Третьяков А.С. АТ-02
Φ_{3}-10, 10 задач

№10.92) $xy' + y = 2x^2 y \ln y \cdot y'$

$$y' = -\frac{y}{x - 2x^2 y \ln y}$$

$x = uv \quad x' = u'v + v'u$

$$x' = -\frac{x}{y} + 2x^2 \ln y$$

$$u'v + v'u = -\frac{uv}{y} + 2u^2v^2 \ln y$$

$$v\left(u' + \frac{u}{y}\right) + v'u = 2u^2v^2 \ln y$$

$$\frac{du}{dy} = -\frac{u}{y} \Rightarrow u = \frac{1}{y}$$

$$\frac{1}{y} v' = \frac{2v^2}{y^2} \ln y \quad \Big| \cdot y$$

$$\frac{dv}{dy} = \frac{2v^2 \ln y}{y} \Rightarrow \int \frac{\ln y \, dy}{y} = \frac{1}{2}\int \frac{dv}{v^2}$$

$$\frac{\ln^2 y}{2} = \frac{-1}{2v} \Rightarrow v = -\frac{1}{\ln^2 y} + C$$

$$x = \frac{1}{y}\left(-\frac{1}{\ln^2 y} + C\right)$$

Ответ: $xy(C - \ln^2 y) = 1$

№10.93) $y' x^3 \sin y + 2y = xy'$

$$y'(x^3 \sin y - x) = -2y$$

$$y' = \frac{-2y}{x^3 \sin y - x}; \quad x' = \frac{x^3 \sin y - x}{-2y}$$

$$x = uv; \quad x' = u'v + v'u$$

$$u'v + v'u = -\frac{u^3v^3 \sin y}{2y} + \frac{uv}{2y}$$

$$v\left(u' - \frac{u}{2y}\right) + v'u = -\frac{u^3v^3}{2y}$$

$\boxed{y = 0}$

$$\frac{du}{dy} = \frac{u}{2y} \Rightarrow u = \sqrt{y}$$

$\sqrt{y}\, v' = \frac{\sqrt{y}\, v^3 \sin y}{2} \qquad 2\int \frac{dv}{v^3} = \int \sin y \, dy$

$v = \pm \frac{1}{c - \cos y}$

$x = \frac{\sqrt{y}}{c - \cos y} \qquad y = x^2 (c - \cos y)$

Ответ: $y = x^2(c - \cos y)$, $y = 0$

№10.99) $\overset{=P}{\left(y + \frac{2}{x^2}\right)} dx + \overset{=Q}{\left(x - \frac{3}{y^2}\right)} dy = 0$

$\frac{\partial P}{\partial Q} = 1 = \frac{\partial Q}{\partial x} = 1 \Rightarrow ПД$

а) $\frac{\partial u}{\partial x} = y + \frac{2}{x^2}$; $\frac{\partial u}{\partial y} = x - \frac{3}{y^2}$

$u = \int \left(y + \frac{2}{x^2}\right) dx + \varphi(y) = -\frac{2}{x} + \varphi(y) + xy$

б) $x - \frac{3}{y^2} = x + \varphi'(y) \Rightarrow \varphi'(y) = \frac{-3}{y^2} \Rightarrow \varphi = \frac{3}{y}$

$c = xy - \frac{2}{x} + \frac{3}{y} = u$

Ответ: $xy - \frac{2}{x} + \frac{3}{y} = c$

№10.100) $\overset{=P}{\frac{3x^2 + y}{y^2}} dx - \overset{=Q}{\frac{2x^3 + xy + 2y^3}{y^3}} dy = 0$

$\frac{\partial P}{\partial y} = -\frac{6x^2}{y^3} - \frac{1}{y^2} = \frac{\partial Q}{\partial x} = \frac{-6x^2}{y^3} - \frac{1}{y^2} \Rightarrow ПД$

$\int \left(\frac{3x^2}{y^2} + \frac{1}{y}\right) dx + \left(-\frac{2x^3}{y^3} - \frac{x}{y^2} - 2\right) dy =$

$= \frac{x^3}{y^2} + \frac{x}{y} + \frac{x^3}{y^2} + \frac{x}{y} - 2y$

Ответ: $\frac{x}{y} + \frac{x^3}{y^2} - 2y = c$

с10.101) $\left(\frac{x}{\sqrt{x^2-y^2}}+y\right)dx+\left(x+\frac{1}{y^2}-\frac{y}{\sqrt{x^2-y^2}}\right)dy=0$

$$\frac{\partial P}{\partial x}=\frac{\sqrt{x^2-y^2}-\frac{x^2}{\sqrt{x^2-y^2}}}{x^2-y^2}+yx=\frac{\partial Q}{\partial y}=\frac{-2}{y^3}+xy-\frac{\sqrt{x^2-y^2}+\frac{y^3}{\sqrt{x^2-y^2}}}{x^2-y^2}$$

