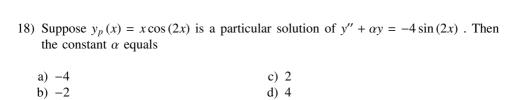
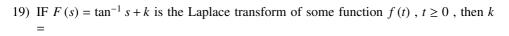
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EE24BTECH11036 - Krishna Patil





a) $-\pi$ c) 0 b) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

b) 1

- 20) Let $S = \{(0,1,1), (1,0,1), (-1,2,1)\} \subseteq \mathbb{R}^3$. Suppose \mathbb{R}^3 is endowed with the standard inner product \langle, \rangle . Define $M = \{x \in \mathbb{R}^3 : \langle x, y \rangle \forall x \in S\}$. Then the dimension of M is
- of *M* is

 a) 0

 c) 2
- 21) Let X be an uncountable set and let $\mathcal{T} = \{U \subseteq X : U = \emptyset \text{ or } U^c \text{ is finite}\}$. Then the topological space (X, \mathfrak{J})

d) 3

- topological space (X, \mathfrak{J})
 - a) is separableb) is Hausdorffc) has a countable basisd) has a countable basis at each point
- 22) Suppose (X,\mathfrak{F}) is a topological space. Let $\{S_n\}_{n\geq 1}$ be a sequence of subsets of X. Then

a)
$$(S_1 \cup S_2)^\circ = S_1^\circ \cup S_2^\circ$$
 c) $\overline{\mid \mid_n S_n} = |\mid_n \overline{S_n}$

a)
$$(S_1 \cup S_2)^\circ = S_1^\circ \cup S_2^\circ$$

b) $(\bigcup_n S_n)^\circ = \bigcup_n S_n^\circ$
c) $\overline{\bigcup_n S_n} = \bigcup_n \overline{S_n}$
d) $\overline{S_1 \cup S_2} = \overline{S_1} \cup \overline{S_2}$

23) Let (X, d) be a metric space. Consider the metric p on X defined by

$$p(x,y) = \min\left\{1, \frac{1}{2}d(x,y)\right\}, \quad x, y \in X.$$

Suppose \mathfrak{J}_1 and \mathfrak{J}_2 are topologies on X defined by d and p, respectively. Then

- a) \mathfrak{J}_1 is a proper subset of \mathfrak{J}_2
- c) neither $\mathfrak{J}_1 \subseteq \mathfrak{J}_2$ nor $\mathfrak{J}_2 \subseteq \mathfrak{J}_1$
- b) \mathfrak{J}_2 is a proper subset of \mathfrak{J}_1
- d) $\mathfrak{J}_1 = \mathfrak{J}_2$
- 24) A basis of $V = \{(x, y, z, w) \in \mathbb{R}^4 : x + y z = 0, y + z + w = 0, 2x + y 3z w = 0\}$ is
 - a) $\{(1,-1,0,0),(0,1,1,1),(2,1,-3,1)\}$ c) $\{(1,0,1,-1)\}$

b) $\{(1,-1,0,1)\}$

- d) $\{(1,-1,0,1),(1,0,1,-1)\}$
- 25) Consider \mathbb{R}^3 with the standard inner product. Let $S = \{(1,1,1),(2,-1,2),(1,-2,1)\}$ For a subset W of \mathbb{R}^3 , let L(W) denote the linear span of W in \mathbb{R}^3 . Then an orthonormal set T with L(S) = L(T) is
 - a) $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{6}} (1, -2, 1) \right\}$ b) $\left\{ (1, 0, 0), (0, 1, 0), (0, 0, 1) \right\}$

- c) $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{2}} (1, -1, 0) \right\}$ d) $\left\{ \frac{1}{\sqrt{3}} (1, 1, 1), \frac{1}{\sqrt{2}} (0, 1, -1) \right\}$
- 26) Let A be a 3×3 matrix. Suppose that the eigenvalues of A are -1,0,1 with respective eigenvectors (1, -1, 0)', (1, 1, -2)' and ((1, 1, 1)'. Then 6A equals
 - a) $\begin{pmatrix} 1 & -3 & 2 \\ 5 & -1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$

