Name ____ Date ________________

Lesson 1: Ferris Wheels—Tracking the Height of a Passenger Car

Exit Ticket

1. Create a graph of a function that represents the height above the ground of the passenger car for a 225-foot diameter Ferris wheel that completes three turns. Assume passengers board at the bottom of the wheel, which is 5 feet above the ground, and that the ride begins immediately afterward. Provide appropriate labels on the axes.

2. Explain how the features of your graph relate to this situation.

Name ____ Date ____________________

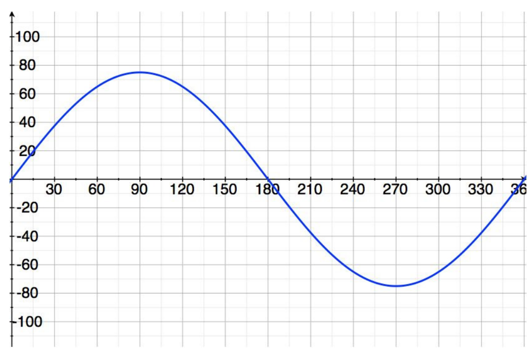
Lesson 2: The Height and Co-Height Functions of a Ferris Wheel

Exit Ticket

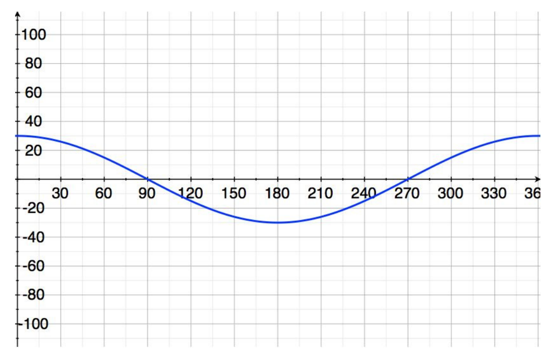
Zeke Memorial Park has two different sized Ferris wheels, one with a radius of 75 feet and one with a radius of 30 feet. Indicate which graph (a)–(d) represents the following functions for the larger and the smaller Ferris wheels. Explain your reasoning.

	Wheel with 75-foot radius		Wheel with 30-foot radius	
Height function:	__________		Height function:	__________
Co-height function:	__________		Co-height function:	__________

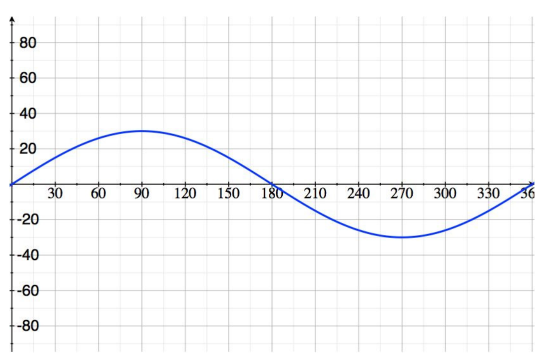
(a)



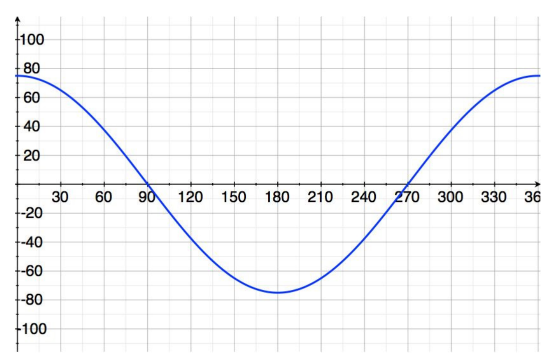
(b)



(c)



(d)



Name ____ Date ____________________

Lesson 3: The Motion of the Moon, Sun, and Stars—Motivating Mathematics

Exit Ticket

1. Explain why counterclockwise is considered to be the positive direction of rotation in mathematics.

2. Suppose that you measure the angle of elevation of your line of sight with the sun to be $67.5°$. If we use the value of 1 astronomical unit (abbreviated AU) as the distance from the earth to the sun, use the portion of the jya table below to calculate the sun's apparent height in astronomical units.

$\boldsymbol{\theta}°$	$\mathbf{jya}(\boldsymbol{\theta})$
$48\frac{3}{4}°$	2585
$52\frac{1}{2}°$	2728
$56\frac{1}{4}°$	2859
$60°$	2978
$63\frac{3}{4}°$	3084
$67\frac{1}{2}°$	3177
$71\frac{1}{4}°$	3256

Name ____ Date ____________________

Lesson 4: From Circle-ometry to Trigonometry

Exit Ticket

1. How did we define the sine function for a number of degrees of rotation θ, where $0 < \theta < 360$?

2. Explain how to find the value of $\sin(210°)$ without using a calculator.

Name ____ Date ____________________

Lesson 5: Extending the Domain of Sine and Cosine to All Real Numbers

Exit Ticket

1. Calculate $\cos(480°)$ and $\sin(480°)$.

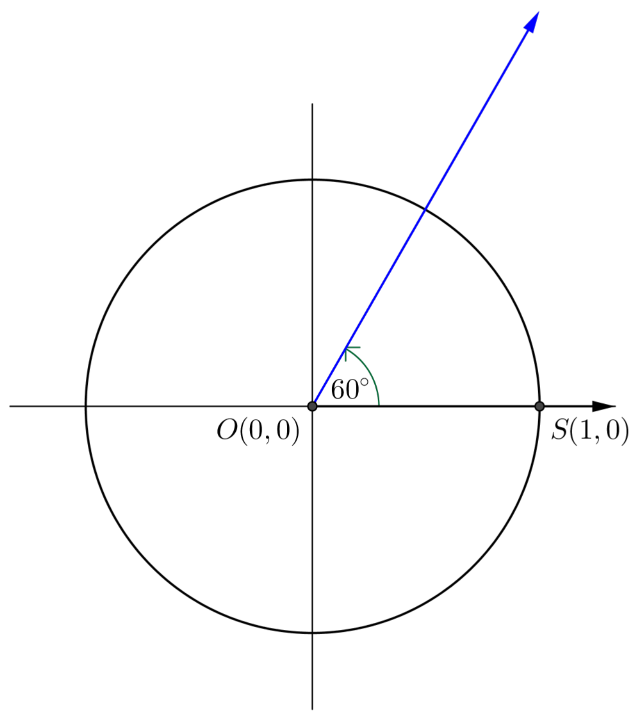
2. Explain how we calculate the sine and cosine functions for a value of θ so that $540 < \theta < 630$.

