# Numerical methods and integral equations

# MATLAB: tips and tricks


‘help <function name>’, ‘help <classname>’, ‘help <classname>/<function name>’ provide useful information about how to use well-documented codes in MATLAB

https://getfullyfunded.com/10-fundraising-lessons-from-harry-potter/

# MATLAB: tips and tricks

Don’t write buggy codes!

But if you do, and when in doubt, just plot: Printing and plotting are the most efficient ways of debugging your code

# MATLAB: tips and tricks

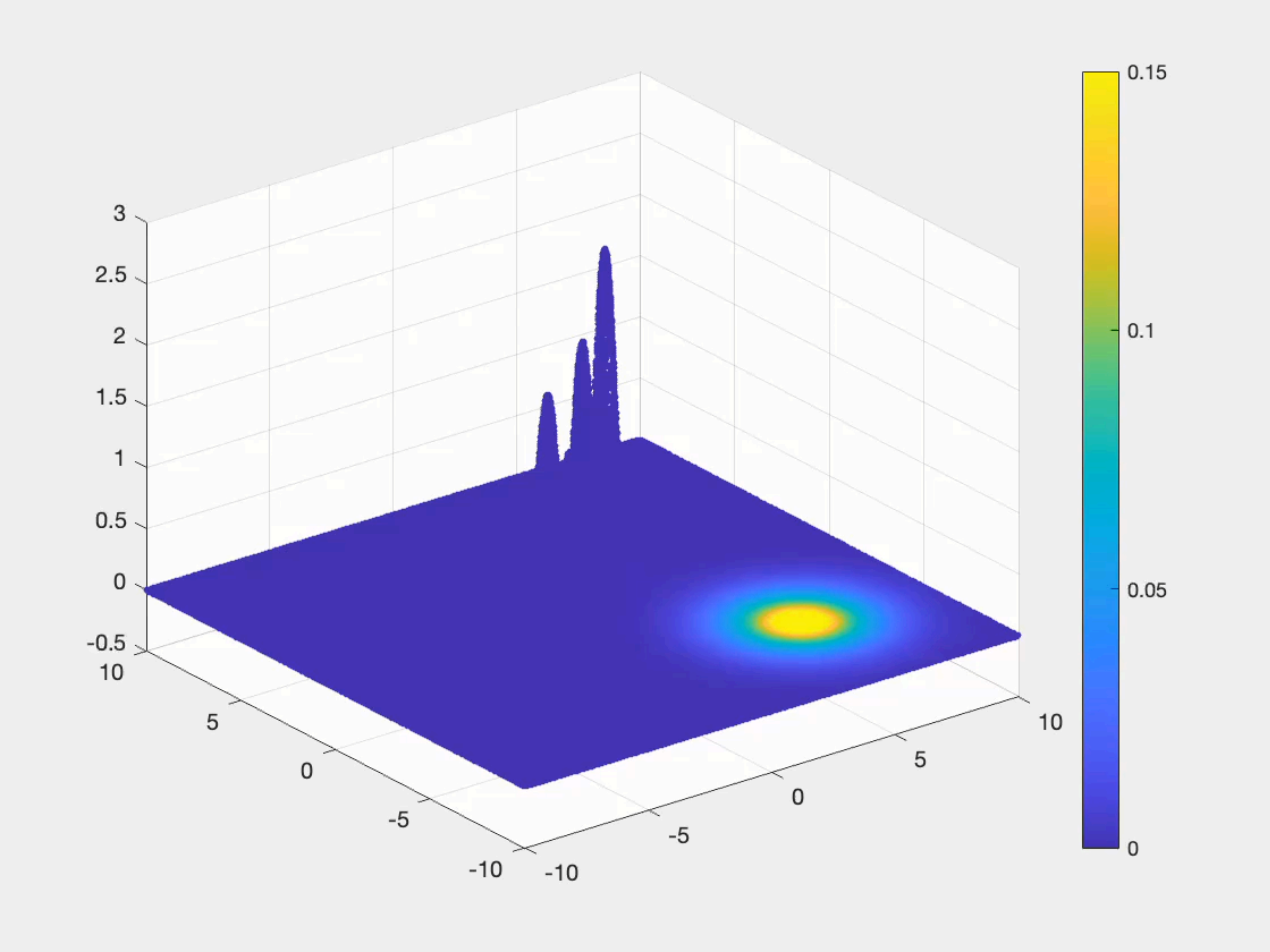
## Dot doesn’t dot, but not dot dots

Be mucho careful with MATLAB’s pointwise multiplication (.*) vs dot product (*)

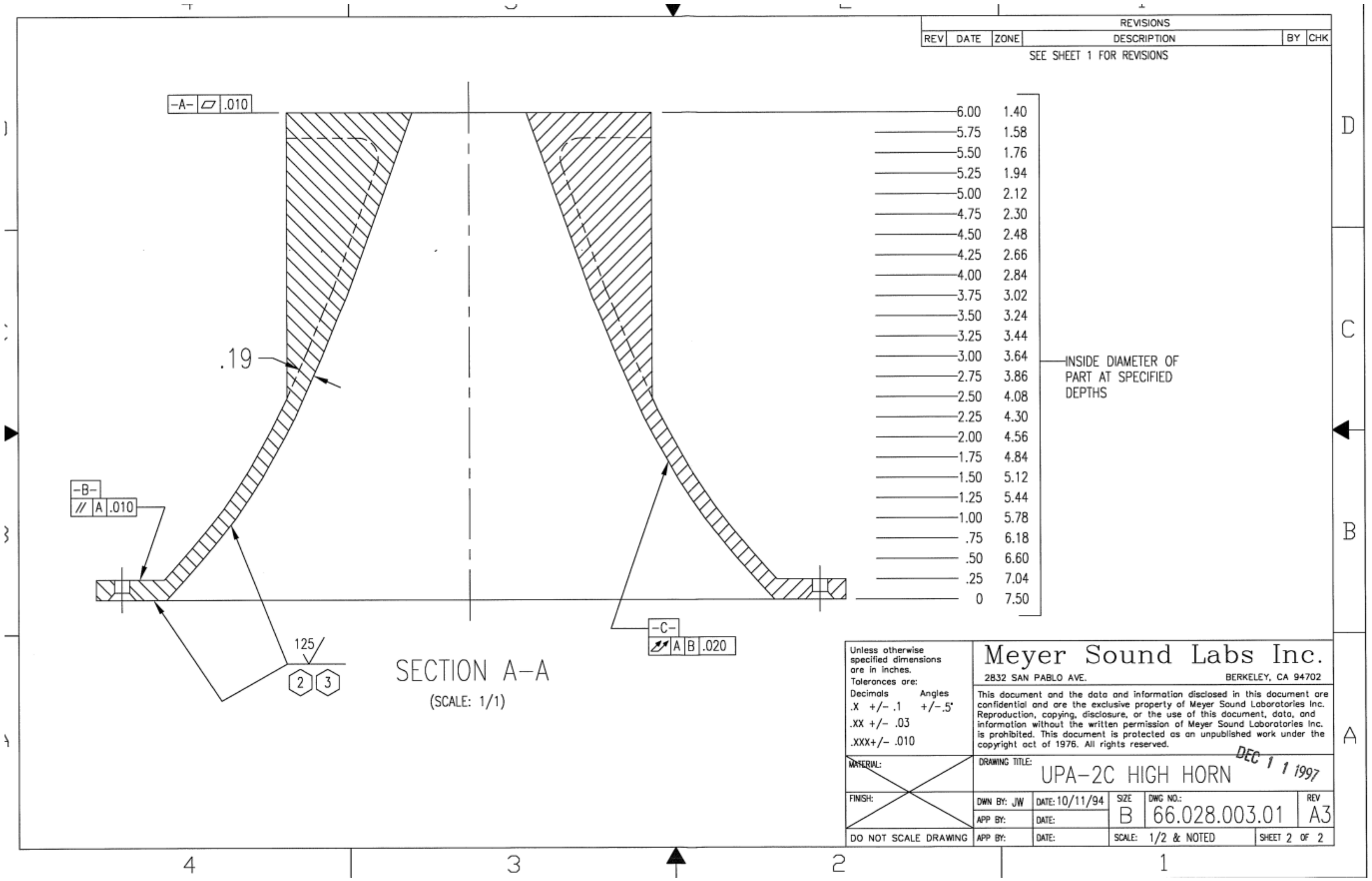
https://imageresizer.com/meme-generator/edit/jackie-chan-confused

