Planck Time, “Photon On/Off”, and Measurement Context: Rigorous Operational Analysis

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Abstract

Claims like “a photon is on or off” become ambiguous (and sometimes self-contradictory) when classical binary predicates are applied to quantum optical systems without specifying (i) the *mode* defining “the photon” and (ii) the *measurement* defining “on/off.” This paper develops a fully operational account using quantized field modes, number states, coherent states, and explicit detector POVMs (ideal and inefficient on/off detectors with dark counts). We compute detection statistics for several scenarios—vacuum vs. one-photon states, vacuum–one superpositions, coherent and thermal states, Mach–Zehnder interference with and without which-path information (including partial which-path overlap), time-bin superpositions, emitter–field entanglement, and mode-basis dependence—and show that “contradictions” typically arise from implicitly assuming non-contextual, measurement-independent truth values for incompatible observables. We derive Planck units and present a standard heuristic bound suggesting that attempting to operationalize time resolution $\Delta t \ll t_P$ pushes required energies toward the Planck scale, where gravitational backreaction cannot be neglected; we distinguish this from claims that time is discrete in steps of t_P.

1 Notation and assumptions

- We use SI units unless otherwise stated.
- c is the speed of light, G Newton’s constant, $\hbar$ reduced Planck’s constant.
- $\mathcal{H}$ is a Hilbert space; density operators are $\rho \geq 0$ with $\mathrm{Tr}(\rho) = 1$.
- A single bosonic mode has annihilation/creation operators $a, a^\dagger$ with $[a, a^\dagger] = 1$.
- Number operator: $N = a^\dagger a$; Fock states satisfy $N|n\rangle = n|n\rangle$.
- “Photon on/off” will mean a *specific measurement* (typically an on/off threshold detector) applied to a *specific mode* (a chosen wavepacket/spatial-temporal mode). Without that, the question is ill-posed.

2 Introduction: why “photon on/off” can look contradictory

Classical reasoning quietly assumes all of the following:

1. **Definiteness:** a property (e.g., “photon present”) has a definite value at all times.
2. **Non-contextuality:** that value does not depend on how one chooses to measure it.
3. **Measurement revelation:** measurement reveals a pre-existing value rather than creating an outcome via interaction.

Quantum theory rejects this package. A "photon on/off" statement becomes meaningful only after specifying:

- (i) a **mode decomposition** (what counts as "the photon"), and
- (ii) an **observable/POVM** (what counts as "on/off" operationally).

Once those are fixed, the theory makes unambiguous predictions; "contradictions" typically reflect mixing incompatible contexts (e.g., demanding which-path facts *and* interference fringes as if both were simultaneously definite).

2.1 "Square circles" from higher-dimensional projections (explicit construction)

"Square circle" is a useful diagnostic phrase: in a single 2D world, "the same planar figure is both a perfect square and a perfect circle (in the same sense, at the same time)" is a contradiction. But **projection** lets a single higher-dimensional object yield different lower-dimensional appearances.

Example: a right circular cylinder whose shadows are a circle and a square. Let the 3D solid cylinder be

$$C := \{(x, y, z) \in \mathbb{R}^3 : \ x^2 + y^2 \le R^2, \ 0 \le z \le h\}.$$

Define orthographic (drop-coordinate) projections:

$$\pi_{xy}(x, y, z) = (x, y), \qquad \pi_{yz}(x, y, z) = (y, z).$$

Then the image of C under π_{xy} is

$$\pi_{xy}(C) = \{(x, y) \in \mathbb{R}^2 : \exists z \text{ s.t. } (x, y, z) \in C\} = \{(x, y) : x^2 + y^2 \le R^2\},$$

which is a **filled circle** (disk) of radius R. Similarly,

$$\pi_{yz}(C) = \{(y, z) \in \mathbb{R}^2 : \exists x \text{ s.t. } (x, y, z) \in C\} = \{(y, z) : y^2 \le R^2, \ 0 \le z \le h\} = [-R, R] \times [0, h],$$

which is a **filled rectangle** of width $2R$ and height h. If we choose $h = 2R$, then $\pi_{yz}(C)$ is a **filled square** of side length $2R$. Thus:

- viewing along the cylinder axis gives a **circle**,
- viewing orthogonally gives a **square** (for $h = 2R$).

