Assignment 1

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1) If $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then
Statement 1 : $f(-x)$ is inverse of $f(x)$.
Statement 2 :$f(x+y) = f(x)f(y)$.

a) Both are true
b) Both are false
c) Only statement 1 is true
d) Only statement 2 is true

2) If $S = \{z \in \mathbb{C} : |z-i| = |z-1| = |z+i|\}$, then the $n(S)$ is :

a) 2 b) 3 c) 1 d) 0

3) If $a = \lim_{x\to 0} \frac{\left(\sqrt{1+\sqrt{1+x^4}}\right)-\sqrt{2}}{x^4}$ and $b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, find ab^3

a) 36 b) 32 c) 25 d) 30

4) Let $\int_0^1 \frac{1}{\sqrt{x+1}+\sqrt{x+3}}\,dx = a+b\sqrt{2}+c\sqrt{3}$ where a,b,c are rational numbers, then $2a+3b-4c$ is equal to :

a) 10 b) 8 c) 4 d) 7

5) The distance of the point $(7,-2,11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-4}{-3} = \frac{z-5}{6}$, is :

a) 14 b) 21 c) 12 d) 18

6) The length of chord of thw ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose midpoint is $\left(1, \frac{2}{5}\right)$, is equal to:

a) $\frac{\sqrt{1691}}{5}$ b) $\frac{\sqrt{2009}}{5}$ c) $\frac{\sqrt{1741}}{5}$ d) $\frac{\sqrt{1541}}{5}$

7) Find number of common terms in the two given series ;
$4, 9, 14, 19, \cdots$ up to 25 terms and $3, 9, 15, 21, \cdots$ up to 37 terms

a) 9 b) 8 c) 5 d) 7

8) If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2+y^2-4x-16y+64 = 0$ is d ,then d^2 is equal to :

a) 36 b) 20 c) 16 d) 24

9) Let $S = \{1, 2, 3, ..., 10\}$. Suppose M is the set of all subsets of S , the relation $R = \{(A, B) : A \cap B \neq \phi; A, B \in M\}$ is :

a) symmetric and transitive only
b) symmetric only
c) symmetric and reflexive only
d) reflexive only

10) Let $x = x(t)$ and $y = y(t)$ be the solutions of the diffrential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in \mathbb{R}$. Given that $x(0) = 2$, $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t , for which $x(t) = y(t)$, is :

a) $\log_3 4$ c) $\log_4 3$
b) $\log_{\frac{4}{3}} 2$ d) $\log_2 2$

11) If ${}^{n-1}C_r = \left(k^2 - 8\right){}^nC_{r+1}$, then the range of 'k' is

a) $\sqrt{2} < k \leq 3$ c) $2 \leq k < 3)$
b) $2\sqrt{2} < k < 3$ d) $2\sqrt{2} < k < 8$

12) If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$, is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

a) 10 b) 5 c) 8 d) 7

13) Let $a = \hat{i} + 2\hat{j} + \hat{k}$, $b = 3(\hat{i} - \hat{j} + \hat{k})$ · Let c be the vector such that axc=b and $a \cdot c = 3$. Let $a \cdot ((b \text{x} c) - b - c)$ is equal to :

a) 24 b) 36 c) 32 d) 20

14) If A denotes the sum of all the coeeficients in the expansion of $\left(1 - 3x + 10x^2\right)^n$ and B enotes the sum of all the coefficients in the expansion of $\left(1 + x^2\right)^n$, then :

a) $A = B^3$ b) $A = 3B$ c) $B = A^3$ d) $3A = B$

15) Consider the line $L : 4x + 5y = 20$. Let two other lines are L_1 and L_2 which trisect the line L and pass through origin, then tangent of angle between lines L_1 and L_2 is

a) $\frac{25}{41}$ b) $\frac{30}{41}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$