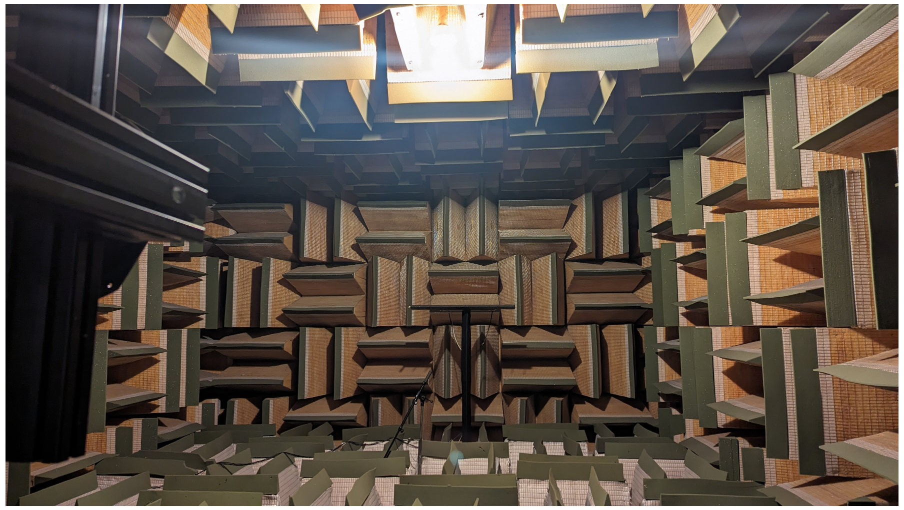
# Applications: Kelvin waves

$$(-\Delta + \omega_\pm^2)u = f_\pm \quad x \in \Omega_\pm$$
$$[[n \cdot \nabla u]] = -mu \quad x \in \partial\Omega$$