There is no contradiction because the "circle" and "square" are properties of *different projections* $\pi_{xy}(C)$ and $\pi_{yz}(C)$, not two simultaneous properties of a single 2D object.

Moral. Projections are many-to-one maps: they discard information. Two different projections of the same higher-dimensional object can look mutually incompatible if one incorrectly assumes the projection is "the whole object."

2.2 Why this maps tightly to quantum measurement (Born rule as projection geometry)

This "projection resolves apparent contradiction" story is not merely metaphorical in quantum theory: **ideal (projective) quantum measurement is literally built from projection operators.**

Let an observable have spectral decomposition

$$A = \sum_a a\, P_a,$$

where P_a are orthogonal projectors ($P_a P_{a'} = \delta_{aa'} P_a$, $\sum_a P_a = I$). For a pure state $|\psi\rangle$, the Born rule can be written as a squared projection norm:

$$p(a) = \langle\psi|P_a|\psi\rangle = \|P_a|\psi\rangle\|^2.$$

The post-measurement state (Lüders rule) is the normalized projection:

$$|\psi\rangle \;\longrightarrow\; \frac{P_a|\psi\rangle}{\sqrt{p(a)}}.$$

So a measurement is, in a precise mathematical sense, a map from a "higher-dimensional" object (state vector or density operator) to a lower-dimensional classical outcome, and it generally discards phase/coherence information relative to other incompatible decompositions. Attempting to demand that a system simultaneously carry measurement-independent truth values for multiple incompatible "views" is analogous to demanding one 2D shadow be both a circle and a square in the same projection.

3 Planck units and Planck time

3.1 Dimensional analysis derivation

We seek a time scale constructed from $\hbar$, G, and c. Write

$$t_P \propto \hbar^a G^b c^d.$$

Using base dimensions:

$$[\hbar] = \mathrm{M}\,\mathrm{L}^2\,\mathrm{T}^{-1}, \quad [G] = \mathrm{L}^3\,\mathrm{M}^{-1}\,\mathrm{T}^{-2}, \quad [c] = \mathrm{L}\,\mathrm{T}^{-1}.$$

We require T^1 overall. Compute:

- Mass exponent: $a - b = 0 \Rightarrow a = b$.
- Length exponent: $2a + 3b + d = 0 \Rightarrow 2a + 3a + d = 0 \Rightarrow d = -5a$.
- Time exponent: $-a - 2b - d = 1 \Rightarrow -a - 2a - (-5a) = 1 \Rightarrow 2a = 1 \Rightarrow a = b = \frac{1}{2}$.

Thus $d = -\frac{5}{2}$ and

$$t_P = \sqrt{\frac{\hbar G}{c^5}}.$$

3.2 Numerical value and associated scales

Using (approximately):

$$\hbar \approx 1.054571817 \times 10^{-34}\,\mathrm{J\,s}, \quad G \approx 6.67430 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}, \quad c = 2.99792458 \times 10^{8}\,\mathrm{m\,s^{-1}},$$

we obtain:

$$t_P \approx 5.39 \times 10^{-44}\,\mathrm{s}.$$

Related Planck units:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\,\mathrm{m}, \quad m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-8}\,\mathrm{kg},$$

$$E_P = m_P c^2 = \sqrt{\frac{\hbar c^5}{G}} \approx 1.96 \times 10^{9}\,\mathrm{J} \approx 1.22 \times 10^{19}\,\mathrm{GeV}.$$

Note the identity $E_P = \hbar/t_P$.

3.3 What Planck time does (and does not) imply

- **Does imply:** a natural scale where the dimensionless combination of couplings suggests quantum-gravity effects may be important.
- **Does not (by itself) imply:** that time is fundamentally discrete in steps of t_P, or that physical systems must "update" at that cadence, or that any particular "on/off" predicate must flip on Planck timescales.

