LETTER

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A stronger null hypothesis for crossing dependencies

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Abstract – The syntactic structure of a sentence can be modeled as a tree where vertices are words and edges indicate syntactic dependencies between words. It is well known that those edges normally do not cross when drawn over the sentence. Here a new null hypothesis for the number of edge crossings of a sentence is presented. That null hypothesis takes into account the length of the pair of edges that may cross and predicts the relative number of crossings in random trees with a small error, suggesting that a ban of crossings or a principle of minimization of crossings are not needed in general to explain the origins of non-crossing dependencies. Our work paves the way for more powerful null hypotheses to investigate the origins of non-crossing dependencies in Nature.

editor's choice

Introduction. – The syntactic structure of a sentence can be defined as a network where vertices are words and edges indicate syntactic dependencies [1,2] as in fig. 1. The most common assumption is that this structure is a tree (an acyclic connected graph) (*e.g.*, [1,3]). In the 1960s, a striking pattern of syntactic dependency trees of sentences was reported: dependencies between words normally do not cross when drawn over the sentence [4,5] (*e.g.*, fig. 1). C, the number of different pairs of edges that cross, is small in real sentences. In fig. 1, $C = 0$ for sentence (a) and $C = 1$ for sentence (b). Interestingly, the tree structure of both sentences is the same but C varies, showing that C depends on the linear arrangement of the vertices.

Imagine that $\pi(v)$ is defined as the position of the vertex v in a linear arrangement of n vertices (the 1st vertex has position 1, the second vertex has position 2 and so on ...) and thus $1 \leq \pi(v) \leq n$. $u \sim v$ is used to refer to an edge formed by the vertices u and v. The length of the edge $u \sim v$ in words is $d(u \sim v) = |\pi(u) - \pi(v)|$ (here $|\dots|$ is the absolute value operator). $s(u \sim v)$ and $e(u \sim v)$ are defined, respectively, as the initial and the end position of the edge $u \sim v$, *i.e.* $s(u \sim v) = \min(\pi(u), \pi(v))$ and $e(u \sim v) = \max(\pi(u), \pi(v))$. $u_1 \sim v_1$ and $u_2 \sim v_2$ cross if and only if one of the following conditions is met:

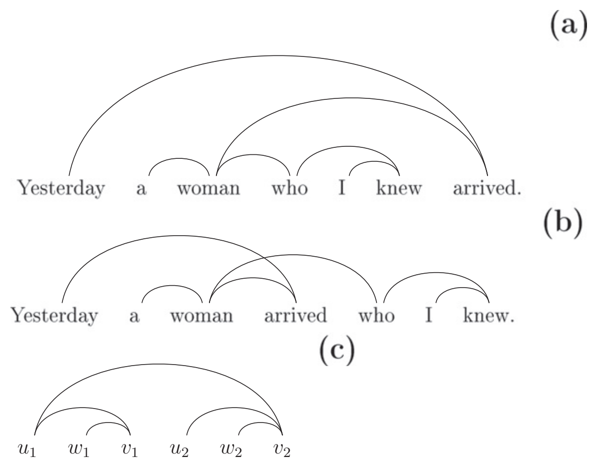

Fig. 1: (a) A sentence without crossings. (b) An alternative ordering yielding one crossing: the link *yesterday* ∼ *arrived* crosses the link *woman* ∼ *who* and vice versa. (c) An abstract structure. Panels (a) and (b) are adapted from [3].

– $s(u_1 \sim v_1) < s(u_2 \sim v_2)$ and $s(u_2 \sim v_2) < e(u_1 \sim v_1)$ and $e(u_1 \sim v_1) < e(u_2 \sim v_2)$;

– $s(u_1 \sim v_1) > s(u_2 \sim v_2)$ and $s(u_1 \sim v_1) < e(u_2 \sim v_2)$ and $e(u_2 \sim v_2) < e(u_1 \sim v_1)$.

