Universal Specificity Investigation 3: Revisiting the Mass Model Assumed by $\mathbf{E = mc^2}$

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The results from the previous investigations into the nature of time revealed that time properly conceptualized is the interval over which change occurs, as opposed to being a property of the Universe as an aspect of spacetime, and a change in inertial time differential (ITD) is caused by specific work done (or change in specific kinetic energy), which results in a change of the interval over which all change in a given reference frame occurs. The last investigation ended with the realization that Newtonian specific kinetic energy model relates more cleanly to changes in ITD than does the relativistic specific kinetic energy model. Now we turn to investigate which model is correct to use.

1. Relativistic Kinetic Energy

Relativistic kinetic energy was first introduced in a thought experiment devised by Einstein, by which he derived a relationship between the mass and internal energy of an object. In this derivation he tacitly made a relativistic mass model assumption that is worth revisiting because the assumed mass model implies the relativistic kinetic energy model.

In this thought experiment, Einstein considered an object at rest that emits energy, in the form of radiation, in two opposite directions, but in equal amounts. Then he considered this same object with the same emission, but viewed from a different inertial reference frame that is moving relative to the object along some arbitrary axis. Figure 1 depicts this thought experiment [1].

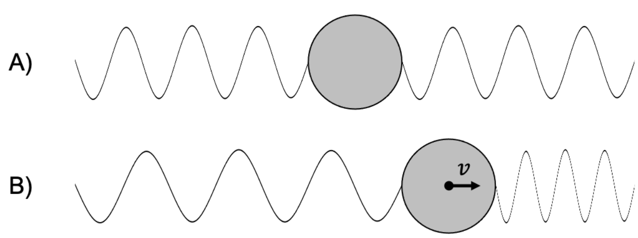

Figure 1. A) Object's inertial reference frame; B) Inertial reference frame with relative motion.

Before relating energy lost to mass lost, Einstein first compared the total energies measured by the two reference frames. He let E represents the total energy of the object as measured from the object's inertial reference frame, and H represents the total energy of the object as measured from the reference frame with relative motion. Einstein said, "[t]hus it is clear that the difference $H - E$ can differ from the kinetic energy K of the body, with respect to the other [reference frame with relative motion], only by an additive constant C..." The resulting model is: $H - E = K + C$ [1].

What is the total energy model and kinetic energy model used by Einstein, and upon what do these models depend? The answer to the first part of the question is best expressed by Feynman in his famous lectures. In short, Feynman derives the total energy relation to kinetic energy from the commonly accepted relativistic mass model as shown in Equation (1) [2].

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{1a}$$

$$m = m_0 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots\right) \tag{1b}$$

$$m \approx m_0 + \frac{1}{2} m_0 \frac{v^2}{c^2} \tag{1c}$$

$$mc^2 \approx m_0 c^2 + \frac{1}{2} m_0 v^2 \ \blacksquare \tag{1d}$$

According to this total energy model, the total energy, mc^2, is approximately the kinetic energy, $\frac{1}{2} m_0 c^2$, plus the internal energy, $m_0 c^2$. This result relates to $H - E = K + C$ as follows:

$$H = mc^2 \tag{2a}$$

$$E = m_0 c^2 \tag{2b}$$

$$K = H - E = mc^2 - m_0 c^2 \tag{2c}$$

$$\therefore\ C = 0 \text{ in this case} \tag{2d}$$

To now answer the second part of the earlier question, these models ultimately depend on a relativistic mass model where rest mass, m_0, of an object remains invariant, while relative mass, m, increases as the object's kinetic energy increases.

