

NCERT 12.9.6.4

EE24BTECH11036 - Krishna Patil

Question: :Solve the following differential equation

$$\frac{dy}{dx} + y \sec x = \tan x \quad (1)$$

with initial conditions :

$$x = 0, \quad y = 1. \quad (2)$$

Solution:

- 1) **Recognize the Linear Form:** This is a linear first-order differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (3)$$

Here:

$$P(x) = \sec x, \quad (4)$$

$$Q(x) = \tan x. \quad (5)$$

- 2) **Find the Integrating Factor (IF):** The integrating factor is given by:

$$\mu(x) = e^{\int P(x)dx} = e^{\int \sec x dx}. \quad (6)$$

The integral of $\sec x$ is:

$$\int \sec x dx = \ln(\sec x + \tan x). \quad (7)$$

Thus:

$$\mu(x) = e^{\ln(\sec x + \tan x)} = (\sec x + \tan x). \quad (8)$$

- 3) **Multiply Through by the Integrating Factor:** Multiply the entire differential equation by $\mu(x)$:

$$(\sec x + \tan x) \frac{dy}{dx} + y(\sec x + \tan x) \sec x = (\sec x + \tan x) \tan x. \quad (9)$$

This simplifies to:

$$\frac{d}{dx} (y(\sec x + \tan x)) = (\sec x + \tan x) \tan x. \quad (10)$$

- 4) **Integrate Both Sides:** Integrate both sides with respect to x :

$$\int \frac{d}{dx} (y(\sec x + \tan x)) dx = \int (\sec x + \tan x) \tan x dx. \quad (11)$$

The integral of $(\sec x + \tan x) \tan x$ simplifies to $\sec x + \tan x - x$:

$$(y (\sec x + \tan x)) = \sec x + \tan x - x + C, \quad (12)$$

where C is the constant of integration.

5) **Solve for y :** Divide through by $(\sec x + \tan x)$:

$$y = \frac{\sec x + \tan x - x + C}{(\sec x + \tan x)}. \quad (13)$$

This is the general solution to the differential equation.

6) **Determining the value of C :** Using initial conditions we can easily determine the value of C

$$1 = \frac{\sec 0 + \tan 0 - x + C}{\sec 0 + \tan 0} \quad (14)$$

$$1 = 1 + c \quad (15)$$

$$\therefore c = 0 \quad (16)$$

$$\therefore y = \frac{\sec x + \tan x - x}{(\sec x + \tan x)} \quad (17)$$

7) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$x_0 = 0 \quad (18)$$

$$y_0 = 1 \quad (19)$$

$$h = 0.001 \quad (20)$$

$$y_{n+1} = y_n + h \cdot (\tan x - y \sec x) \quad (21)$$

$$x_{n+1} = x_n + h \quad (22)$$

Below is verification 7 :

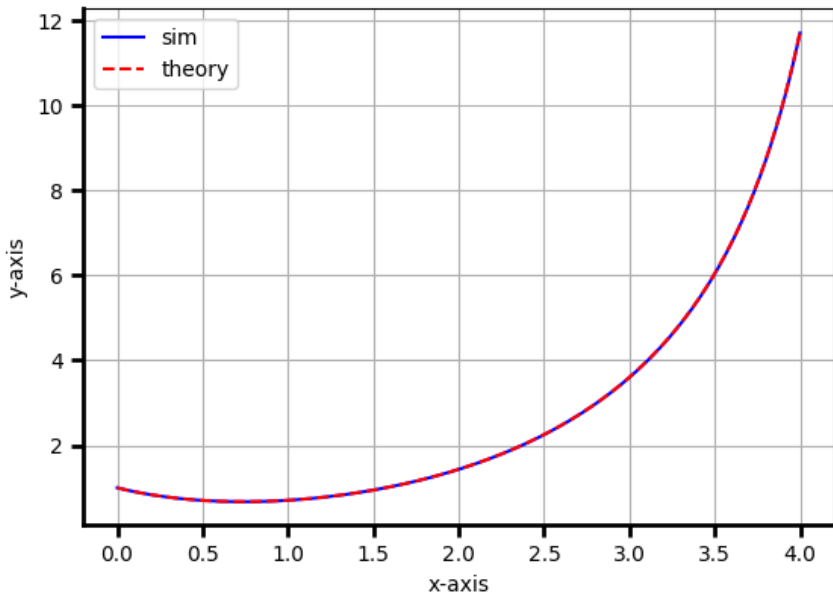


Fig. 7: Verification