

Universal Specificity Investigation 8: Determining Which Frame is Universally Stationary

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Prior investigations into universal specificity found that time properly conceptualized is the interval over which change occurs, and is not a property of the Universe apart from physical changes to things in the Universe; which led to the proper conception of time dilation as a common change in the interval over which change occurs to things. In addition, it was found that a universally stationary frame (USF) must exist; which led to discovering the cause of total time dilation, shown in Equation (1); which ultimately led to a total specific energy model complete with specific internal energy, specific kinetic energy and specific potential energy terms, shown in Equation (2).

$$\begin{aligned}\frac{dt'}{dt} &= \frac{c_0}{c} = \sqrt{1 - \frac{w}{e_T}} \\ &= \sqrt{1 - \frac{\Delta e_t}{e_T}} = \sqrt{1 - \frac{\Delta e_K + \Delta e_P}{e_T}}\end{aligned}\quad (1)$$

$$\begin{aligned}e_T &= e_I + \Delta e_K + \Delta e_P \\ \frac{1}{2}c^2 &= \frac{1}{2}c_0^2 + \frac{1}{2}v^2 + \int_{\infty}^r g(r)dr\end{aligned}\quad (2)$$

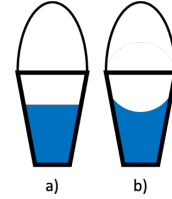
dt' is the time rate of change measured by a clock undergoing time dilation; dt represents the time rate of change measured by an identical clock in the USF infinitely away from gravitational sources; c_0 is the average effective speed of light in the objects reference frame; c is the speed of light in the USF in a vacuum not under any gravity potential; Δe_t is the object's change in specific total energy in the USF; Δe_K is the object's change in specific kinetic energy in the USF; Δe_P is the object's change in specific potential energy; w is the specific work done to the object in the USF; e_T is total specific energy of an object, $\frac{1}{2}c^2$; and e_I is the specific internal energy of an object, $\frac{1}{2}c_0^2$. The ratio of time derivatives is termed *inertial time differential* (ITD), which remains constant for any object until specific work, w , is done.

The focus of the next investigation is to circle back to one of the original questions in this series: is there a way to objectively determine which inertial reference frame is the USF?

1. UNIVERSAL INERTIAL FRAME

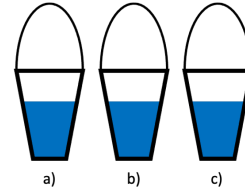
A *universally stationary frame* (USF) is the only inertial reference frame that is “still” (no velocity) in the universe,

which is defined to be the frame where the speed of light (in the absence of gravity) is the same in all directions—commonly referred to as the preferred frame. Such a frame exists for rotation, where it is easy to measure which frame has no angular velocity. A simple water in a bucket experiment can tell you if the bucket reference frame is rotating or not. If the bucket is rotating, then the surface of the water will create a bowl shape; otherwise, if it is not rotating, then the surface will be flat, as shown in Figure 1.



**Figure 1. a) Non-rotating bucket of water.
b) Rotating bucket of water.**

A similar experiment cannot determine if a reference frame has velocity relative to the USF, as shown in Figure 2.



**Figure 2. a) Universally stationary.
b) Velocity is 0.5c.
c) Velocity is 0.9c.**

Michelson and Morely tried (and failed) to determine the relative motion of earth with respect to the USF by measuring the difference in the velocity of light from the same source at different points in earth's orbit, but instead demonstrated the interesting fact that the speed of light measured in any inertial frame is constant [1], or at least the average relative speed of light appears to be constant [2] (also see investigation 2). Something is special about translational velocity, as compared to rotational velocity, where any inertial frame at any translational velocity appears to be stationary, while frames with rotational velocity are immediately noticeable.

Several experiments have been devised [3] that might measure motion relative to the USF, given the correctness of different theories [2]. Our aim in this investigation, given the correctness of specificity, is to devise another experiment that can objectively measure motion relative to the USF.

To begin, you might have noticed a peculiarity about kinetic

time dilation, which is that despite the base unit (or just unit for short) changes to miscalibrated instruments, all pair-wise reference frames seem to agree on their respective relative velocities between each other. This is our first clue as to why translational velocity is special.

