Graph Theory

Dr. Irfan Yousuf
Department of Computer Science (New Campus)
UET, Lahore

(Lecture # 22; April 07, 2023)

Outline

- Graph Coloring

Graph Coloring Algorithm

- Different Types of Algorithms
  - Heuristics
  - Greedy

Welsh Powell Algorithm

- It introduced an upper bound to the chromatic number of a graph .
- It provides a greedy algorithm that runs on a static graph.

Welsh Powell Algorithm

Step 1: The vertex degree of each vertex is calculated and the vertex degrees are added to the degree set $Deg(v_i)$, such that $i = 1,2,\dots,n$.

Step 2: The uncolored vertex that has the largest degree in the degree set $Deg(v_i)$ is selected for coloring. Initially, the first color in the color set is selected as the active color.

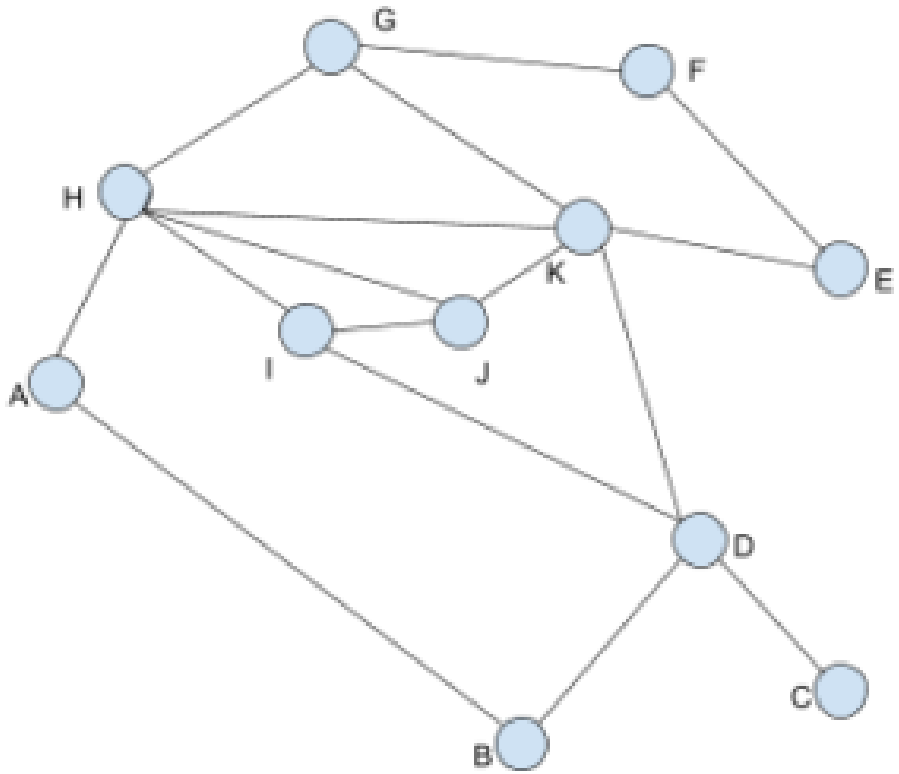
Welsh Powell Algorithm

Step 3: The selected vertex is colored with active color. After that, find the uncolored vertices from adjacency matrix which are not adjacent vertices of the colored vertex and these vertices are added to the V' set ($V' = \{v'_1, v'_2, \ldots, v'_n\}$).

- The uncolored vertex that has the largest degree in the V is selected for coloring. This vertex is colored with active color. After that, the adjacent vertices of the this vertex deleted from V'. This step is repeated until all vertices colored in the set of V'.

Step 4: If the uncolored vertex exists, next color in the color set is selected as active color and it is returned to the step 2. Otherwise the program is terminated, because all vertices in the graph are colored.

