Fractal Dimension of the Diffusion Limited Aggregation Model on and off Lattice

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Contents

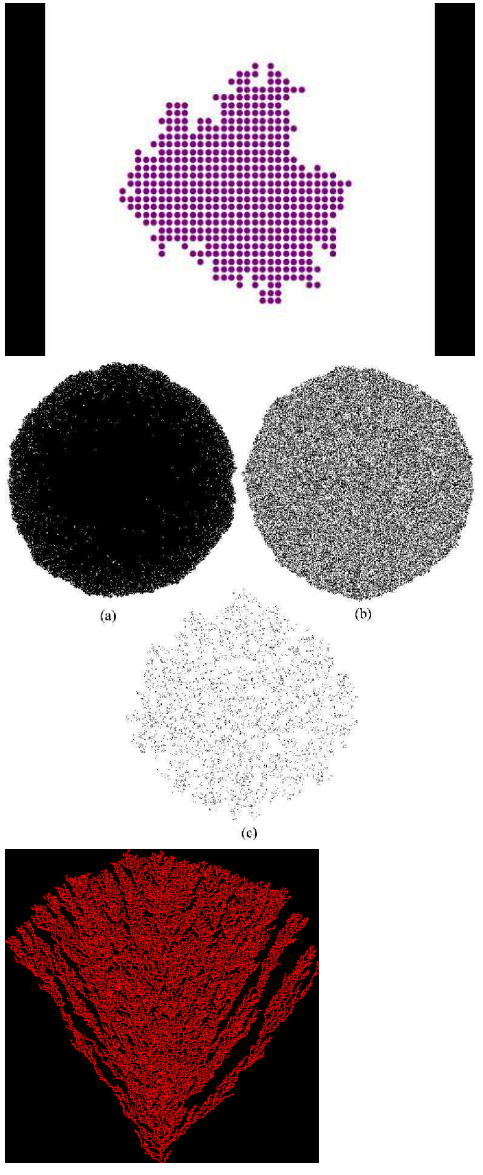
What is Fractal Growth

There are variety of complicated patterns in nature. An enormous amount of these patterns is reproducible by modelling with fractal growth model. Fractal growth processes are a class of phenomena which produce self-similar, disordered objects in the course of development far from equilibrium [7].

Figure: Frost Crystal

Such as . . .

- The Eden model: grow by random accumulation of material on their boundary
- The William and Bjerknes Model: a stochastic model for the spread of cancer cells.
- The Ballistic aggregation models: allows particles to attach themselves to the aggregate if a neighboring lattice point is occupied [1].



Previous Works

The initial model of diffusion-limited aggregation was proposed by Witten and Sander in 1983 [8]. Studies following this fundamental paper can be categorized into several ways:

- Simulation on certain particles or substances using this model, such as Baki and Badr's Electroless, diffusion-limited aggregation of lead dendrites[2]
- Simulation of the diffusion-limited aggregation model on different surfaces, Choi, Crowdy, and Bazant applied this model on curved surfaces [3]
- DLA model in multi-dimensions Sander, Cheng, and Richter used this model in three dimensions [6].
- Investigating some scaling characteristics of the model: I.R. Nogueira, S.G. Alves, S.C. Ferreira explore the Scaling laws in the diffusion-limited aggregation of persistent random walkers [5]

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Contents

Basic On-Lattice Rules

First set of rules is for on-lattice simulation of DLA model, and it is described in Witten and Sander's paper [8].

- Initially, the 2D lattice L is initialized with a cluster containing one particle in the center of the lattice.
- Launch a random particle at a circle of radius r_{launch} which is larger the largest distance from the seed of a particle belonging to the cluster $r_{\text{launch}} = r_{\text{aggr}} + 2$.
- Undergo a random walk steps with each step being UP, DOWN, LEFT, RIGHT with equal probability [4] (See Fig. **??**)