Name ______________________________ Date ______________

Lesson 6: Why Call It Tangent?

Exit Ticket

Draw and label a figure on the circle below that illustrates the relationship of the trigonometric tangent function $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ and the geometric tangent line to a circle through the point $(1,0)$, when $\theta = 60$. Explain the relationship, labeling the figure as needed.

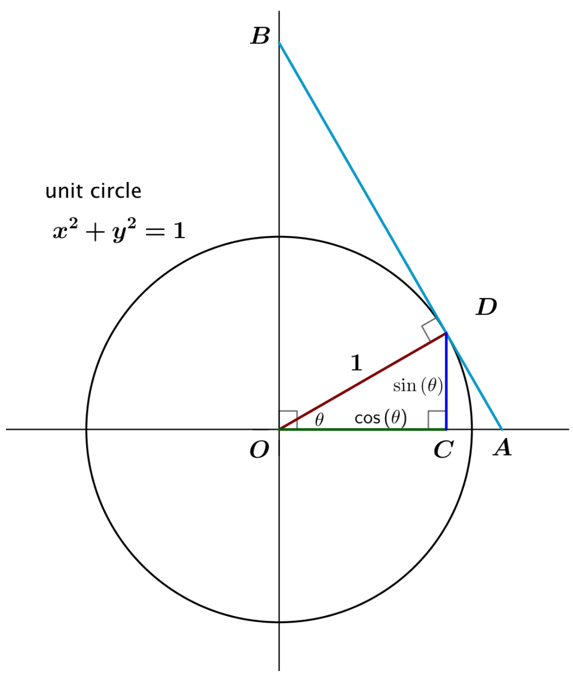


Name ______________________________ Date ________________

Lesson 7: Secant and the Co-Functions

Exit Ticket

Consider the following diagram, where segment $\overline{AB}$ is tangent to the circle at D. Right triangles ΔBAO, ΔBOD, ΔOAD, and ΔODC are similar. Identify each length AD, OA, OB, and BD as one of the following: $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.



Name ____ Date ____________________

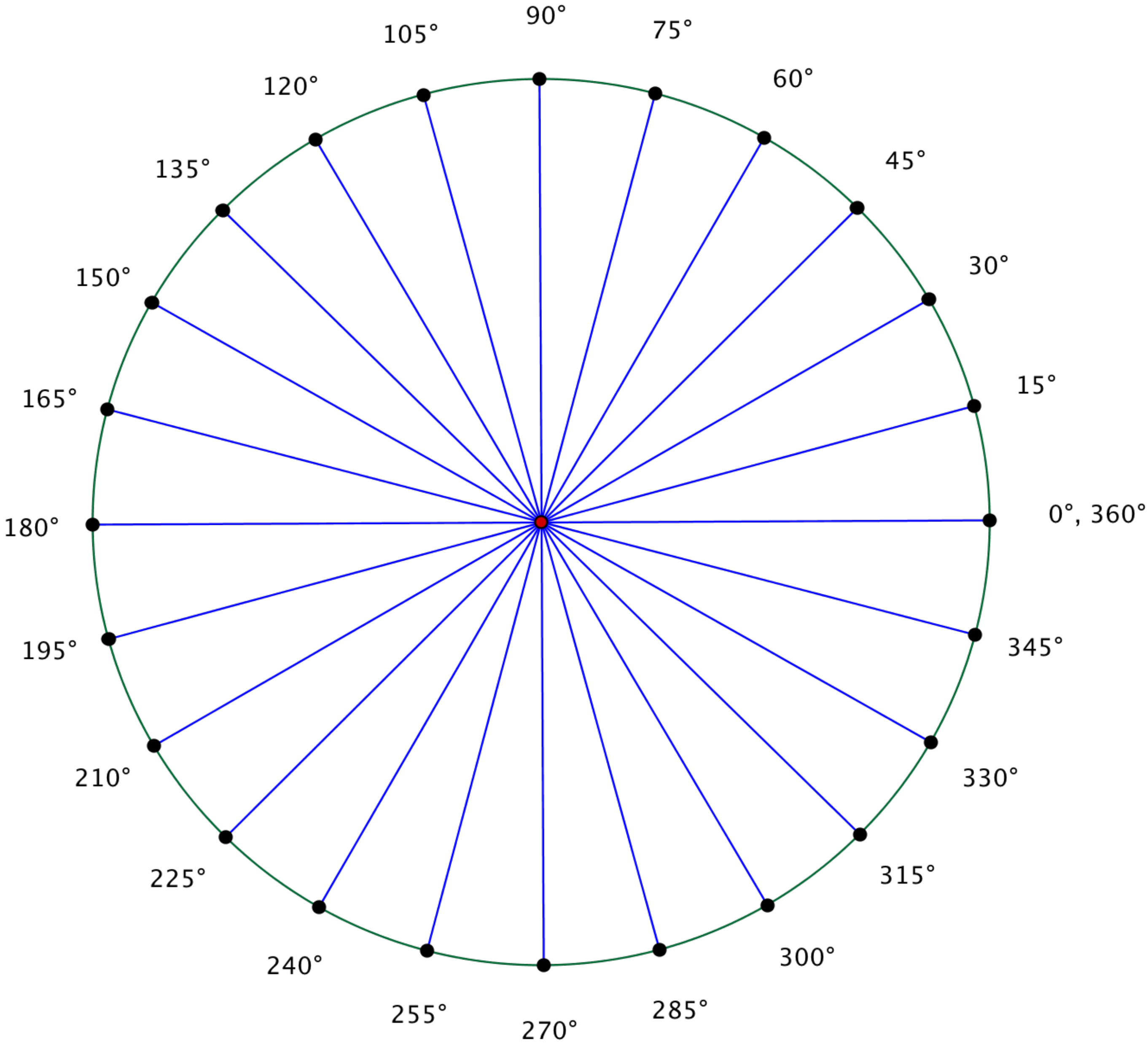
Lesson 8: Graphing the Sine and Cosine Functions

