Graph Theory

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Outline

- Eulerian and Hamiltonian Digraphs

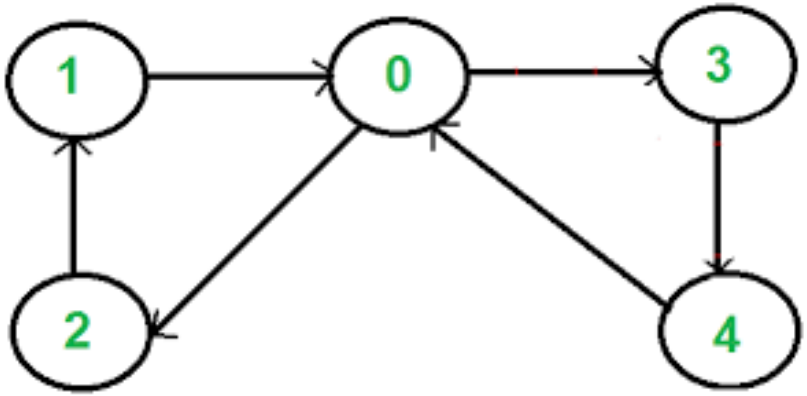
Euler Digraph

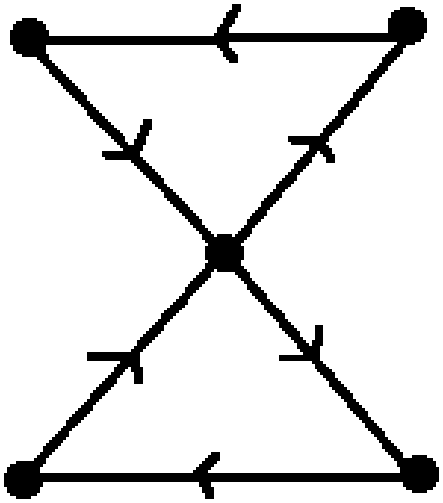
- A digraph that has a directed Eulerian circuit is called an Eulerian digraph.

- A directed trail of a digraph G that contains exactly one copy of each arc of G is called a directed Eulerian trail.

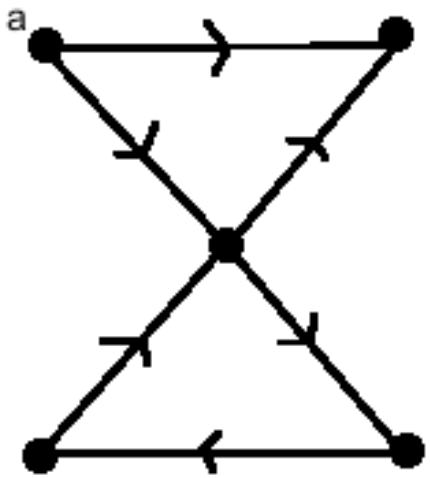
- A closed directed Eulerian trail of a digraph G is called a directed Eulerian circuit.

Euler Digraph





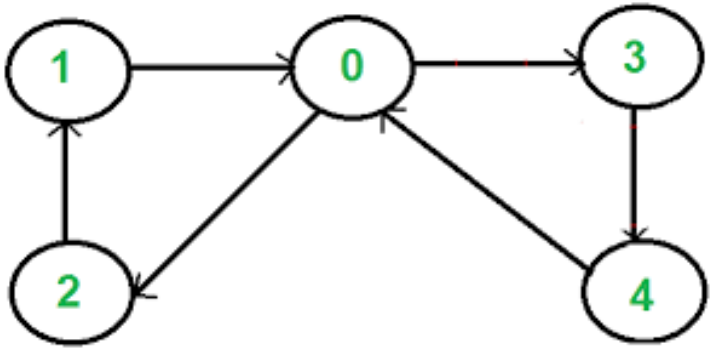
Eulerian

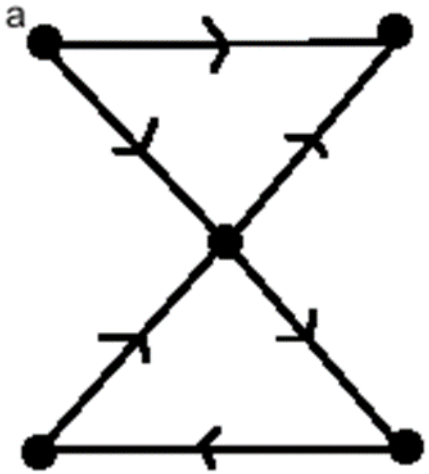


NOT Eulerian

Euler Digraph

- A connected digraph is Eulerian if and only if the in-degree of each vertex equals the out-degree of each vertex.



