Numerical methods and integral equations

MATLAB: tips and tricks

'help <function name>', 'help <classname>', 'help <classname>/<function name>' provide useful information about how to use well-documented codes in MATLAB

https://getfullyfunded.com/10-fundraising-lessons-from-harry-potter/

MATLAB: tips and tricks

Don’t write buggy codes!

But if you do, and when in doubt, just plot: Printing and plotting are the most efficient ways of debugging your code

MATLAB: tips and tricks

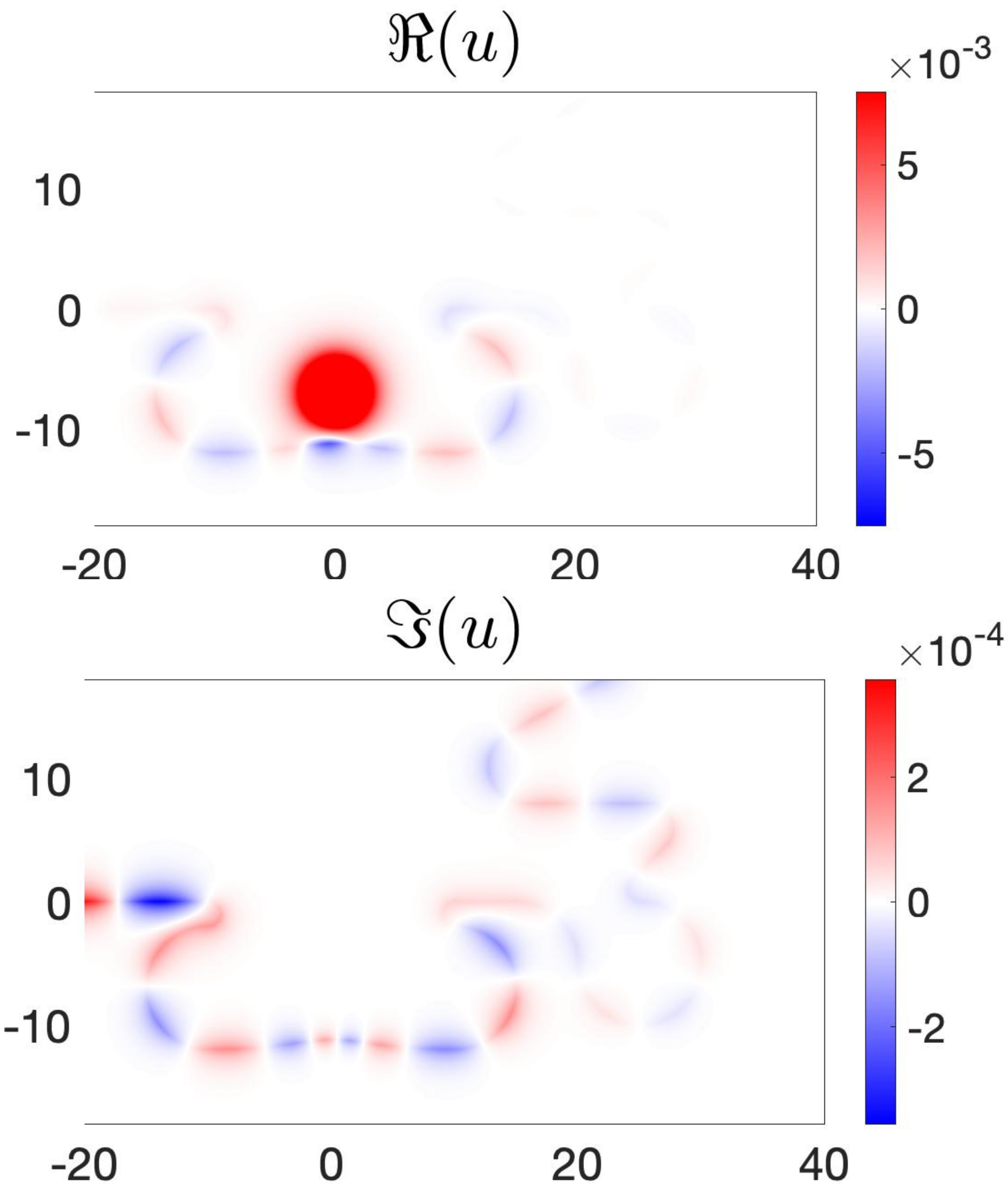
Dot doesn’t dot, but not dot dots

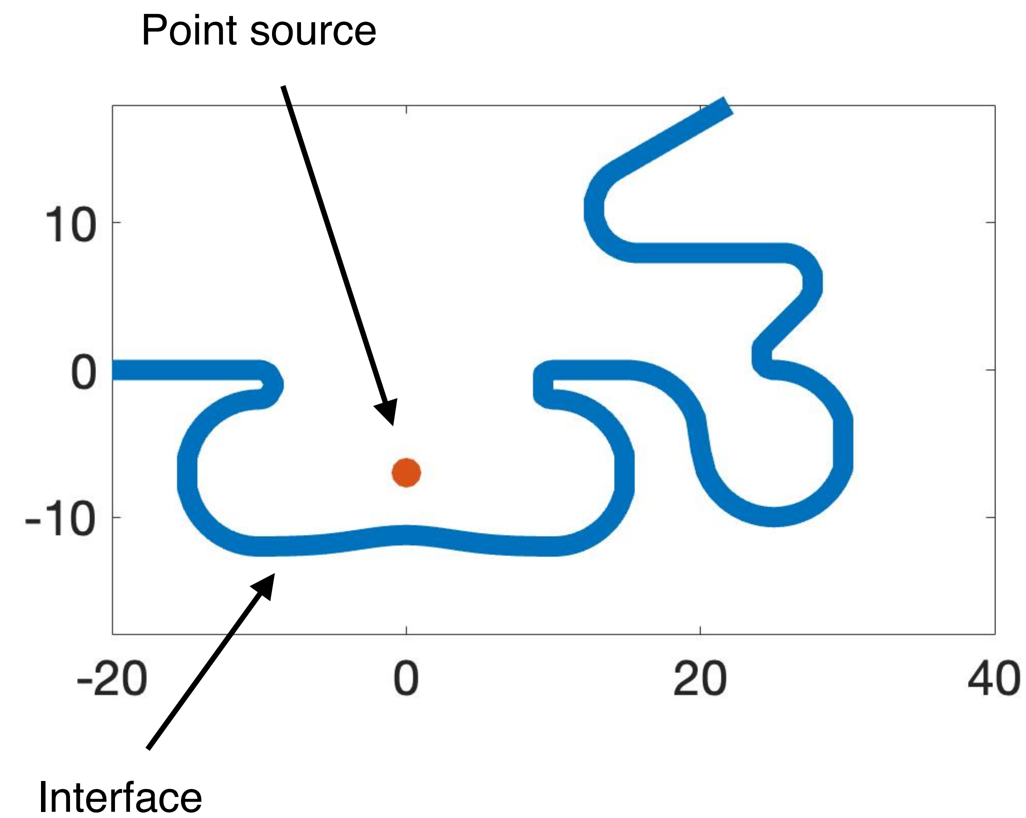
Be mucho careful with MATLAB’s pointwise multiplication (.*) vs dot product (*)

https://imageresizer.com/meme-generator/edit/jackie-chan-confused

Applications: Kelvin waves

$$(-\Delta + \omega^2)u = f_\pm \quad x \in \Omega_\pm$$
$$[[n \cdot \nabla u]] = -mu \quad x \in \partial\Omega$$