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$$E[C] = \frac{1}{2} \sum_{u_1 \sim v_1} \sum_{u_2 \sim v_2, \{u_1, v_1\} \cap \{u_2, v_2\} = \emptyset} E[C(u_1 \sim v_1, u_2 \sim v_2)]. \quad (5)$$

It has been hypothesized that $C \approx 0$ in real sentences [1,6] could be due to a principle of minimization of the length of edges [7–10]. Although the minimization of

$$D = \sum_{u \sim v} d(u \sim v) \quad (1)$$

reduces crossings to practically zero [7], this does not provide a full explanation about the low frequency of crossings in real sentences: (a) minimum D does not imply $C = 0$ [11], (b) the actual value of D in real sentences is located between the minimum and that of a random ordering of vertices [12] and (c) the word order that minimizes D might be in a serious conflict with other linguistic or cognitive constraints [13]. Here the problem of the reduction of D that is required for explaining $C \approx 0$ in real sentences is avoided by means of a null hypothesis that predicts C by considering the actual length of the edges that may cross. With this null hypothesis, one can shed light on a fundamental question: how much surprising is it that $C \approx 0$ given the lengths of edges? That null hypothesis is vital for the development of a general but minimal theory of crossing dependencies in Nature. First, $C \approx 0$ in sentences might also be due to a ban of crossings by grammar [2] or a principle of minimization of C [8]. Second, crossings have also been investigated in networks of nucleotides [14]. Here it will be shown that a simple null hypothesis based on actual dependency lengths would suffice *a priori* for predicting $C \approx 0$ in short enough sentences.

Crossing theory. –

The expected number of crossings. $C(u \sim v)$ is defined as the number of edge crossings where the edge formed by u and v is involved. C can be defined as

$$C = \frac{1}{2} \sum_{u \sim v} C(u \sim v), \quad (2)$$

where the $1/2$ factor is due to the fact that if two edges $u_1 \sim v_1$ and $u_2 \sim v_2$ cross, their crossing will be counted twice, one through $C(u_1 \sim v_1)$ and another through $C(u_2 \sim v_2)$. $C(u_1 \sim v_1)$ can be defined as

$$C(u_1 \sim v_1) = \sum_{u_2 \sim v_2, \{u_1, v_1\} \cap \{u_2, v_2\} = \emptyset} C(u_1 \sim v_1, u_2 \sim v_2), \quad (3)$$

where $C(u_1 \sim v_1, u_2 \sim v_2)$ indicates if u_1, v_1 and u_2, v_2 define a couple of edges that cross, *i.e.* $C(u_1 \sim v_1, u_2 \sim v_2) = 1$ if they cross, $C(u_1 \sim v_1, u_2 \sim v_2) = 0$ otherwise. Applying the definition of $C(u \sim v)$ in eq. (3), C becomes

$$C = \frac{1}{2} \sum_{u_1 \sim v_1} \sum_{u_2 \sim v_2, \{u_1, v_1\} \cap \{u_2, v_2\} = \emptyset} C(u_1 \sim v_1, u_2 \sim v_2). \quad (4)$$

- Assume that the vertices are labeled with integers from 1 to n.
- Produce a uniformly random spanning tree with the Aldous-Brother algorithm [15,16], assuming a complete graph as the basis of the random walk.
- Take vertex labels as vertex positions ($\pi(v) = v$ for every vertex v).

Fig. 2: Procedure to generate a random labeled tree and a random linear arrangement of its vertices.