Exit Ticket

1. Sketch a graph of the sine function on the interval $[0, 360]$ showing all key points of the graph (horizontal and vertical intercepts and maximum and minimum points). Mark the coordinates of the maximum and minimum points and the intercepts.

2. Sketch a graph of the cosine function on the interval $[0, 360]$ showing all key points of the graph (horizontal and vertical intercepts and maximum and minimum points). Mark the coordinates of the maximum and minimum points and the intercepts.

Exploratory Challenge Unit Circle Diagram



Name ______________________________ Date ________________

Lesson 9: Awkward! Who Chose the Number 360, Anyway?

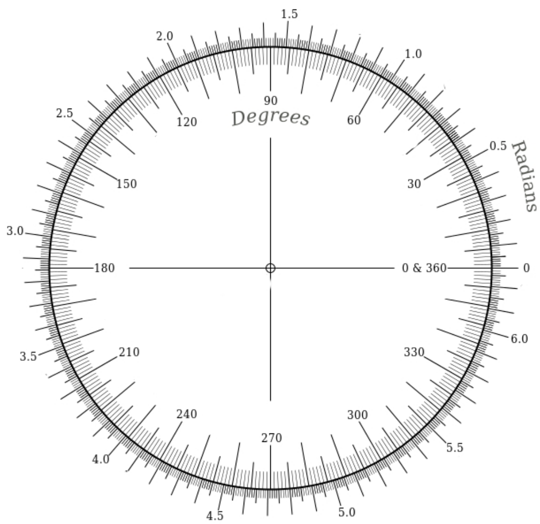
Exit Ticket

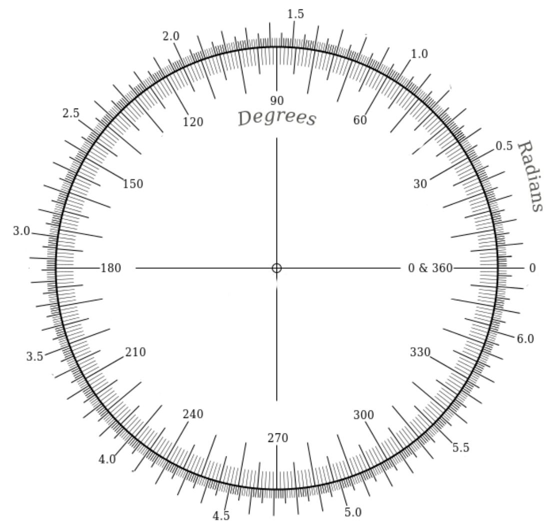
1. Convert $60°$ to radians.

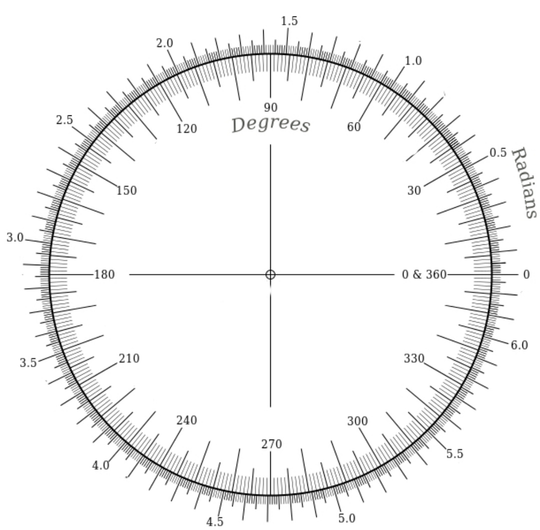
2. Convert $-\frac{\pi}{2}$ rad to degrees.

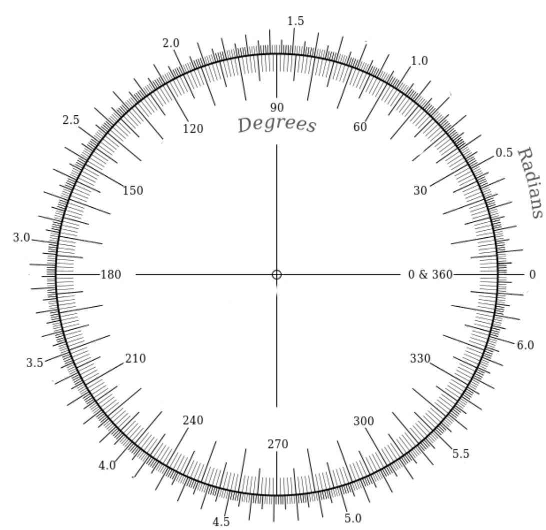
3. Explain how radian measure is related to the radius of a circle. Draw and label an appropriate diagram to support your response.