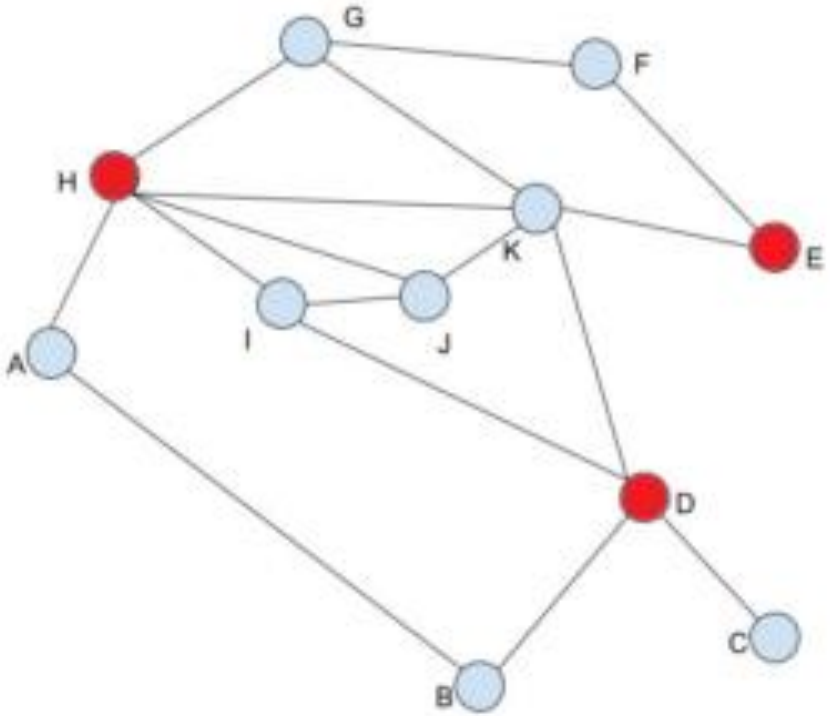
Welsh Powell Algorithm



For H:

