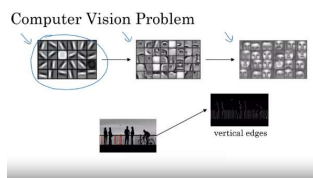
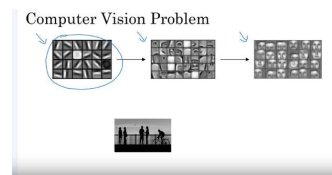
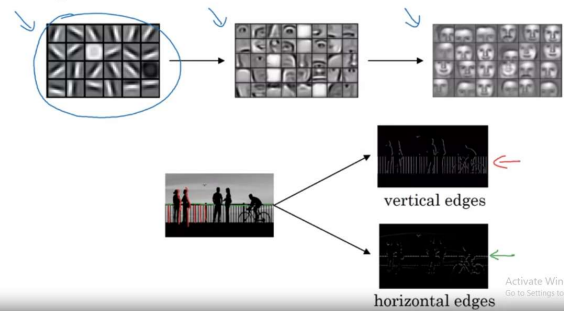


Convolution Neural Networks

Week 1



Computer Vision Problem



Vertical edge detection

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

 6×6

\ast

 3×3
filter

Vertical edge detection

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

 6×6

\ast

1	0	-1
1	0	-1
1	0	-1

 3×3
filter

Vertical edge detection

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

 6×6

\ast

1	0	-1
1	0	-1
1	0	-1

 3×3
filter

What would be dimensions of resulting matrix ??

Vertical edge detection

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

 6×6

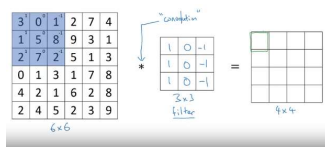
\ast

1	0	-1
1	0	-1
1	0	-1

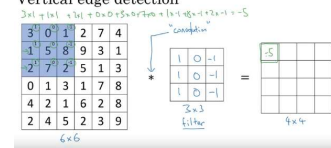
 3×3
filter

$=$

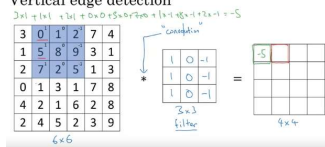
 4×4



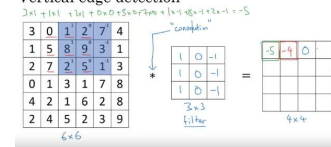
Vertical edge detection



Vertical edge detection



Vertical edge detection



Vertical edge detection

$$3 \times 1 + 1 \times 1 + 3 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times 1 + 0 \times 1 + 7 \times 1 + 1 \times 1 = -5$$

6x6

3x3 filter

4x4

Vertical edge detection

$$3 \times 1 + 1 \times 1 + 3 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times 1 + 0 \times 1 + 7 \times 1 + 1 \times 1 = -5$$

6x6

3x3 filter

4x4

Vertical edge detection

$$3 \times 1 + 1 \times 1 + 3 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times 1 + 0 \times 1 + 7 \times 1 + 1 \times 1 = -5$$

6x6

3x3 filter

4x4

Vertical edge detection

$$3 \times 1 + 1 \times 1 + 3 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times 1 + 0 \times 1 + 7 \times 1 + 1 \times 1 = -5$$

6x6

3x3 filter

4x4

Fill remaining cells

Vertical edge detection

$$3 \times 1 + 1 \times 1 + 2 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times -1 + 8 \times -1 + 2 \times -1 = -5$$

6x6

3x3 filter

4x4

Vertical edge detection

$$3 \times 1 + 1 \times 1 + 2 \times 1 + 0 \times 0 + 5 \times 0 + 7 \times 0 + 1 \times -1 + 8 \times -1 + 2 \times -1 = -5$$

6x6

3x3 filter

4x4

python: conv-forward
tensorflow: tf.nn.conv2d
keras: Conv2D

Activate Windows
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Vertical edge detection