# Applications: Speaker design and modeling

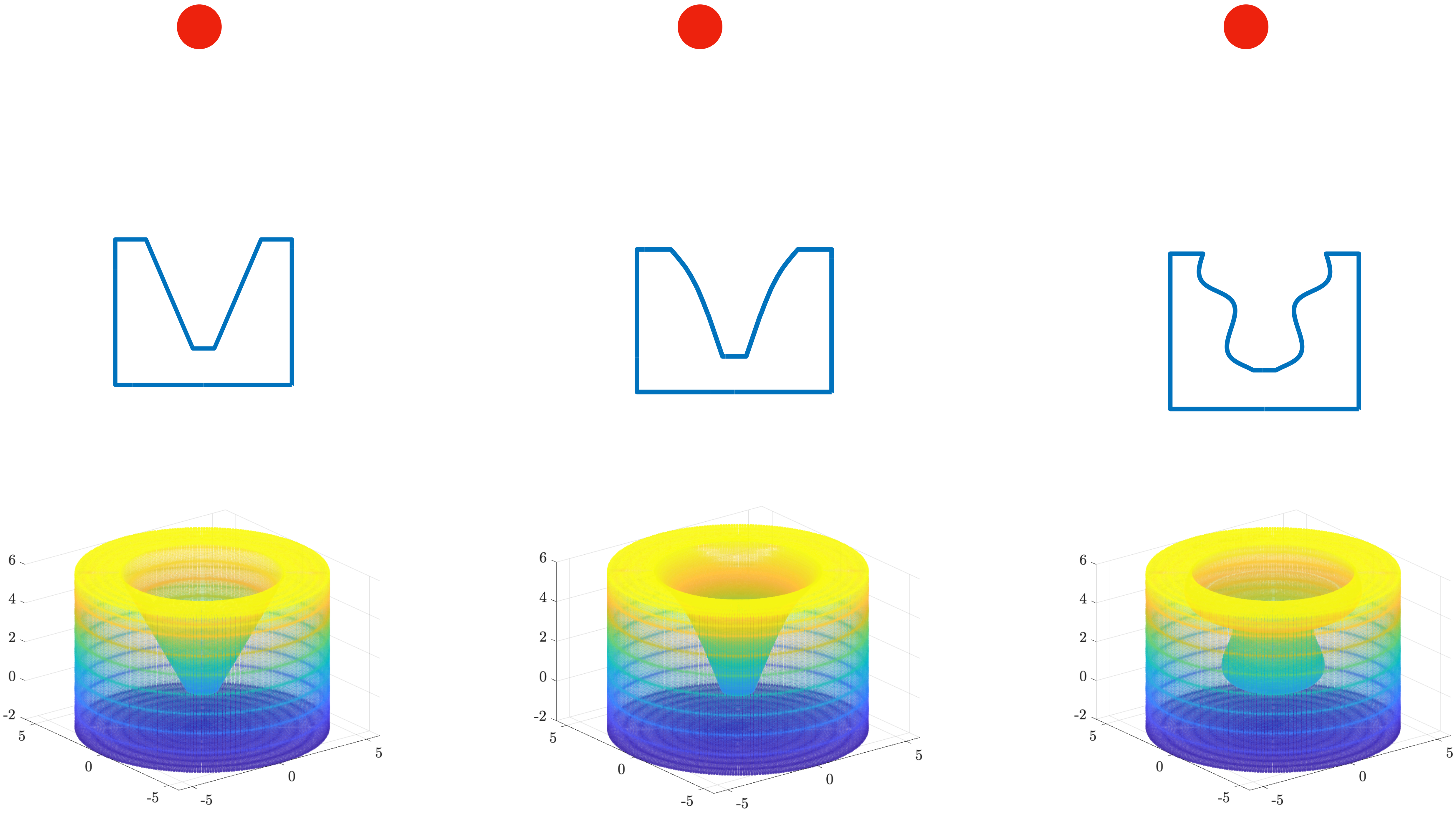


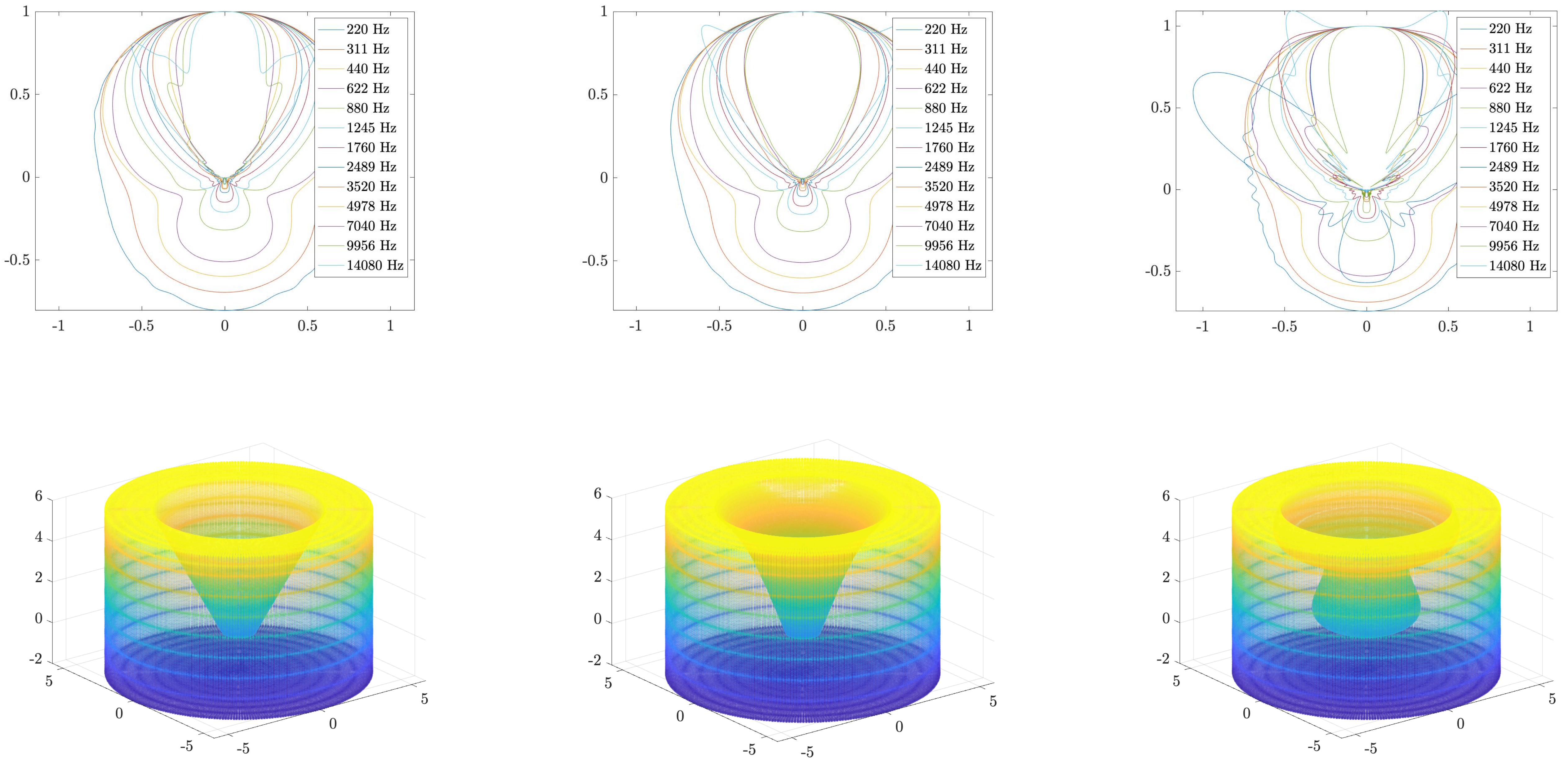
Thanks to Perrin Meyer, Meyer Sound

# Applications: Speaker design and modeling

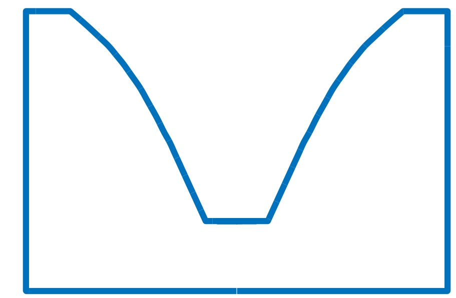
$$(\Delta + k^2)u = 0 \quad \text{in } \mathbb{R}^{2/3}\backslash\Omega$$
$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega$$