2. A SURVEY OF UNIT CHANGES

First, it is important to know that any two observers always agree on their relative speed [4]. We know that the units of measurement change for space and time when kinetic work is done, manifesting in miscalibrated instruments, but why does relative velocity remain unaffected (as in perfectly calibrated)? Velocity, being a ratio of a change in distance to a change in time gives us Equation (3).

$$|v_1| = |-v_2| \quad (3a)$$

$$\frac{dx_1}{dt_1} = \frac{dx_2}{dt_2} \quad (3b)$$

$$\frac{dt_2}{dt_1} = \frac{dx_1}{dx_2} \quad (3c)$$

Mind you, $\frac{dx_1}{dx_2}$ is not length contraction. If we are to use a laser to measure a remote object's velocity, then dx_1 is the measure of distance light appears to travel to the second frame and back (assuming that the speed of light is constant and c since we do not yet know which frame is the USF). Likewise, dx_2 is the measure of distance light appears to travel to the first frame and back. This is shown in Figure 3.

In the example in Figure 3, suppose the blue object is estimating the range velocity of the red object using laser returns, so two returns are needed to estimate the range rate. $t_1 - t_0$ is the round trip elapse time, and assuming the one-way speed of the laser pulse is c for there and back, the ping1 occurred at a distance, s_1 given by:

$$s_1 = (t_1 - t_0)c \quad (4)$$

Given the Galilean geometry of absolute space and time, the red object (if it sent the laser signal at t_0 as well) experiences a round trip time of $t_1 - t_0$ as well, but due to time dilation, its measure of t_1 and t_0 are off, represented by the primed symbols, t'_1 and t'_0 respectively. Their relationship is as expected, $t'_1 \gamma_K = t_1$ and $t'_0 \gamma_K = t_0$; therefore, the red object measures an elapsed time of $\gamma_K^{-1}(t_1 - t_0)$, which is less than the blue object measures. Therefore, assuming the one-way speed of the laser pulse is c for there and back in the red object's frame, the ping1 is measured at a distance, s'_1 given by:

$$s'_1 = \gamma_K^{-1}(t_1 - t_0)c = \gamma_K^{-1}s_1 \quad (5)$$

This same process is repeated to estimate the distance of ping2:

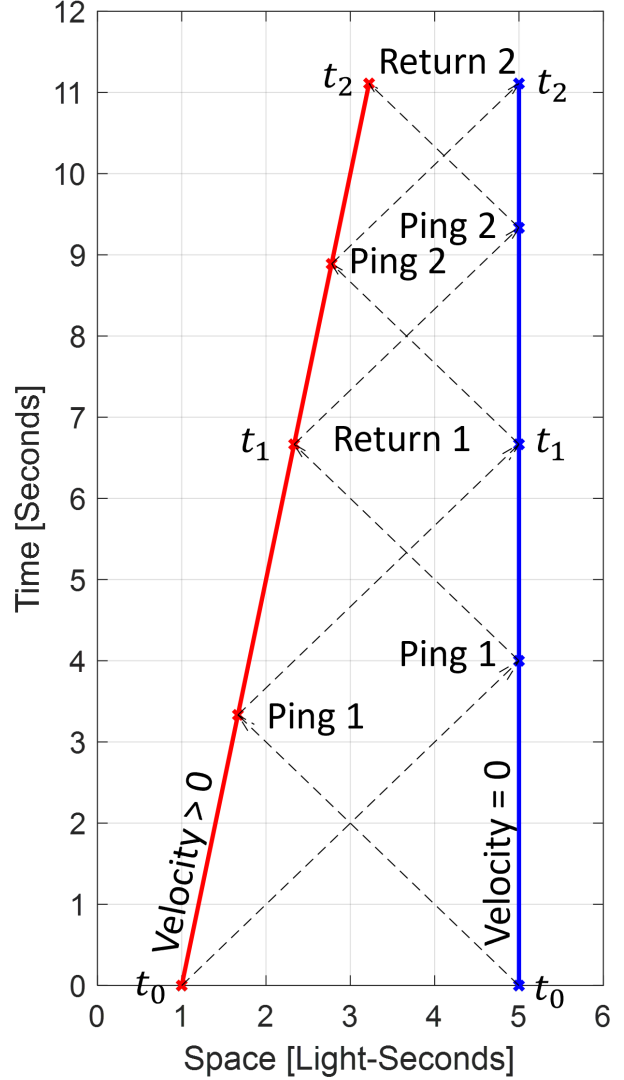


Figure 3. Estimating velocity using pings and returns.