6x6

3x3 filter

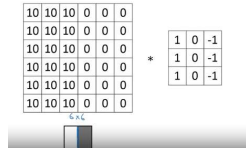
Vertical edge detection

6x6

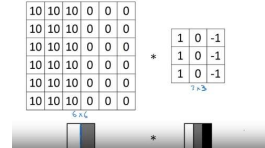
3x3 filter

Acti
Go to

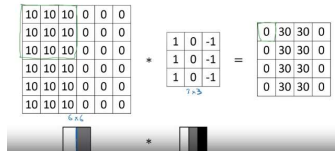
Vertical edge detection



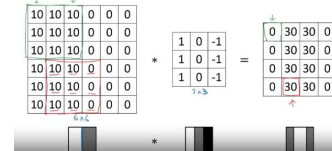
Vertical edge detection



Vertical edge detection



Vertical edge detection



More on Edge Detection

Vertical edge detection examples

00	00	00	00
00	00	00	00
00	00	00	00
00	00	00	00
00	00	00	00

+

1	0	1
1	0	1
1	0	1

=

0	00	00
0	00	00
0	00	00
0	00	00
0	00	00

+
=

7

Vertical and Horizontal Edge Detection

$$\begin{array}{c}
 \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\
 \text{Vertical}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \\
 \text{Horizontal}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} \\
 \text{Input Image}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \\
 \text{Kernel}
 \end{array}
 =
 \begin{array}{c}
 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \text{Output Image}
 \end{array}$$

Vertical and Horizontal Edge Detection

$$\begin{array}{c}
 \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\
 \text{Vertical}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \\
 \text{Horizontal}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} \\
 \text{Input Image}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \\
 \text{Kernel}
 \end{array}
 =
 \begin{array}{c}
 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \text{Output Image}
 \end{array}$$

Learning to detect edges

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \text{ Sobel Filter}$$

Learning to detect edges

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \text{ Sobel Filter} \quad \begin{bmatrix} 1 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix} \text{ Sobel Filter}$$

Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

→

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Activate Windows
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Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

*

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

=

Padding

1	0	-1
1	0	-1
1	0	-1

↑

→

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

*

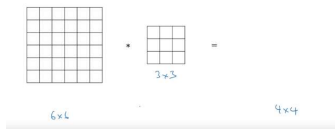
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

=

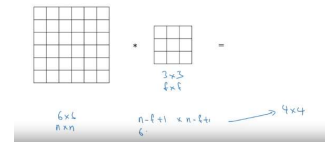
45°
70°
73°

Activate Windows
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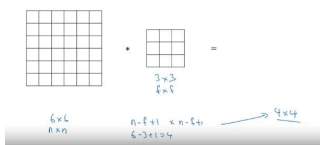
Padding



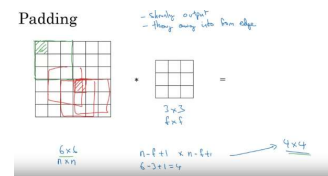
Padding

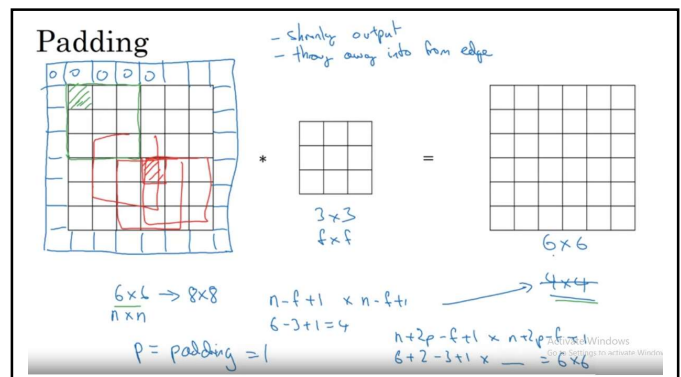
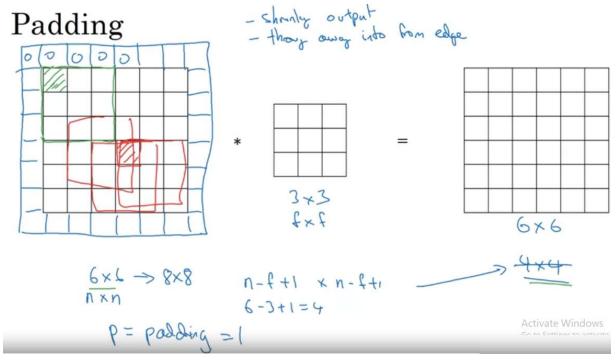
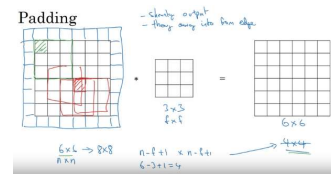
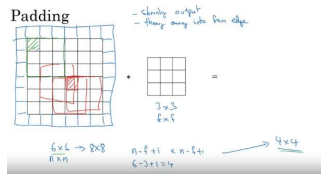


