NCERT 9.1.4

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Question: Solve the ODE $(\frac{d^2y}{dx^2})^2 + \cos\left(\frac{dy}{dx}\right) = 0$.
Solution: The given equation is:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$$

1) **Reformulate the equation:** Define $v = \frac{dy}{dx}$ and $u = \frac{d^2y}{dx^2}$. The equation becomes:

$$u^2 + \cos(v) = 0.$$

Solving for u, we get:

$$u = \pm\sqrt{-\cos(v)}, \quad \text{valid only for } \cos(v) < 0.$$

Thus, the system of equations is:

$$\frac{dy}{dx} = v, \quad \frac{dv}{dx} = u = \pm\sqrt{-\cos(v)}.$$

2) **Update equations for y:** Using the RK4 method, the update for y is:

$$\begin{aligned}
k_{y,1} &= h \cdot v, \\
k_{y,2} &= h \cdot \left(v + \frac{k_{v,1}}{2}\right), \\
k_{y,3} &= h \cdot \left(v + \frac{k_{v,2}}{2}\right), \\
k_{y,4} &= h \cdot (v + k_{v,3}), \\
y_{n+1} &= y_n + \frac{1}{6}\left(k_{y,1} + 2k_{y,2} + 2k_{y,3} + k_{y,4}\right).
\end{aligned}$$

3) **Update equations for v:** Using the RK4 method for v, we have:

$$\begin{aligned}
k_{v,1} &= h \cdot \sqrt{-\cos(v)}, \\
k_{v,2} &= h \cdot \sqrt{-\cos\left(v + \frac{k_{v,1}}{2}\right)}, \\
k_{v,3} &= h \cdot \sqrt{-\cos\left(v + \frac{k_{v,2}}{2}\right)}, \\
k_{v,4} &= h \cdot \sqrt{-\cos(v + k_{v,3})}, \\
v_{n+1} &= v_n + \frac{1}{6}\left(k_{v,1} + 2k_{v,2} + 2k_{v,3} + k_{v,4}\right).
\end{aligned}$$

4) **Numerical Implementation:** The numerical solution proceeds by alternating updates for y and v, using the equations above. Choose initial values $y(0) = y_0$ and $v(0) = v_0$, and iterate using a step size h.

Summary of the Two-Step RK4 Process: - Update y using $k_{y,1}, k_{y,2}, k_{y,3}, k_{y,4}$. - Update v using $k_{v,1}, k_{v,2}, k_{v,3}, k_{v,4}$.

Note: Ensure that the step size h is small enough to maintain accuracy and stability.

Below is the plot Fig. 4.

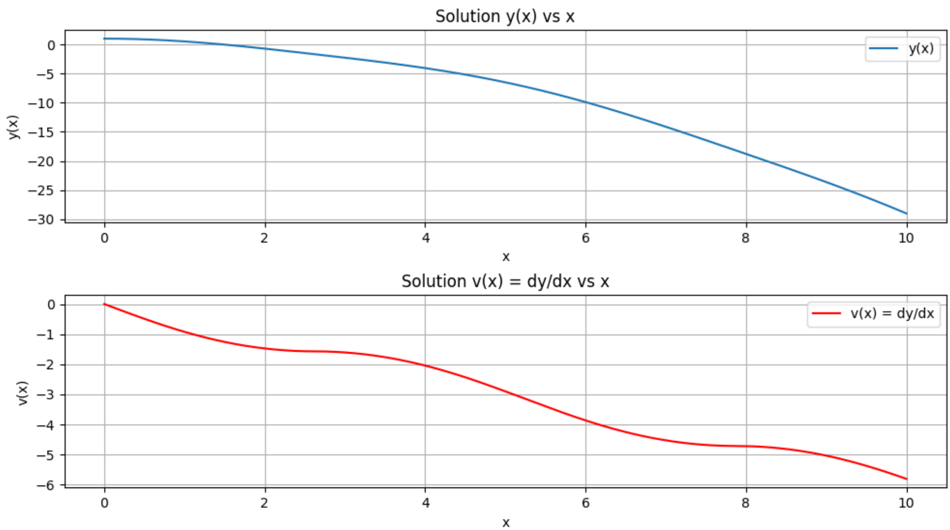

Fig. 4: Verification