EE24BTECH11036 - Krishna Patil

Question:

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Solution:

Equation of a conic in matrix form is:

$$x^{\mathsf{T}}Vx + 2u^{\mathsf{T}}x + f = 0 \tag{1}$$

For given ellipse,

$$V = \begin{pmatrix} \frac{4}{9} & 0\\ 0 & 1 \end{pmatrix} \tag{2}$$

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

$$f = -1 \tag{4}$$

:. Area of the ellipse is

$$A = 4 \int_0^2 3\sqrt{1 - \frac{x^2}{4}} \, dx \tag{5}$$

$$A = 6\pi \tag{6}$$

Parameter	Description	Values
V	$\begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \frac{4}{9} & 0 \\ 0 & 1 \end{pmatrix}$
A	Area under Curve	6π

TABLE I: Parameters Used

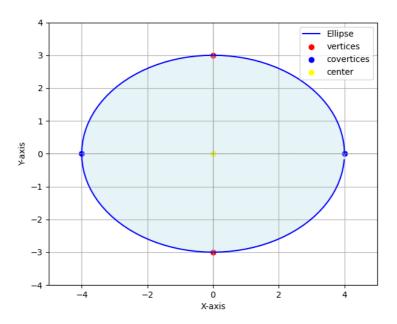


Fig. 1: Plot of ellipse