Basic On-Lattice Rules

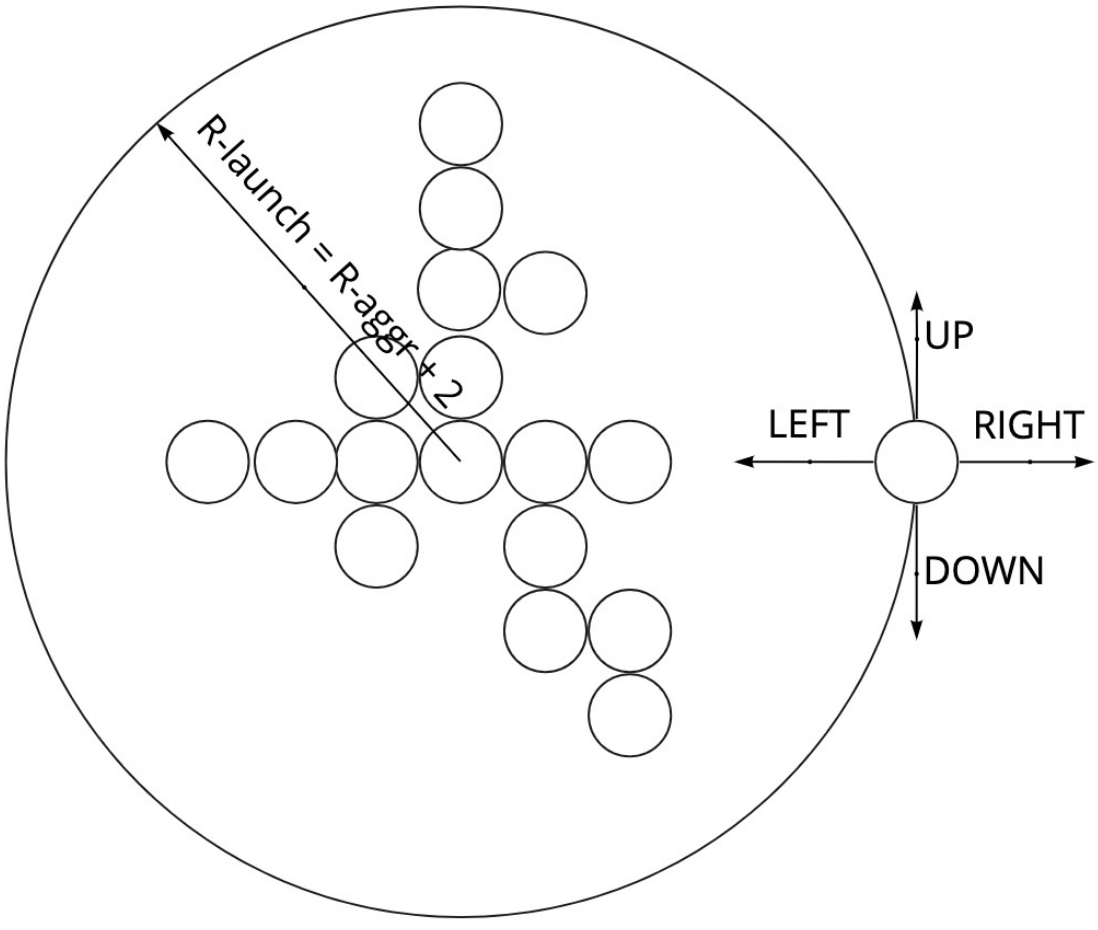

Figure: Random Walk in Diffusion-Limited Aggregation

Basic On-Lattice Rules

- The trajectory is stopped whenever the following happens:
  1. The random walk particle sticks to the cluster when it arrives at a perimeter site of the cluster (The UP, DOWN, LEFT, RIGHT site of random walker is nearest neighbor of site which already belongs to cluster)
  2. The random walk particle is beyond a circle that has 2 times the aggregation size (See Fig. 3).:

$$r_{\mathrm{max}} = r_{\mathrm{aggr}} \times 2 \tag{1}$$

- Steps 2 to 4 will be repeated until the total number of particles in the cluster reaches N particles.