3.4 Heuristic "minimum time" from quantum localization + gravity (careful)

A standard heuristic combines:

1. To operationally probe time resolution Δt, one needs frequency components with $\Delta\omega \gtrsim 1/\Delta t$ (Fourier limitation).
2. Quanta at angular frequency ω have energy $\sim \hbar\omega$, so resolving Δt pushes energies toward $E \sim \hbar/\Delta t$ (order-of-magnitude).
3. Concentrating energy E within a region of size $R \sim c\Delta t$ yields an associated Schwarzschild radius
$$r_s = \frac{2GE}{c^4}.$$

Requiring $r_s \lesssim R$ (avoid immediate horizon formation in this crude model) gives:

$$\frac{2GE}{c^4} \lesssim c\Delta t.$$

Insert $E \sim \hbar/\Delta t$:

$$\frac{2G\hbar}{c^4\,\Delta t} \lesssim c\Delta t \;\Rightarrow\; \Delta t^2 \gtrsim \frac{2\hbar G}{c^5} \;\Rightarrow\; \Delta t \gtrsim \sqrt{2}\,t_P.$$

Interpretation: this is a plausibility argument that sub-Planckian operational time resolution may require energies at which gravitational backreaction becomes non-negligible. It is not a derivation of time discreteness or a theorem of quantum theory alone.

4 Quantized optical modes: "what is a photon?"

In quantum optics, a "photon" is most cleanly defined as an excitation of a **mode** of the electromagnetic field. A "mode" is not unique: it depends on boundary conditions (cavity/waveguide), filtering, pulse shaping, detection mode-matching, etc.

4.1 Single-mode field algebra

For a single bosonic mode:

$$[a, a^\dagger] = 1, \quad N = a^\dagger a.$$

The Fock basis $\{|n\rangle\}_{n=0}^{\infty}$ satisfies:

$$N|n\rangle = n|n\rangle, \quad a|n\rangle = \sqrt{n}\,|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle.$$

4.2 Canonical states

- **Vacuum:** $|0\rangle$.
- **Number (Fock) state:** $|n\rangle$ with definite photon number n.
- **Coherent state:** $|\alpha\rangle$ defined by $a|\alpha\rangle = \alpha|\alpha\rangle$. Expansion:

  $$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

  Photon number distribution:

  $$P(n) = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!},$$

  i.e. Poisson with mean $\mu = |\alpha|^2$.
- **Thermal state:** diagonal in number basis with Bose–Einstein distribution. If mean photon number is $\bar{n}$:

  $$\rho_{\text{th}} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} |n\rangle\langle n|.$$

4.3 Wavepacket modes (time/frequency localized photons)

For a continuum of frequency modes with operators $a(\omega)$ satisfying

$$[a(\omega), a^\dagger(\omega')] = \delta(\omega - \omega'),$$

define a normalized wavepacket mode $f(\omega)$ with $\int d\omega\, |f(\omega)|^2 = 1$ and

$$a_f = \int d\omega\, f(\omega)\, a(\omega), \quad [a_f, a_f^\dagger] = 1.$$

Then $|1_f\rangle = a_f^\dagger|0\rangle$ is "one photon in mode f." Crucially, different choices of f define different "photon number" questions.

5 What does "on/off" mean? Explicit measurement models

5.1 Photon number projective measurement vs. on/off detection

An ideal number-resolving measurement uses projectors $\Pi_n = |n\rangle\langle n|$. An **ideal on/off detector** (threshold detector) distinguishes only:

- "off" = vacuum ($n = 0$),
- "on" = any nonzero photon number ($n \geq 1$).