Suppose that the vertices are arranged linearly at random (being all the permutations of the vertex sequence equally likely). Then, the expectation of C is

see eq. (5) *above*

As $C(u_1 \sim v_1, u_2 \sim v_2)$ is and indicator variable, $E[C(u_1 \sim v_1, u_2 \sim v_2)]$ can be replaced by $p(\text{cross}) = 1/3$, the probability that two arbitrary edges that to not share any vertex cross when their vertices are arranged linearly at random, which yields [17]

$$E_0[C] = C_{max}/3 \quad (6)$$

with

$$C_{max} = \frac{n}{2}\left(n - 1 - \langle k^2 \rangle\right) \quad (7)$$

being the number of edge pairs that can potentially cross and $\langle k^2 \rangle$ the degree 2nd moment of the tree [10]. $\langle k^2 \rangle$ is the mean of squared degrees, *i.e.*

$$\langle k^2 \rangle = \sum_v k_v^2, \quad (8)$$

where k_v is the degree of vertex v. In uniformly random labeled trees, the expected $\langle k^2 \rangle$ is [18,19]

$$E\left[\langle k^2 \rangle\right] = \left(1 - \frac{1}{n}\right)\left(5 - \frac{6}{n}\right). \quad (9)$$

Thus, the expectation of $E_0[C]$ for those trees is

$$\begin{aligned} E[E_0[C]] &= \frac{n}{6}\left(n - 1 - E\left[\langle k^2 \rangle\right]\right) \\ &= \frac{n^2}{6} - n + \frac{11}{6} - \frac{1}{n}. \end{aligned} \quad (10)$$

This analytical result is easy to check numerically by generating random linear arrangements of vertices of random trees with the procedure in fig. 2.

Here we aim to improve $E_0[C]$ introducing information about the actual length of the dependencies. Suppose that

$$p(u_1 \sim v_1 \text{ and } u_2 \sim v_2 \text{ cross} | d) \quad (11)$$

$$E[C|d] = \frac{1}{2} \sum_{u_1 \sim v_1} \sum_{u_2 \sim v_2, \{u_1,v_1\} \cap \{u_2,v_2\} = \emptyset} E[C(u_1 \sim v_1, u_2 \sim v_2)|d] \tag{12}$$

$$= \frac{1}{2} \sum_{u_1 \sim v_1} \sum_{u_2 \sim v_2, \{u_1,v_1\} \cap \{u_2,v_2\} = \emptyset} p(u_1 \sim v_1 \text{ and } u_2 \sim v_2 \text{ cross}|d). \tag{13}$$

$$E_2[C] = \frac{1}{2} \sum_{u_1 \sim v_1} \sum_{u_2 \sim v_2, \{u_1,v_1\} \cap \{u_2,v_2\} = \emptyset} p(\text{cross}|d(u_1 \sim v_1), d(u_2 \sim v_2)). \tag{15}$$

is the probability that the edges $u_1 \sim v_1$ and $u_2 \sim v_2$ cross in a random linear arrangement of vertices where edge lengths are given by the function d above. Then, $E[C|d]$, the expected number of crossings given full knowledge about edge lengths, can be defined as

see eqs. (12) *and* (13) *above*

The calculation of $E[C|d]$ for a given sentence is not straightforward: it requires the calculation of all the permutations of the words of the sentence preserving the edge lengths of the original sentence. Besides, $E[C|d]$ makes a prediction about the crossings of a dependency tree involving a lot of information: the edges of the tree and their length. In contrast, $E_0[C]$ can be computed just from knowledge about the degree sequence or simply the values of n and $\langle k^2 \rangle$, as eqs. (6) and (7) indicate. Here we aim to predict the number of crossings reducing the computational and informational demands of $E[C|d]$ while beating the predictions of $E_0[C]$.

$p(\text{cross}|d(u_1 \sim v_1), d(u_2 \sim v_2))$ is defined as the probability that two edges that are arranged linearly at random cross knowing that their lengths are $d(u_1 \sim v_1)$ and $d(u_2 \sim v_2)$ and that they do not share any vertex. Replacing