Supplementary Transparency Materials









Name ____ Date ____________________

Lesson 10: Basic Trigonometric Identities from Graphs

Exit Ticket

1. Demonstrate how to evaluate $\cos\left(\frac{8\pi}{3}\right)$ by using a trigonometric identity. Explain how you used the identity.

2. Determine if the following statement is true or false, without using a calculator.

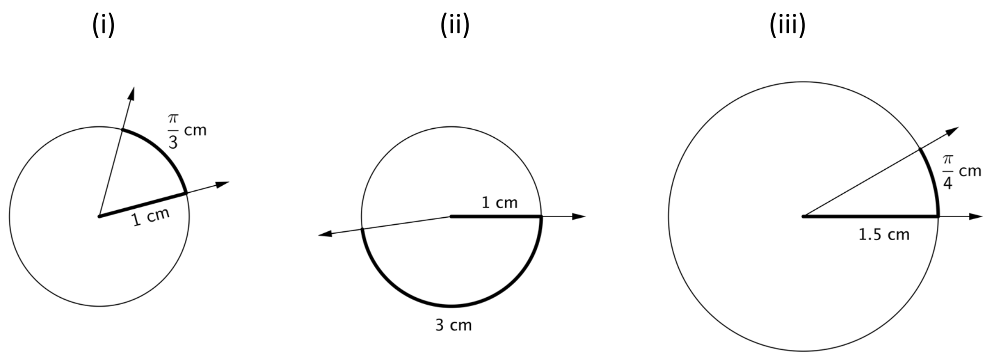
$$\sin\left(\frac{8\pi}{7}\right) = \sin\left(\frac{\pi}{7}\right)$$

3. If the graph of the cosine function is translated to the right $\frac{\pi}{2}$ units, the resulting graph is that of the sine function, which leads to the identity: For all x, $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$. Write another identity for $\sin(x)$ using a different horizontal shift.

Name ______________________________ Date ____________________

1.

a. For each arc indicated below, find the degree measure of its subtended central angle to the nearest degree. Explain your reasoning.



b. Elmo drew a circle with a radius of 1 cm. He drew two radii with an angle of $60°$ between them and then declared that the radian measure of that angle was $\frac{\pi}{3}$ cm. Explain why Elmo is <u>not</u> correct in saying this.

c. Elmo next drew a circle with a radius of 1 cm. He drew two radii that formed a $60°$ angle and then declared that the radian measure of that angle is $\frac{10\pi}{3}$. Is Elmo correct? Explain your reasoning.

d. Draw a diagram that illustrates why $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ using the unit circle. Explain how the unit circle helps us to make this calculation.

2. For each part, use your knowledge of the definition of radians and the definitions of sine, cosine, and tangent to place the expressions in order from least to greatest without using a calculator. Explain your reasoning.

   a. $\sin(1°)$ $\sin(1)$ $\sin(\pi)$ $\sin(60°)$

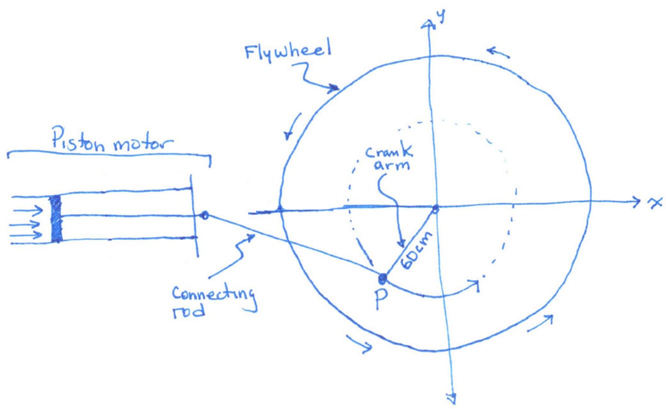
   b. $\sin(25°)$ $\cos(25°)$ $\sin\left(\frac{3\pi}{8}\right)$ $\cos\left(\frac{3\pi}{8}\right)$

c. $\sin(100°)$ $\cos(15°)$ $\tan(15°)$ $\tan(100°)$

d. $\sin(x)$ $\sin\left(x - \frac{3\pi}{2}\right)$ $\sin\left(\frac{11\pi}{4} + x\right)$ $\sin\left(\frac{109\pi}{107}\right)$

where x is a very small positive number with $x < 0.01$

3. An engineer was asked to design a powered crank to drive an industrial flywheel for a machine in a factory. To analyze the problem, she sketched a simple diagram of the piston motor, connecting rod, and crank arm attached to the flywheel, as shown below.



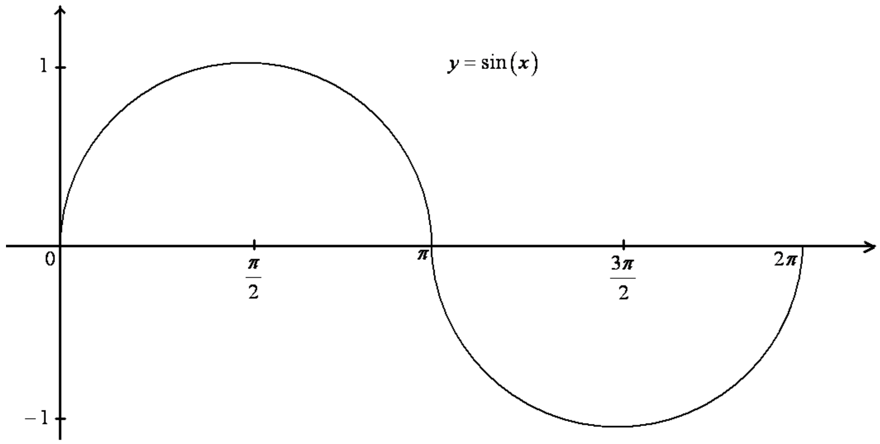
To make her calculations easier, she drew a coordinate axes with origin at the center of the flywheel, and she labeled the joint where the crank arm attaches to the connecting rod by the point P. As part of the design specifications, the crank arm is 60 cm in length, and the motor spins the flywheel at a constant rate of 100 revolutions per minute.

a. With the flywheel spinning, how many radians will the crank arm/connecting rod joint rotate around the origin over a period of 4 seconds? Justify your answer.

b. With the flywheel spinning, suppose that the joint is located at point $P_0 = (0,60)$ at time $t = 0$ seconds; i.e., the crank arm and connecting rod are both parallel to the x-axis. Where will the joint be located 4 seconds later?