$$s_2 = (t_2 - t_1)c \quad (6)$$

$$s'_2 = \gamma_K^{-1}(t_2 - t_1)c = \gamma_K^{-1}s_2 \quad (7)$$

Since speed is estimated as the measured change in distance over the measured change in time each object (red and blue) estimates the same velocity²:

$$|v| = \frac{s_2 - s_1}{t_2 - t_1} \quad (8)$$

$$|v'| = \frac{s'_2 - s'_1}{t'_2 - t'_1} = \frac{\gamma_K^{-1}(s_2 - s_1)}{\gamma_K^{-1}(t_2 - t_1)} = |V| \blacksquare \quad (9)$$

²Speed can also be estimated as a change in frequency due to the Doppler effect, but the measured frequency due to the Doppler effect is already known to be a function of relative velocity and not velocity in the medium.

This ratio of apparent distance traveled to apparent duration of travel cancels any noticeable effect that a change in units might otherwise create. This is what makes velocity special. All attempts to date to measure motion relative to a USF has relied on propagating light in some novel fashion, and thus, uses the speed of light to measure other quantities, e.g., the one way speed of light experiments [3][5], which have thus far proved incapable of detecting the USF. It is not difficult to see why such attempts had to fail in determining which frame is the USF, since the effect of miscalibrated instruments canceled out.

What is required to objectively measure the USF is to use a measurement that does not nullify the effect of being in a reference frame different from the USF. What is required is a means to observe the unit change caused by specific work done, so that we can calculate an object's velocity in the USF. Then we can relate that object's velocity to everything else using known methods.

3. HOW TO OBJECTIVELY MEASURE THE UNIVERSALLY STATIONARY FRAME

Experimenting with acceleration appears to be where we must first look to detect the USF, since relative observers do not agree on pairwise acceleration estimates [6]. In fact, if one takes a closer look at the bucket experiment, one notices that this test also involved acceleration.

We, therefore, need a similar test involving translational acceleration. Only two forms of translational acceleration that involve a unit change are known: kinetic and gravitational. Only gravitational acceleration ends up being useful.

A kinetic acceleration experiment might involve studying unit changes caused by accelerating a rocket to Alpha Centauri and back, like in the twins paradox setup. The problem with this experiment is that it would rely on remote measurements, which takes time and distance to make these kinds of measurements. The best form of remote measurement involves using photons, like LADAR, to make estimates of speed (via Doppler frequency shifts) and estimates of distance (via lap time of returns). Therefore, this experiment depends on the speed of light traveling for some time and distance, which nullifies the effects we are attempting to measure. This experiment, therefore, has to fail in detecting the USF.

A gravitational acceleration experiment, on the other hand, only has to rely on local measurements. As an example, one such experiment might involve using six identical gravimeters, like the one shown in Figure 4. If this experiment is set up appropriately, and given that specificity is correct, then it will allow us to calculate which frame is the USF.

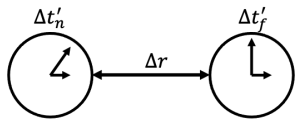


Figure 4. Gravimeter.

Given specificities gravity model, derived in investigation 5 and shown in Equation (10), instrument's measurements relate to gravity as shown in Equation (11):

$$g(r) = -\nabla e_I = -e_T \nabla \frac{e_I}{e_T} \quad (10)$$

$$\nabla \frac{e_I}{e_T} = \nabla \frac{dt'^2}{dt^2} = \lim_{\Delta r \rightarrow 0} \frac{\left(\left(\frac{\Delta t'_f}{\Delta t} \right)^2 - \left(\frac{\Delta t'_n}{\Delta t} \right)^2 \right)}{\Delta r} \quad (11a)$$

$$g(r) = -e_T \lim_{\Delta r \rightarrow 0} \frac{\left(\left(\frac{\Delta t'_f}{\Delta t} \right)^2 - \left(\frac{\Delta t'_n}{\Delta t} \right)^2 \right)}{\Delta r} \quad (11b)$$