The corresponding POVM elements are:

$$\Pi_{\text{off}} = |0\rangle\langle 0|, \quad \Pi_{\text{on}} = I - |0\rangle\langle 0|.$$

For a state ρ, predicted probabilities:

$$p_{\text{off}} = \text{Tr}(\rho\,\Pi_{\text{off}}), \quad p_{\text{on}} = \text{Tr}(\rho\,\Pi_{\text{on}}) = 1 - p_{\text{off}}.$$

5.2 Inefficiency and dark counts (standard quantum-optics model)

Let $\eta \in [0, 1]$ be detection efficiency. A common model treats each photon as independently detected with probability η. Then the probability of **no click** given n photons is $(1-\eta)^n$. This corresponds to the POVM:

$$\Pi_{\text{off}}^{(\eta)} = \sum_{n=0}^{\infty} (1-\eta)^n\, |n\rangle\langle n|, \quad \Pi_{\text{on}}^{(\eta)} = I - \Pi_{\text{off}}^{(\eta)}.$$

Add a dark-count probability p_d per detection window by mixing with a classical "false click":

$$p_{\text{off}} = (1-p_d)\,\text{Tr}(\rho\,\Pi_{\text{off}}^{(\eta)}), \quad p_{\text{on}} = 1 - p_{\text{off}}.$$

5.3 Immediate consequence: phases can be invisible to on/off

Because $\Pi_{\text{off}}^{(\eta)}$ is diagonal in the number basis, any relative phase between $|0\rangle$ and $|1\rangle$ in a superposition $\alpha|0\rangle + \beta|1\rangle$ will not affect $p_{\text{on/off}}$. To see that phase, one needs a different measurement (e.g. homodyne).

6 Scenarios with explicit calculations

6.1 Vacuum vs. one-photon Fock state

Let $\rho = |0\rangle\langle 0|$. With ideal on/off:

$$p_{\text{off}} = 1, \quad p_{\text{on}} = 0.$$

With inefficiency and dark counts:

$$\text{Tr}(\rho\,\Pi_{\text{off}}^{(\eta)}) = (1-\eta)^0 = 1 \Rightarrow p_{\text{on}} = 1 - (1-p_d)\cdot 1 = p_d.$$

So even vacuum can "click" due to dark counts.

Now take $\rho = |1\rangle\langle 1|$. Then

$$\text{Tr}(\rho\,\Pi_{\text{off}}^{(\eta)}) = (1-\eta)^1 = 1-\eta \Rightarrow p_{\text{on}} = 1 - (1-p_d)(1-\eta) = p_d + (1-p_d)\eta.$$

For $p_d = 0$, $p_{\text{on}} = \eta$: a one-photon state does not guarantee a click unless $\eta = 1$.

6.2 The clearest "on/off superposition": $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Let $|\alpha|^2 + |\beta|^2 = 1$ and $\rho = |\psi\rangle\langle\psi|$. Because $\Pi_{\text{off}}^{(\eta)}$ is diagonal in $|n\rangle$,

$$\mathrm{Tr}(\rho\,\Pi_{\text{off}}^{(\eta)}) = |\alpha|^2(1-\eta)^0 + |\beta|^2(1-\eta)^1 = |\alpha|^2 + (1-\eta)|\beta|^2 = 1 - \eta|\beta|^2.$$

Thus (with $p_d = 0$):

$$p_{\text{on}} = \eta|\beta|^2.$$

With ideal detection $\eta = 1$, $p_{\text{on}} = |\beta|^2$.

6.2.1 On/off cannot see coherence; homodyne (quadrature) can (explicit calculation)

The on/off POVM depends only on the populations ρ_{nn}. This means a coherent superposition and an incoherent mixture can have identical on/off statistics. Define the mixture

$$\rho_{\text{mix}} := |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|.$$

Then for either $\rho = |\psi\rangle\langle\psi|$ or ρ_{mix},

$$p_{\text{on}} = 1 - \mathrm{Tr}(\rho\,\Pi_{\text{off}}^{(\eta)}) = \eta|\beta|^2.$$

To see the difference, consider a homodyne (quadrature) measurement. Define the dimensionless quadrature operator

$$X := \frac{a + a^\dagger}{\sqrt{2}},$$

with eigenstates $|x\rangle$ satisfying $X|x\rangle = x|x\rangle$. In the x-representation, the first two harmonic-oscillator wavefunctions are:

$$\psi_0(x) := \langle x|0\rangle = \pi^{-1/4}e^{-x^2/2}, \qquad \psi_1(x) := \langle x|1\rangle = \pi^{-1/4}\sqrt{2}\,x\,e^{-x^2/2}.$$