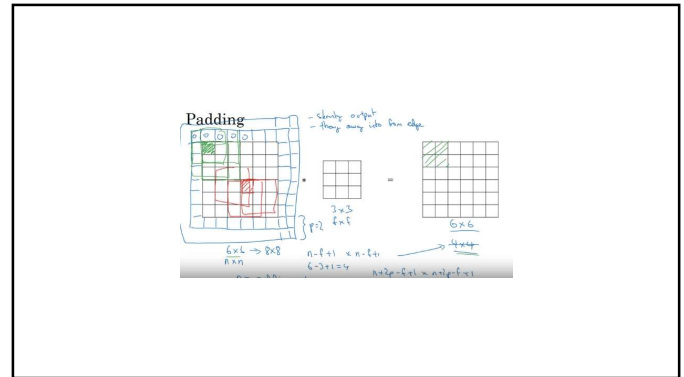
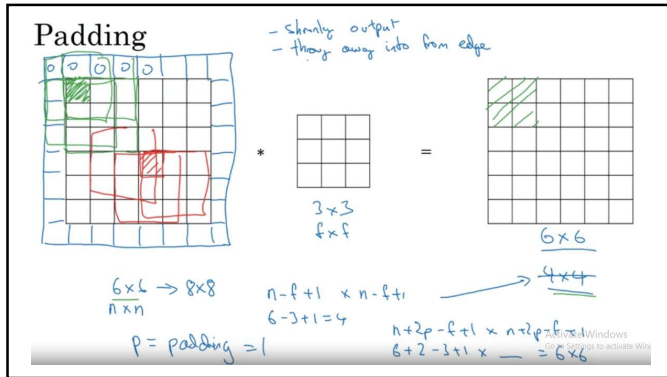
Padding



Padding







Valid and Same convolutions

"Valid": $n \times n \times f \times f \rightarrow n - f + 1 \times n - f + 1$

Valid and Same convolutions

"Valid": $n \times n \times f \times f \rightarrow n - f + 1 \times n - f + 1$

$6 \times 6 \times 3 \times 3 \rightarrow 4 \times 4$

Valid and Same convolutions

→ no padding

"Valid": $n \times n$ \times $f \times f \rightarrow \underline{n-f+1} \times n-f+1$
 6×6 \times $3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.
 $n+2p-f+1 \times n+2p-f+1$

Valid and Same convolutions

→ no padding

"Valid": $n \times n$ \times $f \times f \rightarrow \underline{n-f+1} \times n-f+1$
 6×6 \times $3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.
 $n+2p-f+1 \times n+2p-f+1$
 $6+2p-3+1 = 6 \Rightarrow p = \frac{3-1}{2}$

Valid and Same convolutions

→ no padding

"Valid": $n \times n$ \times $f \times f \rightarrow \underline{n-f+1} \times n-f+1$
 6×6 \times $3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.

$$n+2p-f+1 \times n+2p-f+1$$

$$6+2p-3+1 = 6 \Rightarrow p = \frac{3-1}{2}$$

$$3 \times 3 \quad p = \frac{3-1}{2} = 1$$

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Valid and Same convolutions

→ no padding

"Valid": $n \times n$ \times $f \times f \rightarrow \underline{n-f+1} \times n-f+1$
 6×6 \times $3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.