Since Δt cannot directly be calculated, a laser range finder can estimate the distance, r_f , to the farthest clock, which allows use to estimate $\frac{dt'_f}{dt}$, which then allows us to simplify Equation (11) as follows:

$$\frac{dt'_f}{dt} = \frac{\Delta t'_f}{\Delta t} \quad (12a)$$

$$g(r) = -e_T \lim_{\Delta r \rightarrow 0} \frac{\left(\frac{dt}{dt'_f} \right)^2 \left(1 - \left(\frac{\Delta t'_n}{\Delta t'_f} \right)^2 \right)}{\Delta r} \quad (12b)$$

You can assume for instrumentation purposes, Δr is never zero, and you can even drop the $\frac{dt}{dt'_f}$ term entirely (when it's small) and measure an approximation of gravity, as shown in Equation (13). The measurement will only be an approximation of gravity, but it will be able to estimate the desired effect—motion in the USF.

$$\text{Let : } \tau^2 = 1 - \left(\frac{dt'_n}{dt'_f} \right)^2$$

$$\hat{g}(r) = -e_T \frac{\left(1 - \left(\frac{\Delta t'_n}{\Delta t'_f} \right)^2 \right)}{\Delta r} = -e_T \frac{\tau^2}{\Delta r} \quad (13)$$

The instruments in this experiment would measure the approximated gravitational acceleration, $\hat{g}(r)$, of a massive object, but in three orthogonal dimensions (two gravimeters per dimension). The speed of each gravimeter measured in the massed object's reference frame (MOF) would be equivalent and directed towards its center of mass. Two of three dimensions are shown in Figure 5.³

Suppose each clock is counting the number of cycles a light bounces back and forth in identically constructed light clocks, and the counts are continuously sent from the far side of the instrument to the near side. How this experiment works is that once the nearest clock reaches a marked threshold some radius, r , away from the massed object (as measured in the MOF) the front clock keeps track of the total counts made by the front clock, Δt_n , and the total counts that reach the front

³Note: in and out of paper dimension is not show.

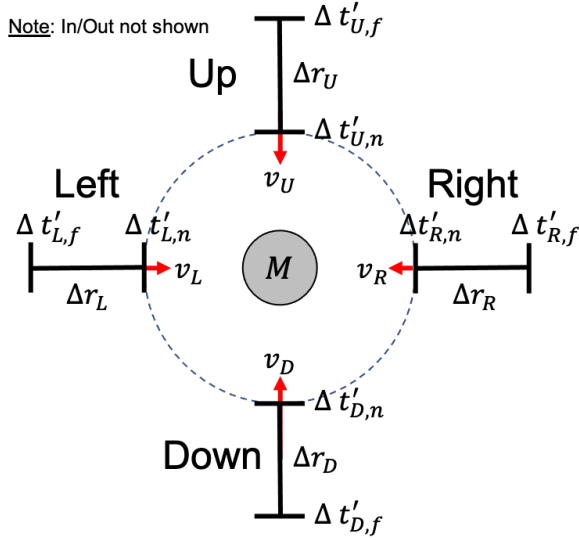


Figure 5. USF detection experimental setup.

clock from the rear, Δt_f . The count stops when the front of the gravimeter reaches another threshold some distance closer to the massed object.

Each gravimeter, being identical and calibrated to the same initial inertial reference frame, assumes Δr is the same under any condition; however, the radial location, r_f , where $\Delta t'_f$ is measured, could change due to length contraction (or its removal). Additionally, any changes to $\Delta t'_n$ and $\Delta t'_f$ in Equation (11) due to kinematic time dilation are nullified, because both change by the same rate, γ_K^{-1} , due to kinematic time dilation and the effect cancels out when you take their ratio, $\frac{\Delta t'_n \gamma_K^{-1}}{\Delta t'_f \gamma_K^{-1}} = \frac{\Delta t'_n}{\Delta t'_f}$. The ratio $\frac{\Delta t'_n}{\Delta t'_f}$ will depend only on the location where $\Delta t'_f$ is measured, which is ultimately governed by the gravimeter's velocity in the USF due to length contraction (or its removal).

Once gravitational acceleration is measured by each gravimeter, an analytical solution for the velocity of the massed object relative to the USF might not be possible because of the set of non-linear equations. However, a simulation can run the set of possible velocities relative to the USF and experimental results can be compared to simulated results. That way the simulated results that line up with experimental results will indicate the MOF's relative velocity in the USF for a given dimension. Subtracting that velocity from the MOF tells us which frame, relative to the MOF, is the USF.