To see this dependency, one can derive this kinetic energy model in Equation (2c) from Newtonian first principles, as shown in some detail in Equation (3).[2]

[2]The full derivation is presented here [3].

$$\text{Let}: \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3a}$$

$$m\gamma^{-1} = m_0 = \text{invariant} \tag{3b}$$

$$\Delta E_K = \int F(s)ds \tag{3c}$$

$$\Delta E_K = \int \frac{dp}{dt}ds \tag{3d}$$

$$\Delta E_K = \int vd(mv) \tag{3e}$$

$$\Delta E_K = \int vd(\gamma m_0 v) = m_0 \int vd(\gamma v) \tag{3f}$$

$$\Delta E_K = m_0(\gamma - 1)c^2 = mc^2 - m_0c^2 \blacksquare \tag{3g}$$

If, on the other hand, m turns out to be invariant, then the Newtonian first principles derivation would instead produce the familiar Newtonian kinetic energy model: $\Delta E_K = \frac{1}{2}m\Delta v^2$. If this kinetic energy model is correct, then it implies that the total energy model shown in Equation (1d) is incorrect. A new total energy model would be required; however, before getting ahead of ourselves, I first want to answer which mass model is correct by bringing in the proper conception of time, and with the aid of another thought experiment.

2. Which Mass Model is Correct?

We know from our previous investigations that the units of time changes with specific work done—our instruments become miscalibrated. Is it not possible that specific work done also causes measures of mass to become miscalibrated as well? The quantity of matter will remain invariant as it cannot be created or destroyed, but the units can easily change without notice as has been observed with the units of time. As a simple example of mass units changing, suppose a spring scale, designed to measure in grams, were to move to a higher altitude, then what it measures as a "gram" would be less than a gram.

If rest mass is not invariant as work is done, because imperceptible unit change is occurring (as with time and space), then it must mean the Newtonian kinetic energy model is correct and we need a new total energy model. Let us return to the twins paradox setup, since we know how the units of time change in that situation, to setup another thought experiment to help us determine if (and how) rest mass units change.

For this next thought experiment, consider relativistic effects on gravitational forces. Suppose we managed to craft four Osnium[3] orbs, each having the same shape and size. Assuming each orb has a radius of 0.1 $[m]$, then the mass of each would be identical and roughly 92 $[kg]$. The first pair of orbs are setup in an inertial frame in empty space with an initial distance of 100 $[m]$ between their center of masses. It would take about four months, in the orb's proper time, for their gravitational forces to bring them into contact.

Now, suppose the other pair of orbs were sent away, in the twins paradox fashion, but otherwise the same initial conditions.[4] Supposing they returned at the moment the stationary orbs touched, then the traveling orbs, being younger due to kinetic time dilation, would not be touching, as shown in Figure 2.

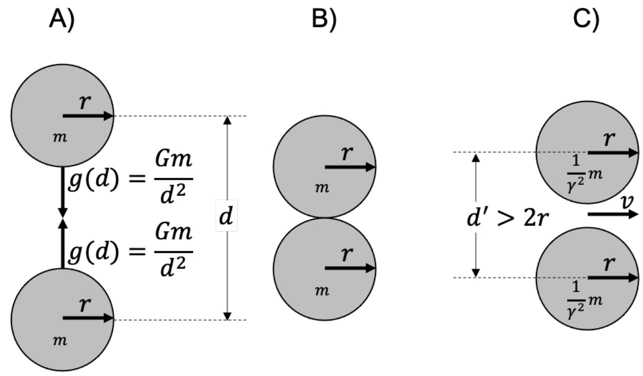

Figure 2. A) Initial Conditions In Stationary Frame; B) State of stationary orbs at the end of four months; C) State of traveling orbs at the end of four months.

These results suggest to me that the mass of the traveling orbs reduces, but to what? Through testing, it was found that the resulting mass reduction is $\gamma^{-2}m$. This result makes intuitive sense, because distance traversed during gravitational acceleration is related to time squared, and the proper time recorded for the traveling orbs is smaller than the proper time recorded for the stationary orbs by a factor of γ^{-1}.

Suppose we conducted another similar experiment; however, for this experiment two new orbs are crafted just like the others, except their radii are reduced to grantee that their mass is equal to $\gamma^{-2}m$. Let these new smaller orbs take the place of the stationary orbs. Given this new setup, we would find that upon the traveling orbs return, both pairs of orbs would be the same distance closer.

