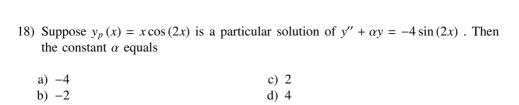
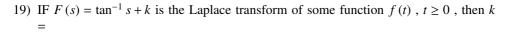
## MA-2007-18-34

1

## EE24BTECH11036 - Krishna Patil





a)  $-\pi$  c) 0 b)  $-\frac{\pi}{2}$  d)  $\frac{\pi}{2}$ 

b) 1

- 20) Let  $S = \{(0,1,1), (1,0,1), (-1,2,1)\} \subseteq \mathbb{R}^3$ . Suppose  $\mathbb{R}^3$  is endowed with the standard inner product  $\langle , \rangle$ . Define  $M = \{x \in \mathbb{R}^3 : \langle x, y \rangle \forall x \in S\}$ . Then the dimension of M is
- a) 0 c) 2
- 21) Let X be an uncountable set and let  $\mathcal{T} = \{U \subseteq X : U = \emptyset \text{ or } U^c \text{ is finite}\}$ . Then the topological space  $(X, \mathfrak{J})$

d) 3

- topological space  $(X,\mathfrak{J})$ 
  - a) is separableb) is Hausdorffc) has a countable basisd) has a countable basis at each point
- 22) Suppose  $(X, \mathfrak{J})$  is a topological space. Let  $\{S_n\}_{n\geq 1}$  be a sequence of subsets of X. Then

a) 
$$(S_1 \cup S_2)^\circ = S_1^\circ \cup S_2^\circ$$
  
b)  $(\bigcup_n S_n)^\circ = \bigcup_n S_n^\circ$   
c)  $\overline{\bigcup_n S_n} = \bigcup_n \overline{S_n}$   
d)  $\overline{S_1 \cup S_2} = \overline{S_1} \cup \overline{S_2}$ 

23) Let (X, d) be a metric space. Consider the metric p on X defined by

$$p(x,y) = \min\left\{1, \frac{1}{2}d(x,y)\right\}, \quad x, y \in X.$$

Suppose  $\mathfrak{J}_1$  and  $\mathfrak{J}_2$  are topologies on X defined by d and p, respectively. Then

- a)  $\mathfrak{J}_1$  is a proper subset of  $\mathfrak{J}_2$
- c) neither  $\mathfrak{J}_1 \subseteq \mathfrak{J}_2$  nor  $\mathfrak{J}_2 \subseteq \mathfrak{J}_1$
- b)  $\mathfrak{J}_2$  is a proper subset of  $\mathfrak{J}_1$
- d)  $\mathfrak{J}_1 = \mathfrak{J}_2$
- 24) A basis of  $V = \{(x, y, z, w) \in \mathbb{R}^4 : x + y z = 0, y + z + w = 0, 2x + y 3z w = 0\}$  is
  - a)  $\{(1,-1,0,0),(0,1,1,1),(2,1,-3,1)\}$  c)  $\{(1,0,1,-1)\}$

b)  $\{(1,-1,0,1)\}$ 

- d)  $\{(1,-1,0,1),(1,0,1,-1)\}$
- 25) Consider  $\mathbb{R}^3$  with the standard inner product. Let  $S = \{(1,1,1),(2,-1,2),(1,-2,1)\}$ For a subset W of  $\mathbb{R}^3$ , let L(W) denote the linear span of W in  $\mathbb{R}^3$ . Then an orthonormal set T with L(S) = L(T) is
  - a)  $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{6}} (1, -2, 1) \right\}$ b)  $\left\{ (1, 0, 0), (0, 1, 0), (0, 0, 1) \right\}$

- c)  $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{2}} (1, -1, 0) \right\}$ d)  $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{2}} (0, 1, -1) \right\}$
- 26) Let A be a  $3\times3$  matrix. Suppose that the eigenvalues of A are -1,0,1 with respective eigenvectors (1, -1, 0)', (1, 1, -2)' and ((1, 1, 1)'. Then 6A equals
  - a)  $\begin{pmatrix} 1 & -3 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ b)  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$

