NCERT 9.4.7

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Question Solve the differential equation:

$$y \ln(y)\, dx - x\, dy = 0$$

with initial conditions $x = 1$ and $y = e$.

Solution:

Step 1: Rearranging the Equation

First, we rewrite the equation in a more convenient form:

$$y \ln(y)\, dx = x\, dy$$

Next, we divide both sides by $x \ln(y)$:

$$\frac{dy}{dx} = \frac{y \ln(y)}{x}$$

Step 2: Separation of Variables

Now, we separate the variables to prepare for integration:

$$\frac{dy}{y \ln(y)} = \frac{dx}{x}$$

Step 3: Integration

We now integrate both sides.

$$u = \ln(y) \implies du = \frac{1}{y}\, dy$$
$$\int \frac{1}{u}\, du = \int \frac{1}{x}\, dx$$

This leads to:

$$\ln|u| = \ln|x| + C$$

Substituting $u = \ln(y)$, we have:

$$\ln|\ln(y)| = \ln|x| + C$$

Exponentiating both sides:

$$|\ln(y)| = C'|x|$$

where $C' = e^C$. Now solving for y, we get:

$$\ln(y) = C' x$$

Exponentiating again:

$$y = e^{C' x}$$

Step 4: Applying the Initial Conditions To determine the constant C', we apply the initial condition $x = 1$ and $y = e$:

$$e = e^{C' \cdot 1}$$

Thus, $C' = 1$, and the solution to the differential equation is:

$$y = e^x$$

The solution for the differantial equation can be graphically solved using coding by using below logic :

$$x_0 = 1$$
$$y_0 = e$$
$$h = 0.1$$
$$y_{n+1} = y_n + h \cdot \left(\frac{y_n \ln y_n}{x_n} \right)$$
$$x_{n+1} = x_n + h$$

Below is verification **??** :