I actually simulated several trials of such an experiment, numerically solving for the dynamics in each reference frame, and the code is in the Appendix. Each trial had the same relative velocities, but each trial tested different return times. At the end of each trial, I compared the two pairs of orbs to see if each traveled the same distance. The results are presented in Figure 3.

The two pairs of orbs had the same gravitational behavior. Therefore, invoking the method of agreement, whereby the same gravitational effect occurred in both cases, proves inductively that the rest mass of the traveling orbs is equivalent to the mass of the stationary orbs.

We can see that the units of rest mass for the traveling orb changes, just as time and space do. The relation I derived is $m = \gamma^2 m_0$, but the relation Feynman derived in his exchange of momentum example is $m = \gamma m_0$ [2]; however, this makes no difference in determining which kinetic energy model is correct since m is invariant. The correct kinetic energy model's derivation from first principles remains $\frac{1}{2}mv^2$ either way, which means two things: (a) the correct relationship between ITD changes and specific work done is shown in Equation (4), and (b) we need a new total energy model.

[3] Atomic number 76.

[4] With the distance between their center of masses being orthogonal to the velocity direction.

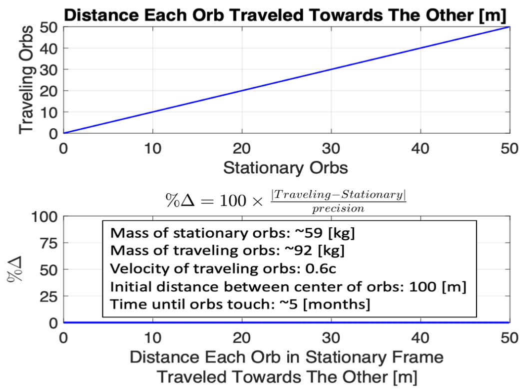

Figure 3. **Results for how far the orbs traveled towards each other.**

$$\frac{dt}{dt'} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{2\Delta e_K}{c^2}} \tag{4a}$$

$$= \sqrt{1 - \frac{2\int a(r)dr}{c^2}} = \sqrt{1 - \frac{2w}{c^2}} \blacksquare \tag{4b}$$

3. A New Total Energy Model

I will use my derived mass model over Feynman's because I know mine was using the proper conception of changes in units due to specific work done. The respective total energy model derivation that relates to this kinetic energy model is shown in Equation (5).

$$\frac{m}{m_0} = \left(\frac{dt'}{dt}\right)^2 = \frac{1}{1 - \frac{2\Delta e_K}{c^2}} = \gamma_K^2 \tag{5a}$$

$$m\left(1 - \frac{2\Delta e_K}{c^2}\right) = m_0 \tag{5b}$$

$$m = m_0 + m\frac{2\Delta e_K}{c^2} \tag{5c}$$

$$\frac{1}{2}mc^2 = \frac{1}{2}m_0c^2 + m\Delta e_K \tag{5d}$$

$$E_T = \frac{1}{\gamma_K^2}\frac{1}{2}mc^2 + \Delta E_K \blacksquare \tag{5e}$$

According to this total energy model, the total energy, $\frac{1}{2}mc^2$, is the kinetic energy, $\frac{1}{2}mv^2$, plus the internal energy, $\frac{1}{\gamma_K^2}\frac{1}{2}mc^2$. Other than the terms being different, this is like Einstein's total energy model in that total energy is kinetic plus internal energy. Unlike Einstein's model, however, the internal energy of an object diminishes as it gains kinetic energy, all the while its total energy is conserved. For example, when the object is at rest, then all of its energy is internal (the inner happenings is all that is happening). However, if object achieves c, then all of its energy is in its motion (there is no inner happenings meaning proper time stands still). In either extreme, or at any point in between, the object's total energy and mass remains the same.