- c)  $\begin{pmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{pmatrix}$ d)  $\begin{pmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$
- 27) Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be a linear transformation defined by T((x,y,z)) =(x + y - z, x + y + z, y - z). Then the matrix of the linear transformation T with respect to the ordered basis  $B = \{(0, 1, 0), (0, 0, 1), (1, 0, 0)\}$  of  $\mathbb{R}^3$  is
  - a)  $\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ b)  $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

c)  $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$  $d) \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ 

- 28) Let  $Y(x) = (y_1(x), y_2(x))^T$  and let

$$A = \begin{pmatrix} -3 & 1 \\ k & -1 \end{pmatrix}$$

Further, let S be the set of values of k for which all the solutions of the system of equations Y'(x) = AY(x) tend to zero as  $x \to \infty$ . Then S is given by

|   | 3  |
|---|--|
| a) $\{k : k \le -1\}$   | c) $\{k: k < -1\}$   |
| b) $\{k : k \le 3\}$  | d) $\{k : k < 3\}$   |
| 29) Let $u(x, y) = f(xe^y) + g(y^2 \cos \theta)$<br>Then the partial differential $\cos \theta$ | (sy), where $f$ and $g$ are infinitely differentiable functions. Equation of minimum order satisfied by $u$ is |
| $a) \ u_{xy} + xu_{xx} = xu_x$  | c) $u_{yy} - xu_{xy} = u_x$  |
| $b) \ u_{xx} + xu_{xx} = u_x$   | $d) \ u_{yy} - xu_{xx} = xu_x$   |

30) Let C be the boundary of the triangle formed by the points (1,0,0), (0,1,0), (0,0,1). Then the value of the line integral

$$\oint_C (-2y)dx + (3x - 4y^2)dy + (z^2 + 3y)dz$$

is

a) 0

c) 2

b) 1

- d) 4
- 31) Let X be a complete metric space and let  $E \subseteq X$ . Consider the following statements:
  - $(S_1)$  E is compact,
  - $(S_2)$  E is closed and bounded,
  - $(S_3)$  E is closed and totally bounded,
  - $(S_4)$  Every sequence in E has a subsequence converging in E. Which one of the above statements does NOT imply all the other statements?
  - a)  $S_1$

c)  $S_3$ 

b)  $S_2$ 

d)  $S_4$ 

32) Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \sin(nx).$$

Then the series

- a) converges uniformly on  $\mathbb{R}$
- b) converges pointwise but NOT uniformly on
- c) converges in  $L^1$  norm to an integrable function on  $[0, 2\pi]$  but does NOT converge uniformly on  $\mathbb{R}$
- d) does NOT converge pointwise
- 33) Let f(z) be an analytic function. Then the value of

$$\int_0^{2\pi} f\left(e^{it}\right) \cos t. dt$$

equals

a) 0

c)  $2\pi f'(0)$ 

b)  $2\pi f(0)$ 

d)  $\pi f'(0)$ 

- 34) Let  $G_1$  and  $G_2$  be the images of the disc  $\{z \in \mathbb{C} : |z+1| < 1\}$  under the transformations  $w = \frac{(1-i)z+2}{(1+i)z+2}$  and  $w = \frac{(1+i)z+2}{(1-i)z+2}$ , respectively. Then
  - a)  $G_1 = \{ w \in \mathbb{C} : \text{Im}(w) < 0 \}$  and  $G_2 = \{ w \in \mathbb{C} : \text{Im}(w) > 0 \}$
  - b)  $G_1 = \{ w \in \mathbb{C} : \text{Im}(w) > 0 \}$  and  $G_2 = \{ w \in \mathbb{C} : \text{Im}(w) < 0 \}$
  - c)  $G_1 = \{ w \in \mathbb{C} : |w| > 2 \}$  and  $G_2 = \{ w \in \mathbb{C} : |w| < 2 \}$
  - d)  $G_1 = \{ w \in \mathbb{C} : |w| < 2 \}$  and  $G_2 = \{ w \in \mathbb{C} : |w| > 2 \}$