Basic On-Lattice Rules

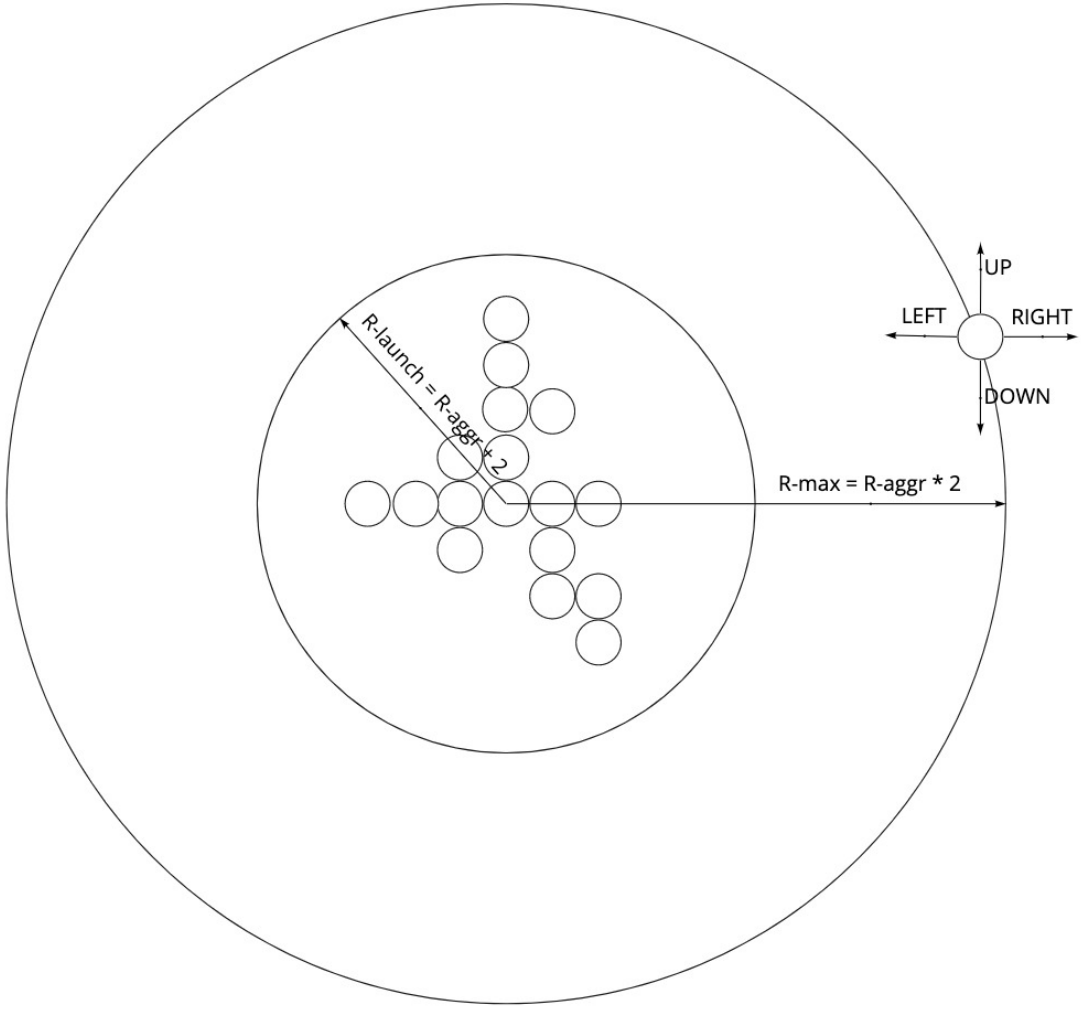

Figure: Beyond the $R_{\max}$

Off-lattice Rules

The second set of rules is for off-lattice simulation of DLA model[5],

- Initially, there is a two dimensional space initialized with a cluster containing one particle of radius r_{p} on the center of this area. (All the particle in the cluster have a radius of r_{p})
- Launch a random particle with radius of r_{p} at a circle of radius greatly larger than the largest distance from the seed of a particle belonging to the cluster $r_{\mathrm{launch}} = 5 \times (r_{\mathrm{aggr}} + 2)$

Off-lattice Rules

- The particle undergoes a random walk, where the position of the n-th step is given by:

$$x_n = x_{n-1} + \alpha \cos \phi_n \tag{2}$$

$$y_n = y_{n-1} + \alpha \sin \phi_n \tag{3}$$

where the direction of the nth step depends on the preceding ones as

$$\phi_n \ = \phi_{n-1} \ + \ \eta_n \tag{4}$$

where η_n is a random variable uniformly distributed in the interval $(-\delta_\theta/2, \delta_\theta/2)$ and the δ_θ limits the next move direction inside an angular opening of size $0 \leq \delta_\theta \leq 2\pi$. The initial value of ϕ_0 is a random angle uniformly distributed n the interval $(0, 2\pi)$.