Now if Feynman's mass model were used instead of mine, then Equation (5e) would still be the result. Even mass and rest mass where fixed and constant, Equation (5e) would still be the result. In fact, Equation (5e) would be the result, if m were invariant, regardless of the rest mass model. Something else is at work here, and it has to do with the relative speed of light in the moving reference frame. If the internal happenings is dependant on how fast light moves in that reference frame, then light has actually slowed down to $c_0 = \sqrt{c^2 - v^2}$, as shown in Figure 4.

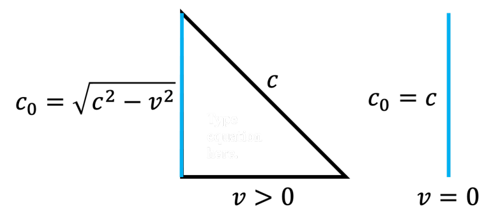

Figure 4. **Changes in local frame's relative speed of light.**

This means specific internal energy is a function of how fast light moves relative to that frame, as measured in a universally stationary frame. This makes logical sense because specific total energy is a function of how fast light moves relative to the universally stationary frame; therefore, in this way internal energy can be thought of as total rest energy. Updating Equation (5) with this knowledge yields:

$$\frac{1}{2}mc^2 = \frac{1}{2}mc_0^2 + \frac{1}{2}mv^2 \tag{6a}$$

$$E_T = E_I + \Delta E_K \tag{6b}$$

Integrating this discovery back into our inertial time differential equation from a previous investigation we discover a new implication, which is c^2 is really twice specific total energy as shown in Equation (7).

$$\frac{dt}{dt'} = \sqrt{1 - \frac{2\Delta e_K}{c^2}} = \sqrt{1 - \frac{\Delta e_K}{e_T}} \tag{7a}$$

$$= \sqrt{1 - \frac{\int a(r)dr}{e_T}} = \sqrt{1 - \frac{w}{e_T}} \blacksquare \tag{7b}$$

4. Conclusion

In conclusion, we found that the relativistic kinetic energy model was not accounting for a change in units caused by work done, which meant the Newtonian kinetic energy model is correct, which resulted in a new (more intuitive) total energy model. This new model relates to the inertial time differential equation, where the ratio of velocity to speed of light squared in the equation is interpreted as a ratio of change in specific kinetic energy to specific total energy.

The next investigation will look into the question: if a change in specific kinetic energy of an object causes its time differential to change, then what happens to an object that exists in a time dilation gradient, such as those gradients that exist around massed objects?

REFERENCES

[1] A. Einstein, *Does the inertia of a body depend upon its energy-content?*. [Online]. Available: https://www.fourmilab.ch/etexts/einstein/E_mc2/e_mc2.pdf. [Accessed: 21-Aug-2022].

[2] R. Feynman, *The Feynman Lectures on Physics*, 2012. [Online]. Available: https://www.feynmanlectures.cal tech.edu [Accessed: 20-Aug-2022].

[3] *Relativistic kinetic energy: Derivation, formula, definition* Mech Content, 23-Aug-2022. [Online]. Available: https://mechcontent.com/relativistic-kinetic-energy/. [Accessed: 09-Sep-2022].

