① $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$P_A = Q\Sigma Q^T \; ; \quad \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \; Q = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P_A = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

$$P_B = B(B^T B)^{-1} B^T$$

$$P_B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \overset{\begin{pmatrix} 5/6 & 1/3 \\ -1/3 & 1/3 \end{pmatrix}}{\begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}^{-1}} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}$$

QR decomposition:

$$A = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

first column of B

$$B: \quad \bar{q}_1 = \frac{\bar{B}_1}{|B_1|} = \frac{\bar{B}_1}{\sqrt{2}} \; ; \quad \bar{q}_2 = \bar{B}_2 - (\bar{B}_2, \bar{q})q_1 = \bar{B}_2 - \sqrt{2}\bar{q}_1 = \bar{B}_2 - \bar{B}_1 =$$

$$= \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1/\sqrt{2} & 1 \\ 0 & 1 \\ 1/\sqrt{2} & -1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 1 \end{pmatrix}$$