IRIS Quadrotor Model

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1 CAD Model

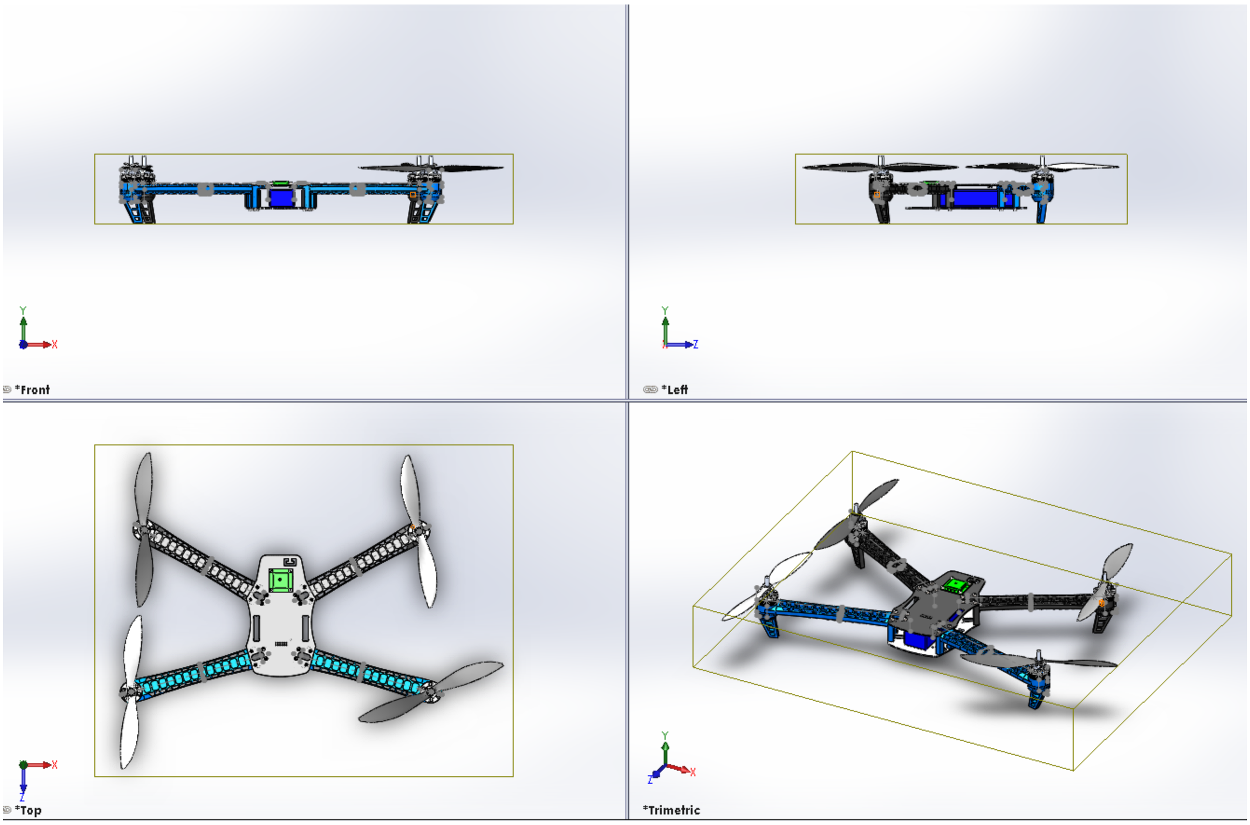

Figure 1: IRIS CAD Model

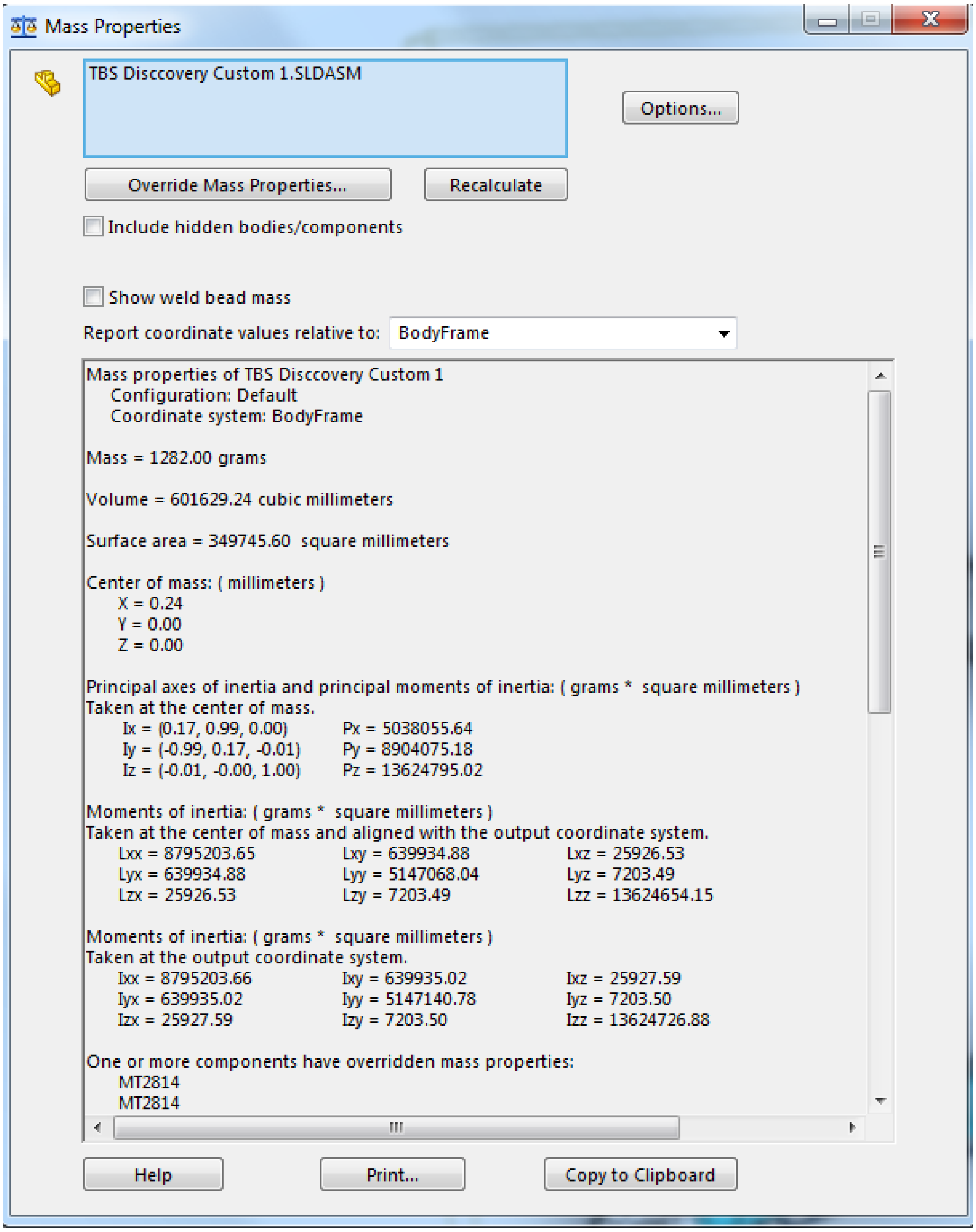

Figure 2: Inertial Matrix of IRIS Quadrotor from Solidworks

2 Motion Equations

2.1 Parameters for Quadrotor Model

m: Mass of Quadrotor;
$\mathbf{I}$: Inertial Matrix with respect to Body Frame;
L: Distance from the axis of rotation of the rotors to the center of mass of the Quadrotor;
r: Propeller Radius;
rpm_min: Minimum Rotation Speed of motor;
rpm_max: Maximum Rotation Speed of motor;
k_F: Thrust Coefficient;
k_M: Moment Coefficient;
k_m: Motor Gain;
Controller Gains: $\mathbf{k_x}, \mathbf{k_v}, \mathbf{k_\Omega}, \mathbf{k_R}$;

