9.ex.3

EE24BTECH11036 - Krishna Patil

Question : Find two numbers whose sum is 24 and whose product is as large as possible.

Solution:

Let the two numbers be x and y. We are given the following conditions:

1) The sum of the numbers is 24:

$$x + y = 24 \tag{1}$$

2) We need to maximize the product $P = x \cdot y$.

Express y in terms of x From the equation $x + y = 24$, we can solve for y:

$$y = 24 - x \tag{2}$$

Write the product in terms of x Substitute $y = 24 - x$ into the product equation:

$$P(x) = x \cdot (24 - x) = 24x - x^2 \tag{3}$$

We want to maximize this quadratic expression for $P(x)$.

Find the value of x that maximizes $P(x)$ To maximize $P(x) = 24x - x^2$, we first take the derivative of $P(x)$ with respect to x:

$$P'(x) = 24 - 2x \tag{4}$$

Next, we set the derivative equal to zero to find the critical points:

$$24 - 2x = 0 \tag{5}$$

Solving for x:

$$2x = 24 \implies x = 12 \tag{6}$$

Verify that this is a maximum To ensure that $x = 12$ gives a maximum, we take the second derivative of $P(x)$:

$$P''(x) = -2 \tag{7}$$

Since $P''(x) = -2$ is negative, the function $P(x)$ is concave down, confirming that $x = 12$ is indeed a maximum.

Find the value of y Since $x = 12$, substitute this into the equation $y = 24 - x$:

$$y = 24 - 12 = 12 \tag{8}$$

Calculate the maximum product Now, we calculate the product of the two numbers:

$$P = 12 \cdot 12 = 144 \tag{9}$$

Conclusion: The two numbers are 12 and 12, and their product is 144. Therefore, the maximum product of two numbers whose sum is 24 is 144.

Gradient Descent Algorithm: We use the method of gradient descent to find the maximum of the given function, since the objective function is convex. Since the coefficient of $\left(24x - x^2\right) < 0$, we expect to find the maximum.

$$x_{n+1} = x_n - \mu f'(x_n) \tag{10}$$

$$f'(x_n) = 24 - 2x_n \tag{11}$$

$$\rightarrow x_{n+1} = x_n - \mu (24 - 2x_n) \tag{12}$$

$$= (1 + 2\mu) x_n - 24\mu \tag{13}$$

Applying unilateral Z-transform, 1. Apply $Z[\lambda_n] = Y(z)$:

$$zY(z) - z\lambda_0 = (1 + 2\mu) Y(z) - 24\mu \frac{z}{z-1} \tag{14}$$

2. Rearrange:

$$Y(z)(z - (1 + 2\mu)) = z\lambda_0 - 24\mu \frac{z}{z-1} \tag{15}$$

3. Solve for $Y(z)$:

$$Y(z) = \frac{\lambda_0 z}{z - (1 + 2\mu)} - \frac{24\mu z}{(z-1)(z - (1 + 2\mu))} \tag{16}$$

Take the inverse Z-transform:

$$\lambda_n = (1 + 2\mu)^n \lambda_0 + 1 - (24\mu + 12)(1 + 2\mu)^{n-1} \tag{17}$$

$$\lambda_n = (1 + 2\mu)^n \lambda_0 + 12 - 12(1 + 2\mu)^n \tag{18}$$

But say initial guess , $\lambda_0 = 0$ so,

$$\lambda_n = 12(1 - (1 + 2\mu)^n) \tag{19}$$

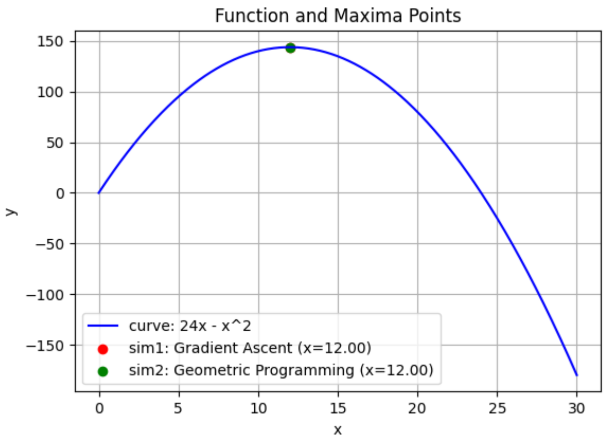
For Radius of convergence , using ratio test , we get ,

$$\mu < 0 \tag{20}$$

Taking initial guess = 0 , step size = 0.01 ,tolerance(minimum value of gradient) = 1e-5 , We get $x_{min} \approx 12$.

Geometric Programming :

We generally use geometric programming for minimize so instead of maximizing $24x - x^2$,we minimize $x^2 - 24x$. We solve it using *cvxpy* module in python. On running the code we, get maximum value at x is, 12.00000, so, maximum product value is 144.00000.



Function and Maxima Points
150
100
50
0
−50
−100
−150
y
0
5
10
15
20
25
30
x
curve: 24x - x^2
sim1: Gradient Ascent (x=12.00)
sim2: Geometric Programming (x=12.00)