10.3.3.3.5

EE24BTECH11036 - Krishna Patil

Question : A fraction becomes $\frac{9}{11}$, if 2 Is added to both the numerator and the denominator. If, 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction .

Solution: Let the fraction be $\frac{x}{y}$ then, according to the statements in the question ,

$$\frac{x+2}{y+2} = \frac{9}{11} \tag{1}$$

$$11x + 22 = 9y + 18 \tag{2}$$

$$11x - 9y = -4 \tag{3}$$

and

$$\frac{x+3}{y+3} = \frac{5}{6} \tag{4}$$

$$6x + 18 = 5y + 15 \tag{5}$$

$$6x - 5y = -3 \tag{6}$$

CODING LOGIC

Let us assume the given system of equations are consistent and we will try solving using LU decomposition

Given the system of linear equations:

$$11x - 9y = -4 \tag{7}$$

$$6x - 5y = -3 \tag{8}$$

We rewrite the equations as:

$$x_1 = x, \tag{9}$$

$$x_2 = y, \tag{10}$$

giving the system:

$$11x_1 - 9x_2 = -4, \tag{11}$$

$$6x_1 - 5x_2 = -3. \tag{12}$$

Step 1: Convert to Matrix Form

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{13}$$

where:

$$A = \begin{bmatrix} 11 & -9 \\ 6 & -5 \end{bmatrix}, \tag{14}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{15}$$

$$\mathbf{b} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}. \tag{16}$$

Step 2: LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

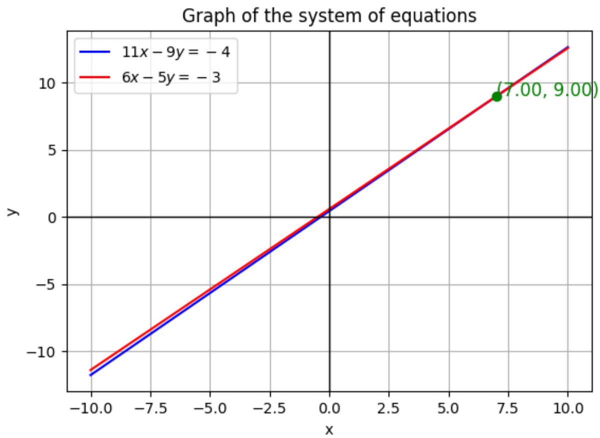
This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.

Using a code we get L,U as

$$L = \begin{bmatrix} 1.00 & 0.00 \\ \frac{6}{11} & 1.00 \end{bmatrix} \tag{17}$$

$$U = \begin{bmatrix} 11.00 & -9.00 \\ 0.00 & -\frac{1}{11} \end{bmatrix} \tag{18}$$

So, solving the $A\mathbf{x} = \mathbf{b}$ we get, $x = 7$ and $y = 9$, the fraction is $\frac{7}{9}$.



Graph of the system of equations
$11x - 9y = -4$
$6x - 5y = -3$
(7.00, 9.00)
10
5
0
−5
−10
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x