4. When plotting the graph of $y = \sin(x)$, with x measured in radians, Fanuk draws arcs that are semicircles. He argues that semicircles are appropriate because, in his words, "Sine is the height of a point on a circle."

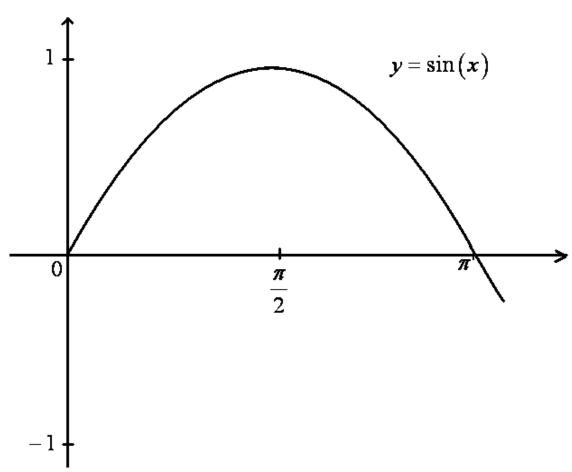
Here is a picture of a portion of his incorrect graph.



Fanuk claims that the first semicircular arc comes from a circle with center $\left(\frac{\pi}{2}, 0\right)$.

a. Explain why Fanuk's claim is incorrect.

JoJo knows that the arcs in the graph of the sine function are not semicircles, but she suspects each arc might be a section of a parabola.



b. Write down the equation of a quadratic function that crosses the x-axis at $x = 0$ and $x = \pi$, and has vertex $\left(\frac{\pi}{2}, 1\right)$.

c. Does the arc of a sine curve between $x = 0$ and $x = \pi$ match your quadratic function for all values between $x = 0$ and $x = \pi$? Is Jojo correct in her suspicions about the shape of these arcs? Explain.

5.

a. Graph the function $f(x) = 3\cos(2x) + 1$ between 0 and 2π.

b. Graph and label the midline on your graph. Draw and label a segment to represent the period and specify its length.

c. Explain how you can find the midline, period, and amplitude in part (b) from the function $f(x) = 3\cos(2x) + 1$.

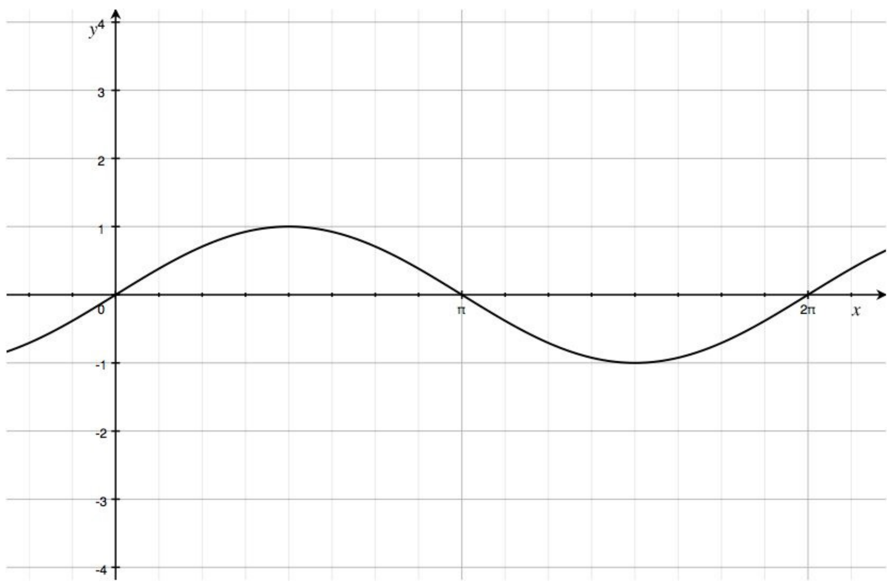
d. Construct a periodic function that has period 8π, a midline given by the equation $y = 5$, and an amplitude of $\frac{1}{2}$.

Name ____ Date ______________

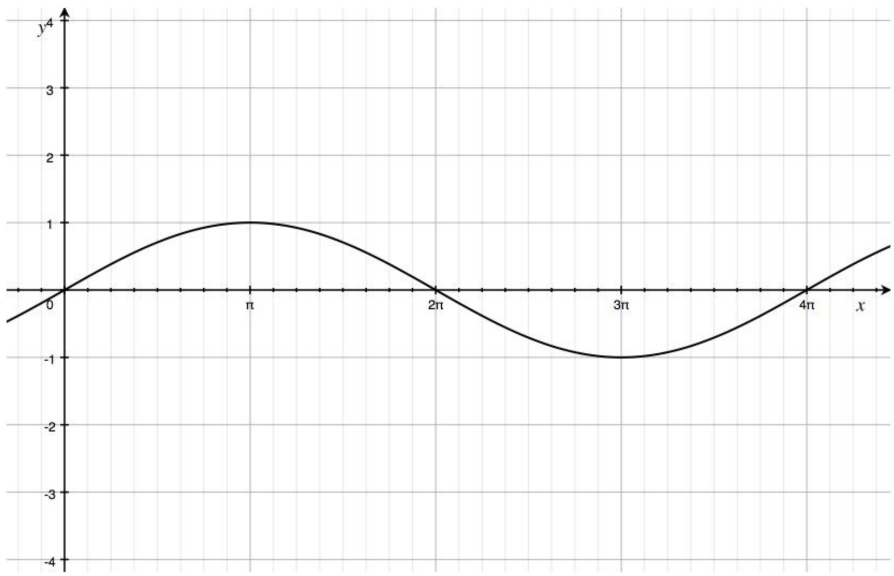
Lesson 11: Transforming the Graph of the Sine Function