For the pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$,

$$\langle x|\psi\rangle = \alpha\psi_0(x) + \beta\psi_1(x) = \pi^{-1/4}e^{-x^2/2}\left(\alpha + \sqrt{2}\,x\,\beta\right),$$

so the probability density is

$$P_\psi(x) = |\langle x|\psi\rangle|^2 = \pi^{-1/2}e^{-x^2}\left(|\alpha|^2 + 2x^2|\beta|^2 + 2\sqrt{2}\,x\,\mathrm{Re}(\alpha^*\beta)\right).$$

For the mixture,

$$P_{\text{mix}}(x) = \mathrm{Tr}(\rho_{\text{mix}}|x\rangle\langle x|) = \pi^{-1/2}e^{-x^2}\left(|\alpha|^2 + 2x^2|\beta|^2\right),$$

which lacks the linear "interference" term proportional to $\mathrm{Re}(\alpha^*\beta)$. Thus coherence (and relative phase) is operationally accessible in quadrature statistics even though it is invisible to on/off detection. In practice one measures a phase-rotated quadrature $X_\varphi = (ae^{-\mathrm{i}\varphi} + a^\dagger e^{\mathrm{i}\varphi})/\sqrt{2}$ by varying the local-oscillator phase φ.

6.3 Coherent state $|\alpha\rangle$: Poisson counting $\Rightarrow$ closed-form click probability

For $\rho = |\alpha\rangle\langle\alpha|$ with mean photon number $\mu = |\alpha|^2$,

$$\mathrm{Tr}(\rho\,\Pi_{\mathrm{off}}^{(\eta)}) = \sum_{n=0}^{\infty}(1-\eta)^n\,P(n) = \sum_{n=0}^{\infty}(1-\eta)^n\,e^{-\mu}\frac{\mu^n}{n!}.$$

Recognize the exponential series:

$$\mathrm{Tr}(\rho\,\Pi_{\mathrm{off}}^{(\eta)}) = e^{-\mu}\sum_{n=0}^{\infty}\frac{((1-\eta)\mu)^n}{n!} = e^{-\mu}\,e^{(1-\eta)\mu} = e^{-\eta\mu}.$$

So (with dark counts p_d):

$$p_{\mathrm{on}} = 1-(1-p_d)e^{-\eta\mu}.$$

6.4 Thermal state ρ_{th}: click probability and heavy tails

For $\rho = \rho_{\mathrm{th}}$ with mean $\bar{n}$,

$$\mathrm{Tr}(\rho\,\Pi_{\mathrm{off}}^{(\eta)}) = \sum_{n=0}^{\infty}(1-\eta)^n\,\frac{\bar{n}^n}{(1+\bar{n})^{n+1}} = \frac{1}{1+\bar{n}}\sum_{n=0}^{\infty}\left(\frac{(1-\eta)\bar{n}}{1+\bar{n}}\right)^n.$$

This is a geometric series with ratio $r = \frac{(1-\eta)\bar{n}}{1+\bar{n}}$, so

$$\mathrm{Tr}(\rho\,\Pi_{\mathrm{off}}^{(\eta)}) = \frac{1}{1+\eta\bar{n}}.$$

Thus (with $p_d = 0$):

$$p_{\mathrm{on}} = 1-\frac{1}{1+\eta\bar{n}} = \frac{\eta\bar{n}}{1+\eta\bar{n}}.$$

6.5 Mach–Zehnder interferometer: single-photon interference vs. which-path information

We model two spatial modes (arms) A and B with creation operators $a^\dagger, b^\dagger$.

