Universidade Federal do Espirito Santo
Digital Image Processing
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1. **What is aliasing and why does it happen? How is it possible to reduce its effects?**

When a continuous signal is sampled at a rate that is too low to capture the changes in the signal accurately, the resulting samples become indistinguishable or appear as aliases of one another.

Aliasing happens when there is a violation of the Nyquist-Shannon sampling theorem, which states that a continuous signal can be accurately reconstructed from its samples if the samples are taken at a rate greater than twice the highest frequency present in the signal (the Nyquist rate).

In order to reduce aliasing, it's necessary to ensure that the sampling rate it's at least twice the highest frequency in the signal or image (Nyquist rate). Also, it's possible to apply anti-aliasing filters as follows: apply a low-pass filter to the continuous signal to remove high-frequency components that cannot be accurately sampled when processing images, there are algorithms that smooth images to reduce high-frequency content that could cause aliasing.

This information can be translated to applying techniques as supersampling, where the image it's set a higher resolution and then downsampled, the other tool would be applying pre-filtering methods to smooth out high-frequency content.

2. **Why We Use DFT for Digital Images Instead of DTFT. What are the implications of applying DFT to digital images?**

First, we need to understand that digital images are inherently discrete and finite in both spatial dimensions and DFT was specifically designed for discrete and finite sequences, making it suitable for digital images. In contrast, DTFT is defined for discrete but infinite sequences, which is not applicable to the finite nature of digital images.

Applying the DFT to digital images has several important implications. The DFT transforms an image from the spatial domain to the frequency domain, enabling the analysis of the image's frequency components. This transformation results in a finite set of frequency components, which simplifies manipulation and analysis. However, DFT assumes that the image is periodic in both spatial dimensions, potentially causing anomalies such as ringing or Gibbs phenomenon, especially in images with sharp edges or high-frequency components. To mitigate these anomalies, windowing or zero-padding can be applied before performing the DFT.

Moreover, using DFT facilitates various image processing tasks, such as filtering, enhancement, and compression. In the frequency domain, it's easier to apply filters like low-pass or high-pass to selectively enhance or suppress specific frequency components.

3. **Refer to the contraharmonic filter given in Section 5.**
   (a) Explain why the filter is effective in elimination pepper noise when Q is positive.

The contraharmonic mean filter is effective in eliminating pepper noise when Q is positive because pepper noise appears as dark pixels (black spots) in an image. When Q is positive, the filter tends to emphasize the brighter pixels within the filter window while diminishing the influence of the dark pixels. The formula for the contraharmonic mean filter is:

$$\tilde{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q+1}}{\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q}}$$

With $Q > 0$, the numerator $\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q+1}$ and the denominator $\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q}$ give more weight to the higher intensity values (brighter pixels). Thus, the filter effectively reduces the impact of dark pixels (pepper noise) within the filter window, resulting in a restored image with reduced pepper noise.

(b) Explain why the filter is effective in eliminating salt noise when Q is negative.

With $Q < 0$, the numerator $\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q+1}$ and the denominator $\sum_{(s,t)\in S_{xy}} g_{(s,t)}^{Q}$ give more weight to the lower intensity values (darker pixels). Thus, the filter effectively reduces the impact of bright pixels (salt noise) within the filter window, resulting in a restored image with reduced salt noise.

(c) Explain why the filter gives poor results (such as the results shown in Section 5) when the wrong polarity is chosen for Q.

The contraharmonic mean filter gives poor results when the wrong polarity is chosen for Q because it emphasizes the wrong type of pixels, thereby failing to properly address the noise type present in the image. If Q is positive in an image corrupted by salt noise (bright pixels), the filter will emphasize the bright pixels, thus failing to eliminate the noise and possibly even amplifying it. Conversely, if Q is negative in an image corrupted by pepper noise (dark pixels), the filter will emphasize the dark pixels, failing to eliminate the noise and possibly amplifying it. This misalignment between the filter parameter and the noise type leads to ineffective noise reduction and poor image restoration quality, as shown in the results discussed in Section 5.

4. **Explain the problem related to the inverse filtering applied for image restoration.**

Here are some problems with the inverse filtering restoration method, considering what is presented in section 5.7 of the book "Digital Image Processing, Third Edition." Initially, the mathematical description of inverse filtering is given by the function:

$$\tilde{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

where $\tilde{F}(u,v)$ represents the recovered image, and $G(u,v)$ is the matrix to which the inverse filtering procedure is applied. This implies that it is necessary to know the function $H(u,v)$ to fully recover the original image. If is unknown, the corrupted image would be represented by

$N(u,v)$, which is unknown to the equation. A function H (degradation function) would be applied and added to the original image F, resulting in the equation:

$$\tilde{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

If the degradation function $H(u,v)$ has values that are **zero or very small**, the term $\frac{N(u,v)}{H(u,v)}$ becomes significant, potentially dominating the estimation $\tilde{F}(u,v)$. This increases the noise component, leading to poor image restoration.

This can be seen, for example, in blur functions, where the function has values very close to zero for certain frequencies. If the terms are regularized for $H(0,0)$, there may be a loss of image information compared to the original image.

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