# Applications: Speaker design and modeling

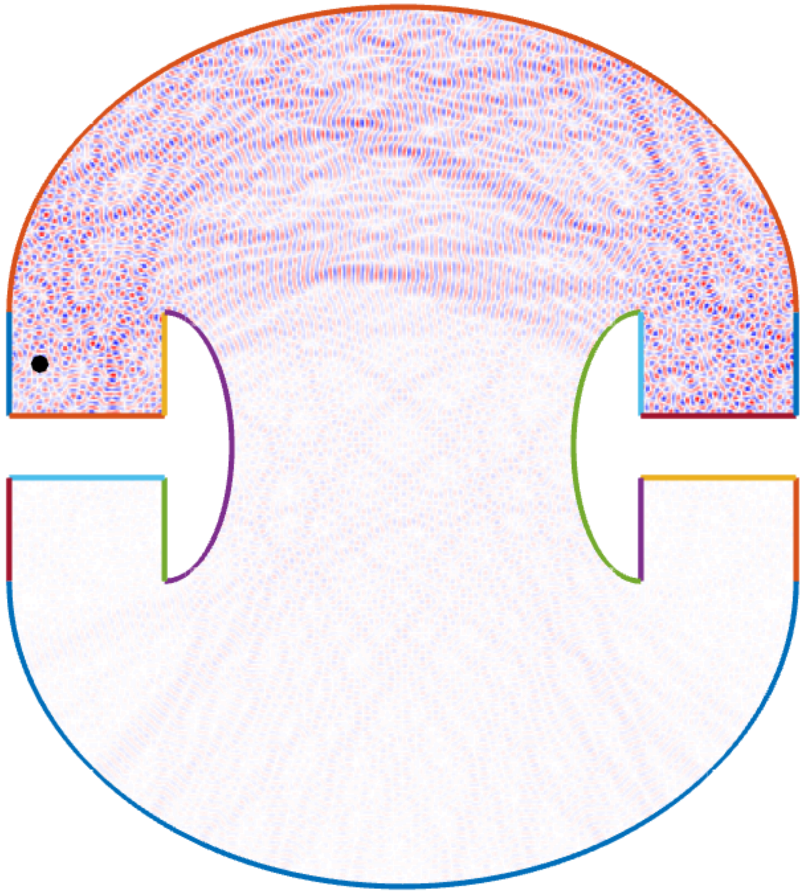


# Applications: Speaker design and modeling



# 6 step to live the good life through integral equations

1. Identify a PDE and boundary conditions
2. Convert it into an integral equation (Lecture 2)
3. Discretize geometry (Lecture 1)
4. Discretize integral operators (Lecture 3)
5. Solve the integral equation (Lecture 4+5)
6. Post-process/recover PDE solution (Lecture 4+5)