Exit Ticket

1. Given the graph of $y = \sin(x)$ below, sketch the graph of the function $f(x) = \sin(4x)$ on the same set of axes. Explain the similarities and differences between the two graphs.



2. Given the graph of $y = \sin\left(\frac{x}{2}\right)$ below, sketch the graph of the function $g(x) = 3\sin\left(\frac{x}{2}\right)$ on the same set of axes. Explain the similarities and differences between the two graphs.

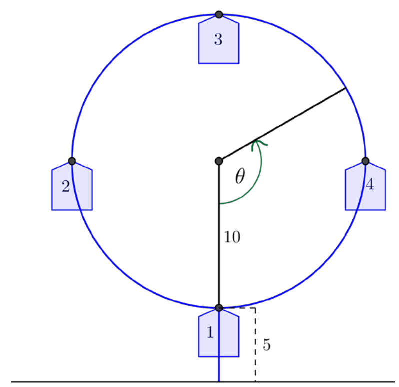


Name ____ Date ______________

Lesson 12: Ferris Wheels—Using Trigonometric Functions to Model Cyclical Behavior

Exit Ticket

The Ferris Wheel Again



In an amusement park, there is a small Ferris wheel, called a kiddie wheel, for toddlers. We will use the points on the circle in the diagram at right to represent the position of the cars on the wheel. The kiddie wheel has four cars, makes one revolution every minute, and has a diameter of 20 feet. The distance from the ground to a car at the lowest point is 5 feet. Assume $t = 0$ corresponds to a time when car 1 is closest to the ground.

1. Sketch the height function for car 1 with respect to time as the Ferris wheel rotates for two minutes.

2. Find a formula for a function that models the height of car 1 with respect to time as the kiddie wheel rotates.

3. Is your equation in Question 1 the only correct answer? Explain how you know.

Name ______________________________ Date ______________

Lesson 13: Tides, Sound Waves, and Stock Markets

Exit Ticket

Tidal data for New Canal Station, located on the shore of Lake Pontchartrain, LA, and Lake Charles, LA are shown below.

New Canal Station on Lake Pontchartrain, LA Tide Chart

Date	Day	Time	Height	High/Low
2014/05/28	Wed.	07:22 a.m.	0.12	L
2014/05/28	Wed.	07:11 p.m.	0.53	H
2014/05/29	Thurs.	07:51 a.m.	0.11	L
2014/05/29	Thurs.	07:58 p.m.	0.53	H

Lake Charles, LA Tide Chart

Date	Day	Time	Height	High/Low
2014/05/28	Wed.	02:20 a.m.	-0.05	L
2014/05/28	Wed.	10:00 a.m.	1.30	H
2014/05/28	Wed.	03:36 p.m.	0.98	L
2014/05/28	Wed.	07:05 p.m.	1.11	H
2014/05/29	Thurs.	02:53 a.m.	-0.06	L
2014/05/29	Thurs.	10:44 a.m.	1.31	H
2014/05/29	Thurs.	04:23 p.m.	1.00	L
2014/05/29	Thurs.	07:37 p.m.	1.10	H

1. Would a sinusoidal function of the form $f(x) = A\sin\big(\omega(x-h)\big) + k$ be appropriate to model the given data for each location? Explain your reasoning.