Off-lattice Rules

- The trajectory is stopped whenever the following happens:
  1. The particle visits a position adjacent to the cluster where it sticks, that is – two particles i and k are neighbors if $|\vec{r}_i - \vec{r}_k| < 2r_{\mathrm{p}}$, where particle i is a member of cluster and k is a random walker particle (See Fig. 4).:
  2. The particle is discarded whenever it crosses a distance $r_{\mathrm{max}} \gg 10 \times (r_{\mathrm{aggr}} + 2)$
- Steps 2 to 4 will be repeated until the total number of particles in cluster reaches N particles.

Off-Lattice Rules

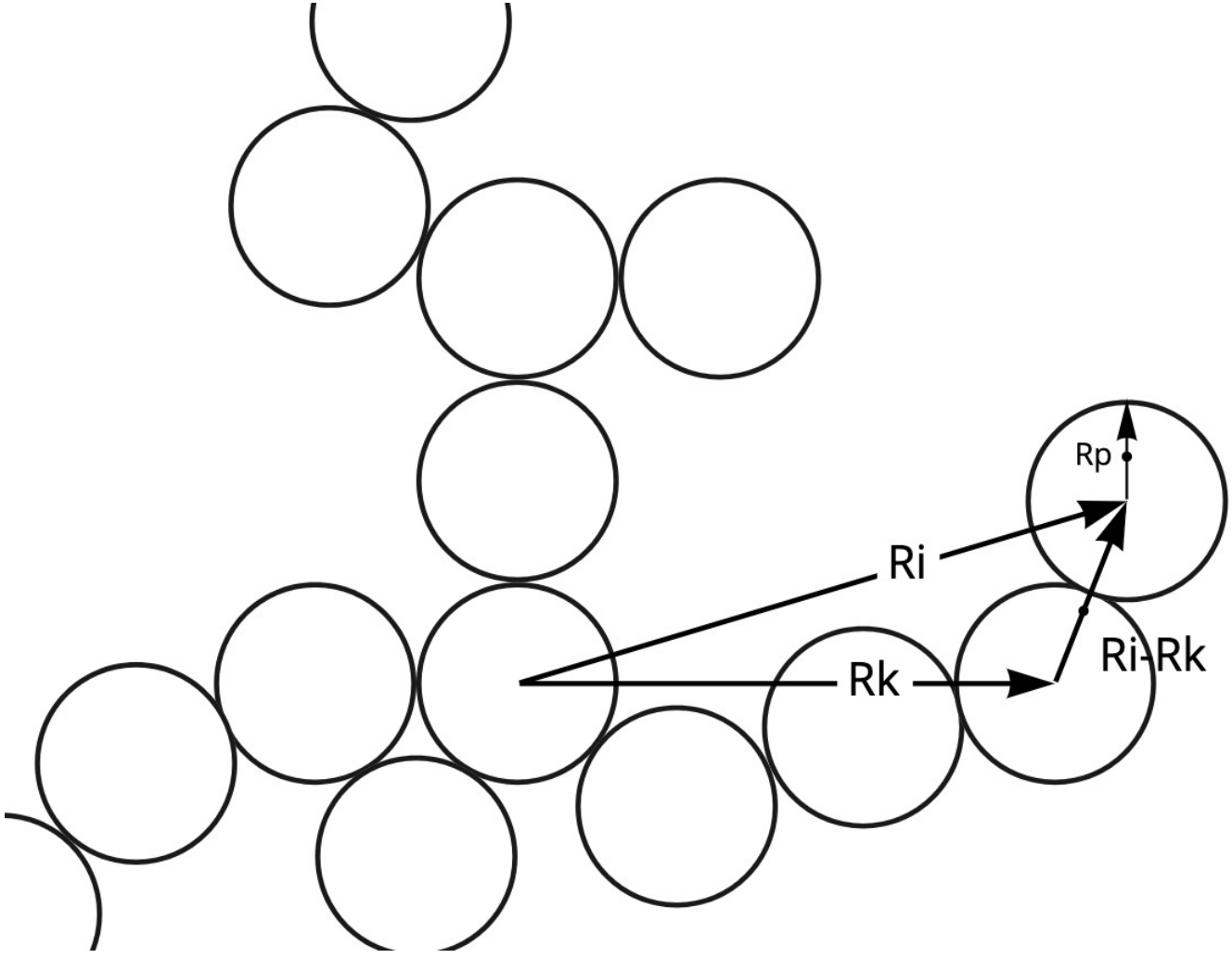

Figure: Off-lattice neighbor

Contents

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