$$p(u_1 \sim v_1 \text{ and } u_2 \sim v_2 \text{ cross}|d) \tag{14}$$

by $p(\text{cross}|d(u_1 \sim v_1), d(u_2 \sim v_2))$ in eq. (13), one obtains

see eq. (15) *above*

$E_x[C]$ refers to an approximation to the expected value of C knowing the length of x edges in every potential crossing (giving priority to the knowledge about the lengths of the pair of edges that may cross in every potential crossing as in eq. (15)). $E_2[C]$ is an approximation to $E[C|d]$ that is based on a stronger null hypothesis than that of $E_0[C]$ for the probability that two edges cross. $E_0[C]$ and $E_{n-1}[C]$ are true expectations (notice $E_{n-1}[C] = E[C|d]$). While $E[C|d]$ conditions globally with the function d, *i.e.* the same conditioning for every pair of edges that may cross, $E_2[C]$ conditions locally with two edge lengths that depend on the pair of edges under consideration (eq. (13) *vs.* eq. (15)). In the remainder of the paper two virtues of $E_2[C]$ over $E[C|d]$ will be shown. First, $E_2[C]$ is easier to calculate. Second, it predicts C with small error in spite of discarding, for every pair of edges that may potentially cross, the lengths of other edges. The point is: if such a rough but simple predictor of crossing works, is it necessary to believe that crossings are forbidden by grammars [2] or postulate an independent principle of minimization of C [8]?

The probability that two edges cross knowing their lengths. The set $S(n,d)$ is defined as the set of possible initial positions for an edge of length d in a sequence of length n, *i.e.*

$$S(n,d) = \{s | 1 \le s \le n - d\}. \tag{16}$$

We say that s_1 and s_2 are a valid pair of initial positions if they define the initial positions of two edges that have lengths d_1 and d_2, respectively, and that do not share vertices, *i.e.* $s_1 \in S(n, d_1)$, $s_2 \in S(n, d_2)$ and $\{s_1, s_1 + d_1\} \cap \{s_2, s_2 + d_2\} = \emptyset$.

$p(\text{cross} = 1|d_1, d_2)$ can be defined as a proportion, *i.e.*

$$p(\text{cross}|d_1, d_2) = \frac{|\alpha(d_1, d_2)|}{|\beta(d_1, d_2)|}, \tag{17}$$

where here $|\ldots|$ is the cardinality operator, $\alpha(d_1, d_2)$ is the set of valid pairs of initial position of two edges of lengths d_1 and d_2 that involve a crossing and $\beta(d_1, d_2)$ is simply the set of valid pairs of initial positions of edges of lengths d_1 and d_2. More formally,

$$\beta(d_1, d_2) = \{s_1, s_2 | s_1 \text{ and } s_2 \text{ are valid initial positions}\} \tag{18}$$

and

see eq. (19) *on the next page*

The definition of $\alpha(d_1, d_2)$ is based on an adapted version of the formal definition of crossing in the introduction section (notice that $e(u \sim v) = s(u \sim v) + d(u \sim v)$). Figure 3 shows $p(\text{cross}|d_1, d_2)$ for two different numbers of vertices. If $\beta(d_1, d_2) = 0$ then $\alpha(d_1, d_2) = 0$ and then $p(\text{cross}|d_1, d_2)$ is undefined (notice that $\beta(n-1, n-1) = \beta(n-2, n-1) = \beta(n-1, n-2) = 0$). If that happens, the reasonable convention that $p(\text{cross}|d_1, d_2) = 0$ is adopted. The order of edge length information is irrelevant, *i.e.* $p(\text{cross}|d_1, d_2) = p(\text{cross}|d_2, d_1)$, as fig. 3 shows. Some crossings are impossible *a priori*, *i.e.* $p(\text{cross}|1, d_2) = p(\text{cross}|n-1, d_2) = 0$ and

$$\begin{aligned}\alpha(d_1, d_2) = \{&s_1, s_2 | s_1 \text{ and } s_2 \text{ are valid initial positions and} \\ &(s_1 < s_2 \text{ and } s_2 < s_1 + d_1 \text{ and } s_1 + d_1 < s_2 + d_2) \text{ or} \\ &(s_1 > s_2 \text{ and } s_1 < s_2 + d_2 \text{ and } s_2 + d_2 < s_1 + d_1)\}.\end{aligned} \quad (19)$$

