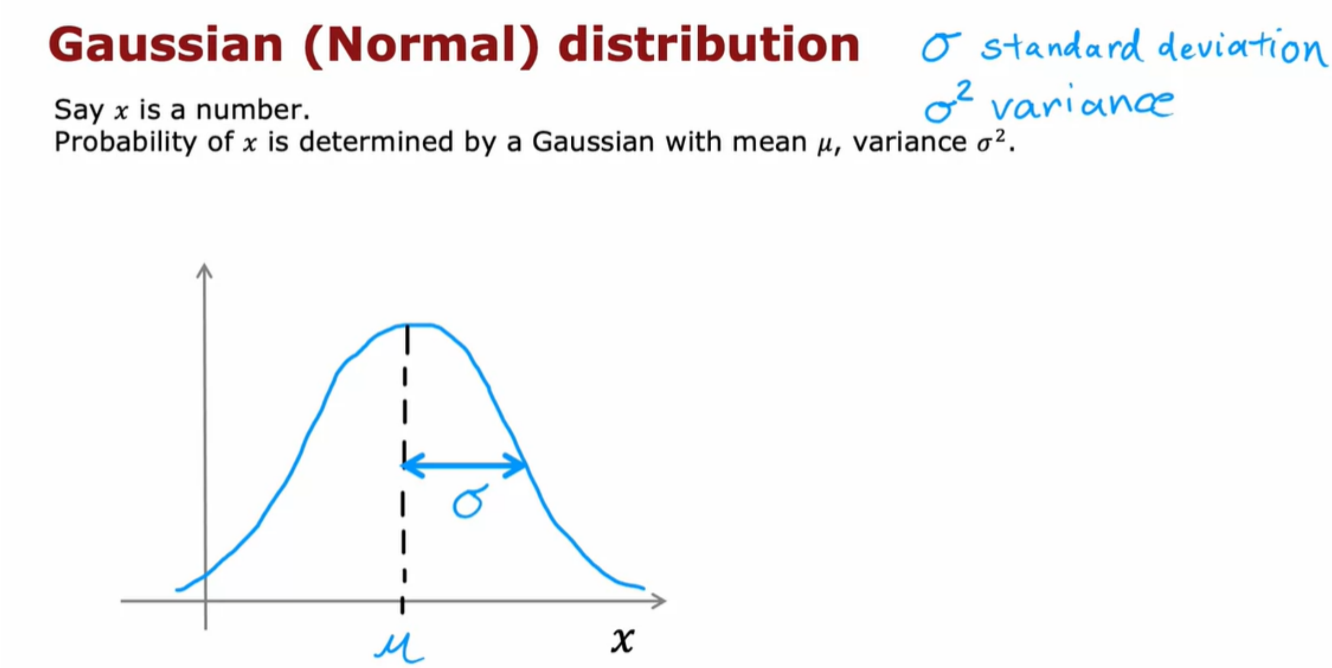
Gaussian Distribution and Anomaly Detection

Gaussian Distribution Overview

- **Gaussian Distribution:** Also known as the normal distribution, it is a probability distribution characterized by a bell-shaped curve. The curve is defined by two parameters: mean (Mu) and variance (sigma squared).



- **Probability of x:** The probability of a random variable x following a Gaussian distribution is given by the formula:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$$

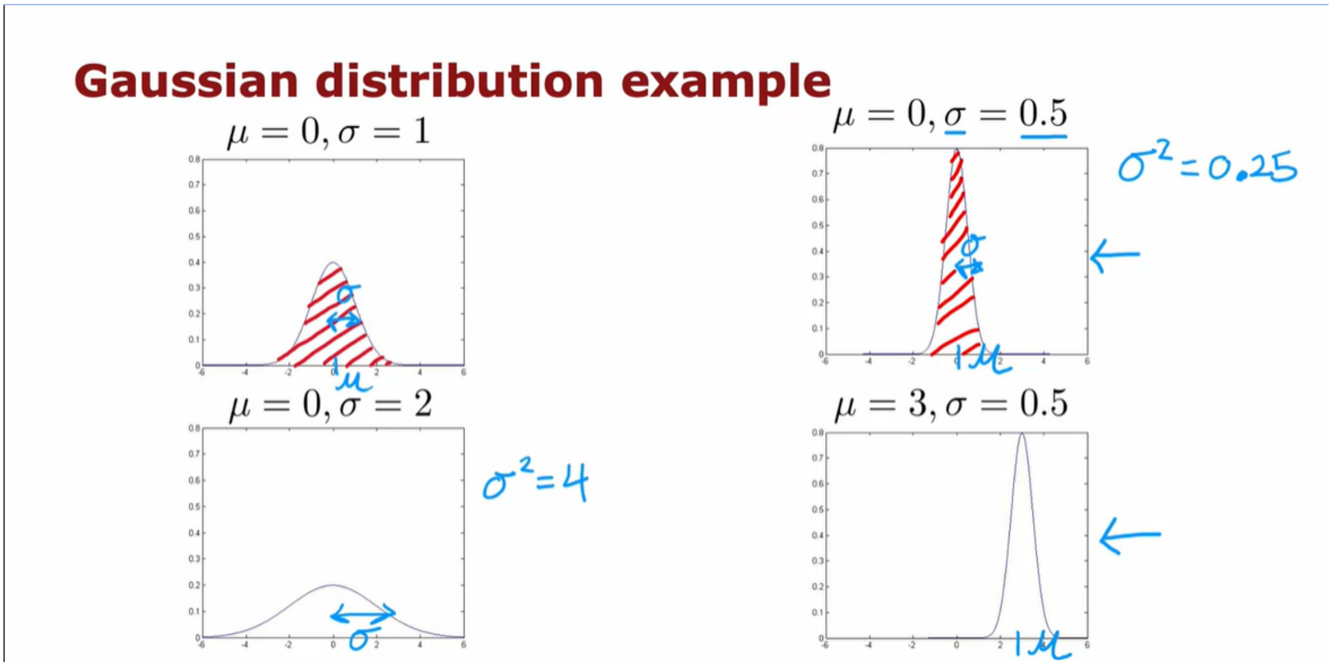
- **Interpretation:** The probability density function $p(x)$ represents the likelihood of observing a specific value of x. The bell-shaped curve illustrates the distribution of a large number of samples drawn from this distribution.

