NCERT 9.1.4: Solving a Nonlinear ODE

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Outline

Problem Statement

Solve the following nonlinear ODE:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$$

Reformulating the Equation

Define:

$$v = \frac{dy}{dx},$$
$$u = \frac{d^2y}{dx^2}.$$

Substituting into the equation gives:

$$u^2 + \cos(v) = 0.$$

Solve for u:

$$u = \pm\sqrt{-\cos(v)}, \quad \text{valid only for } \cos(v) < 0.$$

Numerical Method: RK4

System of Equations:

$$\frac{dy}{dx} = v,$$
$$\frac{dv}{dx} = \pm\sqrt{-\cos(v)}.$$

RK4 Updates for y:

$$k_{y,1} = h \cdot v,$$
$$k_{y,2} = h \cdot \left(v + \frac{k_{v,1}}{2} \right),$$
$$k_{y,3} = h \cdot \left(v + \frac{k_{v,2}}{2} \right),$$
$$k_{y,4} = h \cdot (v + k_{v,3}),$$
$$y_{n+1} = y_n + \frac{1}{6}\left(k_{y,1} + 2k_{y,2} + 2k_{y,3} + k_{y,4}\right).$$

Numerical Method: RK4 (contd.)

RK4 Updates for v:

$$
\begin{aligned}
k_{v,1} &= h \cdot \sqrt{-\cos(v)}, \\
k_{v,2} &= h \cdot \sqrt{-\cos\left(v + \frac{k_{v,1}}{2}\right)}, \\
k_{v,3} &= h \cdot \sqrt{-\cos\left(v + \frac{k_{v,2}}{2}\right)}, \\
k_{v,4} &= h \cdot \sqrt{-\cos\left(v + k_{v,3}\right)}, \\
v_{n+1} &= v_n + \frac{1}{6}\left(k_{v,1} + 2k_{v,2} + 2k_{v,3} + k_{v,4}\right).
\end{aligned}
$$

Implementation Steps

1. Choose initial values $y(0) = y_0$ and $v(0) = v_0$.
2. Select a step size h to control accuracy.
3. Alternate updates for y and v using RK4 equations.
4. Ensure $\cos(v) < 0$ during integration for validity.

Results and Visualization

Below is the plot of y vs x obtained using the RK4 method:

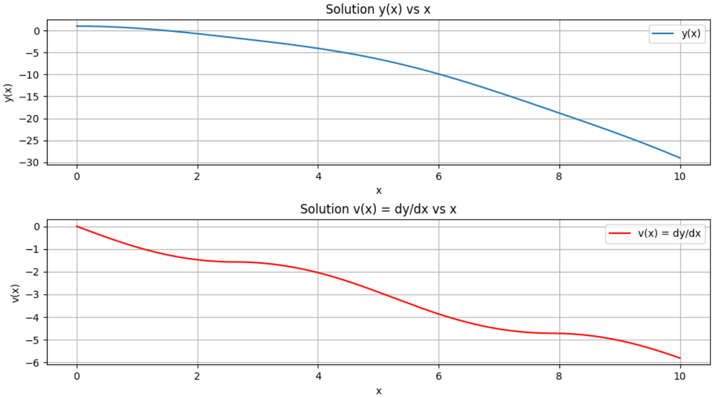

Figure: Numerical Solution of the ODE

Conclusion

- Solved the nonlinear ODE using the RK4 method.
- Demonstrated the solution's dependence on initial conditions and step size.
- Plots confirm the validity of the numerical solution.