$$n+2p-f+1 \times n+2p-f+1$$

$$6+2p-3+1 = 6 \Rightarrow p = \frac{3-1}{2}$$

$$3 \times 3 \quad p = \frac{3-1}{2} = 1 \quad \left| \begin{array}{l} 5 \times 5 \\ f=5 \end{array} \right. \quad p=2$$

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Valid and Same convolutions

$\nearrow n = \text{padding}$
 "Valid": $n \times n \times f \times f \rightarrow \frac{n-f+1}{1} \times \frac{n-f+1}{1}$
 $6 \times 6 \times 3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$
 $n+2p-f+1 = n \Rightarrow p = \frac{f-1}{2}$
 $3 \times 3 \quad p = \frac{3-1}{2} = 1 \quad \left| \begin{array}{c} 5 \times 5 \\ f=5 \end{array} \right. \quad p=2$

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Valid and Same convolutions

$\nearrow n = \text{padding}$
 "Valid": $n \times n \times f \times f \rightarrow \frac{n-f+1}{1} \times \frac{n-f+1}{1}$
 $6 \times 6 \times 3 \times 3 \rightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$
 $n+2p-f+1 = n \Rightarrow p = \frac{f-1}{2}$
 $3 \times 3 \quad p = \frac{3-1}{2} = 1 \quad \left| \begin{array}{c} 5 \times 5 \\ f=5 \end{array} \right. \quad p=2$

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Strided Convolution

Strided convolution

$$\begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 5 & 6 \\ \hline 6 & 7 & 8 & 9 & 10 \\ \hline 3 & 4 & 5 & 6 & 7 \\ \hline 7 & 8 & 9 & 10 & 11 \\ \hline 4 & 5 & 6 & 7 & 8 \\ \hline 8 & 9 & 10 & 11 & 12 \\ \hline 5 & 6 & 7 & 8 & 9 \\ \hline 9 & 10 & 11 & 12 & 13 \\ \hline \end{array}
 \times
 \begin{array}{|c|c|c|} \hline 3 & 4 & 5 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

5×5 3×3 3×3
 $\text{stride} = 2$

Strided convolution

$$\begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 7 & 4 \\ \hline 6 & 6 & 9 & 8 & 7 \\ \hline 3 & 4 & 5 & 8 & 9 \\ \hline 7 & 8 & 3 & 6 & 3 \\ \hline 4 & 2 & 1 & 8 & 3 \\ \hline 3 & 2 & 4 & 1 & 9 \\ \hline 0 & 1 & 3 & 9 & 2 \end{array}
 \begin{array}{|c|c|c|} \hline 3 & 4 & 4 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|} \hline 9 & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$\xrightarrow{\text{stride}=2}$ $\text{stride}=2$

Strided convolution

$$\begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 7 & 4 \\ \hline 6 & 6 & 9 & 8 & 7 \\ \hline 3 & 4 & 5 & 8 & 9 \\ \hline 7 & 8 & 3 & 6 & 3 \\ \hline 4 & 2 & 1 & 8 & 3 \\ \hline 3 & 2 & 4 & 1 & 9 \\ \hline 0 & 1 & 3 & 9 & 2 \end{array}
 \begin{array}{|c|c|c|} \hline 3 & 4 & 4 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|} \hline 9 & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$\xrightarrow{\text{stride}=2}$ $\text{stride}=2$

Strided convolution

$$\begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 7 & 4 \\ \hline 6 & 6 & 9 & 8 & 7 \\ \hline 3 & 4 & 5 & 8 & 9 \\ \hline 7 & 8 & 3 & 6 & 3 \\ \hline 4 & 2 & 1 & 8 & 3 \\ \hline 3 & 2 & 4 & 1 & 9 \\ \hline 0 & 1 & 3 & 9 & 2 \end{array}
 \begin{array}{|c|c|c|} \hline 3 & 4 & 4 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|} \hline 9 & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$\xrightarrow{\text{stride}=2}$ $\text{stride}=2$