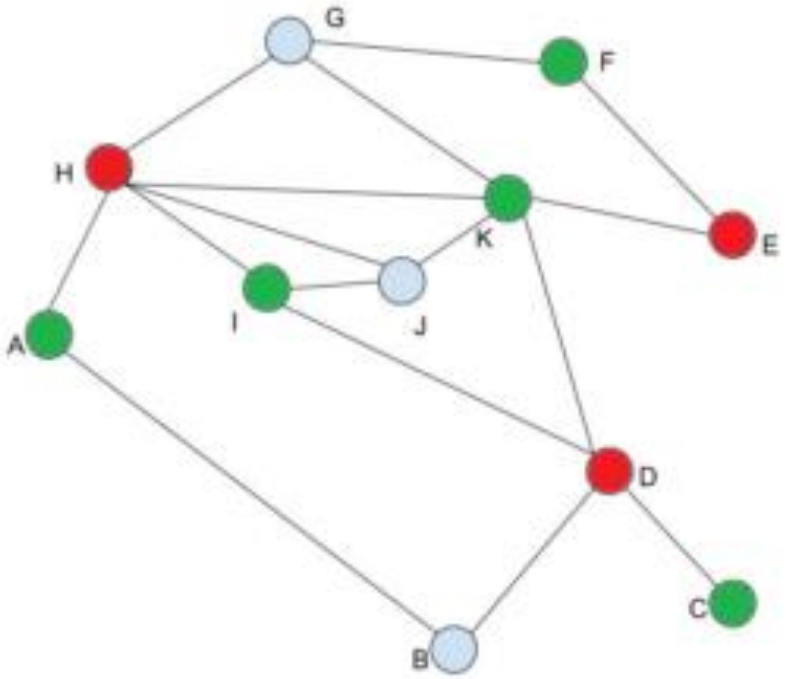
V’ = {B,C,D,E,F}

Welsh Powell Algorithm

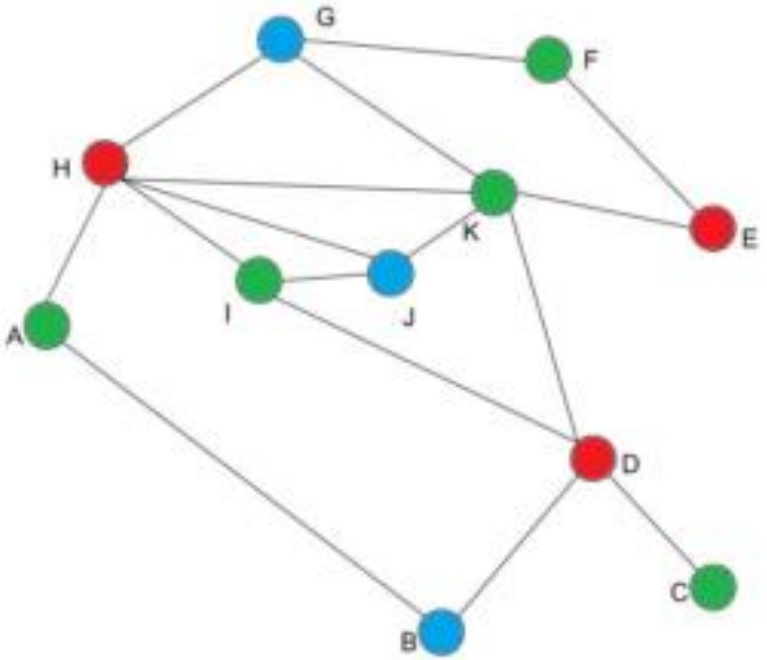


First remove D (B,C) and then E (F)
Color1: {H,D,E}

Welsh Powell Algorithm



Walsh Powell Algorithm



Recursive Largest First (RLF) Algorithm

- The Recursive Largest First (RLF) algorithm is a heuristic algorithm.
- The RLF algorithm assigns colors to a graph's vertices by constructing each color class one at a time.
- It does this by identifying a maximal independent set of vertices in the graph, assigning these to the same color, and then removing these vertices from the graph

Recursive Largest First (RLF) Algorithm

Step 1: Vertex degree is calculated for each vertex and the degrees of vertices added to the set of $Deg(v_i)$. Initially, the first color in the color set is selected as the active color. Select the uncolored vertex which has the largest degree from set of $Deg(v_i)$ for coloring.

Recursive Largest First (RLF) Algorithm

Step 2: The selected vertex is colored with active color. Adjacent vertices of the selected vertex can not color with active color. But the uncolored vertices which are not adjacent vertices of the colored vertex can be colored with active color. So RLF uses a recursive structure for select the uncolored vertices to color with active color. During this process the below steps should be followed:

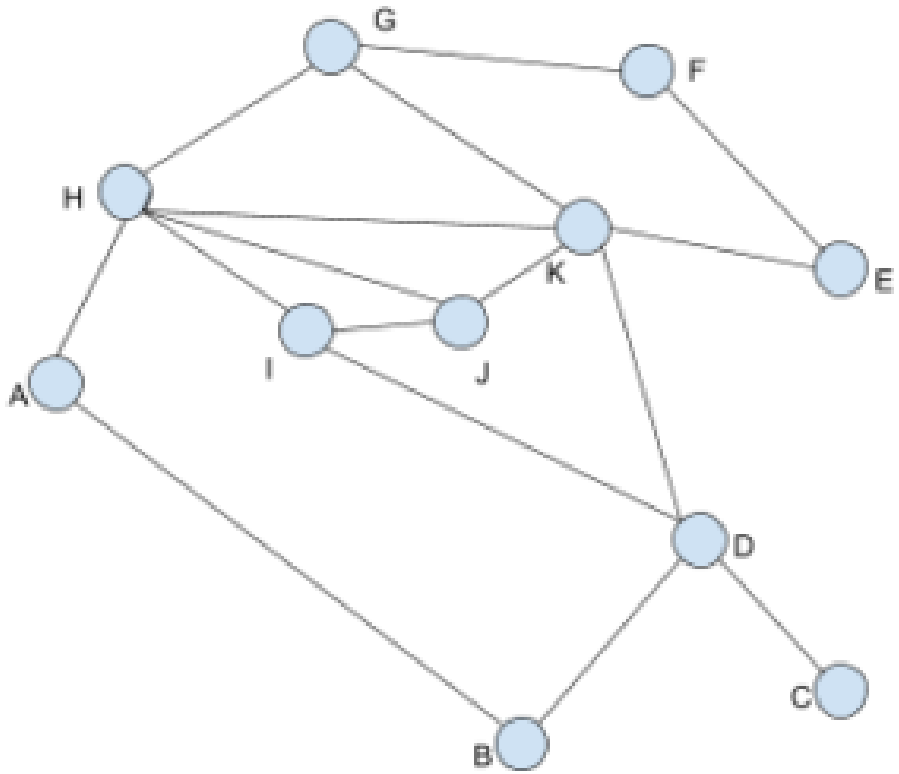
- Adjacent vertices of the selected vertex vi are found from adjacency matrix. Adjacent vertices are added to the adjacent set U. ($U = \{u_1, u_2, \dots\dots, u_t\}$)
- The vertices which are not adjacent vertices of the selected vertex vi are found from adjacency matrix. These vertices are added to the set of V' . Calculate the number adjacent vertices which are in the set of U for every vertex in set of V′. After that, the uncolored vertex whose has maximum adjacent vertices (which are in the set of U) in the set of V' is selected for coloring. The selected vertex is colored with active color.
- The colored vertex and the adjacent vertices of the colored vertex are deleted from V' and added to the set of U.
- If the set of V′ is not empty, it is returned to the step 2. Otherwise move to step 3.

Recursive Largest First (RLF) Algorithm

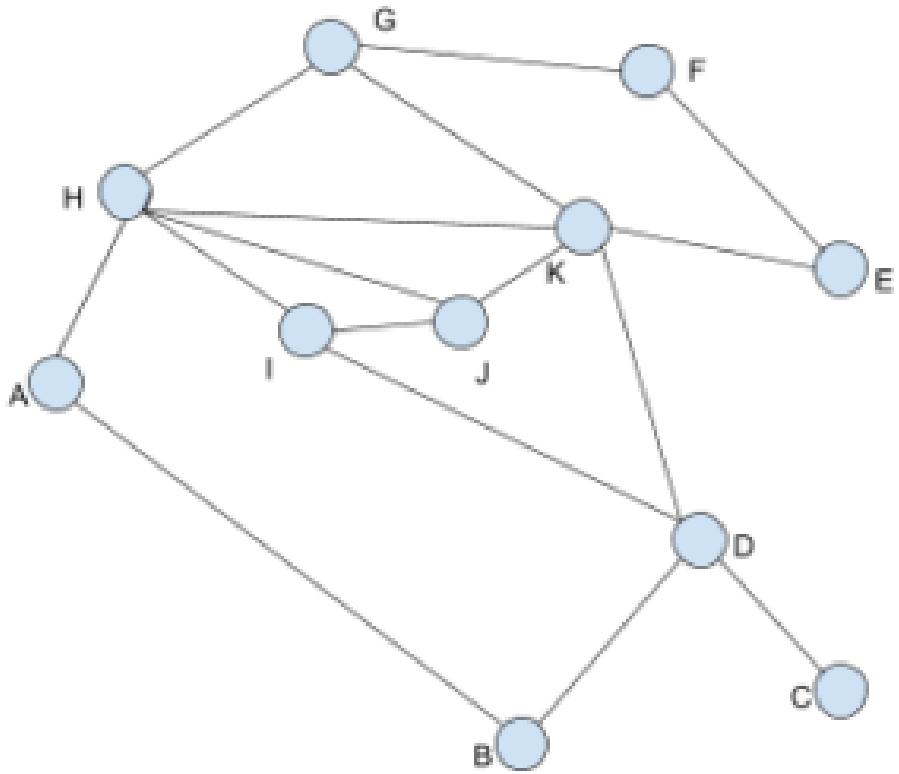
Step 3: If the uncolored vertex exists, next color in the color set is selected as active color. Otherwise the program is terminated.