а) $\frac{\partial u}{\partial x}=\frac{x}{\sqrt{x^2-y^2}}+y$; $\frac{\partial u}{\partial y}=x+\frac{1}{y^2}-\frac{y}{\sqrt{x^2-y^2}}$

$$u=\frac{1}{2}\int\left(\frac{d(x^2-y^2)}{\sqrt{x^2-y^2}}+\int y\right)dx+\varphi(y)=\sqrt{x^2-y^2}+xy+\varphi(y)$$

б) $x+\frac{1}{y^2}-\frac{y}{\sqrt{x^2-y^2}}=-\frac{y}{\sqrt{x^2-y^2}}+x+\varphi'(y)$

$\varphi=-\frac{1}{y}$

Ответ: $\sqrt{x^2-y^2}+xy-\frac{1}{y}=C$

с10.102) $(2x-ye^{-x})dx+e^{-x}dy=0$

$$\frac{\partial P}{\partial x}=2+ye^{-x}=\frac{\partial Q}{\partial y}$$

а) $\frac{\partial u}{\partial x}=2x-ye^{-x}$

$$u=\int(2x-y\cdot e^{-x})dx+\varphi(y)=x^2+ye^{-x}+\varphi(y)$$

б) $\cancel{e^{-x}}=\cancel{x^2}+e^{-x}+\varphi(y)\Rightarrow\varphi(y)=0$

Ответ: $x^2+ye^{-x}=0$

с10.103) $(2x+e^{x/y})dx+(1-\frac{x}{y})e^{x/y}dy=0$

а) $\frac{\partial u}{\partial x}=2x+e^{x/y}$ $\quad\frac{\partial u}{\partial y}=e^{x/y}-\frac{x}{y}e^{x/y}$

$$u=\int(2x+e^{x/y})dx+\varphi(y)=x^2+ye^{x/y}+\varphi(y)$$

б) $e^{x/y}-\frac{x}{y}e^{x/y}=e^{x/y}+\frac{x}{y}e^{x/y}+\varphi'(y)$

$\varphi=0$

Ответ: $x^2+ye^{x/y}=C$

№ 10.149) $y' = \frac{1-2x}{y^2}$

$\int y^2 dy = \int (1-2x)\, dx$

$\frac{y^3}{3} = x - x^2 + C \quad | \cdot 3$

Ответ: $y = \sqrt[3]{3x - 3x^2 + C}$

№ 10.150) $xy' + y = y^2 \ln x$

$y' = \frac{y^2 \ln x}{x} - \frac{y}{x}$

$y = uv \Rightarrow y' = u'v + v'u$

$u'v + v'u = \frac{u^2 v^2 \ln x}{x} - \frac{uv}{x}$

$v \underbrace{\left(u' + \frac{u}{x}\right)}_{0} + v'u = \frac{u^2 v^2 \ln x}{x}$

$\frac{du}{dx} = \frac{-u}{x} \Rightarrow u = -x$

$v'x = xv^2 \ln x \quad | : x \qquad \Rightarrow \boxed{y = 0}$

$\int \frac{dv}{v^2} = \int \ln x\, dx \qquad -\frac{1}{v} = x \ln x - x + C$

$v = \frac{1}{-x \ln x + x - C}$

Ответ: $y = \frac{1}{\ln x + 1 + Cx}, \quad y = 0$

№ 10.151) $3x + y - 2 + y'(x-1) = 0$

$y' = \frac{2 - 3x - y}{x - 1} \qquad y = uv \Rightarrow y' = u'v + v'u$

$u'v + v'u = \frac{2}{x-1} - \frac{3x}{x-1} - \frac{uv}{x-1}$

$v\left(u' + \frac{u}{x-1}\right) + v'u = \frac{2 - 3x}{x-1}$

$\frac{du}{dx} = \frac{-u}{x-1} \Rightarrow \ln u = -\ln|x-1| \Rightarrow u = \frac{1}{x-1}$

$\frac{v'}{x-1} = \frac{2-3x}{x-1} \Rightarrow v = 2x - \frac{3x^2}{2} + C$

Ответ: $(2y + 3x - 1)(x - 1) = C$