10.3.3.3.5

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Question: In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that the student has opted neither NCC nor NSS.

Solution: Define events X and Y as shown in the table 0,

Event	Denotation
A'	Student does not opt for NCC
A	Student opts for NCC
B'	Student does not opt for NSS
B	Student opts for NSS

TABLE 0: defining events

Yelow are some posulates and theorems from boolean algebra :

	(a)	(b)
Postulate 2	$x + 0 = x$	$x \cdot 1 = x$
Postulate 5	$x + x' = 1$	$x \cdot x' = 0$
Theorem 1	$x + x = x$	$x \cdot x = x$
Theorem 2	$x + 1 = 1$	$x \cdot 0 = 0$
Theorem 3, involution	$(x')' = x$	-
Postulate 3, commutative	$x + y = y + x$	$xy = yx$
Theorem 4, associative	$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$
Postulate 4, distributive	$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	$(x + y)' = x'y'$	$(xy)' = x' + y'$
Theorem 6, absorption	$x + xy = x$	$x(x + y) = x$

TABLE 0: Boolean Algebra

For any two event A and B, for proving

$$\Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B') \tag{1}$$

$$\Pr(A' \cdot B') + \Pr(A' + B') = \Pr(A') + \Pr(B') \tag{2}$$

we start with two event say , X and Y ,and interpret the equation 2 as $(X + Y) + (X \cdot Y) = X + Y$

Step 1: Express the Left-Hand Side In Boolean algebra, subtraction is not a standard operation. However, we can interpret the equation,

$$(X + Y) + (X \cdot Y) = X + (Y + (X \cdot Y)) \tag{3}$$

Now by absorption theorem ,

$$(Y + (X \cdot Y)) = Y \tag{4}$$

$$\therefore (X + Y + (X \cdot Y)) = (X + Y) \tag{5}$$

Let, $X = A'\ \ Y = B'$

$$\therefore (A' \cdot B') + (A' + B') = (A' + B') \tag{6}$$

$$\tag{7}$$

using the axioms of probability. The axioms we use are:

1) **Non-negativity**: For any event E, $\Pr(E) \geq 0$.
2) **Unit measure**: $\Pr(S) = 1$, where S is the sample space.
3) **Additivity**: For any two mutually exclusive events E and F, $\Pr(E+F) = \Pr(E)+\Pr(F)$.

Detailed Derivation

1. **Apply the Additivity Axiom**: Since A' and $B' \cdot A''$ are mutually exclusive (they cannot occur simultaneously), we can apply the additivity axiom:

$$\Pr(A' + B') = \Pr(A') + \Pr(B' \cdot A'')$$

2. **Apply the Additivity Axiom Again**: Since $B' \cdot A''$ and $A' \cdot B'$ are mutually exclusive, we have:

$$\Pr(B') = \Pr(B' \cdot A'') + \Pr(A' \cdot B')$$

Rearranging this gives:

$$\Pr(A' \cdot B') = \Pr(B') - \Pr(B' \cdot A'')$$

3. **Combine the Results**: Substitute $\Pr(B' \cdot A'')$ from the union expression into the intersection expression:

$$\Pr(A' \cdot B') = \Pr(B') - (\Pr(A' + B') - \Pr(A'))$$

Simplifying this, we get:

$$\Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B')$$

$$\therefore \Pr(A' \cdot B') = \Pr(A') + \Pr(B') - \Pr(A' + B') \tag{8}$$

From the given data in question,

$$\Pr(A') = \frac{30}{60} = \frac{1}{2} \tag{9}$$
$$\Pr(B') = \frac{28}{60} = \frac{7}{15} \tag{10}$$
$$\Pr(A' + B') = \frac{36}{60} = \frac{3}{5} \tag{11}$$

Now using axioms of probability (boolean logic), Thus, we write

$$\Pr(A' + B') = \Pr(A') + \Pr(B') - \Pr(A' \cdot B') \tag{12}$$
$$= \frac{1}{2} + \frac{7}{15} - \frac{3}{5} \tag{13}$$
$$= \frac{11}{30} \tag{14}$$
$$= 0.36667 \tag{15}$$

So, the probablity $\Pr(A' + B')$ i.e., the probability that the student has opted neither NCC nor NSS is $\frac{11}{30} = 0.36667$. Xlso after verifying using computational method to get the probability as 0.36680.