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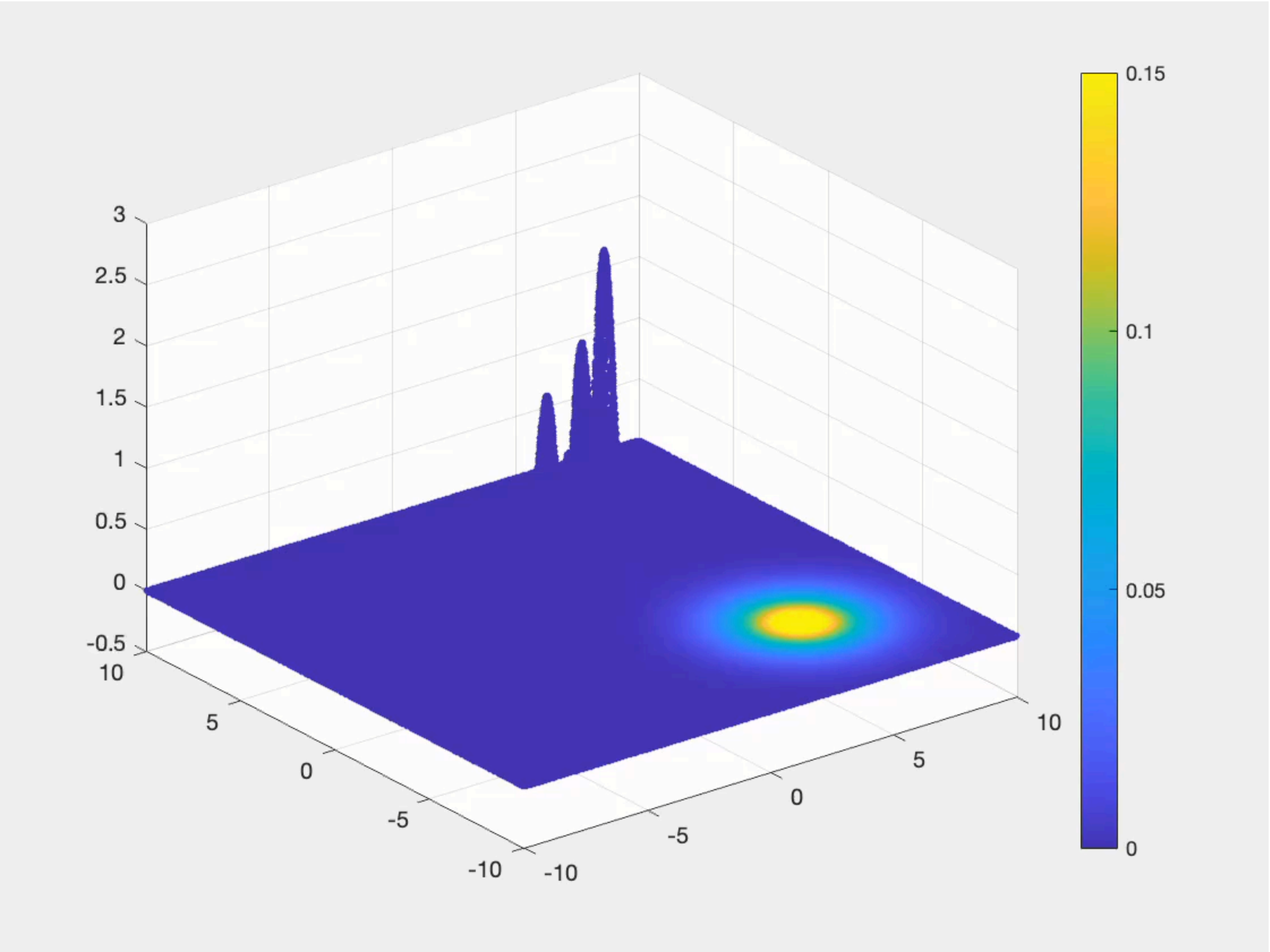