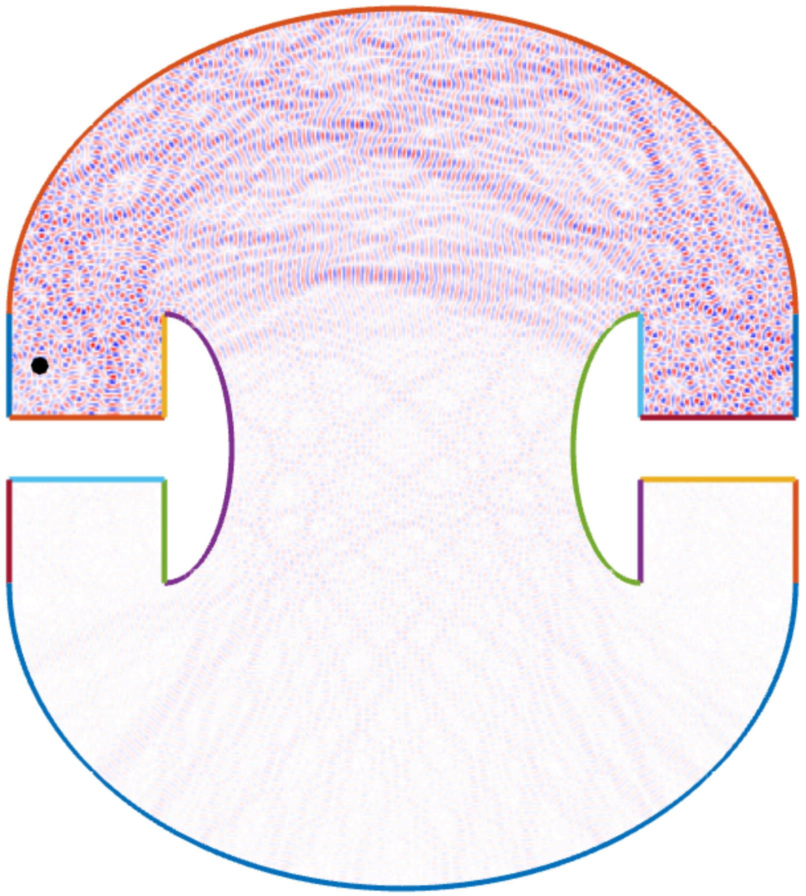


# 6 step to live the good life through integral equations

1. **Identify a PDE and boundary conditions**

$$(\Delta + k^2)u = 0 \quad \text{in } \Omega$$

$$u = f \quad \text{on } \partial\Omega$$



$$(\Delta + k^2)u = 0 \quad \text{in } \Omega$$
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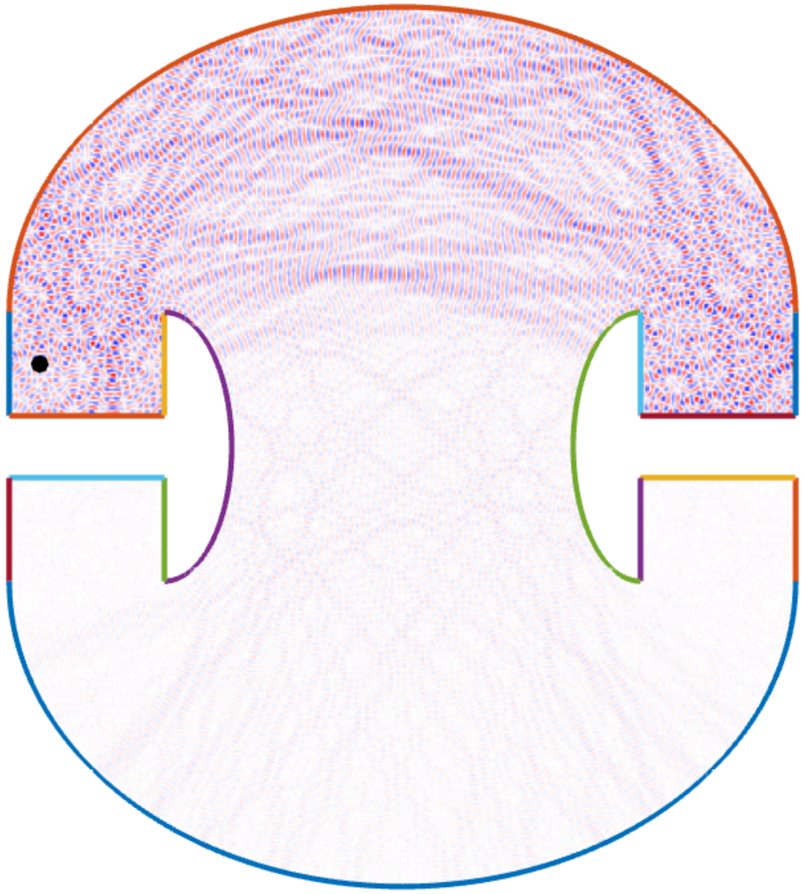
# 6 step to live the good life through integral equations



1. Identify a PDE and boundary conditions
2. **Convert it into an integral equation**
   - Integral representations use Green's functions/ fundamental solutions to represent the solution

