NCERT 12.9.6.4

EE24BTECH11036 - Krishna Patil

Question: :Solve the following differential equation

$$\frac{dy}{dx} + y\sec x = \tan x \tag{1}$$

with initial condtitions :

$$x = 0, \quad y = 1. \tag{2}$$

Solution:

1) **Recognize the Linear Form:** This is a linear first-order differential equation of the form:

$$\frac{dy}{dx} + P(x)\,y = Q(x) \tag{3}$$

Here:

$$P(x) = \sec x, \tag{4}$$

$$Q(x) = \tan x. \tag{5}$$

2) **Find the Integrating Factor** (IF): The integrating factor is given by:

$$\mu(x) = e^{\int P(x)dx} = e^{\int \sec x\, dx}. \tag{6}$$

The integral of $\sec x$ is:

$$\int \sec x\, dx = \ln(\sec x + \tan x). \tag{7}$$

Thus:

$$\mu(x) = e^{\ln(\sec x + \tan x)} = (\sec x + \tan x). \tag{8}$$

3) **Multiply Through by the Integrating Factor**: Multiply the entire differential equation by $\mu(x)$:

$$(\sec x + \tan x)\frac{dy}{dx} + y(\sec x + \tan x)\sec x = (\sec x + \tan x)\tan x. \tag{9}$$

This simplifies to:

$$\frac{d}{dx}\left(y(\sec x + \tan x)\right) = (\sec x + \tan x)\tan x. \tag{10}$$

4) **Integrate Both Sides:** Integrate both sides with respect to x:

$$\int \frac{d}{dx}\left(y(\sec x + \tan x)\right)dx = \int (\sec x + \tan x)\tan x\, dx. \tag{11}$$

The integral of $(\sec x + \tan x)\tan x$ simplifies to $\sec x + \tan x - x$:

$$(y\,(\sec x + \tan x)) = \sec x + \tan x - x + C, \tag{12}$$

where C is the constant of integration.

5) **Solve for** y: Divide through by $(\sec x + \tan x)$:

$$y = \frac{\sec x + \tan x - x + C}{(\sec x + \tan x)}. \tag{13}$$

This is the general solution to the differential equation.

6) **Determining the value of C**: Using initial conditions we can easily determine the value of C

$$1 = \frac{\sec 0 + \tan 0 - x + C}{\sec 0 + \tan 0} \tag{14}$$

$$1 = 1 + c \tag{15}$$

$$\therefore c = 0 \tag{16}$$

$$\therefore y = \frac{\sec x + \tan x - x}{(\sec x + \tan x)} \tag{17}$$

7) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$x_0 = 0 \tag{18}$$

$$y_0 = 1 \tag{19}$$

$$h = 0.001 \tag{20}$$

$$y_{n+1} = y_n + h \cdot (\tan x_n - y_n \sec x_n) \tag{21}$$

$$x_{n+1} = x_n + h \tag{22}$$

Below is verification 7 :

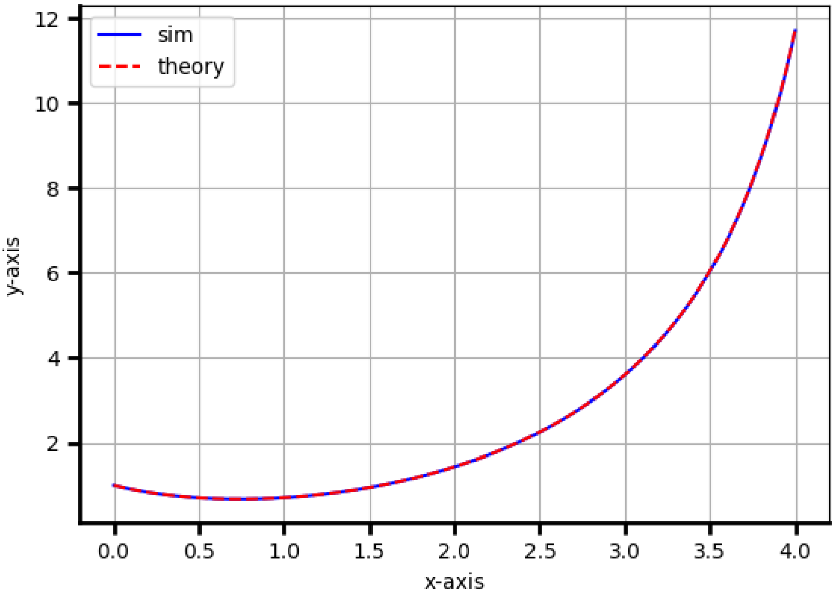

Fig. 7: Verification