Strided convolution

$$\begin{array}{|c|c|c|c|c|} \hline 2 & 3 & 4 & 7 & 4 \\ \hline 6 & 6 & 9 & 8 & 7 \\ \hline 3 & 4 & 5 & 8 & 9 \\ \hline 7 & 8 & 3 & 6 & 3 \\ \hline 4 & 2 & 1 & 8 & 3 \\ \hline 3 & 2 & 4 & 1 & 9 \\ \hline 0 & 1 & 3 & 9 & 2 \end{array}
 \begin{array}{|c|c|c|} \hline 3 & 4 & 4 \\ \hline 1 & 0 & 2 \\ \hline -1 & 0 & 3 \\ \hline \end{array}
 =
 \begin{array}{|c|c|c|} \hline 9 & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$\xrightarrow{\text{stride}=2}$ $\text{stride}=2$

Strided convolution

Fill remaining values

Strided convolution

Strided convolution

$n \times n$ * $f \times f$
 padding p stride s
 $s=2$

$$\frac{n+p-f}{s} + 1 \times \frac{n+p-f}{s} + 1$$

$$\frac{7+0-3}{2} + 1 = \frac{4}{2} + 1 = 3$$

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Strided convolution

$n \times n$ * $f \times f$
 padding p stride s
 $s=2$

$$\left\lfloor \frac{n+p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+p-f}{s} + 1 \right\rfloor$$

$$\left\lfloor \frac{7+0-3}{2} + 1 \right\rfloor = \frac{4}{2} + 1 = 3$$

$\lfloor z \rfloor = \text{floor}(z)$
 Activate Windows
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Summary of convolutions

$n \times n$ image $f \times f$ filter

padding p stride s

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Activate Wind

Technical note on cross-correlation vs. convolution

Convolution in math textbook:

2	3	7	4	6	2
6	6	9	8	7	4
3	4	8	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

3	4	5
1	0	2
-1	9	7

Technical note on cross-correlation vs. convolution

Convolution in math textbook:

2	3	7	4	6	2
6	6	9	8	7	4
3	4	8	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

3	4	5
1	0	2
-1	9	7

5	4	3

Activate 1
Go to Settings

Technical note on cross-correlation vs. convolution

Convolution in math textbook:

2	3	7	4	6	2
6	6	9	8	7	4
3	4	8	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

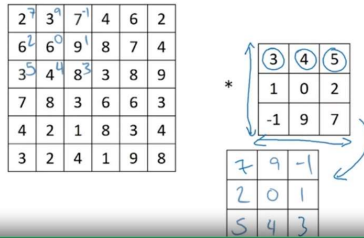
3	4	5
1	0	2
-1	9	7

7	9	-1
2	0	1
5	4	3

Activate
Go to 3

Technical note on cross-correlation vs. convolution

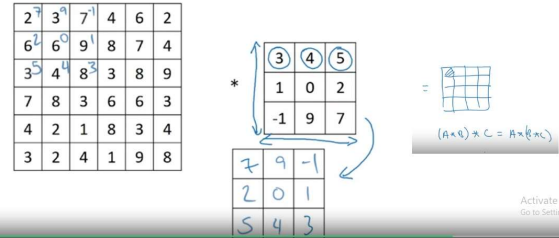
Convolution in math textbook:



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Technical note on cross-correlation vs. convolution

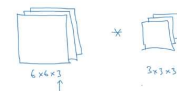
Convolution in math textbook:



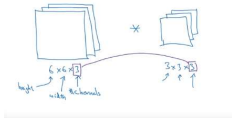
Activate Windows
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Convolution over Volume

