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Дз-8, 10 задач

№10.4) $y(\ln|x^2-1|+C)=1, \quad y(0)=1$

$1\cdot(\ln|\underbrace{0-1}_{0}|+C)=1 \Rightarrow C=1$

Ответ: $y(\ln|x^2-1|+1)=1$

№10.12) $x^2-y^2=2ax \Rightarrow a=\dfrac{x^2-y^2}{2x}$

$2x-2yy'=2a$

$2x-2yy'=\dfrac{x^2-y^2}{x} \quad \Big| \cdot x$

$2yy'\cdot x=y^2+x^2$

Ответ: $2xyy'=x^2+y^2$

№10.32) $ye^{2x}dx-(1+e^{2x})dy=0$

$ye^{2x}dx=(1+e^{2x})dy$

$\int\dfrac{e^{2x}+1-1}{(1+e^{2x})}dx=\int\dfrac{dy}{y}$

$\frac{1}{2}\int\dfrac{d(e^{2x})}{1+e^{2x}}=\int\dfrac{dy}{y}$

$\dfrac{\ln|e^{2x}+1|}{2}=\ln|y|+\ln|c|$

Ответ: $y=C\sqrt{e^{2x}+1}$

№10.36) $dy-2\sqrt{y}\ln x\,dx=0$

$\int\dfrac{dy}{\sqrt{y}}=2\int\ln x\,dx$

$\sqrt{y}=x\ln x-x+C$

Ответ: $\sqrt{y}=x\ln x-x+C$

№ 10.33) $2e^x \operatorname{tg} y\, dx + (1+e^x)\sec^2 y\, dy = 0$

$$\int \frac{\sec^2 y\, dy}{\operatorname{tg} y} = -2\int \frac{e^x dx}{1+e^x}$$

$$\int \frac{\frac{1}{\cos^2 y}}{\frac{\sin y}{\cos y}} dy = -2 \ln|1+e^x| + c$$

$$\ln|\operatorname{tg} y| = -2\ln|1+e^x| + c$$

Ответ: $\operatorname{tg} y (1+e^x)^2 = c$

№ 10.34) $(1+y)(e^x dx - e^{2y} dy) - (1+y^2) dy = 0$

$$e^x dx - e^{2y} dy = \frac{1+y^2}{1+y} dy$$

$$\int e^x dx = \int \frac{1+y^2}{1+y} dy + \int e^{2y} dy$$

$$e^x + c = \ln|1+y| + \int \frac{y^2}{1+y} dy + \frac{e^{2y}}{2}$$

$$e^x + c = \ln|1+y| + \frac{(1+y)^2}{2} - 2(1+y) + \ln|1+y| + \frac{e^{2y}}{2}$$

$$e^x - \frac{1}{2}e^{2y} - 2\ln|1+y| - \frac{(y-1)^2}{2} = C$$

Ответ: ↗

№ 10.35) $(1+x^2) dy + y\sqrt{1+x^2}\, dx - xy\, dx = 0$

$$\int \frac{dy}{y} = \int \frac{x - \sqrt{1+x^2}}{1+x^2} dx$$

$$\ln|y| = \frac{1}{2}\int \frac{x\,dx^2}{1+x^2} - \int \frac{dx}{\sqrt{1+x^2}}$$

$$\ln|y| = \frac{1}{2}\ln|1+x^2| - \ln|x+\sqrt{x^2+1}| + C$$

Ответ: $y = \dfrac{C\sqrt{1+x^2}}{x+\sqrt{x^2+1}}$

№10.38) $y' = \dfrac{1}{2x+y}$

$u = 2x+y$

$u' = 2+y'$

$y' = u'-2$

$u'-2 = \dfrac{1}{y} \Rightarrow u' = \dfrac{1+2u}{u}$

$$\int \frac{u\,du}{1+2u} = \int dx$$

$$\frac{1}{2}\int \frac{u+1/2-1/2}{1/2+u}\,du = x+C$$

$$\frac{1}{2}u - \frac{1}{4}\ln\left|\frac{1}{2}+u\right| = x+C$$

Ответ: $1+4x+2y = Ce^{2y}$

№10.40) $y' = \sin(y-x-1)$

$u = y-x-1 \Rightarrow u' = y'-1 \Rightarrow y' = u'+1$

$u'+1 = \sin u$

$\dfrac{du}{dx} = \sin u - 1$ $\qquad \displaystyle\int \frac{du}{\sin u - 1} = \int dx$

$$\left\{\begin{aligned} & t = tg\frac{u}{2} \\ & \sin u = \frac{2t}{t^2+1} \\ & du = \frac{2dt}{t^2+1} \end{aligned}\right\} = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t-t^2-1}{t^2+1}} = -2\int\frac{dt}{(t-1)^2} =$$

$$= \frac{-2}{t-1} = \frac{-2}{tg\frac{u}{2}-1}$$

$$\frac{-2}{tg\frac{y-x-1}{2}-1} = x+c$$

особое: $\sin u - 1 = 0$

$u = \pi/2 + 2\pi n$

$y - x - 1 = \pi/2 + 2\pi n$

Ответ: $(x+c)\left(tg\frac{y-x-1}{2}-1\right)=2$; особое: $y - x - 1 = \frac{\pi}{2} + 2\pi n$

№10.44) $(xy^2+x)dy + (x^2y-y)dx = 0$; $y(1)=1$

Решили на практике

$c = 1$

Ответ: $\frac{y^2}{2} + \ln|y| = \frac{-x}{2} + \ln|x| + 1$