For IRIS Quadrotor:

$$\begin{aligned}
m &= 1282\ g \\
\mathbf{I} &= \begin{bmatrix} 8795203.66 & 639935.02 & 25927.59 \\ 639935.02 & 5147140.78 & 7203.50 \\ 25927.59 & 7203.50 & 13624726.88 \end{bmatrix} g \cdot mm^2 \\
L &= 252.33\ mm \\
r &= 127\ mm \\
k_F &= 1.5693 \times 10^{-7}\ N/RPM^2 \\
k_M &= -2.7848 \times 10^{-6}\ N \cdot mm/RPM^2 \\
k_m &= 20\ s^{-1}
\end{aligned}$$

2.2 Newton's Equations of Motion

$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

Define the first input u_1 to be

$$u_1 = \sum_{i=1}^{4} F_i$$

Here in the equation:

r is Position Vector in Inertial Frame;

R is Rotation Matrix which we use $Z - X - Y$ Euler angles to model the rotation of the Quadrotor in the World Frame;

$F_i = k_f w_i^2$ is Thrust produced by each motor;

w_i is rotor speed;

2.3 Euler's Equations of Motion

$$\mathbf{I}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}F_1 l_{1x} - F_2 l_{2x} + F_3 l_{3x} - F_4 l_{4x}\\ -F_1 l_{1y} + F_2 l_{2y} + F_3 l_{3y} - F_4 l_{4y}\\ M_1 + M_2 - M_3 - M_4\end{bmatrix} - \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \mathbf{I}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

with

$$\mathbf{I} = \begin{bmatrix}I_{xx} & I_{xy} & I_{xz}\\ I_{yx} & I_{yy} & I_{yz}\\ I_{zx} & I_{zy} & I_{zz}\end{bmatrix}$$

or we could write

$$\mathbf{I}\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}l_{1x} & -l_{2x} & l_{3x} & -l_{4x}\\ -l_{1y} & l_{2y} & l_{3y} & -l_{4y}\\ \gamma & \gamma & -\gamma & -\gamma\end{bmatrix}\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{bmatrix} - \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times \mathbf{I}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

Define the second input $\mathbf{u}_2$ to be

$$\mathbf{u_2} = \begin{bmatrix}l_{1x} & -l_{2x} & l_{3x} & -l_{4x}\\ -l_{1y} & l_{2y} & l_{3y} & -l_{4y}\\ \gamma & \gamma & -\gamma & -\gamma\end{bmatrix}\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{bmatrix} = \begin{bmatrix}u_2\\ u_3\\ u_4\end{bmatrix}$$

Here is the equation:

p, q, r: Components of Angular Velocity of the robot in Body Frame;

$l_i x$: The Arm of Force of thrust F_i about x axis;

$l_i y$: The Arm of Force of thrust F_i about y axis

$M_i = k_m w_i^2$: Moment produced by each motor;

$\gamma = \frac{k_m}{k_f}$: Relationship between Lift and Drag;

3 State Space Equation

A nonlinear system can be described by the following differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}, \mathbf{u})$$

We could linearize it to get state space equation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

with initial conditions $\mathbf{x}(0) = \mathbf{x}_0$

For quadrotor, we define its state vector $\mathbf{X}$ to be

$$\mathbf{X} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & \dot{x} & \dot{y} & \dot{z} & p & q & r \end{bmatrix}^T$$

(x, y, z) are Position in Inertial frame, (ϕ, θ, ψ) are Orientation, $(\dot{x}, \dot{y}, \dot{z})$ are Linear Velocity in Inertial frame, (p, q, r) are components of Angular Velocity in Body frame. Especially,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & -\cos(\phi)\sin(\theta) \\ 0 & 1 & \sin(\phi) \\ \sin(\theta) & 0 & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

