10.4.4.2.2

EE24BTECH11036 - Krishna Patil

Question: Find the values of k for following quadratic equations, so that they have two equal roots :

$$kx(x-2)+6=0 \tag{1}$$

Solution: For equal roots of a quadratic equation, the discriminant is zero i.e., For the quadratic $ax^2+bx+c=0$ the discriminant $b^2-4ac=0$.

$$kx(x-2)+6=0 \tag{2}$$
$$kx^2-2kx+6=0 \tag{3}$$
$$\therefore a=k, b=-2k, c=6 \tag{4}$$
$$\therefore 4k^2-24k=0 \tag{5}$$
$$k=0 \quad or \quad k=6 \tag{6}$$

but $k=0$ doesn't make sense as no solution is obtained so, $k=6$ is the answer.

So, the equation becomes

$$6x(x-2)+6=0 \tag{7}$$
$$6x^2-12x+6=0 \tag{8}$$
$$x^2-2x+1=0 \tag{9}$$

1) **Thereotical Solution** :

$$x^2-2x+1=0 \tag{10}$$
$$(x-1)^2=0 \implies x=1 \tag{11}$$

2) **Numerical Solution** :

   a) **Fixed point iteration Method** :
   Say , we have to find roots of

$$f(x)=0 \tag{12}$$

   using algebra, we first solve for x i.e., $x=g(x)$ where, $g(x)$ is some other function formed after solving. then we select an initial guess x_0, iterate using the formula

$$x_{n+1}=g(x_n) \tag{13}$$

   Repeat this step until the difference between successive approximations $|x_{n+1}-x_n|$ is less than a specified tolerance ϵ.

say for our case , we choose $\epsilon = 10^{-6}$ and $x_0 = 1.5$ and $f(x) = x^2 - 2x + 1$,

$$\because f(x) = 0 \tag{14}$$

$$x^2 - 2x + 1 = 0 \tag{15}$$

$$\sqrt{2x-1} = x \tag{16}$$

$$\therefore g(x) = \sqrt{2x-1} \tag{17}$$

So, the iterative equation is ,

$$x_{n+1} = \left(\sqrt{2x_n - 1}\right) \tag{18}$$

after computing , we obtain $x = 1.0014142$ which is pretty near to the theoretical solution.

b) **Newton-Raphson Method**
The Newton-Raphson method is an iterative technique to find the roots of a real-valued function $f(x)$. The update formula is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{19}$$

For the function $f(x) = x^2 - 2x + 1$, the derivative is:

$$f'(x) = 2x - 2 \tag{20}$$

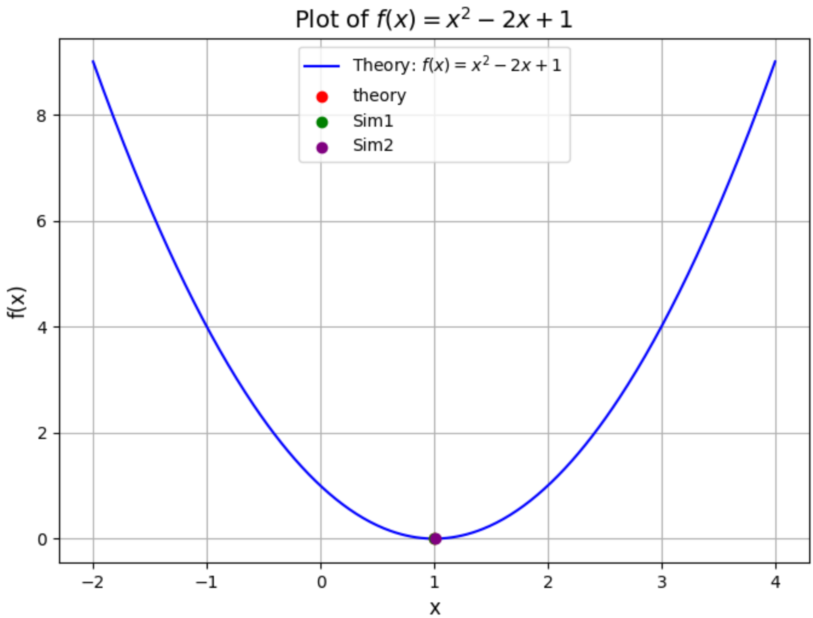
$$f(x) = x^2 - 2x + 1 \tag{21}$$

$$f'(x) = 2x - 2 \tag{22}$$

The Newton-Raphson update formula becomes:

$$x_{n+1} = x_n - \frac{x_n^2 - 2x_n + 1}{2x_n - 2} \tag{23}$$

After computing , we obtain $x = 0.999999$ which is pretty near to the solution.



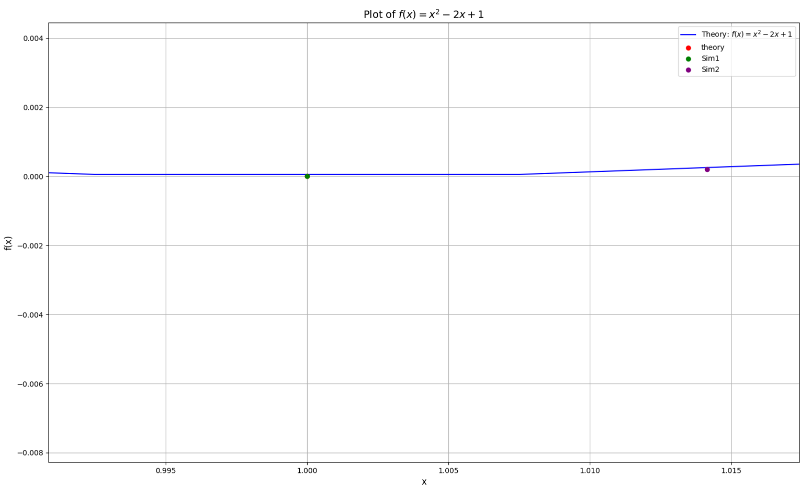


Fig. 2: Zoomed Form