

Assignment 1

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1) If $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then

Statement 1 : $f(-x)$ is inverse of $f(x)$.

Statement 2 : $f(x+y) = f(x)f(y)$.

- a) Both are true
- b) Both are false
- c) Only statement 1 is true
- d) Only statement 2 is true

2) If $S = \{z \in \mathbb{C} : |z - i| = |z - 1| = |z + i|\}$, then the $n(S)$ is :

- a) 2
- b) 3
- c) 1
- d) 0

3) If $a = \lim_{x \rightarrow 0} \frac{(\sqrt{1+\sqrt{1+x^4}}) - \sqrt{2}}{x^4}$ and $b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, find ab^3

- a) 36
- b) 32
- c) 25
- d) 30

4) Let $\int_0^1 \frac{1}{\sqrt{x+1} + \sqrt{x+3}} dx = a+b\sqrt{2}+c\sqrt{3}$ where a, b, c are rational numbers, then $2a + 3b - 4c$ is equal to :

- a) 10
- b) 8
- c) 4
- d) 7

5) The distance of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-4}{-3} = \frac{z-5}{6}$, is :

- a) 14
- b) 21
- c) 12
- d) 18

6) The length of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose midpoint is $(1, \frac{2}{5})$, is equal to:

- a) $\frac{\sqrt{1691}}{5}$
- b) $\frac{\sqrt{2009}}{5}$
- c) $\frac{\sqrt{1741}}{5}$
- d) $\frac{\sqrt{1541}}{5}$

7) Find number of common terms in the two given series ;
 $4, 9, 14, 19, \dots$ up to 25 terms and $3, 9, 15, 21, \dots$ up to 37 terms

- a) 9
- b) 8
- c) 5
- d) 7

8) If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d ,then d^2 is equal to :

- a) 36 b) 20 c) 16 d) 24

9) Let $S = \{1, 2, 3, \dots, 10\}$. Suppose M is the set of all subsets of S , the relation $R = \{(A, B) : A \cap B \neq \phi; A, B \in M\}$ is :

- a) symmetric and transitive only
b) symmetric only
c) symmetric and reflexive only
d) reflexive only

10) Let $x = x(t)$ and $y = y(t)$ be the solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in \mathbb{R}$. Given that $x(0) = 2$, $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t , for which $x(t) = y(t)$, is :

- a) $\log_3 4$ c) $\log_4 3$
b) $\log_{\frac{4}{3}} 2$ d) $\log_2 2$

11) If ${}^{n-1}C_r = (k^2 - 8){}^nC_{r+1}$, then the range of k is

- a) $\sqrt{2} < k \leq 3$ c) $2 \leq k < 3$
b) $2\sqrt{2} < k < 3$ d) $2\sqrt{2} < k < 8$

12) If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$, is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

- a) 10 b) 5 c) 8 d) 7

13) Let $a = \hat{i} + 2\hat{j} + \hat{k}$, $b = 3(\hat{i} - \hat{j} + \hat{k})$. Let c be the vector such that $a \times c = b$ and $a \cdot c = 3$. Let $a \cdot ((b \times c) - b - c)$ is equal to :

- a) 24 b) 36 c) 32 d) 20

14) If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

- a) $A = B^3$ b) $A = 3B$ c) $B = A^3$ d) $3A = B$

15) Consider the line $L : 4x + 5y = 20$. Let two other lines are L_1 and L_2 which trisect the line L and pass through origin, then tangent of angle between lines L_1 and L_2 is

- a) $\frac{25}{41}$ b) $\frac{30}{41}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$