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EE24BTECH11036 - Krishna Patil

18) Suppose $y_p(x) = x\cos(2x)$ is a particular solution of $y'' + \alpha y = -4\sin(2x)$. Then the constant α equals

a) -4
b) -2
c) 2
d) 4

19) IF $F(s) = \tan^{-1} s + k$ is the Laplace transform of some function $f(t)$, $t \geq 0$, then k =

a) $-\pi$
b) $-\frac{\pi}{2}$
c) 0
d) $\frac{\pi}{2}$

20) Let $S = \{(0,1,1),(1,0,1),(-1,2,1)\} \subseteq \mathbb{R}^3$. Suppose $\mathbb{R}^3$ is endowed with the standard inner product $\langle , \rangle$. Define $M = \left\{x \in \mathbb{R}^3 : \langle x, y\rangle \forall x \in S\right\}$. Then the dimension of M is

a) 0
b) 1
c) 2
d) 3

21) Let X be an uncountable set and let $\mathcal{T} = \{U \subseteq X : U = \emptyset \text{ or } U^c \text{ is finite}\}$. Then the topological space $(X, \mathcal{T})$

a) is separable
b) is Hausdorff
c) has a countable basis
d) has a countable basis at each point

22) Suppose $(X, \mathfrak{I})$ is a topological space. Let $\{S_n\}_{n\geq 1}$ be a sequence of subsets of X. Then

a) $(S_1 \cup S_2)^\circ = S_1^\circ \cup S_2^\circ$
b) $\left(\bigcup_n S_n\right)^\circ = \bigcup_n S_n^\circ$
c) $\overline{\bigcup_n S_n} = \bigcup_n \overline{S_n}$
d) $\overline{S_1 \cup S_2} = \overline{S_1} \cup \overline{S_2}$

23) Let (X, d) be a metric space. Consider the metric p on X defined by

$$p(x,y) = \min\left\{1, \frac{1}{2}d(x,y)\right\}, \quad x, y \in X.$$

Suppose $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are topologies on X defined by d and p, respectively. Then

a) $\mathfrak{J}_1$ is a proper subset of $\mathfrak{J}_2$
b) $\mathfrak{J}_2$ is a proper subset of $\mathfrak{J}_1$
c) neither $\mathfrak{J}_1 \subseteq \mathfrak{J}_2$ nor $\mathfrak{J}_2 \subseteq \mathfrak{J}_1$
d) $\mathfrak{J}_1 = \mathfrak{J}_2$

24) A basis of $V = \left\{(x, y, z, w) \in \mathbb{R}^4 : x + y - z = 0, y + z + w = 0, 2x + y - 3z - w = 0\right\}$ is

a) $\{(1, -1, 0, 0), (0, 1, 1, 1), (2, 1, -3, 1)\}$
b) $\{(1, -1, 0, 1)\}$
c) $((1, 0, 1, -1))$
d) $\{(1, -1, 0, 1), (1, 0, 1, -1)\}$

25) Consider $\mathbb{R}^3$ with the standard inner product. Let $S = \{(1, 1, 1), (2, -1, 2), (1, -2, 1)\}$ For a subset W of $\mathbb{R}^3$, let $L(W)$ denote the linear span of W in $\mathbb{R}^3$. Then an orthonormal set T with $L(S) = L(T)$ is

a) $\left\{\frac{1}{\sqrt{3}}(1, 1, 1), \frac{1}{\sqrt{6}}(1, -2, 1)\right\}$
b) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
c) $\left\{\frac{1}{\sqrt{3}}(1, 1, 1), \frac{1}{\sqrt{2}}(1, -1, 0)\right\}$
d) $\left\{\frac{1}{\sqrt{3}}(1, 1, 1), \frac{1}{\sqrt{2}}(0, 1, -1)\right\}$

26) Let A be a 3×3 matrix. Suppose that the eigenvalues of A are $-1, 0, 1$ with respective eigenvectors $(1, -1, 0)'$, $(1, 1, -2)'$ and $((1, 1, 1)'$. Then 6A equals

a) $\begin{pmatrix} 1 & -5 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$
b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
c) $\begin{pmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{pmatrix}$
d) $\begin{pmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

27) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation defined by $T((x, y, z)) = (x + y - z, x + y + z, y - z)$.Then the matrix of the linear transformation T with respect to the ordered basis $B = \{(0, 1, 0), (0, 0, 1), (1, 0, 0)\}$ of $\mathbb{R}^3$ is

a) $\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$
b) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$
c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$
d) $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

28) Let $Y(x) = (y_1(x), y_2(x))^T$ and let

$$A = \begin{pmatrix} -3 & 1 \\ k & -1 \end{pmatrix}$$

Further, let S be the set of values of k for which all the solutions of the system of equations $Y'(x) = AY(x)$ tend to zero as $x \rightarrow \infty$. Then S is given by

a) $\{k : k \leq -1\}$
b) $\{k : k \leq 3\}$
c) $\{k : k < -1\}$
d) $\{k : k < 3\}$

29) Let $u(x, y) = f(xe^y) + g\left(y^2 \cos y\right)$,where f and g are infinitely differentiable functions. Then the partial differential equation of minimum order satisfied by u is

a) $u_{xy} + xu_{xx} = xu_x$
b) $u_{xx} + xu_{xx} = u_x$
c) $u_{yy} - xu_{xy} = u_x$
d) $u_{yy} - xu_{xx} = xu_x$

30) Let C be the boundary of the triangle formed by the points (1,0,0), (0,1,0), (0,0,1).Then the value of the line integral

$$\oint_C (-2y)dx + (3x - 4y^2)dy + (z^2 + 3y)dz$$

is

a) 0
b) 1
c) 2
d) 4

31) Let X be a complete metric space and let $E \subseteq X$. Consider the following statements:
(S_1) E is compact,
(S_2) E is closed and bounded,
(S_3) E is closed and totally bounded,
(S_4) Every sequence in E has a subsequence converging in E.
Which one of the above statements does NOT imply all the other statements?

a) S_1
b) S_2
c) S_3
d) S_4

32) Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \sin(nx).$$

Then the series

a) converges uniformly on $\mathbb{R}$
b) converges pointwise but NOT uniformly on
c) converges in L^1 norm to an integrable function on $[0, 2\pi]$ but does NOT converge uniformly on $\mathbb{R}$
d) does NOT converge pointwise

33) Let $f(z)$ be an analytic function. Then the value of

$$\int_0^{2\pi} f\left(e^{it}\right) \cos t.dt$$

equals

a) 0
b) $2\pi f(0)$
c) $2\pi f'(0)$
d) $\pi f'(0)$

34) Let G_1 and G_2 be the images of the disc $\{z \in \mathbb{C} : |z+1| < 1\}$ under the transformations $w = \frac{(1-i)z+2}{(1+i)z+2}$ and $w = \frac{(1+i)z+2}{(1-i)z+2}$, respectively. Then

a) $G_1 = \{w \in \mathbb{C} : \mathrm{Im}(w) < 0\}$ and $G_2 = \{w \in \mathbb{C} : \mathrm{Im}(w) > 0\}$
b) $G_1 = \{w \in \mathbb{C} : \mathrm{Im}(w) > 0\}$ and $G_2 = \{w \in \mathbb{C} : \mathrm{Im}(w) < 0\}$
c) $G_1 = \{w \in \mathbb{C} : |w| > 2\}$ and $G_2 = \{w \in \mathbb{C} : |w| < 2\}$
d) $G_1 = \{w \in \mathbb{C} : |w| < 2\}$ and $G_2 = \{w \in \mathbb{C} : |w| > 2\}$