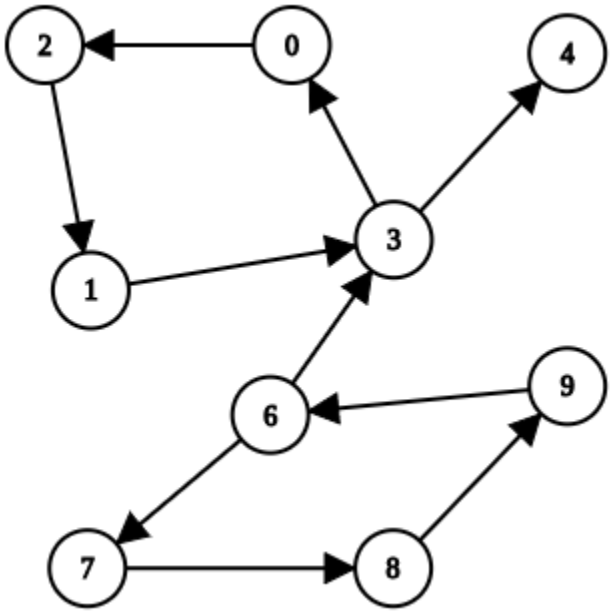


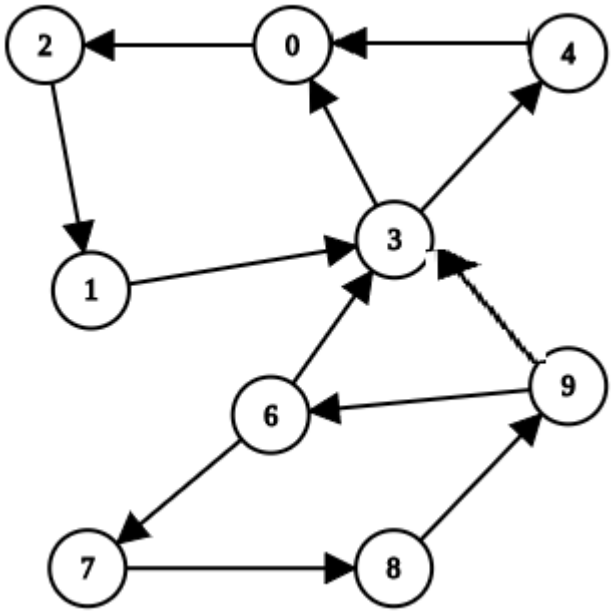
NOT Eulerian

Euler Digraph

$\overrightarrow{G}$ is an Eulerian digraph if and only if for every $u \in V(\overrightarrow{G})$, we have $\deg^-(u) = \deg^+(u)$.

$\overrightarrow{G}$ has a directed Eulerian trail if and only if $\overrightarrow{G}$ contains at most two vertices, say u and v, have different indegree and outdegree with $\deg^+(u) - \deg^-(u) = 1$ *and* $\deg^-(v) - \deg^+(v) = 1$.