6.5.1 50/50 beamsplitter transformation (one convention)

Let the first beamsplitter implement:

$$U_{\mathrm{BS}}^\dagger a\,U_{\mathrm{BS}} = \frac{a+\mathrm{i}b}{\sqrt{2}}, \quad U_{\mathrm{BS}}^\dagger b\,U_{\mathrm{BS}} = \frac{\mathrm{i}a+b}{\sqrt{2}}.$$

Input state: one photon in mode a, vacuum in b:

$$|\psi_{\mathrm{in}}\rangle = |1\rangle_a|0\rangle_b = a^\dagger|0\rangle.$$

After the first beamsplitter:

$$|\psi_1\rangle = U_{\mathrm{BS}}|\psi_{\mathrm{in}}\rangle = \left(\frac{a^\dagger-\mathrm{i}b^\dagger}{\sqrt{2}}\right)|0\rangle = \frac{|1,0\rangle-\mathrm{i}|0,1\rangle}{\sqrt{2}}.$$

Apply a phase shift ϕ in arm B: $b^\dagger \mapsto e^{\mathrm{i}\phi} b^\dagger$:

$$|\psi_\phi\rangle = \frac{|1,0\rangle - \mathrm{i}e^{\mathrm{i}\phi}|0,1\rangle}{\sqrt{2}}.$$

After the second identical beamsplitter, define output modes:

$$c = \frac{a + \mathrm{i}b}{\sqrt{2}}, \quad d = \frac{\mathrm{i}a + b}{\sqrt{2}} \Rightarrow a = \frac{c - \mathrm{i}d}{\sqrt{2}}, \quad b = \frac{-\mathrm{i}c + d}{\sqrt{2}}.$$

Using

$$a^\dagger = \frac{c^\dagger + \mathrm{i}d^\dagger}{\sqrt{2}}, \quad b^\dagger = \frac{\mathrm{i}c^\dagger + d^\dagger}{\sqrt{2}},$$

we have

$$|\psi_\phi\rangle = \frac{1}{\sqrt{2}}\left(a^\dagger - \mathrm{i}e^{\mathrm{i}\phi}b^\dagger\right)|0\rangle = \frac{1}{\sqrt{2}}\left(\frac{c^\dagger + \mathrm{i}d^\dagger}{\sqrt{2}} - \mathrm{i}e^{\mathrm{i}\phi}\frac{\mathrm{i}c^\dagger + d^\dagger}{\sqrt{2}}\right)|0\rangle.$$

Simplify the bracket:

$$\frac{1}{\sqrt{2}}\left(\frac{c^\dagger + \mathrm{i}d^\dagger}{\sqrt{2}} + \frac{e^{\mathrm{i}\phi}c^\dagger - \mathrm{i}e^{\mathrm{i}\phi}d^\dagger}{\sqrt{2}}\right) = \frac{1}{2}\left((1 + e^{\mathrm{i}\phi})c^\dagger + \mathrm{i}(1 - e^{\mathrm{i}\phi})d^\dagger\right).$$

Thus:

$$|\psi_{\mathrm{out}}\rangle = \frac{1 + e^{\mathrm{i}\phi}}{2}|1\rangle_c|0\rangle_d + \frac{\mathrm{i}(1 - e^{\mathrm{i}\phi})}{2}|0\rangle_c|1\rangle_d.$$

Therefore detection probabilities:

$$P(c) = \left|\frac{1 + e^{\mathrm{i}\phi}}{2}\right|^2 = \frac{1 + \cos\phi}{2} = \cos^2\left(\frac{\phi}{2}\right),$$

$$P(d) = \left|\frac{1 - e^{\mathrm{i}\phi}}{2}\right|^2 = \frac{1 - \cos\phi}{2} = \sin^2\left(\frac{\phi}{2}\right).$$

6.5.2 Which-path information as dephasing: interference disappears

Suppose the path becomes entangled with an environment/pointer state $|E_A\rangle, |E_B\rangle$:

$$|\Psi\rangle = \frac{|1,0\rangle|E_A\rangle - \mathrm{i}|0,1\rangle|E_B\rangle}{\sqrt{2}}.$$

If $\langle E_A|E_B\rangle = 0$, tracing out the environment yields:

$$\rho_{AB} = \mathrm{Tr}_E(|\Psi\rangle\langle\Psi|) = \frac{1}{2}\left(|1,0\rangle\langle 1,0| + |0,1\rangle\langle 0,1|\right),$$

with no coherence. Propagating this mixture through the phase shifter and second beamsplitter yields $P(c) = P(d) = \frac{1}{2}$, independent of ϕ.