- c) $\begin{pmatrix} 1 & 5 & 3 \\ 5 & 1 & 3 \\ 3 & 3 & 3 \end{pmatrix}$ d) $\begin{pmatrix} -3 & 9 & 0 \\ 9 & -3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$
- 27) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by T((x,y,z)) =(x + y - z, x + y + z, y - z). Then the matrix of the linear transformation T with respect to the ordered basis $B = \{(0, 1, 0), (0, 0, 1), (1, 0, 0)\}$ of \mathbb{R}^3 is
 - a) $\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ $d) \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

- 28) Let $Y(x) = (y_1(x), y_2(x))^T$ and let

$$A = \begin{pmatrix} -3 & 1 \\ k & -1 \end{pmatrix}$$

Further, let S be the set of values of k for which all the solutions of the system of equations Y'(x) = AY(x) tend to zero as $x \to \infty$. Then S is given by

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a) $\{k : k \le -1\}$	c) $\{k: k < -1\}$
b) $\{k : k \le 3\}$	d) $\{k : k < 3\}$
29) Let $u(x, y) = f(xe^y) + g(y^2 \cos \theta)$ Then the partial differential $\cos \theta$	(sy), where f and g are infinitely differentiable functions. Equation of minimum order satisfied by u is
$a) \ u_{xy} + xu_{xx} = xu_x$	c) $u_{yy} - xu_{xy} = u_x$
$b) \ u_{xx} + xu_{xx} = u_x$	$d) \ u_{yy} - xu_{xx} = xu_x$

30) Let C be the boundary of the triangle formed by the points (1,0,0), (0,1,0), (0,0,1). Then the value of the line integral

$$\oint_C (-2y)dx + (3x - 4y^2)dy + (z^2 + 3y)dz$$

is

a) 0

c) 2

b) 1

- d) 4
- 31) Let X be a complete metric space and let $E \subseteq X$. Consider the following statements:
 - (S_1) E is compact,
 - (S_2) E is closed and bounded,
 - (S_3) E is closed and totally bounded,
 - (S_4) Every sequence in E has a subsequence converging in E. Which one of the above statements does NOT imply all the other statements?
 - a) S_1

c) S_3

b) S_2

d) S_4

32) Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \sin(nx).$$

Then the series

- a) converges uniformly on \mathbb{R}
- b) converges pointwise but NOT uniformly on
- c) converges in L^1 norm to an integrable function on $[0, 2\pi]$ but does NOT converge uniformly on \mathbb{R}
- d) does NOT converge pointwise
- 33) Let f(z) be an analytic function. Then the value of

$$\int_0^{2\pi} f\left(e^{it}\right) \cos t. dt$$

equals

a) 0

c) $2\pi f'(0)$

b) $2\pi f(0)$

d) $\pi f'(0)$

- 34) Let G_1 and G_2 be the images of the disc $\{z \in \mathbb{C} : |z+1| < 1\}$ under the transformations $w = \frac{(1-i)z+2}{(1+i)z+2}$ and $w = \frac{(1+i)z+2}{(1-i)z+2}$, respectively. Then
 - a) $G_1 = \{ w \in \mathbb{C} : \text{Im}(w) < 0 \}$ and $G_2 = \{ w \in \mathbb{C} : \text{Im}(w) > 0 \}$
 - b) $G_1 = \{ w \in \mathbb{C} : \text{Im}(w) > 0 \}$ and $G_2 = \{ w \in \mathbb{C} : \text{Im}(w) < 0 \}$
 - c) $G_1 = \{ w \in \mathbb{C} : |w| > 2 \}$ and $G_2 = \{ w \in \mathbb{C} : |w| < 2 \}$
 - d) $G_1 = \{ w \in \mathbb{C} : |w| < 2 \}$ and $G_2 = \{ w \in \mathbb{C} : |w| > 2 \}$