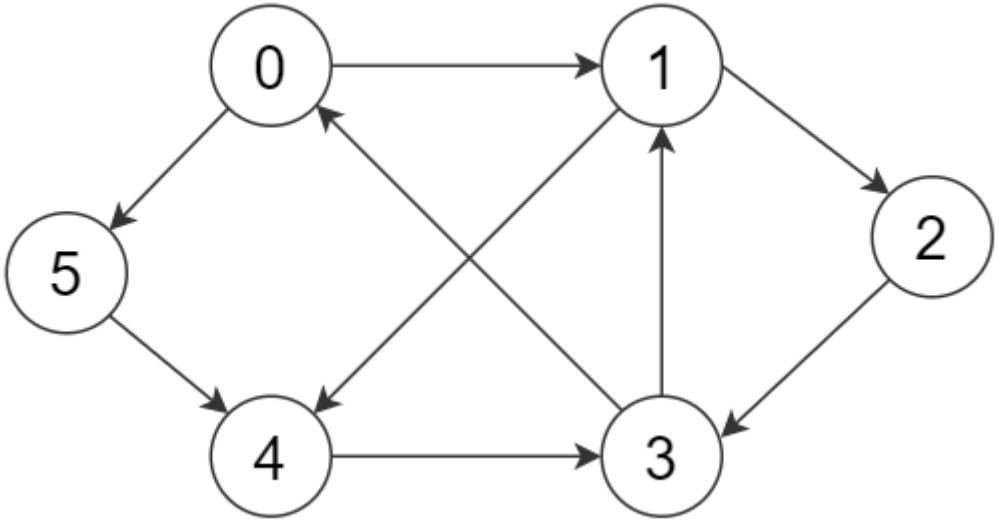


How to find path in Euler Digraph?

- We will be using Hierholzer's algorithm for searching the Eulerian path.

- Select any vertex v and place it on a stack. At first, all edges are unmarked.
- While the stack is not empty, examine the top vertex, u. If u has an unmarked incident edge to a vertex w (say), push w onto the stack and mark the edge uw. If, on the other hand, there are no unmarked incident edges, then remove it from the stack and print.

How to find path in Euler Digraph?



0 —> 1 —> 2 —> 3 —> 0 —> 5 —> 4 —> 3 —> 1 —> 4.

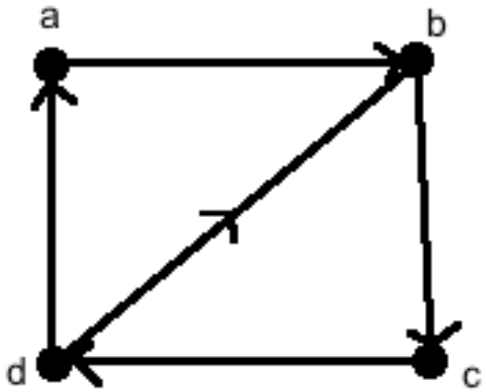
Hamiltonian Digraph?

- A digraph graph that has a directed Hamiltonian cycle is called a Hamiltonian digraph.

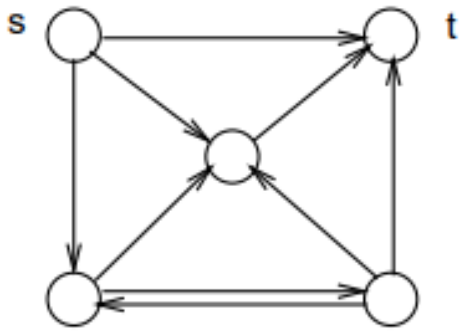
- A directed cycle that contains all the vertices of digraph G is called a directed Hamiltonian cycle.

- A directed path of digraph G that contains all the vertices of G is called a directed Hamiltonian path.

Hamiltonian Digraph?