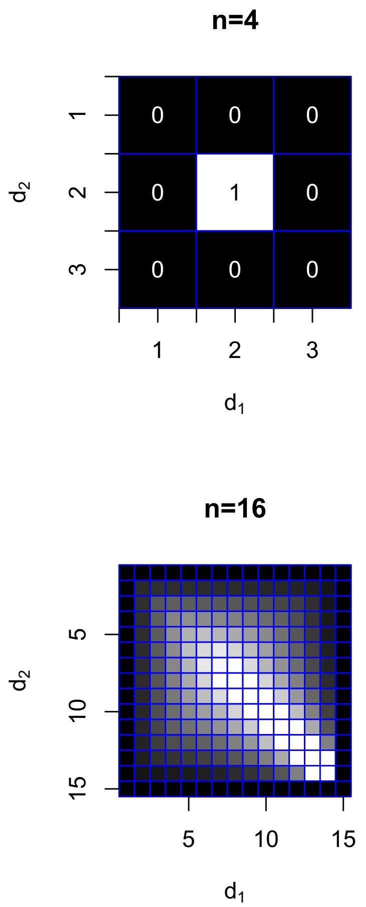

Fig. 3: (Colour on-line) $p(\text{cross}|d_1, d_2)$, the probability that two edges cross when arranged linearly at random knowing their lengths (d_1 and d_2) and that they do not share vertices. Brightness is proportional to $p(\text{cross}|d_1, d_2)$ (black for $p(\text{cross}|d_1, d_2) = 0$ and white for $p(\text{cross}\ |d_1, d_2) = 1$). n is the number of vertices ($C > 0$ needs $n \geq 4$ [10]).

some others are unavoidable, *e.g.*, $p(\text{cross}|n-2, n-2) = 1$ (we are assuming $n \geq 4$).

$p(\text{cross})$ and $p(\text{cross}|d_1, d_2)$ are related through

$$\sum_{d_1=1}^{n-1} \sum_{d_2=1}^{n-1} p(\text{cross}|d_1, d_2) p(d_1, d_2) = p(\text{cross}), \quad (20)$$

where $p(d_1, d_2)$ is the probability that a random linear arrangement of four different vertices, *i.e.* u_1,v_1,u_2 and v_2, produces $|\pi(u_1) - \pi(v_1)| = d_1$ and $|\pi(u_2) - \pi(v_2)| = d_2$.

Results. – The relative number of crossings is defined as $\bar{C}_{true} = C_{true}/C_{max}$ and thus $E_x[\bar{C}] = E_x[C]/C_{max}$. Table 1 shows that $E_2[\ldots]$ makes better predictions about the (absolute or relative) number of crossings than $E_0[\ldots]$ for the real syntactic dependency trees in fig. 1. $\bar{C}_{true}$ and $E_x[\bar{C}]$ allow for a fairer comparison of the real number of crossings and its predictions as they measure crossings in units of the potential number of crossings. We wish to investigate if $E_x[\bar{C}]$ might shed light on the small number of crossings of real sentences abstracting away from the details of a concrete language, in the spirit of a long tradition of research on crossing dependencies [20,21]. Our language neutral perspective is not based on the analysis of real syntactic dependency trees but those of uniformly random labeled trees whose vertex labels are distinctive numbers from 1 to n that also represent the positions of the vertices, *i.e.* $\pi(v) = v$. Here we aim to compare the capacity of $E_0[\bar{C}]$ and $E_2[\bar{C}]$ to predict $\bar{C}_{true}$, the real number of a crossings in uniformly random labeled trees, when C_{true} is small ($C_{true} \leq 3$) as in real sentences [4,5]. The relative error of the prediction is defined as

$$\begin{aligned}\Delta_x &= E_x[\bar{C}] - \bar{C}_{true} \\ &= (E_x[C] - C_{true})/C_{max}.\end{aligned} \quad (21)$$

