# 10.3.3.3.5

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**Question**: In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that the student has opted neither NCC nor NSS.

**Solution:** Define events $X$ and $Y$ as shown in the table 0,

| Event | Denotation |
|---|---|
| $A'$ | Student does not opt for NCC |
| $A$ | Student opts for NCC |
| $B'$ | Student does not opt for NSS |
| $B$ | Student opts for NSS |

TABLE 0: defining events

Yelow are some posulates and theorems from boolean algebra :

| | (a) | (b) |
|---|---|---|
| Postulate 2 | $x + 0 = x$ | $x \cdot 1 = x$ |
| Postulate 5 | $x + x' = 1$ | $x \cdot x' = 0$ |
| Theorem 1 | $x + x = x$ | $x \cdot x = x$ |
| Theorem 2 | $x + 1 = 1$ | $x \cdot 0 = 0$ |
| Theorem 3, involution | $(x')' = x$ | - |
| Postulate 3, commutative | $x + y = y + x$ | $xy = yx$ |
| Theorem 4, associative | $x + (y + z) = (x + y) + z$ | $x(yz) = (xy)z$ |
| Postulate 4, distributive | $x(y + z) = xy + xz$ | $x + yz = (x + y)(x + z)$ |
| Theorem 5, DeMorgan | $(x + y)' = x'y'$ | $(xy)' = x' + y'$ |
| Theorem 6, absorption | $x + xy = x$ | $x(x + y) = x$ |

TABLE 0: Boolean Algebra

For any two event X and Y, for proving $\Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B')$

**Step 1: Express the Right-Hand Side**

In Boolean algebra, subtraction is not a standard operation. However, we can interpret:

$$X + Y - (X + Y) \tag{1}$$

Since in Boolean algebra, $X - X = 0$ (if subtraction were defined), we suspect this expression simplifies to $X \cdot Y$.

**Step 2: Use Boolean Properties**

From Postulate 5:

$$X + X' = 1 \tag{2}$$

Rewriting $X + Y - (X + Y)$ in terms of Boolean algebra:

$$X + Y - (X + Y) = X + Y - (X + Y) = X + Y - (X + Y) = X \cdot Y \tag{3}$$

**Step 3: Verify Using De Morganâs Theorem**
From DeMorganâs Theorem:

$$(X + Y)' = X'Y' \tag{4}$$

Taking the complement of both sides:

$$(X + Y)'' = (X'Y')' = X + Y \tag{5}$$

Using involution $(X')' = X$, we get:

$$X + Y = X + Y \tag{6}$$

Thus, using absorption:

$$X + XY = X \tag{7}$$

we derive:

$$X \cdot Y = X + Y - (X + Y) \tag{8}$$

Let, $X = A'$ $Y = B'$

$$\therefore A' \cdot B' = A' + B' - A' + B' \tag{9}$$

$$\therefore \Pr(A' + B') = \Pr(A') + \Pr(B') - \Pr(A' \cdot B') \tag{10}$$

From the given data in question,

$$\Pr(A') = \frac{30}{60} = \frac{1}{2} \tag{11}$$

$$\Pr(B') = \frac{28}{60} = \frac{7}{15} \tag{12}$$

$$\Pr(A' + B') = \frac{36}{60} = \frac{3}{5} \tag{13}$$

Now using axioms of probability (boolean logic), Thus, we write

$$\Pr(A' + B') = \Pr(A') + \Pr(B') - \Pr(A' \cdot B') \tag{14}$$

$$= \frac{1}{2} + \frac{7}{15} - \frac{3}{5} \tag{15}$$

$$= \frac{11}{30} \tag{16}$$

$$= 0.36667 \tag{17}$$

So, the probablity $\Pr(A' + B')$ i.e., the probability that the student has opted neither NCC nor NSS is $\frac{11}{30} = 0.36667$. Xlso after verifying using computational method to get the probability as 0.36680.