Even though we lack experimental results, a simulation was ran for a notional case to demonstrate how this simulated numerical solution would appear. A simulation for a single dimension was ran (see code in Appendix), and the parameters for the executed simulation were:

- Mass of object: 1000 [Solar Masses]
- Original distance, Δr , clocks were apart in MOF: 1 [km]
- dt measurement distance from center of mass: 0.5 [AU]
- Speed of gravimeters in the MOF: 0.1 [fraction of c]

The results of this simulation can be seen in Figure 6. From the results we can see how the MOF's velocity in the USF (x-axis) affects the gravimeter readings (y-axis).

Three gravimeters were simulated: an orbital gravimeter, a gravimeter traveling faster than the MOF in the USF, and a gravimeter traveling slower.

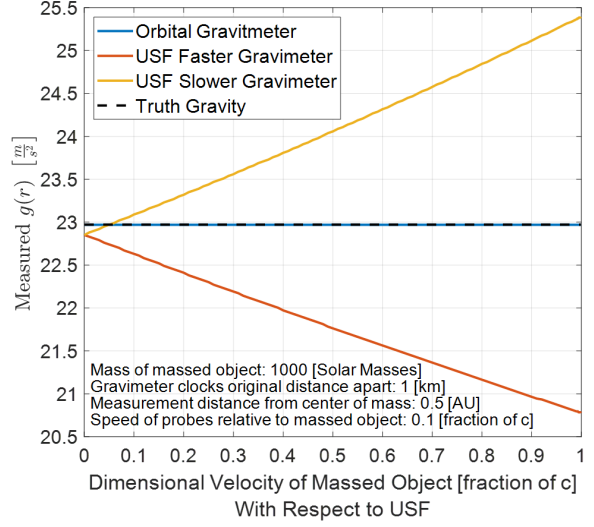


Figure 6. Simulated results.

Suppose we were able to execute a real experiment with such parameters, and found that the measured gravitational acceleration, $g(r)$, were found to be $21.75 [ms^{-2}]$ and $24.05 [ms^{-2}]$ for each gravimeter. That would mean the MOF had a dimensional speed of $0.5c$ relative to the USF in the direction of the gravimeter's velocity that measured $21.75 [ms^{-2}]$.

4. CONCLUSION

In conclusion, it was shown that it is possible to detect the USF if measurements are taken that do not nullify the effects of miscalibrated instruments caused by work done. The designed experiment involving sending gravimeters hurtling towards the center of mass of a massive object is capable of such a detection, if specificity is correct.

The last question to be investigated by this specificity series is: if spacetime is not a real thing, and therefore, cannot be responsible for any time dilation, as an environmental affect on objects, then what causes everything in the same reference frame to be effected by time dilation to the same degree? Addressing that question is the focus of the next (and last) investigation.

REFERENCES

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- [2] K. Szostek and R. Szostek, *Kinematics in the special theory of ether*, Moscow University Physics Bulletin, vol. 73, no. 4, pp. 413–421, 2018.
- [3] K. Szostek and R. Szostek, *The concept of a mechanical system for measuring the one-way speed of light*, Technical Transactions, No. 2023/003, e2023003, 1-9, 2023, ISSN 0011-4561

- [4] R. Buenker, *On the equality of relative velocities between two objects for observers in different rest frames*. Apeiron, Volume 20, No. 2., December 2015.
- [5] *One-way speed of light*. Wikipedia. Retrieved March 15, 2023, from https://en.wikipedia.org/wiki/One-way_speed_of_light
- [6] *Acceleration (special relativity)* Wikipedia, 29-July-2022. [Online]. Available: [https://en.wikipedia.org/wiki/Acceleration_\(special_relativity\)](https://en.wikipedia.org/wiki/Acceleration_(special_relativity)). [Accessed: 26-Sep-2022].

