

Lab Report : Transient Response of LC circuits

Analysing the LC circuit response

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*Experiment conducted as part of ELectric Circuits Lab
Coursework.*

Objective

The objective of this experiment is to investigate the transient response of an LC circuit, analyze its oscillatory behavior, determine the natural frequency (Ω_n), and evaluate the damping ratio (ξ) using both theoretical and experimental methods.

Equipment Required

- 100 μF capacitor
- Largest available inductor (denoted as L)
- Small resistor (for optional damping analysis)
- DC power supply
- Digital oscilloscope
- Function generator
- Connecting wires and probes
- Multimeter for component verification

1 Theory

1.1 Mathematical Analysis

An LC circuit follows a second-order differential equation:

$$\frac{d^2V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + \frac{1}{LC} V = 0 \quad (1)$$

For an **ideal LC circuit** (with zero resistance), the equation reduces to:

$$\frac{d^2V}{dt^2} + \frac{1}{LC} V = 0 \quad (2)$$

The general solution is given by:

$$V(t) = V_0 \cos(\Omega_n t + \phi) \quad (3)$$

where:

- $\Omega_n = \frac{1}{\sqrt{LC}}$ is the natural frequency,
- V_0 is the initial voltage, and
- ϕ is the phase angle.

However, in the **real world**, resistance (R) cannot be ignored. This results in a damped response:

$$\frac{d^2V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + \frac{1}{LC} V = 0 \quad (4)$$

The solution depends on the damping ratio $\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$:

- Underdamped ($\xi < 1$):

$$V(t) = V_0 e^{-\xi \Omega_n t} \cos(\Omega_d t + \phi), \quad \Omega_d = \Omega_n \sqrt{1 - \xi^2} \quad (5)$$

- Critically damped ($\xi = 1$):

$$V(t) = (A + Bt)e^{-\Omega_n t} \quad (6)$$

- Overdamped ($\xi > 1$):

$$V(t) = Ae^{-s_1 t} + Be^{-s_2 t}, \quad s_{1,2} = \Omega_n(\xi \pm \sqrt{\xi^2 - 1}) \quad (7)$$

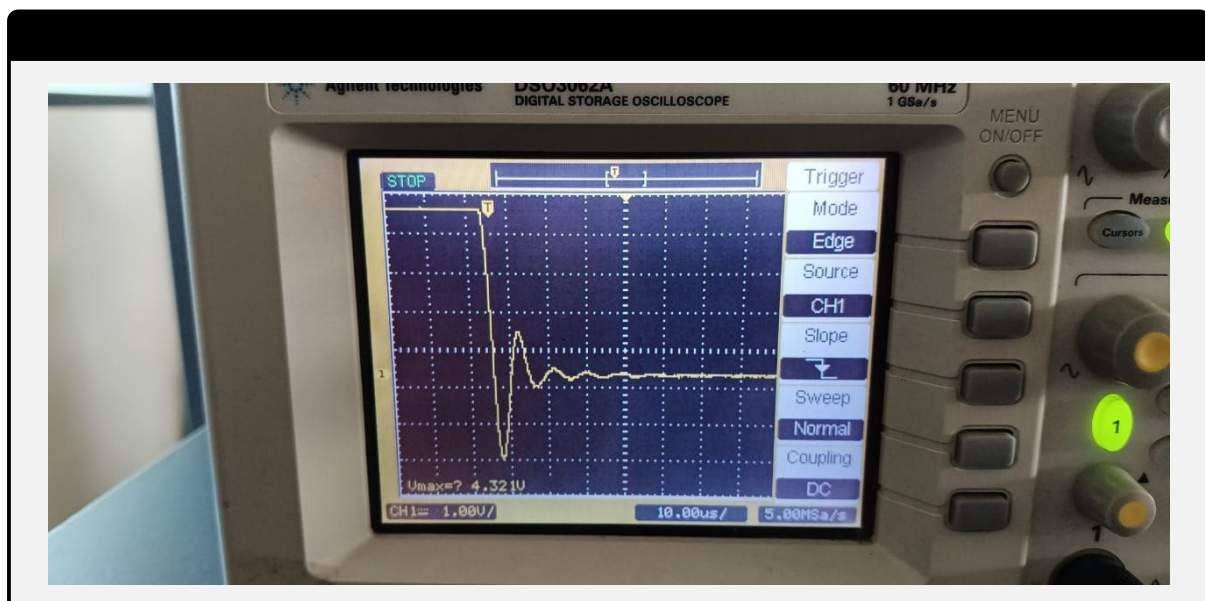
2 Real-World Considerations

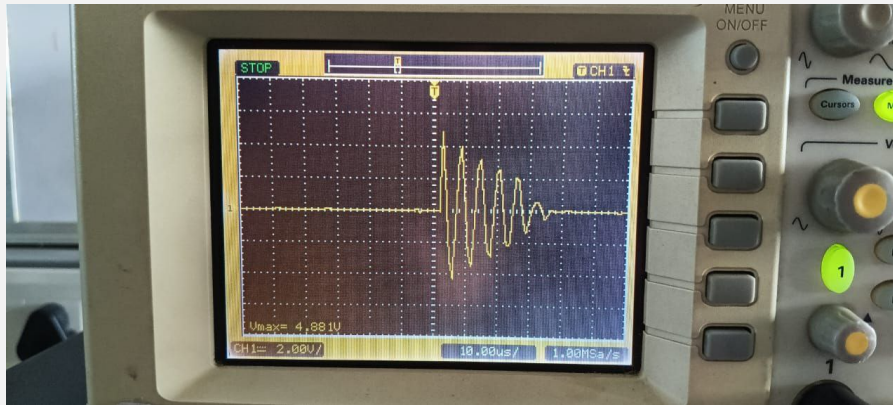
In practical circuits, parasitic resistances from wires, connectors, and components introduce damping. This results in energy loss and prevents perpetual oscillation. The presence of resistance reduces the amplitude over time, transforming the ideal sinusoidal response into a decaying waveform. The real-world transient response is always governed by the damped equation:

$$\frac{d^2 V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + \frac{1}{LC} V = 0 \quad (8)$$

Understanding and accounting for these effects is essential for accurate circuit design and analysis.

3 Images of Responses





Above images depict the transient response captured during the experiment, highlighting differences between ideal and real-world oscillations.

4 Conclusion

The experimentally observed natural frequency closely matched theoretical values. However, real-world resistance caused measurable damping effects, reducing amplitude over time. This experiment validated that while ideal equations predict behavior accurately, practical factors must be accounted for in realistic scenarios.

5 Safety Precautions

- Handle charged capacitors carefully to avoid accidental discharges.
- Ground the oscilloscope properly for accurate and safe measurements.

6 References

- *Transient Response of an LC Circuit.*
- Circuit analysis textbooks and academic literature.