Universidade Federal do Espirito Santo
2nd List of Digital Image Processing
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Theorical questions

1. **Show with examples why the duality between erosion and dilation, shown in Chapter 9, is true.**

R. Considering the explanations in the reference book, chapter 9, section 9.2, most morphological processes in images are derived from these two primitives. Therefore, it is important to understand that, from the definition of the equations of the mentioned processes, we consider that:

Erosion is shown with the notation:

$$A \ominus B = \{z \mid B_z \subseteq A\}$$

Where B_z is the set *B* translated by *z*.

Where it is defined that a set A by a structuring element B is defined as the set of all points where the structuring element BBB, when translated, is completely contained within A.

And dilation is shown as:

$$A \oplus B = \{z \mid (B \cap A_z) \neq \emptyset\}$$

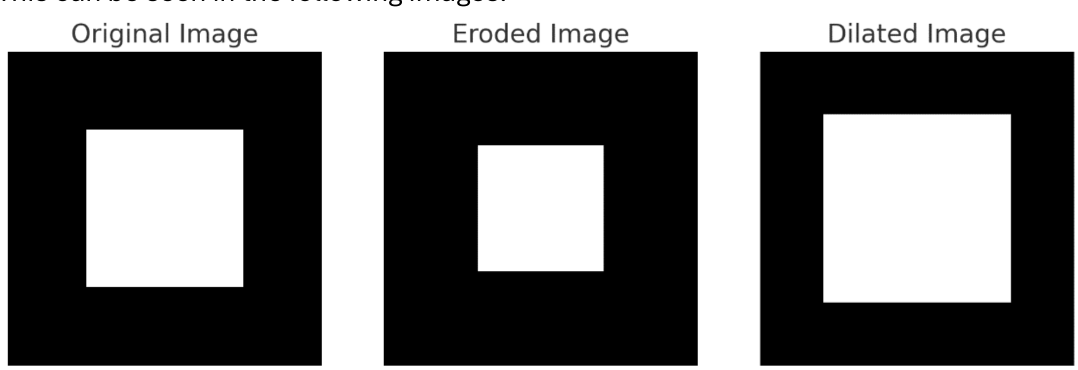
Where A_z is the set *A* translated by *z*.

The set A by a structuring element B is defined as the set of all points where B intersects at least one point of A when B is translated by z.

Therefore, erosion and dilation are dual operations with respect to complementation and reflection. That is, the erosion of a set A by a structuring element B is equal to the complementation of the dilation of the complement of A by the reflected structuring element $\tilde{B}$, and vice versa.

$$(A \ominus B)^c = A^c \oplus \tilde{B}$$
$$and$$
$$(A \oplus B)^2 = A^c \ominus \tilde{B}$$

This can be seen in the following images.



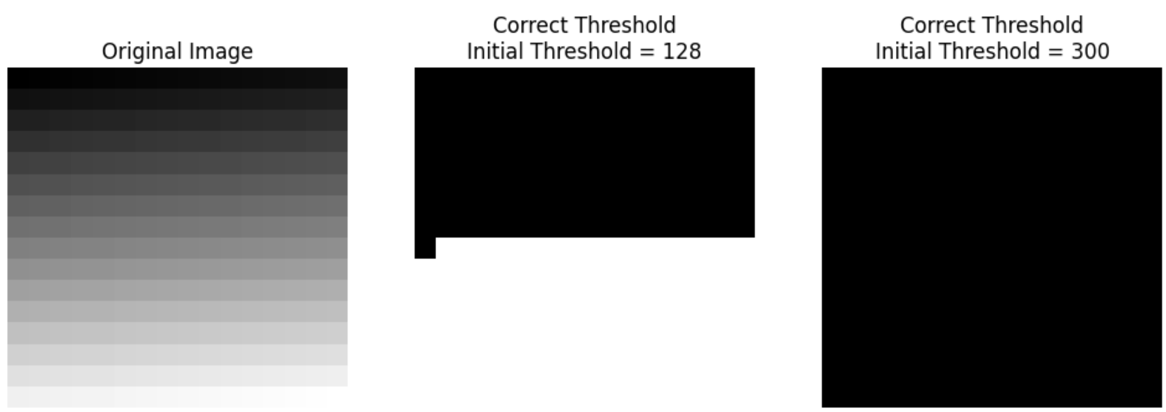
2. **Explain why the initial threshold in the basic global thresholding algorithm must be between the minimum and maximum value of the image. (Hint: Construct an**

example that shows the algorithm setting a selected threshold value outside this range).

R. In the context of the basic global thresholding algorithm, selecting an initial threshold between the minimum and maximum values of the image is crucial to ensure the correct functioning of the algorithm. This requirement is due to the following key reasons:

1. **Definition of Pixel Groups:** The algorithm divides the pixels of the image into two groups: G1, consisting of pixels with intensity values greater than the threshold T, and G2, consisting of pixels with intensity values less than or equal to T. If the initial threshold T is chosen outside the range of intensity values of the image, one of these groups could be empty, making it impossible to calculate the necessary intensity averages to adjust the threshold in subsequent iterations.
2. **Calculation of Intensity Averages:** The algorithm calculates the intensity averages m1 and m2 for the pixels in G1 and G2, respectively. These averages are used to adjust the threshold. If T is less than the minimum value of the image, all pixels will be in G1 and G2 will be empty, making it impossible to calculate m2. If T is greater than the maximum value of the image, all pixels will be in G2 and G1 will be empty, making it impossible to calculate m1.
3. **Algorithm Convergence:** The algorithm relies on iteratively adjusting the threshold until the difference between successive values of T is less than a predefined value ΔT. For the algorithm to correctly adjust the threshold and converge, it is essential that the initial threshold allows a correct initial separation of the pixels into G1 and G2.

This can be seen in the following images.



3. **Explain the region splitting and merging procedure (Section 10.4.2).**

R. The region splitting and merging procedure, described in Section 10.4.2 of the book, involves dividing an image into disjoint regions and then merging and/or splitting these regions to satisfy certain segmentation conditions. Here are the key steps of this procedure:

1. **Region Splitting**:
   - Represent the entire image as a region R and select a predicate Q.
   - The image is successively subdivided into smaller quadrant regions until, for any region R_i, $Q(R_i)$ is true.
   - If Q_i is false, the image is divided into quadrants. If Q is false for any quadrant, that quadrant is further subdivided into sub-quadrants, and so on.
2. **Use of Quadtrees**:
   - This subdivision technique is conveniently represented using "quadtrees," where each node has exactly four descendants.
   - The root of the tree corresponds to the entire image, and each node corresponds to the subdivision of a node into four child nodes.
3. **Region Merging**:
   - If only splitting is used, the final partition typically contains adjacent regions with identical properties.
   - To address this, merging is allowed in addition to splitting. Merging is done between adjacent regions whose combined pixels satisfy the predicate Q.
4. **Complete Procedure**:
   - Split any region R_i into four disjoint quadrants if $Q(R_i)$ is false.
   - When no further splitting is possible, merge any adjacent regions R_i and R_k for which $Q(R_i \cup R_k)$ is true.
   - The process stops when no further merging is possible.

An illustrative example is the segmentation of an image of the Cygnus Loop supernova, where the standard deviation and mean intensity value are used to define the predicate QQQ. In this case, the interested regions were those with a standard deviation greater than the background and a mean intensity greater than the background but less than the center of the dense region.

The basic procedure can vary and be simplified, for example, by allowing the merging of any two adjacent regions that individually satisfy the predicate, resulting in a simpler and faster algorithm.