For every sentence of length $n \geq 4$ (because $C > 0$ needs it [10]), an ensemble of $R = 10^4$ uniformly random labeled trees with $C_{true} \leq 3$ was generated a) following the procedure in fig. 2 and b) rejecting random trees yielding $C_{true} > 3$ till the desired size R was reached. For every relevant value of C_{true} ($0 \leq C_{true} \leq 3$), the mean Δ_2 was calculated over all configurations where $C_{max} > 0$ ($C_{max} = 0$ is only achieved by star trees [10]). $n_{max} = 20$ was the maximum sentence length considered due to the explosion of rejections as n increases. The space of possible trees is huge (there are n^{n-2} labeled trees of n vertices [22]) and trees with $C_{true} \leq 3$ have a number of crossings that is unexpectedly low for that class of random trees (recall eq. (10)). These considerations notwithstanding, n_{max} covers the average length of English sentences (about 17.8 words [23], pp. 37–55), and that of other languages [12].

Figure 4 shows the mean Δ_x over ensembles of random trees with $C_{true} \leq 3$ indicating both $E_0[\bar{C}]$ and $E_2[\bar{C}]$ overestimate $\bar{C}_{true}$ in general. While Δ_2 is small, *i.e.* of the order of 5%, Δ_0 converges to 1/3 as expected from the fact that

$$\begin{aligned}\Delta_0 &= (C_{max}/3 - C_{true})/C_{max} \\ &= 1/3 - C_{true}/C_{max},\end{aligned} \quad (22)$$

which yields $\Delta_0 \approx 1/3$ for sufficiently large n and C_{true} small.

Table 1: The properties and predictions of crossings for the sentences in fig. 1. n is the number of vertices (sentence length in words), $\langle k^2 \rangle$ is the degree 2nd moment, C_{max} is the potential number of crossings, C_{true} and $\bar{C}_{true}$ are, respectively, the absolute and the relative actual number of crossings. $E_0[\ldots]$ is the expectation of crossings ignoring edge lengths and $E_2[\ldots]$ is an approximation to the expectation knowing the lengths of edges. Numbers were rounded to leave two significant decimals.

Example	n	$\langle k^2 \rangle$	C_{max}	C_{true}	$E_0[C]$	$E_2[C]$	$\bar{C}_{true}$	$E_0[\bar{C}]$	$E_2[\bar{C}]$
Fig. 1(a)	7	3.4	9	0	3	0.57	0	0.33	0.063
Fig. 1(b)	7	3.4	9	1	3	1.5	0.11	0.33	0.17