2. Write a sinusoidal function to model the data for New Canal Station.

Name ____ Date ____________________

Lesson 14: Graphing the Tangent Function

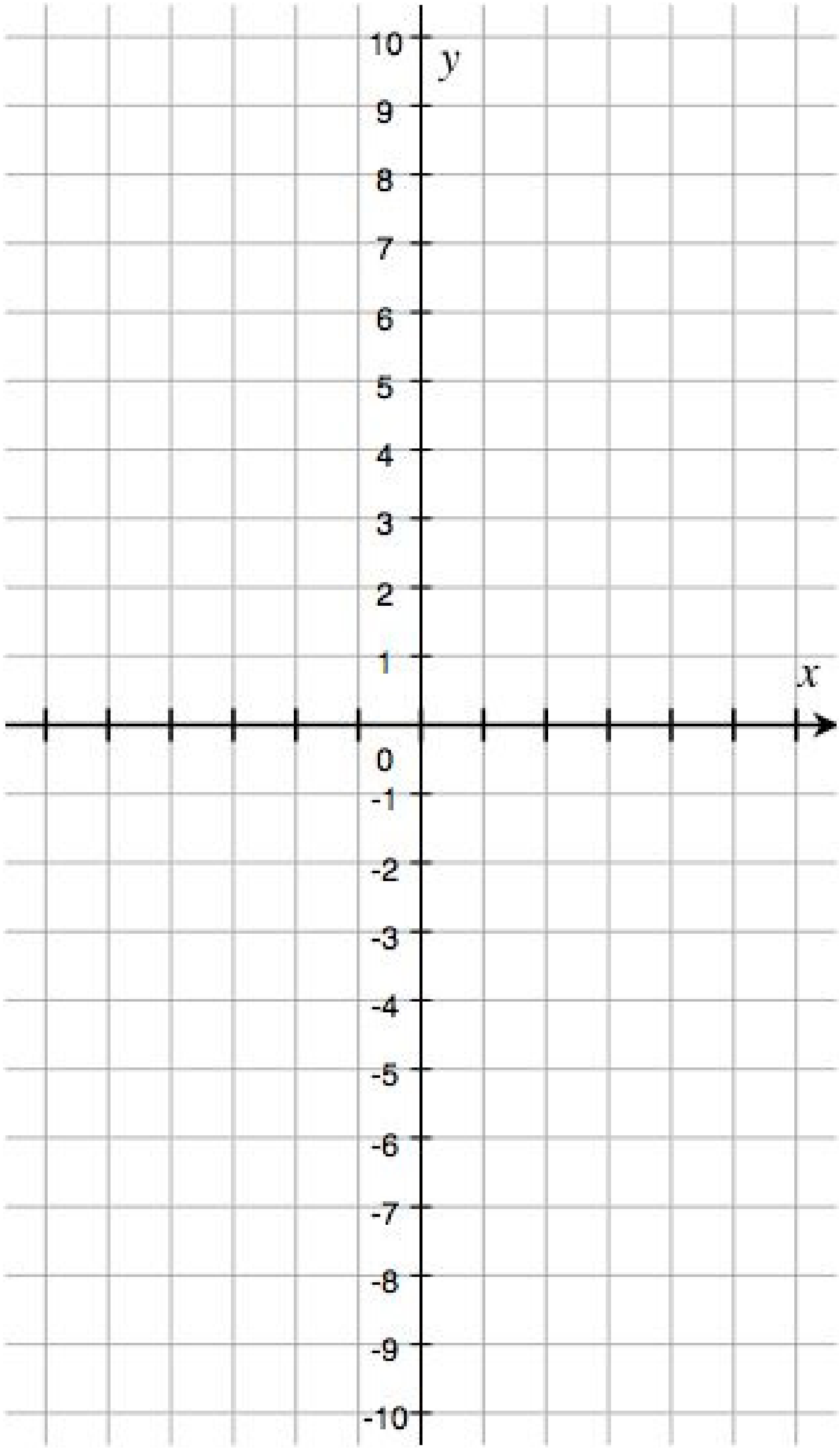
Exit Ticket

1. Use a tangent identity to explain why the tangent function is periodic.

2. Given $\tan(x) = 7$, find the following function values:
   a. $\tan(\pi - x)$

   b. $\tan(\pi + x)$

   c. $\tan(2\pi - x)$


10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
y
x

Name ____ Date ____________________

Lesson 15: What Is a Trigonometric Identity

Exit Ticket

April claims that $1+\frac{\cos^2(\theta)}{\sin^2(\theta)}=\frac{1}{\sin^2(\theta)}$ is an identity for all real numbers θ that follows from the Pythagorean identity.

a. For which values of θ are the two functions $f(\theta)=1+\frac{\cos^2(\theta)}{\sin^2(\theta)}$ and $g(\theta)=\frac{1}{\sin^2(\theta)}$ defined?

b. Show that $1+\frac{\cos^2(\theta)}{\sin^2(\theta)}=\frac{1}{\sin^2(\theta)}$ follows from the Pythagorean identity.

c. Is April correct? Explain why or why not.

d. Write the equation $1+\frac{\cos^2(\theta)}{\sin^2(\theta)}=\frac{1}{\sin^2(\theta)}$ in terms of other trigonometric functions.

Name ______________________________ Date ______________

Lesson 16: Proving Trigonometric Identities

Exit Ticket

Prove the following identity:

$\tan(\theta)\sin(\theta) + \cos(\theta) = \sec(\theta)$ for real numbers θ, where $\theta \neq \frac{\pi}{2} + \pi k$, for all integers k.

Name ____ Date ____________________

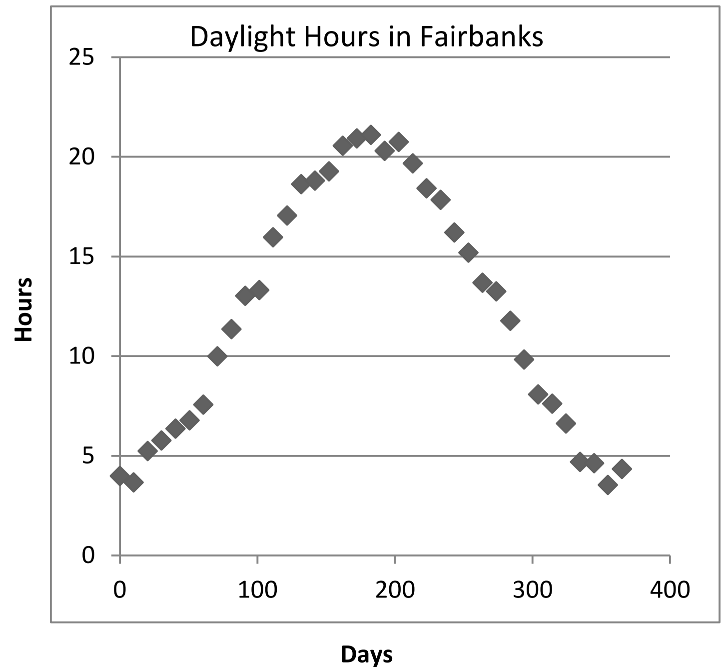
Lesson 17: Trigonometric Identity Proofs

Exit Ticket

Derive a formula for $\tan(\alpha - \beta)$ in terms of $\tan(\alpha)$ and $\tan(\beta)$, where $\alpha \neq \frac{\pi}{2} + k\pi$ and $\beta \neq \frac{\pi}{2} + k\pi$, for all integers k.