6.5.3 Partial which-path information: overlap controls visibility and yields a quantitative tradeoff

Let $\gamma := \langle E_B|E_A\rangle$, with $0 \le |\gamma| \le 1$, and include a phase shift ϕ in arm B:

$$|\Psi_\phi\rangle = \frac{|1,0\rangle|E_A\rangle - \mathrm{i}e^{\mathrm{i}\phi}|0,1\rangle|E_B\rangle}{\sqrt{2}}.$$

Tracing out the environment yields

$$\rho_{AB} = \frac{1}{2}\Big(|1,0\rangle\langle 1,0| + |0,1\rangle\langle 0,1| + \mathrm{i}\gamma e^{-\mathrm{i}\phi}|1,0\rangle\langle 0,1| - \mathrm{i}\gamma^* e^{\mathrm{i}\phi}|0,1\rangle\langle 1,0|\Big).$$

Using $c = (a + \mathrm{i}b)/\sqrt{2}$ so that

$$n_c = c^\dagger c = \frac{a^\dagger a + b^\dagger b + \mathrm{i}a^\dagger b - \mathrm{i}b^\dagger a}{2},$$

one obtains

$$P(c) = \frac{1}{2}\left[1 + |\gamma|\cos(\phi - \arg\gamma)\right],$$

so the fringe visibility is $V = |\gamma|$.

The best possible which-path guess from the environment record is a binary state discrimination problem between $|E_A\rangle$ and $|E_B\rangle$ with equal priors. Helstrom's bound gives

$$P^\star_{\mathrm{succ}} = \frac{1}{2}\left(1 + \sqrt{1 - |\langle E_A|E_B\rangle|^2}\right) = \frac{1}{2}\left(1 + \sqrt{1 - |\gamma|^2}\right).$$

Define distinguishability $D := 2P^\star_{\mathrm{succ}} - 1 = \sqrt{1 - |\gamma|^2}$. Therefore

$$V^2 + D^2 = 1$$

in this pure-state model (more general models give $V^2 + D^2 \le 1$).

6.6 Time-bin superposition: "early or late" is not classical ignorance

Let $|E\rangle$ and $|L\rangle$ be orthonormal time-bin modes. Consider

$$|\psi\rangle = \frac{|E\rangle + e^{\mathrm{i}\theta}|L\rangle}{\sqrt{2}}.$$

Measurement in the $\{|E\rangle, |L\rangle\}$ basis yields each with probability $1/2$, independent of θ. But recombination (measuring in the $|\pm\rangle = (|E\rangle \pm |L\rangle)/\sqrt{2}$ basis) yields:

$$P(+) = \left|\frac{1 + e^{\mathrm{i}\theta}}{2}\right|^2 = \cos^2\left(\frac{\theta}{2}\right), \qquad P(-) = \sin^2\left(\frac{\theta}{2}\right).$$

6.7 Emitter–field entanglement: "photon on/off" can be conditional

Consider

$$|\Psi\rangle = \frac{|e\rangle|0\rangle + |g\rangle|1\rangle}{\sqrt{2}}.$$

Tracing out the atom yields

$$\rho_{\text{field}} = \text{Tr}_{\text{atom}}(|\Psi\rangle\langle\Psi|) = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right).$$

An on/off detector with efficiency η and $p_d = 0$ predicts:

$$p_{\text{on}} = \text{Tr}(\rho_{\text{field}}\,\Pi_{\text{on}}^{(\eta)}) = \frac{\eta}{2}.$$

Conditioning on measuring the atom prepares definite field states $|0\rangle$ or $|1\rangle$; thus "photon on/off" can be a conditional statement relative to correlated degrees of freedom.

