Revisiting the Mass Model Assumed by $\mathbf{E = mc^2}$

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***Abstract*—Universal Specificity quantifies the causal relationship between changes in total specific energy and time dilation. This law was the result of a path taken to induce the cause of kinetic time dilation and relate this cause to gravitational time dilation. These two sources of time dilation, are inductively proven in this paper to be causally related. It is demonstrated that changes in specific energy cause changes in time dilation, and environmental changes in time dilation, from what is termed a time derivative gradient, causes changes in specific energy. In other words, kinetic time dilation and gravitational time dilation are causal reciprocals of each other—the former caused by specific work done and the latter causing specific work to be done. From this causal reciprocal, a change in total energy equation was derived, which is different from Einstein's total energy equation, $\mathbf{E = mc^2}$. $\mathbf{E = mc^2}$ measures the total energy of a body, its internal energy content, and its kinetic energy. The change in total energy derived in this paper, on the other hand, ignores internal energy content, and measures how a change in an object's potential and kinetic energy are causally related to the time dilation it experiences. These two energy equations are similar enough that an investigation in how they are related was pursued, which lead to a shocking discovery.**

1. Introduction

Einstein, in his original proof for his $E = mc^2$ total energy model, made a tacit assumption about the relativistic mass model when he assumed a kinetic energy model. The evidence for the genius of Einstein is numerous, and in this particular case he devised a thought experiment to tease out a means to measure the internal energy of a stationary object. Everyone at that point knew stationary objects were made of particles and these particles had some energy, but the means to measure it had remained elusive until Einstein, using previously established principles and relationships from electromagnetism and relativity, found a way [8].

In this thought experiment, Einstein considered an object that emits energy, in the form of radiation in two equal but opposite directions (so its velocity does not change). Then he considered this same object with the same emission, but from reference frame that is moving relative to it along the axes of emission, as shown in Figure 1.

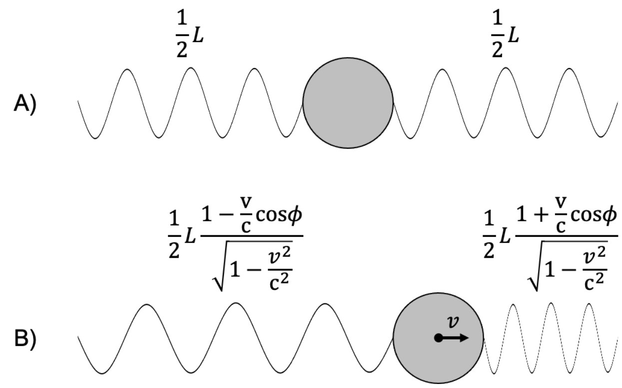

Figure 1. **A) Object's inertial reference frame; B) Inertial reference frame with relative motion.**

Let E_0 and E_1 be the total energy of the object before and after radiation emission, respectively, as measured from the object's inertial reference frame. Let H_0 and H_1 be the total energy of the object before an after radiation emission, respectively, as measured from the inertial reference frame with relative motion. The radiated energy measured from the objects stationary perspective is shown in Equation (1a), while the energy measured from the reference frame that is moving relative to the object is shown in Equation (1b) [8].

$$E_0 - E_1 = L \tag{1a}$$

$$H_0 - H_1 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1b}$$

Einstein relates the two reference frames before and after the emission as $H - E$ to find a difference in total energy between the two reference frames, and finds its relation to an assumed kinetic energy model. He states, "Thus it is clear that the difference $H - E$ can differ from the kinetic energy K of the body, with respect to the other [reference frame with relative motion], only by an additive constant C." This model is shown in in Equation (2) [8].

$$H_0 - E_0 = K_0 + C \tag{2a}$$

$$H_1 - E_1 = K_1 + C \tag{2b}$$

Where did this kinetic energy model's relation to total energy come from, and upon what does it depend? The answer to the first part of the question is best expressed by Feynman in his famous lectures. In short, Feynman derives the total energy relation to kinetic energy from relativistic mass as show in Equation (3) [7].

