace{\sin(\Omega_0 n)[\ldots]}=k\cos(\Omega_0 n)$$

همراستای منطق $=0 \rightarrow \Omega_0=n$

$$\rightarrow A-\frac{1}{2}A+0=k \Rightarrow A=2k$$

$$\rightarrow y_p[n]=2k\cos(\Omega_0 n)u[n]$$

$$\rightarrow y[n]=K'(\tfrac{1}{2})^n u[n]+2k\cos(\Omega_0 n)u[n] \xrightarrow{n=0} y[0]=K'+2k$$

شرایط اولیه $\Rightarrow y[0]=k \rightarrow K'+2k=k \Rightarrow K'=-k$

$$y=-k(\tfrac{1}{2})^n u[n]+2k\cos(\Omega_0 n)u[n]$$

(B-۲ ادامه

$$\sum_{k=n}^{-1} ۴^{n-k} = ۴^{n} \sum_{k=n}^{-1} ۴^{-k} = ۴^{n} \sum_{k=1}^{-n} ۴^{k} = ۴^{n}\left(\sum_{k=0}^{-n} ۴^{k} - 1\right) = ۴^{n}\left(\frac{1-(۴)^{-n+1}}{1-۴}\right)$$

$$= \frac{۴^{n}}{۳}\left((۴)^{-n+1}-1\right)$$

$$\sum_{k=-\infty}^{n-1} \left(\tfrac{1}{۲}\right)^{n-k} = \sum_{k=-\infty}^{0} \left(\tfrac{1}{۲}\right)^{n-k} - \sum_{k=n}^{0} \left(\tfrac{1}{۲}\right)^{n-k} = \left(\tfrac{1}{۲}\right)^{n}\left[\sum_{k=0}^{\infty} \left(\tfrac{1}{۲}\right)^{k} - \sum_{k=0}^{n} \left(\tfrac{1}{۲}\right)^{k}\right]$$

$$= \left(\tfrac{1}{۲}\right)^{n}\left(\frac{1}{1-\frac{1}{۲}} - \frac{1-\left(\frac{1}{۲}\right)^{n+1}}{1-\frac{1}{۲}}\right) = \left(\tfrac{1}{۲}\right)^{۲n}$$

$$F[n] = ۲ - \left(\tfrac{1}{۲}\right)^{n} + \tfrac{1}{۳} + \frac{۴^{(n)}}{۳}\left((۴)^{-n+1}-1\right) + \left(\tfrac{1}{۲}\right)^{۲n}$$