Effect of Changing Mu and Sigma

1. **Standard Gaussian Distribution (Mu = 0, Sigma = 1):**
    - Centered at zero with a standard deviation of one.

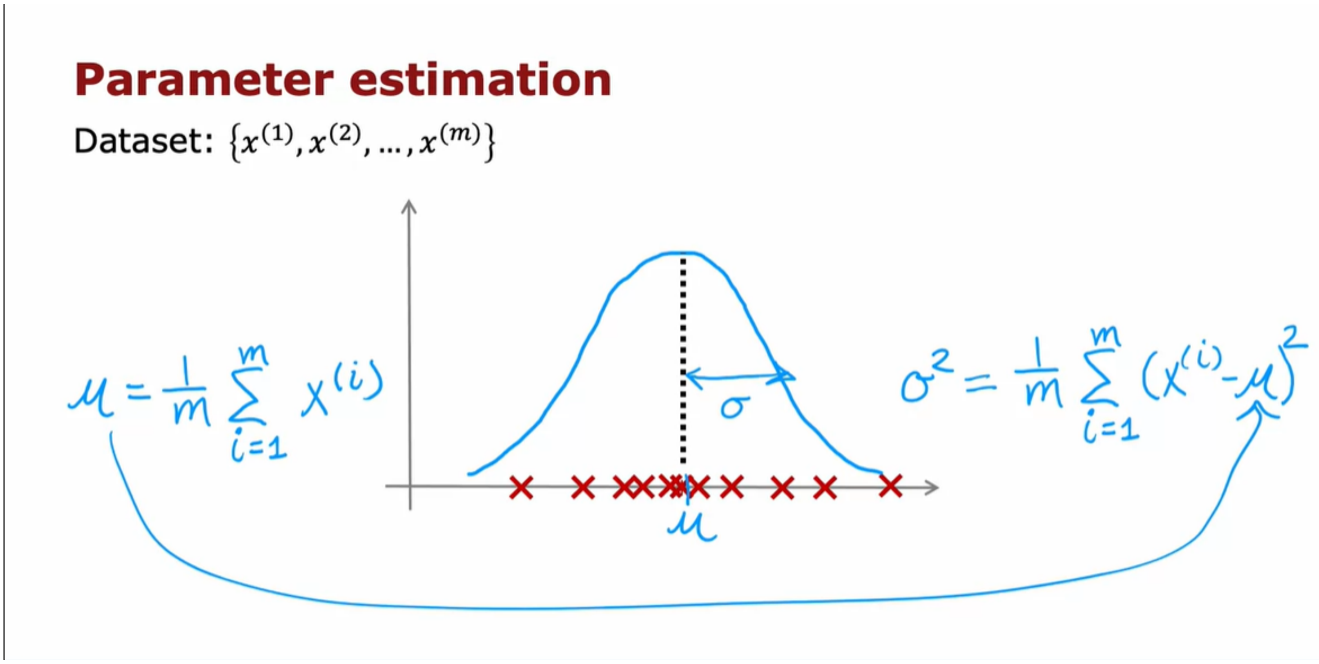
- A common reference for comparison.

2. **Reduced Sigma (Sigma = 0.5):**
   - Thinner curve due to a smaller standard deviation.
   - Taller curve compensates for the reduced width.
3. **Increased Sigma (Sigma = 2):**
   - Wider distribution with a larger standard deviation.
   - Shorter curve to maintain a total area under the curve equal to one.
4. **Change in Mu (Mu = 2, Sigma = 0.5):**
   - Shifts the center of the distribution to the right while maintaining the standard deviation.



Application to Anomaly Detection

- **Anomaly Detection Dataset:** Given a dataset of m examples with a single feature x, the goal is to estimate parameters Mu and Sigma squared for a Gaussian distribution.
- **Estimation Formulas:**
  - $$\mu = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$$
  - $$\sigma^2 = \frac{1}{m} \sum_{i=1}^{m} (x^{(i)} - \mu)^2$$



- **Maximum Likelihood Estimates:** These formulas provide estimates for Mu and Sigma squared and are technically known as maximum likelihood estimates.
- **Anomaly Decision:** Given an example x_{test}:
  - If $p(x_{\text{test}}) < \epsilon$, flag it as an anomaly.
  - If $p(x_{\text{test}}) \geq \epsilon$, consider it normal.

Handling Multiple Features

- **Extension to Multiple Features:** In practical applications, there are usually multiple features (n features).
- **Sophisticated Anomaly Detection:**
  - The principles learned from a single Gaussian distribution can be extended to handle multiple features.
  - Calculate Mu and Sigma for each feature independently.
- **Multivariate Gaussian Distribution:** Utilize a multivariate Gaussian distribution to model the joint probability of multiple features. The extension involves using a covariance matrix in addition to Mu and Sigma.
- **Next Steps:** The next video will explore the application of Gaussian distribution to more sophisticated anomaly detection algorithms, considering scenarios with multiple features.