Appendix

Matlab Code

```matlab
%% constants and functions
G                 = 6.6744e-11; % [m^3/(kg s)] gravitational constant
gamma             = @(v) 1./sqrt(1-v.^2);
seconds2months    = 12/60^2/24/365;

%% Traveling orbs
% initial conditions
rho     = 22000;       % [kg/m^3] density of osmium
r       = 1e-1;        % [m]      radius of each orb
vol     = 4*pi*r^3/3;  % [m^3]    volume of each orb
m       = rho*vol;     % [kg]     mass of each orb
d       = 1e2;         % [m]      initial distance between orbs' surfaces
d_min   = 2*r;         % [m]      minimum distance between center mass of orbs
gd1     = 2*G*m/(d);   % [J/kg]   initial relative xspecific potential energy
v       = 0.6;         % [-]      fraction of the speed of light of orbs
gamma_v = gamma(v);    % [-]      1/sqrt(1-v^2/c^2)

% initialize other variables
dy  = (d-d_min)/1e4;      % increment steps to numerical solution
ds  = d:-dy:d_min;        % all numerical steps
gds = ones(size(ds))*gd1;% specific potential energy
vs  = zeros(size(ds));    % relative velocity of orbs
ts  = zeros(size(ds));    % proper time passed

% incremental solution of orb pairs relative velocity and time passed
for id = 2 : length(ds)
    % this relative specific potential energy for orbs
    gds(id) = 2*G*m/(ds(id));

    % delta relative specific potential energy for orbs
    delta_gd = gds(id)-gd1;

    % relative velocity between them
    vs(id) = sqrt(2*delta_gd);

    % time for distance to close by mean relative velocity
    ts(id) = ts(id-1) + dy/mean([vs(id),vs(id-1)]);
end

% total passage of proper time until orbs contact in years and months
total_time_months = max(ts)*seconds2months;

%% Stationary orbs
my       = m/gamma_v^2; % [kg]   mass of stationary orb is traveling orb's mass
gd1_my   = 2*G*my/(d);  % [J/kg] initial specific potential energy

% time passed, as measured by stationary orbs
ts_gamma = ts*gamma_v;

% total passage of proper time until orbs contact in years and months
total_time_months_my   = max(ts_gamma)*seconds2months;

% initialize stationary orbs with mass my distance steps
dy_my = dy;               % increment steps to numerical solution
ds_my = d:-dy_my:d_min; % all numerical steps

% initialize other variables
vs_my   = zeros(size(ds_my));
gds_my  = ones(size(ds_my))*gd1_my;
ts_my   = zeros(size(ds_my));

% incremental solution of orb pairs relative velocity and time passed
for id = 2 : length(ds_my)
    % this relative specific potential energy
    gds_my(id) = 2*G*my/(ds_my(id));
```

```
    % delta relative specific potential energy
    delta_gd_my = gds_my(id)-gd1_my;

    % relative velocity between them
    vs_my(id) = sqrt(2*delta_gd_my);

    % time for distance to close by mean relative velocity
    ts_my(id) = ts_my(id-1) + dy_my/mean([vs_my(id),vs_my(id-1)]);
end

%% Plot Results
figure(1);
% plot the movement of each orb makes towards its pair
subplot(2,1,1)
plot((d-ds)/2,(d-interp1(ts_my,ds_my,ts_gamma))/2,'-b','LineWidth',1.5)
xlim([0 d/2]);
ylim([0 d/2]);
grid on
xlabel('Stationary Orbs','FontSize',20);
ylabel('Traveling Orbs','FontSize',20);
title({'Distance Each Orb Traveled Towards The Other [m]'},'fontsize',16);

% plot the percent difference in movement between pairs of orbs
percent_difference = 100*abs((d-interp1(ts_my,ds_my,ts_gamma))/2 - (d-ds)/2)./(dy);
subplot(2,1,2)
plot((d-ds)/2,percent_difference,'-b','LineWidth',2)
xlim([0 d/2]);
ylim([0 100]);
grid on
xlabel({'Distance Each Orb in Stationary Frame'...
    ,'Traveled Towards The Other [m]'},'FontSize',20);
ylabel({'\%$\Delta$'},'FontSize',20,'Interpreter','latex');
title({'\%$\Delta=100\times\frac{|Traveling-Stationary|}{precision}$'}...
    ,'Interpreter','latex','fontsize',16);

% print ellapsed proper (AAK wall) time for each pair or orbs
fprintf('Elapsed Time for Traveling Orbs: %0.1f [months]\n',total_time_months);
fprintf('Elapsed Time for Stationary Orbs: %0.1f [months]\n',total_time_months_my);
```