APPENDIX

MATLAB CODE

```

1 % Code designed to demonstrate detection of universally stationary frame (USF)
2 function USF_detection_via_gravitational_acceleration()
3 %% initializations , constants and simple functions
4 % initialization
5 clear all
6 clc
7 close all
8
9 % constants
10 c = 299792458; % [m/s] speed of light
11 G = 6.6744e-11; % [m^3/(kg s)] gravitational constant
12 Me = 5.97219e24; % [kg] earth's mass
13 Ms = 333000*Me; % [kg] sun's mass
14 e_T = 0.5*c^2; % [m^2/s^2] specific total energy
15 AU = 152.03e9; % [m] distance from sun to earth
16
17 % simple functions
18 r_s = @(M) G*M/e_T;
19 gamma_inv_K = @(v) 1./sqrt(1-v.^2);
20 add_vel = @(v1_in,v2_in) (v1_in+v2_in)/(1 + v1_in*v2_in);
21 r_2_gravObj = @(M,r) G*M/r^2;
22 gamma_inv_P = @(M,r) sqrt(1-r_s(M)./r);
23 gravimeter = @(dtnear_dtfar,dr) (e_T/dr)*(1-(dtnear_dtfar)^2);
24 prop_dist = @(M,r1,r2) (r2*gamma_inv_P(M,r2) + 0.5*r_s(M)*log(2*r2*(gamma_inv_P(M,
    r2)+1)-r_s(M))) ...
    - (r1*gamma_inv_P(M,r1) + 0.5*r_s(M)*log(2*r1*(gamma_inv_P(M,r1)+1)-r_s(M)));
25
26 %% experiment: travel two gravimeters (probes) towards center of massed object (MO)
27 % set conditions (in MO's frame)
28 MMO = 1e3*Ms; % [kg] mass of object at center of experiment
29 r_measure = AU/2; % [m] nearest clock distance from center of MO
30 probe_dv = 0.1; % [frac of c] speed of probes relative to MO
31 dr_orb_MO_0 = 1e3; % [m] clocks distance apart when stationary in zero gravity
32
33 % initialize
34 gr_orbit_all = [];
35 gr_probe1_all = [];
36 gr_probe2_all = [];
37
38 % loop through range of MO velocities
39 v_obj_all = [0:0.01:0.99 0.99:0.001:0.999]; % [frac of c] speed of MO (in USF)
40 for ivo = 1 : length(v_obj_all)
41     % (in USF)
42     v_obj = v_obj_all(ivo); % [frac of c] velocity of MO
43     v_p1 = add_vel(v_obj,probe_dv); % [frac of c] velocity of probe1
44     v_p2 = add_vel(v_obj,-probe_dv); % [frac of c] velocity of probe2
45     drUSF_drp_obj = gamma_inv_K(v_obj); % [-] kinetic differential for MO
46     drUSF_drp_p1 = gamma_inv_K(v_p1); % [-] kinetic differential for
47     probe1
48     drUSF_drp_p2 = gamma_inv_K(v_p2); % [-] kinetic differential for
49     probe2
50     drp_p1_drp_obj = drUSF_drp_obj/drUSF_drp_p1; % [-] kinetic differential WRT MO
51     drp_p2_drp_obj = drUSF_drp_obj/drUSF_drp_p2; % [-] kinetic differential WRT MO
52
53 % determine miscalibration effects on grivimeters (in MO frame)
54 r2 = solve_for_r2(MMO,r_measure,dr_orb_MO_0);
55 dr_orb_MO = (r2-r_measure); % [m] clocks distance apart in gravity, no
56 velocity
57 dr_p1_MO = dr_orb_MO*drp_p1_drp_obj; % [m] clocks distance apart in gravity
58 with velocity
59 dr_p2_MO = dr_orb_MO*drp_p2_drp_obj; % [m] clocks distance apart in gravity
60 with velocity
61
62 % determine effects on gravimeter from orbit of MO (in MO frame)
63 r_f_orbit = r_measure+dr_orb_MO; % [m] farthest clock

