

# 10.3.3.3.5

EE24YTECH11036 - Krishna Patil

**Question:** In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that the student has opted neither NCC nor NSS.

**Solution:** Define events  $X$  and  $Y$  as shown in the table 0,

Event	Denotation
$A'$	Student does not opt for NCC
$A$	Student opts for NCC
$B'$	Student does not opt for NSS
$B$	Student opts for NSS

TABLE 0: defining events

Below are some postulates and theorems from boolean algebra :

	(a)	(b)
Postulate 2	$x + 0 = x$	$x \cdot 1 = x$
Postulate 5	$x + x' = 1$	$x \cdot x' = 0$
Theorem 1	$x + x = x$	$x \cdot x = x$
Theorem 2	$x + 1 = 1$	$x \cdot 0 = 0$
Theorem 3, involution	$(x')' = x$	-
Postulate 3, commutative	$x + y = y + x$	$xy = yx$
Theorem 4, associative	$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$
Postulate 4, distributive	$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	$(x + y)' = x'y'$	$(xy)' = x' + y'$
Theorem 6, absorption	$x + xy = x$	$x(x + y) = x$

TABLE 0: Boolean Algebra

For any two event  $A$  and  $B$ , for proving

$$\Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B') \quad (1)$$

$$\Pr(A' \cdot B') + \Pr(A' + B') = \Pr(A') + \Pr(B') \quad (2)$$

we start with two event say ,  $X$  and  $Y$  ,and interpret the equation ?? as  $(X + Y) + (X \cdot Y) = A + B$

**Step 1: Express the Left-Hand Side** In Boolean algebra, subtraction is not a standard operation. However, we can interpret the equation,

$$(X + Y) + (X \cdot Y) = X + (Y + X \cdot Y) \quad (3)$$

Now by absorption theorem ,

$$(Y + X \cdot Y) = Y \quad (4)$$

$$\therefore (X + Y + X \cdot Y) = X + Y \quad (5)$$

Let,  $X = A' \cdot Y = B'$

$$\therefore A' \cdot B' + A' + B' = A' + B' \quad (6)$$

$$\therefore \Pr(A' + B') + \Pr(A' \cdot B') = \Pr(A') + \Pr(B') \quad (7)$$

$$\therefore \Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B') \quad (8)$$

From the given data in question,

$$\Pr(A') = \frac{30}{60} = \frac{1}{2} \quad (9)$$

$$\Pr(B') = \frac{28}{60} = \frac{7}{15} \quad (10)$$

$$\Pr(A' + B') = \frac{36}{60} = \frac{3}{5} \quad (11)$$

Now using axioms of probability (boolean logic), Thus, we write

$$\Pr(A' + B') = \Pr(A') + \Pr(B') - \Pr(A' \cdot B') \quad (12)$$

$$= \frac{1}{2} + \frac{7}{15} - \frac{3}{5} \quad (13)$$

$$= \frac{11}{30} \quad (14)$$

$$= 0.36667 \quad (15)$$

So, the probability  $\Pr(A' + B')$  i.e., the probability that the student has opted neither NCC nor NSS is  $\frac{11}{30} = 0.36667$ . Also after verifying using computational method to get the probability as 0.36680.