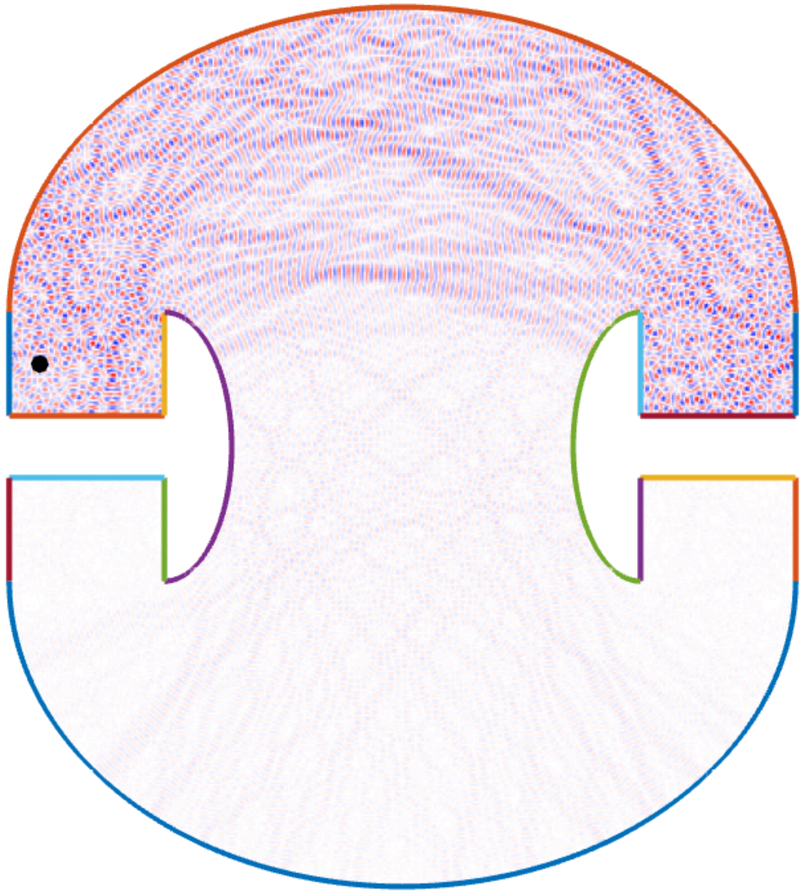
$$(\Delta_x + k^2)G(x,y) = -\delta_{x=y}$$

$$G(x,y) = \frac{i}{4}H_0^{(1)}(k|x-y|)$$

   - Information lives on the boundary alone, construct solution from bunch of Green's functions located on the boundary, i.e. set $u(x) = \int_{\partial\Omega} G(x,y)\sigma(y)ds_y$ for $x \in \Omega$
   - Need to solve for $\sigma$ such that $\int_{\partial\Omega} G(x,y)\sigma(y) = f(x)$ for $x \in \partial\Omega$

# 6 step to live the good life through integral equations

1. Identify a PDE and boundary conditions
2. Convert it into an integral equation (Lecture 2)
3. **Discretize geometry**
    - Need a way to represent functions and integral equations on the computer, thus need to be able to interpolate functions from finite number of samples, compute their integrals…



# What is function interpolation/representation?

Given:

- Discretization/sample points $\{x_j\}_{j=1}^{N}$
- Basis functions $\{\phi_j(x)\}_{j=1}^{M}$ construct an approximation
- Exact samples of a function $f$ at discretization points $\{f(x_j)\}_{j=1}^{N}$

Construct:

$$\tilde{f}(x) = \sum_{j=1}^{M} c_j \phi_j(x) \quad \text{with} \quad \tilde{f}(x_j) = f(x_j)$$

**Function types**

1. Nice and periodic
2. Nice and aperiodic
3. Not so nice

- Structure unknown: Adaptive
- Not niceness known: Generalized chebyshev methods

# Considerations for function interpolation

$$\tilde{f}(x) = \sum_{j=1}^{M} c_j \phi_j(x) \quad \textbf{with} \quad \tilde{f}(x_j) = f(x_j)$$

- Accuracy
  - Small approximation error for ``nice" functions, i.e. interpolant satisfies $\max_x |\tilde{f}(x) - f(x)| < \varepsilon$

- Speed
  - Ability to obtain in $c_j$ given $f(x_j)$ in $O(N+M)$ or $O((N+M)\log(N+M))$ time
  - Rapid evaluation of $\tilde{f}(x)$ at new points $\Longrightarrow$ $\phi_j$ not too complicated to evaluate

- Stability