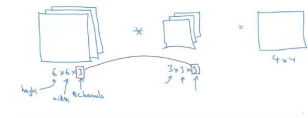
Convolutions on RGB images



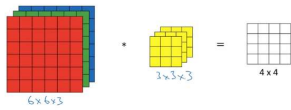
Convolutions on RGB images



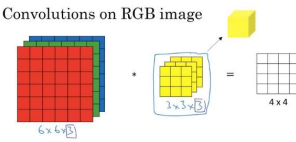
Convolutions on RGB images



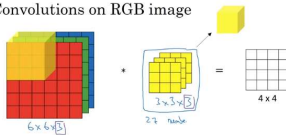
Convolutions on RGB image



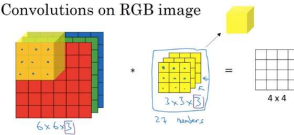
Convolutions on RGB image



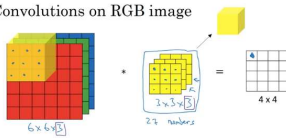
Convolutions on RGB image



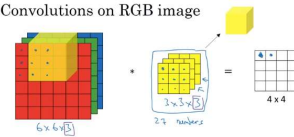
Convolutions on RGB image



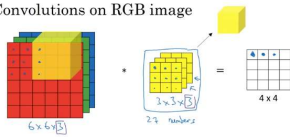
Convolutions on RGB image



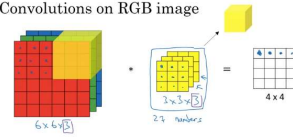
Convolutions on RGB image



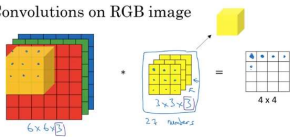
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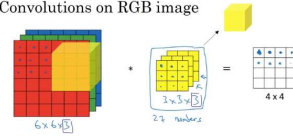
Convolutions on RGB image



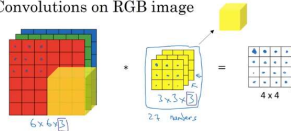
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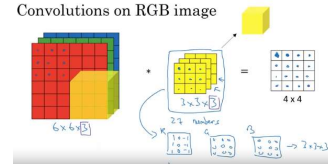
Convolutions on RGB image