The inputs u as defined in section 2.2 and 2.3 are:

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$$

Equilibrium State:

$$\mathbf{X}_0 = \begin{bmatrix} x_0 & y_0 & z_0 & 0 & 0 & \psi_0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$\mathbf{u_0} = \begin{bmatrix} mg & 0 & 0 & 0 \end{bmatrix}^T$$

We first write the state vector as below:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \end{bmatrix}^T$$

Then we write the state equations:

$$
\begin{aligned}
f_1(\mathbf{x},\mathbf{u}) &= \dot{x_1} = x_7 \\
f_2(\mathbf{x},\mathbf{u}) &= \dot{x_2} = x_8 \\
f_3(\mathbf{x},\mathbf{u}) &= \dot{x_3} = x_9 \\
f_4(\mathbf{x},\mathbf{u}) &= \dot{x_4} = x_{10}\cos(x_5) + x_{12}\sin(x_5) \\
f_5(\mathbf{x},\mathbf{u}) &= \dot{x_5} = x_{10}\frac{\sin(x_4)\sin(x_5)}{\cos(x_4)} + x_{11} - x_{12}\frac{\cos(x_5)\sin(x_4)}{\cos(x_4)} \\
f_6(\mathbf{x},\mathbf{u}) &= \dot{x_6} = -x_{10}\frac{\sin(x_5)}{\cos(x_4)} + x_{12}\frac{\cos(x_5)}{\cos(x_4)} \\
f_7(\mathbf{x},\mathbf{u}) &= \dot{x_7} = \frac{u_1}{m}(\cos(x_6)\sin(x_5) + \cos(x_5)\sin(x_4)\sin(x_6)) \\
f_8(\mathbf{x},\mathbf{u}) &= \dot{x_8} = \frac{u_1}{m}(\sin(x_6)\sin(x_5) - \cos(x_6)\cos(x_5)\sin(x_4)) \\
f_9(\mathbf{x},\mathbf{u}) &= \dot{x_9} = -g + \frac{u_1}{m}\cos(x_4)\cos(x_5) \\
f_{10}(\mathbf{x},\mathbf{u}) &= \dot{x_{10}} = \frac{I_{xy}u_3 - I_{yy}u_2}{I_{xy}^2 - I_{xx}I_{yy}} \\
f_{11}(\mathbf{x},\mathbf{u}) &= \dot{x_{11}} = \frac{I_{xy}u_2 - I_{xx}u_3}{I_{xy}^2 - I_{xx}I_{yy}} \\
f_{12}(\mathbf{x},\mathbf{u}) &= \dot{x_{12}} = \frac{u_4}{I_{zz}}
\end{aligned}
$$

Then we get A and B matrix:

$$
\mathbf{A} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & g\sin(\psi_0) & g\cos(\psi_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -g\cos(\psi_0) & g\sin(\psi_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{m} & 0 & 0 & 0 \\
0 & \frac{-I_{yy}}{I_{xy}^2 - I_{xx}I_{yy}} & \frac{I_{xy}}{I_{xy}^2 - I_{xx}I_{yy}} & 0 \\
0 & \frac{I_{xy}}{I_{xy}^2 - I_{xx}I_{yy}} & \frac{-I_{xx}}{I_{xy}^2 - I_{xx}I_{yy}} & 0 \\
0 & 0 & 0 & \frac{1}{I_{zz}}
\end{bmatrix}
$$

For the measurement $\mathbf{y} = \mathbf{C}\mathbf{x}, \mathbf{C}$ is an Identity matrix of dimension 12;

References

[1] T.Lee, M.Leok and N.McClamroch,"Geometric Tracking Control of a Quadrotor UAV on SE(3)," Atlanta, GA, USA, December 2010

[2] N.Michael, D.Mellinger, Q.Lindsey, and V.Kumar, "The grasp multiple micro uav testbed," IEEE Robotics and Automation Magazine, 2010