Name ______________________________ Date ____________________

1. The graph below shows the number of daylight hours each day of the year in Fairbanks, Alaska, as a function of the day number of the year. (January 1st is day 1, January 2nd is day 2, and so on.)



a. Find a function that models the shape of this daylight-hour curve reasonably well. Define the variables you use.

b. Explain how you chose the numbers in your function from part (a): What is the midline? What is the amplitude? What is the period?

c. A friend looked at the graph and wondered, "What was the average number of daylight hours in Fairbanks over the past year?" What might be a reasonable answer to that question? Use the structure of the function you created in part (a) to explain your answer.

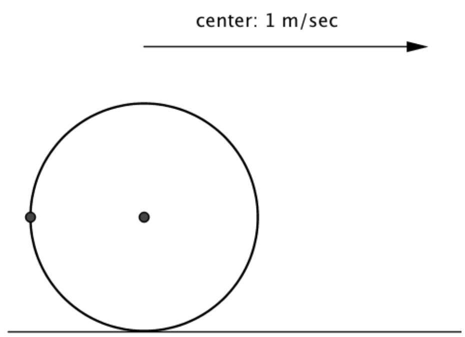
d. According to the graph, around which month of the year did the first day of the year with $17\frac{1}{2}$ hours of daylight occur? Does your function in part (a) agree with your estimation?

e. The scientists who reported this data now inform us that their instruments were incorrectly calibrated; each measurement of the daylight hours is 15 minutes too long. Adjust your function from part (a) to account for this change in the data. How does your function now appear? Explain why you changed the formula as you did.

f. To make very long-term predictions, researchers would like a function that acknowledges that there are, on average, $365\frac{1}{4}$ days in a year. How should you adjust your function from part (e) so that it represents a function that models daylight hours with a period of $365\frac{1}{4}$ days? How does your function now appear?

g. Do these two adjustments to the function significantly change the prediction as to which day of the year first possesses $17\frac{1}{2}$ hours of daylight?

2. On a whim, James challenged his friend Susan to model the movement of a chewed-up piece of gum stuck to the rim of a rolling wheel with radius 1 m. To simplify the situation, Susan drew a diagram of a circle to represent the wheel and imagined the gum as a point on the circle. Furthermore, she assumed that the center of the wheel was moving to the right at a constant speed of 1 m/sec, as shown in the diagram.



At time $t = 0$ seconds, the piece of gum was directly to the left of the center of the wheel, as indicated in the diagram above.

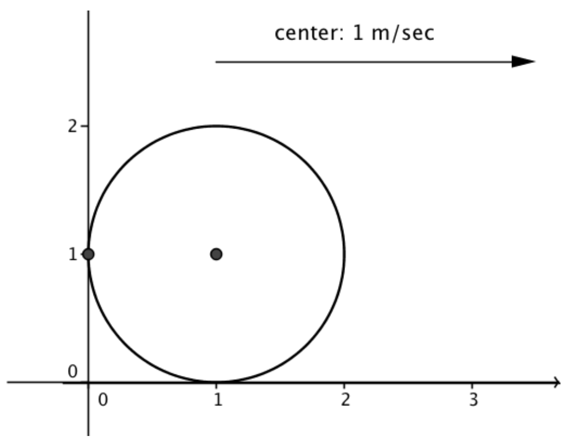
a. What is the first time that the gum was at the top position of the wheel?

b. What is the first time that the gum was again directly to the left of the center of the wheel?

c. After doing some initial calculations as in parts (a) and (b), Susan realized that the height of the gum is a function of time. She let $V(t)$ stand for the vertical height of the gum from the ground at time t seconds. Find a formula for her function.

d. What is the smallest positive value of t for which $V(t) = 0$? What does this value of t represent in terms of the situation?

Next, Susan imagined that the wheel was rolling along the horizontal axis of a coordinate system, with distances along the horizontal axis given in units of meters (and height along the vertical axis also given in units of meter). At time $t = 0$, the center of the wheel has coordinates $(1,1)$ so that the gum was initially at position $(0,1)$.



e. What is the x-coordinate of the position of the gum after $\frac{\pi}{2}$ seconds (when it first arrived at the top of the wheel)? After π seconds (when it was directly to the right of the center)?

f. From the calculations like those in part (e), Susan realized that the horizontal distance, H, of the gum from its initial location is also a function of time t, given by the distance the wheel traveled plus its horizontal displacement from the center of the wheel. Write a formula $H(t)$ for the function, i.e., find a function that specifies the x-coordinate of the position of the gum at time t.

g. Susan and James decide to test Susan's model by actually rolling a wheel with radius 1 m. However, when the gum first touched the ground, it came off the wheel and stuck to the ground at that position. How horizontally far from the initial position is the gum? Verify that your function from part (f) predicts this answer, too.

3. Betty was looking at the Pythagorean Identity: for all real numbers θ,

$$\sin^2(\theta) + \cos^2(\theta) = 1.$$

a. Betty used the Pythagorean identity to make up the equation below. She then stated, “Wow, I've discovered a new identity that is true for all θ.” Do you agree with her? Why or why not?

$$\frac{\sin^2(\theta)}{1 - \cos(\theta)} = 1 + \cos(\theta).$$

b. Prove the Pythagorean identity.

c. The real number θ is such that $\sin(\theta) = 0.6$. Calculate $|\cos(\theta)|$ and $|\tan(\theta)|$.

d. Suppose additional information is given about the number θ from part (c). You are told that θ is in the second quadrant. What are the values of cosine and tangent? Explain.