Step 4: Calculate the number adjacent vertices for every uncolored vertex. After that, the uncolored vertex whose has maximum adjacent vertices is selected for coloring process. If more than one vertex provide this condition, the vertex which has the largest degree among them is selected. Then, it is returned to the step 2

Recursive Largest First (RLF) Algorithm



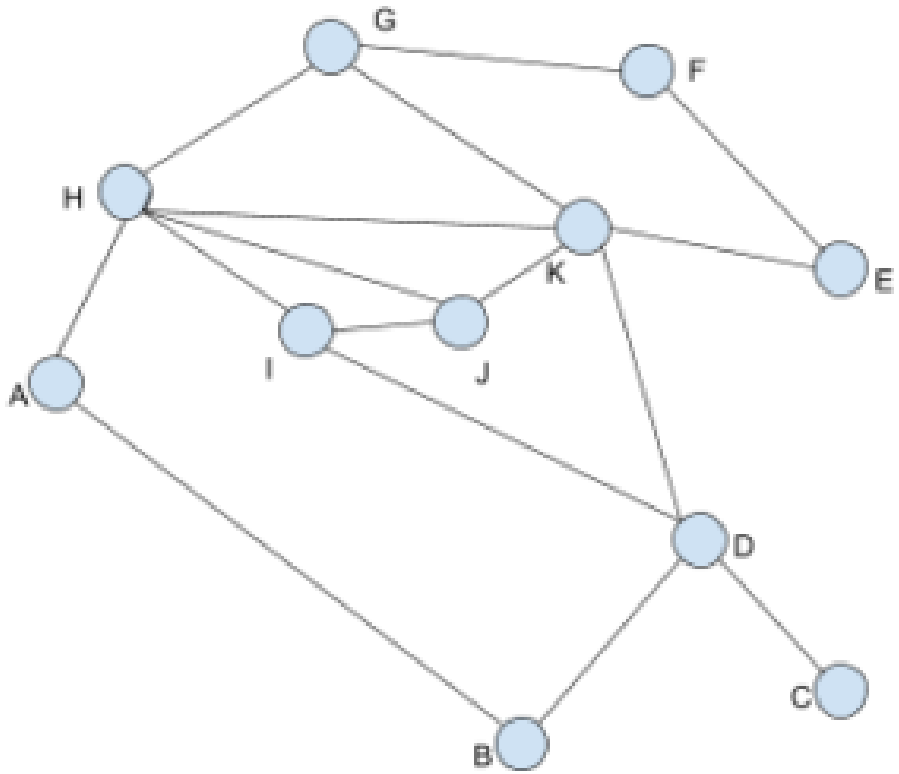
Recursive Largest First (RLF) Algorithm



For H:

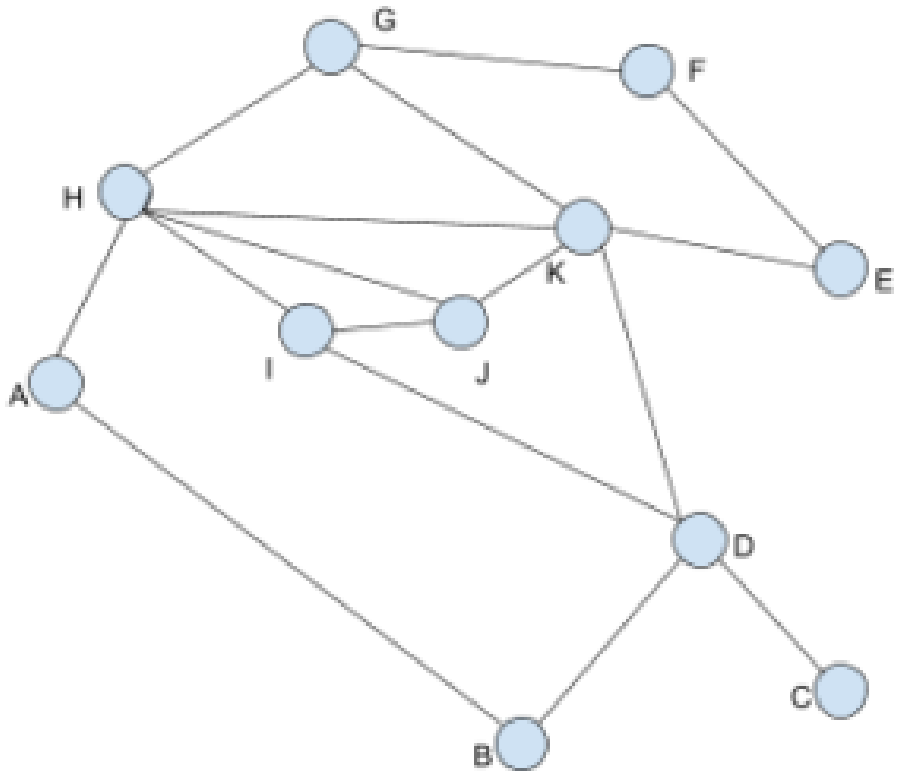
U = {A,G,I,J,K} and V' = {B,C,D,E,F}

Recursive Largest First (RLF) Algorithm



Color 1: H, D, E

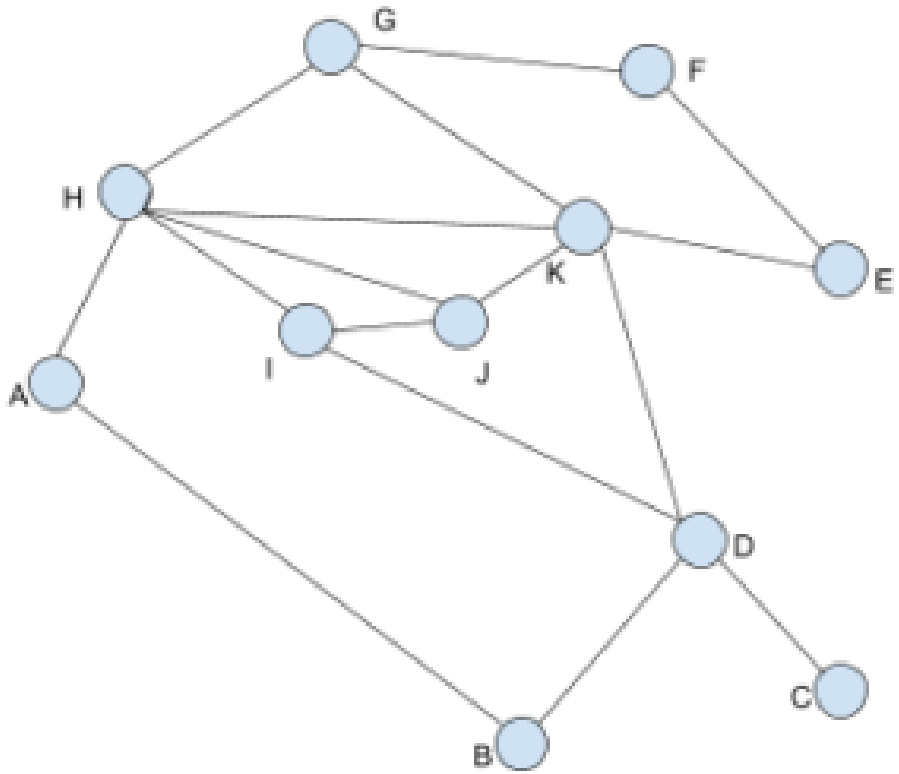
Recursive Largest First (RLF) Algorithm



For K:

U = {J,E,G,H,D} and V' = {I,A,B,C,F}

Recursive Largest First (RLF) Algorithm



Color 2: {K, I, B, F, C}

Summary

- Graph Coloring