6.8 Mode-basis dependence: a definite photon in one mode can be a superposition in another

Let a_u, a_v be annihilation operators for orthonormal modes u, v and define rotated modes:

$$a_\pm = \frac{a_u \pm a_v}{\sqrt{2}}.$$

Then $a_u^\dagger = (a_+^\dagger + a_-^\dagger)/\sqrt{2}$. Acting on vacuum:

$$|1_u, 0_v\rangle = a_u^\dagger|0\rangle = \frac{1}{\sqrt{2}}\left(a_+^\dagger + a_-^\dagger\right)|0\rangle = \frac{|1_+, 0_-\rangle + |0_+, 1_-\rangle}{\sqrt{2}}.$$

So "one photon present" is mode-dependent; any binary on/off claim must specify the detector's mode.

7 Where the "contradictions" come from (and how superposition resolves them)

The recurring pattern is:

- one measurement context asks a yes/no question about a particular basis (e.g., "which path?", "which time bin?", "is $n = 0$?"),
- another context asks about interference/coherence in a complementary basis.

Attempting to assign simultaneous, context-independent truth values to both sets of questions leads to apparent contradictions. Quantum theory avoids contradiction by representing the system by a state (vector or density operator) that can contain coherent superpositions, predicting outcomes via Born-rule probabilities $\text{Tr}(\rho\,\Pi_i)$, and allowing measurement interactions (and entanglement with environments) to change which coherences are accessible.

8 Time resolution, bandwidth, and the Planck scale (operational framing)

8.1 "Time window" vs. "photon present": what a detector really does

A real detector defines a detection **mode** (filtering + mode overlap), a detection **window** of duration T, and a POVM (often thresholding). Thus "photon on in a Planck-time slice" requires a physically realizable detector whose response has support on that timescale *and* couples to the relevant ultra-broadband modes.

8.2 Fourier-limited pulses: explicit bandwidth–duration relation

For a transform-limited pulse with temporal envelope $g(t)$ and spectrum $\tilde{g}(\omega)$, rms widths satisfy

$$\Delta t \, \Delta\omega \geq \frac{1}{2},$$

with equality for Gaussians. If $\Delta t = t_P$, then $\Delta\omega \gtrsim 1/(2t_P) \sim 10^{43}\,\mathrm{s}^{-1}$, corresponding to energies per quantum of order $\hbar\Delta\omega \sim \hbar/t_P = E_P$.

8.3 Why this does not imply "photons flip on/off each t_P"

The reasoning says that probing such short times pushes one to Planckian energies and gravitational backreaction, so naive quantum-optics models likely fail. It does *not* say that physical fields evolve in discrete ticks, that a photon has an intrinsic binary on/off variable that updates at t_P, or that standard quantum superposition ceases to apply.

9 Conclusion

1. "Photon on/off" is shorthand for a mode-defined and measurement-defined predicate.
2. With explicit detector POVMs, on/off outcomes are straightforward to compute and show no inconsistency.
3. Apparent contradictions most often come from importing classical assumptions of measurement-independent, simultaneously definite properties for incompatible observables.
4. Planck time is a natural quantum-gravity scale; heuristic arguments suggest operational probes below t_P require Planckian energies, but this does not by itself imply time discreteness or binary "flip" dynamics.

A On/off POVM derivation for inefficiency

If each photon is independently detected with probability η, then with n photons the probability of *no* detection is $(1-\eta)^n$. For a number-diagonal state $\rho = \sum_n p_n |n\rangle\langle n|$,

$$p_{\text{off}} = \sum_{n=0}^{\infty} p_n (1-\eta)^n = \mathrm{Tr}\left(\rho \sum_{n=0}^{\infty} (1-\eta)^n |n\rangle\langle n| \right).$$

Thus the operator in parentheses is the POVM element $\Pi_{\text{off}}^{(\eta)}$.

B Two-mode basis rotation identity

For orthonormal modes u, v, define $a_\pm = (a_u \pm a_v)/\sqrt{2}$. Then

$$a_u^\dagger = \frac{a_+^\dagger + a_-^\dagger}{\sqrt{2}} \Rightarrow a_u^\dagger |0\rangle = \frac{|1_+, 0_-\rangle + |0_+, 1_-\rangle}{\sqrt{2}}.$$

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