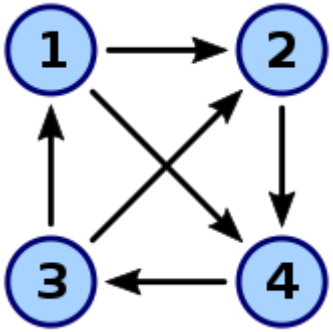


Hamiltonian



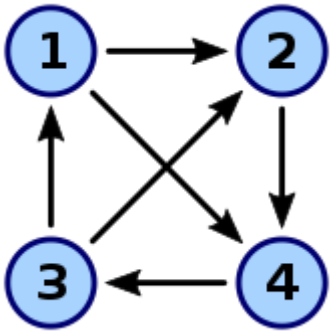
Tournament

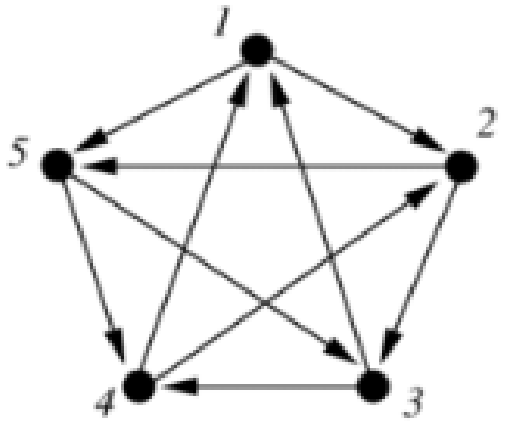
- A tournament on n vertices is a directed graph whose underlying graph is Kn (a complete graph on n vertices).
- A tournament is a directed graph (digraph) obtained by assigning a direction for each edge in an undirected complete graph.

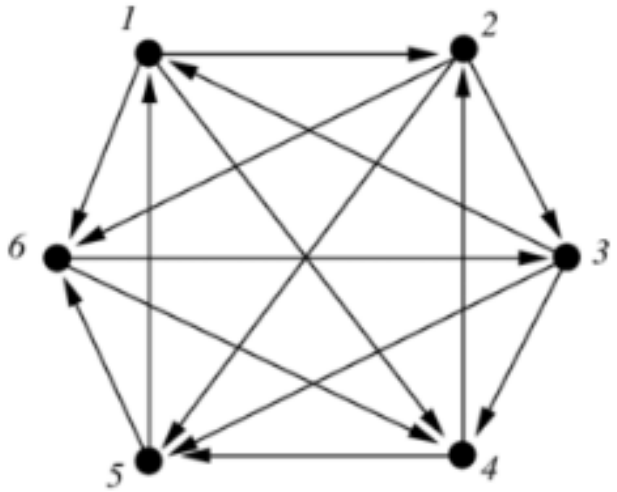


Tournament

A ***tournament*** is a directed graph with exactly one directed edge between any pair of vertices. That is, for any vertices v and w, a tournament contains either an edge $v \to w$ or an edge $w \to v$, but not both.

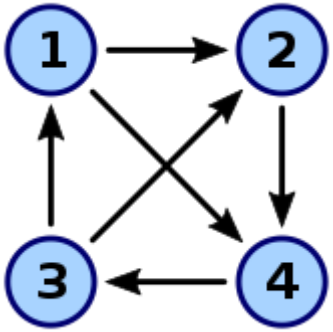


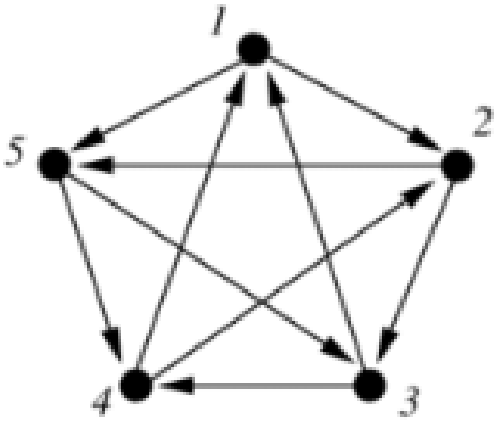


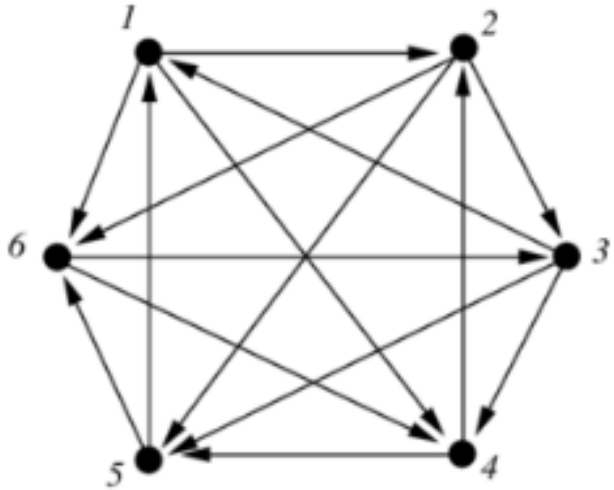


Tournament

- Every tournament has a Hamiltonian path (not necessarily a cycle!).







Summary

- Euler Digraphs
- Hamiltonian Digraphs