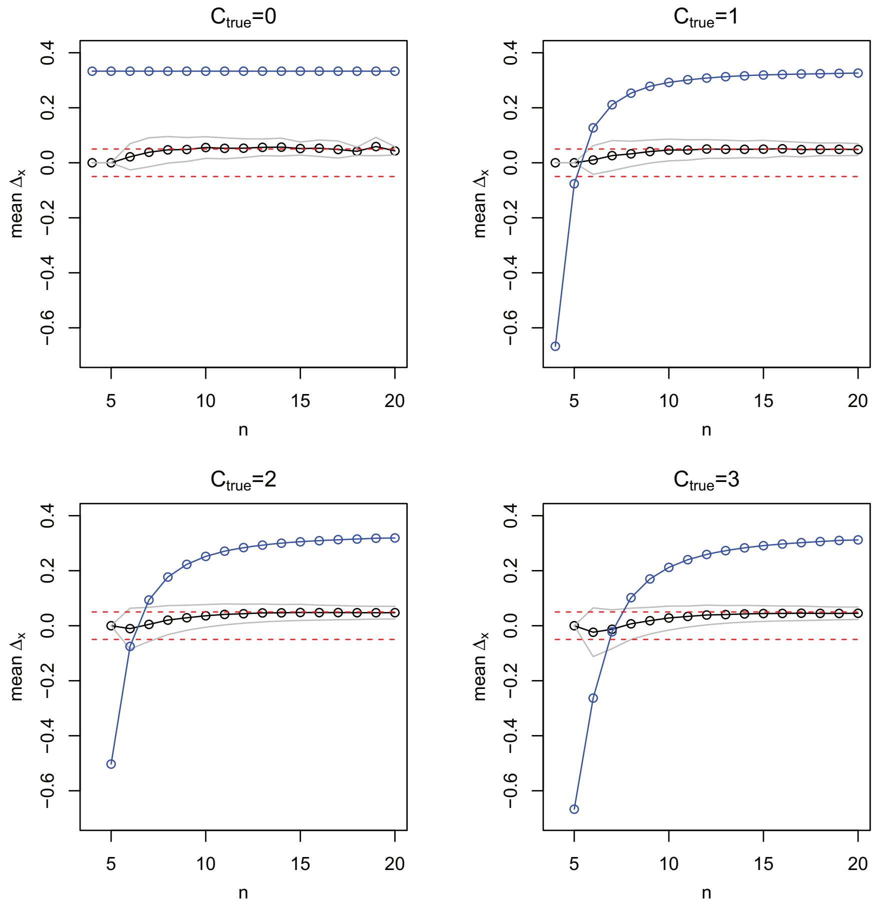

Fig. 4: (Colour on-line) The average relative error Δ_x as a function of the number of vertices n of the random trees conditioning on C_{true} (black for $x = 2$ and blue for $x = 0$). The mean Δ_2 is surrounded by two boundary gray lines: one standard deviation above and one standard deviation below. The two red dashed lines are a guide to the eye for $\Delta_x = \pm 0.05$. $C_{true} > 1$ is impossible for $n < 4$ [10].

Discussion. – It has been shown that $E_2[\bar{C}]$ is able to predict the actual relative number of crossings in random unlabeled trees. This is not very surprising: the edge length does give information on how likely edges are to cross. What is not straightforward is that a method that estimates crossings based exclusively on local dependency length information (just on the length of the pair of edges that can potentially cross) is able to make predictions with a small relative error in trees of the size of real sentences. Our finding has important consequences for language research: it suggests that there is no need *a priori* for banning crossings by grammar [2] or minimizing C [8] to explain $C \approx 0$ in short enough sentences. This is consistent with the view that syntactic constraints, in general, do not imply an internally represented grammar [21].

However, the predictive power of $E_2[\bar{C}]$ decreases slightly as the number of vertices increases (fig. 4). The reason is very simple: $E_2[\ldots]$ departs from an estimation of the probability that two edges cross that is based exclusively on their lengths, thus discarding the length of other edges. $p(\text{cross}|d_1, d_2)$ neglects the length of $n - 3$ edges. As n increases, the amount of information discarded increases and predictions worsen. In the tree in fig. 1(c), the only pairs of edges that could cross in the sense of $p(\text{cross}|d_1, d_2) > 0$ (*i.e.* if dependency lengths of other edges were ignored) are $u_1 \sim v_1$ and $u_2 \sim v_2$ (recall that edges of length 1 or $n-1$ cannot produce crossings). Equation (17) gives $p(\text{cross}|d_1 = d_2 = 2) = 0.75$ but $p(C(u_1 \sim v_1, u_2 \sim v_2) = 1|d(u_1 \sim v_1) = d(u_2 \sim v_2) = 2, d(u_1 \sim v_2) = 5) = 0$ ($d(u_1 \sim v_2) = 5$ can only be achieved placing u_1 and v_2 at the ends of the sequence, which turns $C(u_1 \sim v_1, u_2 \sim v_2) = 1$ impossible). For this reason, $E_{n-1}[\bar{C}]$, the expected relative number of crossings knowing all edge lengths in every potential crossing, should be investigated in the future.

* * *

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Note added in proofs: Reference [14] has been used to show the interest of crossing dependencies beyond human language. A fundamental reference for the issue of crossing/non-crossing dependencies in both language and RNA structure is [24].

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