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3a}$$

$$m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{3b}$$

$$m = m_0 \left(\sum_{i=0}^{\infty} (-1)^i \binom{-1/2}{i} \left(\frac{v^2}{c^2}\right)^i\right) \tag{3c}$$

$$m = m_0 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots\right) \tag{3d}$$

$$m \approx m_0 + \frac{1}{2} m_0 \frac{v^2}{c^2} \tag{3e}$$

$$mc^2 \approx m_0 c^2 + \frac{1}{2} m_0 v^2 \blacksquare \tag{3f}$$

How this result relates to Equation (2) is as follows:

- $H - E = mc^2$
- $C = m_0 c^2$
- $K = \frac{1}{2} mc^2$ for small v
- $K = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) m_0 c^2$ is the full kinetic energy model

That is where the kinetic energy model comes from, but it depends on m_0 being invariant, while m changes as the kinetic energy of an object increases relative to some chosen inertial frame. To see this dependency, one can derive the kinetic energy model from Newtonian first principles relating kinetic energy to force applied over some distance, as shown in Equation (4).

$$\frac{m}{\gamma} = m_0 = \text{invariant} \tag{4a}$$

$$\Delta K = \int F(s)ds \tag{4b}$$

$$\Delta K = \int \frac{dp}{dt} ds \tag{4c}$$

$$\Delta K = \int v d(mv) \tag{4d}$$

$$\Delta K = \int v d(\gamma m_0 v) \tag{4e}$$

$$\Delta K = m_0 \int v d(\gamma v) \tag{4f}$$

$$\Delta K = m_0(\gamma - 1)c^2 = (\gamma - 1) m_0 c^2 \blacksquare \tag{4g}$$

Where :

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4h}$$

If, on the other hand, it turns out the m is invariant while m_0 changes, then the relativistic kinetic energy model changes, and so does its relation to total energy. This alternative kinetic energy model is derived in Equation (5).

$$m = \gamma m_0 = \text{invariant} \tag{5a}$$

$$\Delta K = \int F(s)ds \tag{5b}$$

$$\Delta K = \int \frac{dp}{dt} ds \tag{5c}$$

$$\Delta K = \int v d(mv) \tag{5d}$$

$$\Delta K = m \int v d(v) \tag{5e}$$

$$\Delta K = \frac{1}{2} mv^2 = \gamma \frac{1}{2} m_0 v^2 \blacksquare \tag{5f}$$

How this kinetic energy model relates to total energy is shown in Equation (6).

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0 \tag{6a}$$

$$m^2 \left(1 - \frac{v^2}{c^2}\right) = m_0 \tag{6b}$$

$$m^2 = m_0^2 + m^2 \frac{v^2}{c^2} \tag{6c}$$

$$mc^2 = \frac{1}{\gamma} m_0 c^2 + mv^2 \tag{6d}$$

$$mc^2 = \frac{1}{\gamma^2} mc^2 + 2\frac{1}{2} mv^2 \tag{6e}$$

$$\frac{1}{2} mc^2 = \frac{1}{\gamma^2}\frac{1}{2} mc^2 + \frac{1}{2} mv^2 \blacksquare \tag{6f}$$

Therefore, the total energy difference between the two reference frames is $H - E = K + I$, where K is the kinetic energy portion of the total energy, and I is the internal potential energy that can be converted into kinetic energy. Equation (6) relates to this new model in the following way:

- $H - E = \frac{1}{2} mc^2$
- $K = \frac{1}{2} mv^2$
- $I = \frac{1}{\gamma^2}\frac{1}{2} mc^2$

The internal potential energy of an object diminishes as it gains kinetic energy because this internal potential energy is being converted into kinetic energy, all the while total energy is conserved. Once the object reaches the speed of light, all of its internal potential energy has been converted to kinetic energy, and no more potential remains to gain kinetic energy.

The total energy model rests ultimately on the mass model, as the mass model implies a kinetic energy model, and the kinetic energy model implies a total energy model. The question remains: which mass model is correct?

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