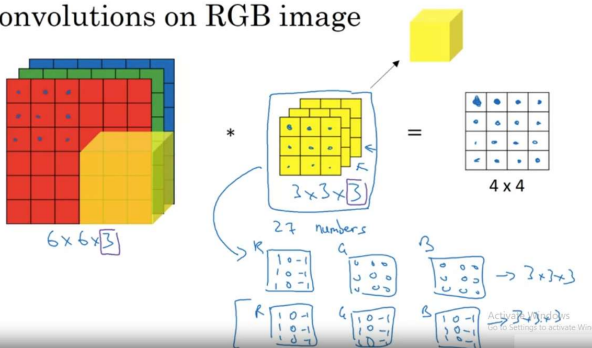
Convolutions on RGB image



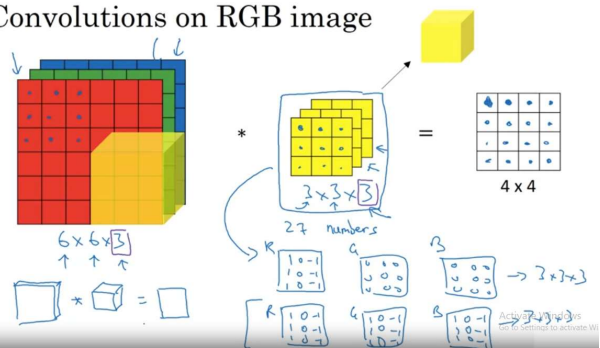
Convolutions on RGB image



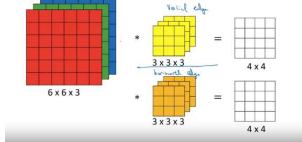
Convolutions on RGB image



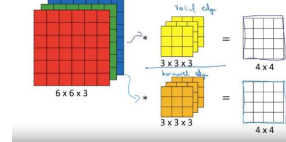
Convolutions on RGB image



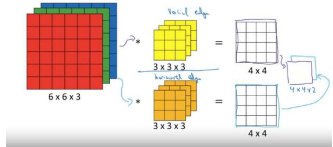
Multiple filters



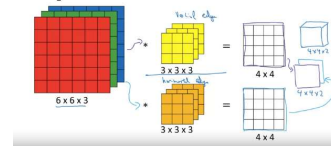
Multiple filters



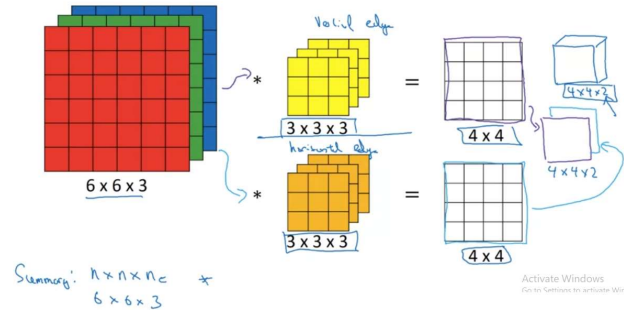
Multiple filters



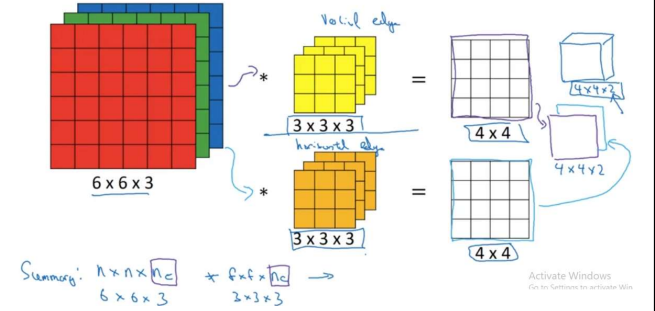
Multiple filters



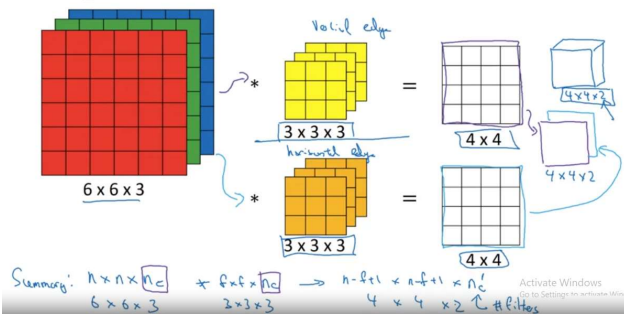
Multiple filters



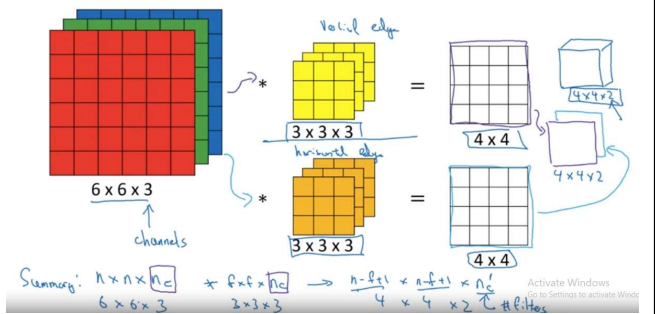
Multiple filters



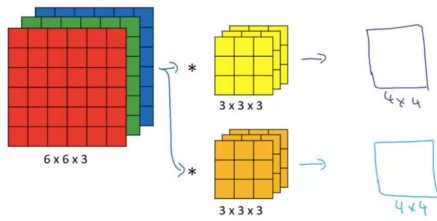
Multiple filters



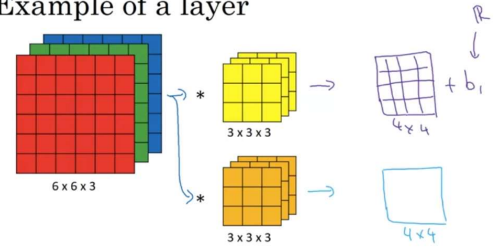
Multiple filters



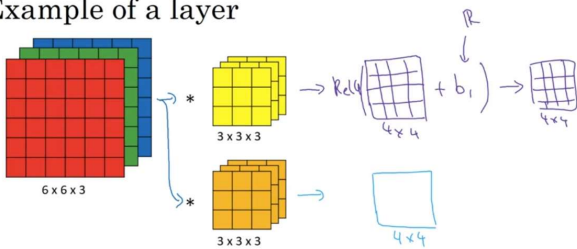
Example of a layer



Example of a layer



Example of a layer



Example of a layer