```

```

        distance to MO
60    r_n_orbit      = r_measure;                                % [m] nearest clock
        distance to MO
61    dtn_dtf_orbit = frames_dtn_dtf(MMO,r_f_orbit ,r_n_orbit); % [-] clock
        differential
62    g_m_orbit      = gravimeter(dtn_dtf_orbit ,dr_orb_MO);      % [m/s^2] measured g
63
64    % determine effects on gravimeter from probe 1 (in MO frame)
65    r_f_probe1     = r_measure+dr_p1_MO;                        % [m] farthest
        clock distance to MO
66    r_n_probe1     = r_measure;                                % [m] nearest clock
        distance to MO
67    dtn_dtf_probe1 = frames_dtn_dtf(MMO,r_f_probe1 ,r_n_probe1); % [-] clock
        differential
68    g_m_probe1     = gravimeter(dtn_dtf_probe1 ,dr_orb_MO);      % [m/s^2] measured
        g
69
70    % determine effects on gravimeter from probe 2 (in MO frame)
71    r_f_probe2     = r_measure+dr_p2_MO;                        % [m] farthest
        clock distance to MO
72    r_n_probe2     = r_measure;                                % [m] nearest clock
        distance to MO
73    dtn_dtf_probe2 = frames_dtn_dtf(MMO,r_f_probe2 ,r_n_probe2); % [-] clock
        differential
74    g_m_probe2     = gravimeter(dtn_dtf_probe2 ,dr_orb_MO);      % [m/s^2] measured
        g
75
76    % store results
77    gr_orbit_all   = [ gr_orbit_all g_m_orbit ];
78    gr_probe1_all  = [ gr_probe1_all g_m_probe1 ];
79    gr_probe2_all  = [ gr_probe2_all g_m_probe2 ];
80 end
81
82 % plot results
83 fig = figure(1);
84 hold off
85 plot(v_obj_all , gr_orbit_all , 'LineWidth',2);
86 hold on
87 plot(v_obj_all , gr_probe1_all , 'LineWidth',2);
88 plot(v_obj_all , gr_probe2_all , 'LineWidth',2);
89 plot([v_obj_all(1) v_obj_all(end)], [r_2_gravObj(MMO,r_measure) r_2_gravObj(MMO,
    r_measure)], 'k--', 'LineWidth',2)
90
91 % clean up plot
92 legend('Orbital Gravimeter','USF Faster Gravimeter','USF Slower Gravimeter','Truth
    Gravity','FontSize',16,'location','NW');
93 xlabel({'Dimensional Velocity of Massed Object [fraction of c]','With Respect to USF
    '},'FontSize',16);
94 ylabel('Measured  $g(r) \sim \left[\frac{m}{s^2}\right]$ ','FontSize',16,'Interpreter','
    latex');
95 grid on
96 xticks([0:.1:1]);
97 a = get(gca,'XTickLabel');
98 set(gca,'XTickLabel',a,'fontsize',16)
99 annotation(fig, 'textbox', [.13 .10 .8 .2], 'String'...
100     ,sprintf('Mass of massed object: %d [Solar Masses]',MMO/Ms) ...
101     , 'EdgeColor','none','FontSize',14);
102 annotation(fig, 'textbox', [.13 .07 .8 .2], 'String'...
103     ,sprintf('Gravimeter clocks original distance apart: %d [km]',dr_orb_MO_0/1e3)
104     , 'EdgeColor','none','FontSize',14);
105 annotation(fig, 'textbox', [.13 .04 .8 .2], 'String'...
106     ,sprintf('Measurement distance from center of mass: %0.1f [AU]',r_measure/AU) ...
107     , 'EdgeColor','none','FontSize',14);
108 annotation(fig, 'textbox', [.13 .01 .8 .2], 'String'...
109     ,sprintf('Speed of probes relative to massed object: %0.1f [fraction of c]',
    probe_dv) ...
110     , 'EdgeColor','none','FontSize',14);

```

```

111
112 %% supporting function
113 function dtn_dtf = frames_dtn_dtf(M,r_f,r_n)
114     % (in MO frame)
115     dt_f      = gamma_inv_P(M,r_f);    % time dilation of clock farthest from MO
116     dt_n      = gamma_inv_P(M,r_n);    % time dilation of clock nearest to MO
117     dtn_dtf   = dt_n/(dt_f);           % relative time differential between closest and
                                         farthest clock
118 end
119
120 function r2 = solve_for_r2(M,r1,dr)
121     % initial guess
122     r2_upper = r1 + 2*dr;
123     r2_lower = r1;
124     r2 = (r2_lower + r2_upper)/2;
125     dr_guess = prop_dist(M,r1,r2);
126     error = dr_guess - dr;
127     while (1e-9 < abs(error) || dr/(2^25) > abs(r2_upper-r2_lower))
128         if 0 < error
129             r2_upper = r2;
130         else
131             r2_lower = r2;
132         end
133         r2 = (r2_lower + r2_upper)/2;
134         dr_guess = prop_dist(M,r1,r2);
135         error = dr_guess - dr;
136     end
137 end
138 end

```