9-9.2-5

EE24BTECH11036 - Krishna Patil

Question:
Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Solution:

Equation of a conic in matrix form is :

$$x^\top V x + 2u^\top x + f = 0 \tag{1}$$

For given ellipse,

$$V = \begin{pmatrix} 1 & 0 \\ 0 & \frac{4}{9} \end{pmatrix} \tag{2}$$

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

$$f = -4 \tag{4}$$

$\therefore$ Area of the ellipse is

$$A = 4\int_0^2 3\sqrt{1 - \frac{x^2}{4}}\,dx \tag{5}$$

$$A = 6\pi \tag{6}$$

Parameter	Description	Values
V	$\begin{pmatrix} 1 & 0 \\ 0 & 1-e^2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & \frac{4}{9} \end{pmatrix}$
u	-	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
f	$b^2(e^2-1)$	-4
A	Area under Curve	6π

TABLE I: Parameters Used

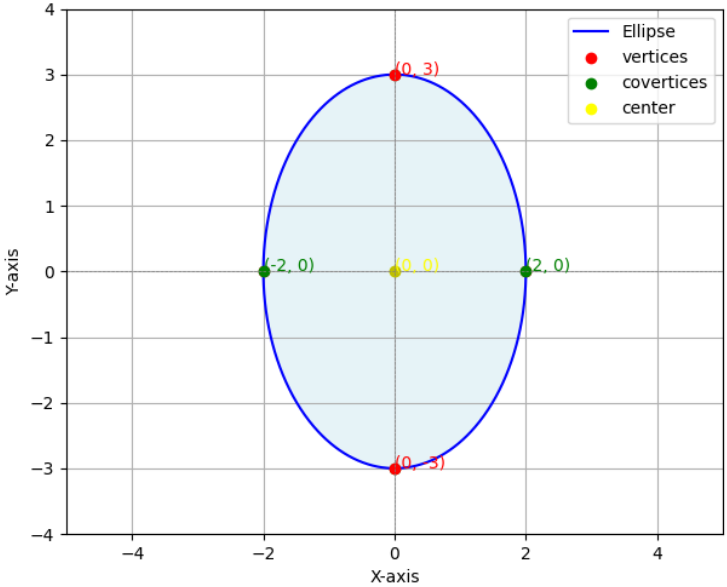

Fig. 1: Plot of ellipse