Speaker Design

Meyer Sound

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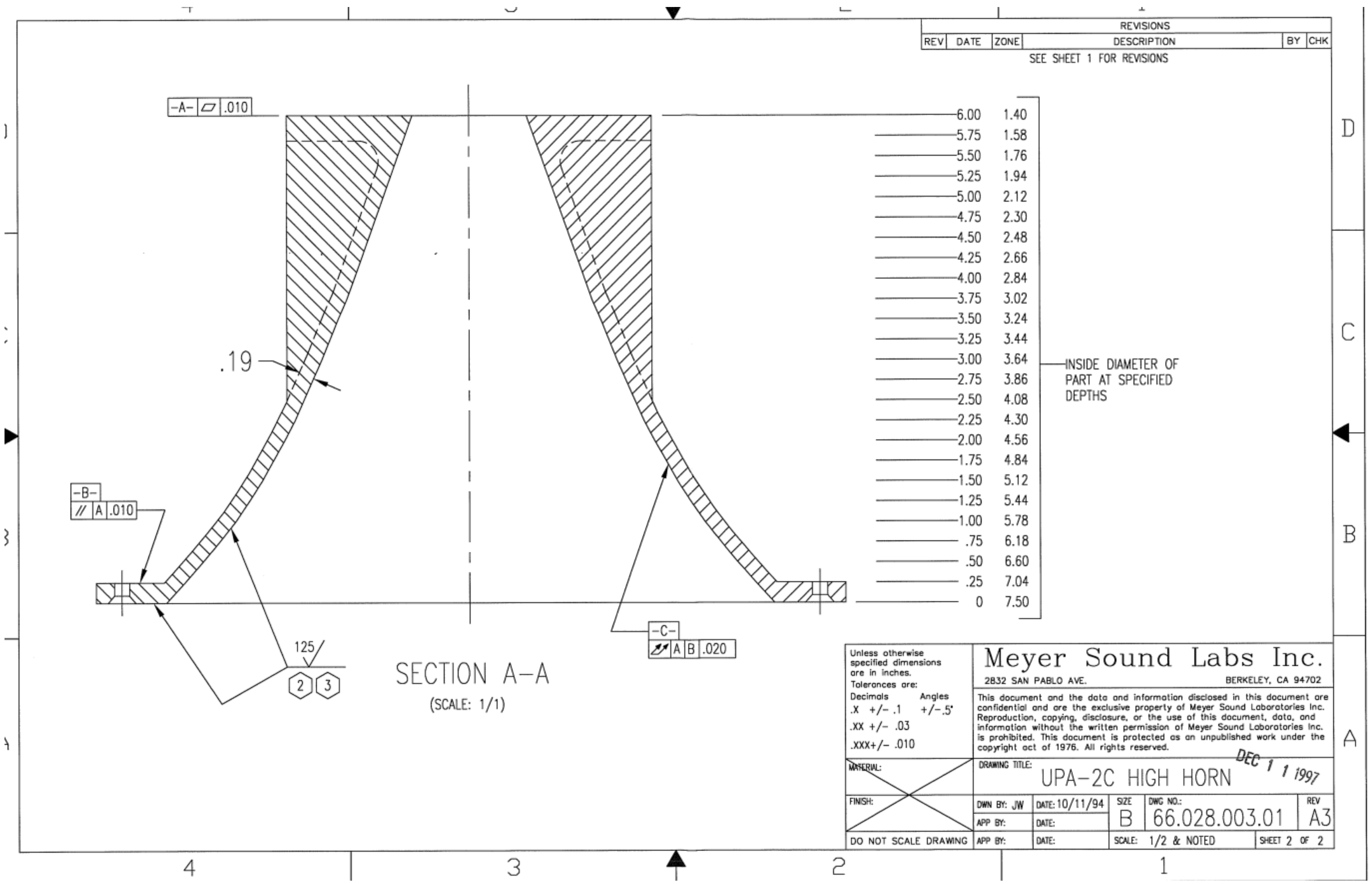
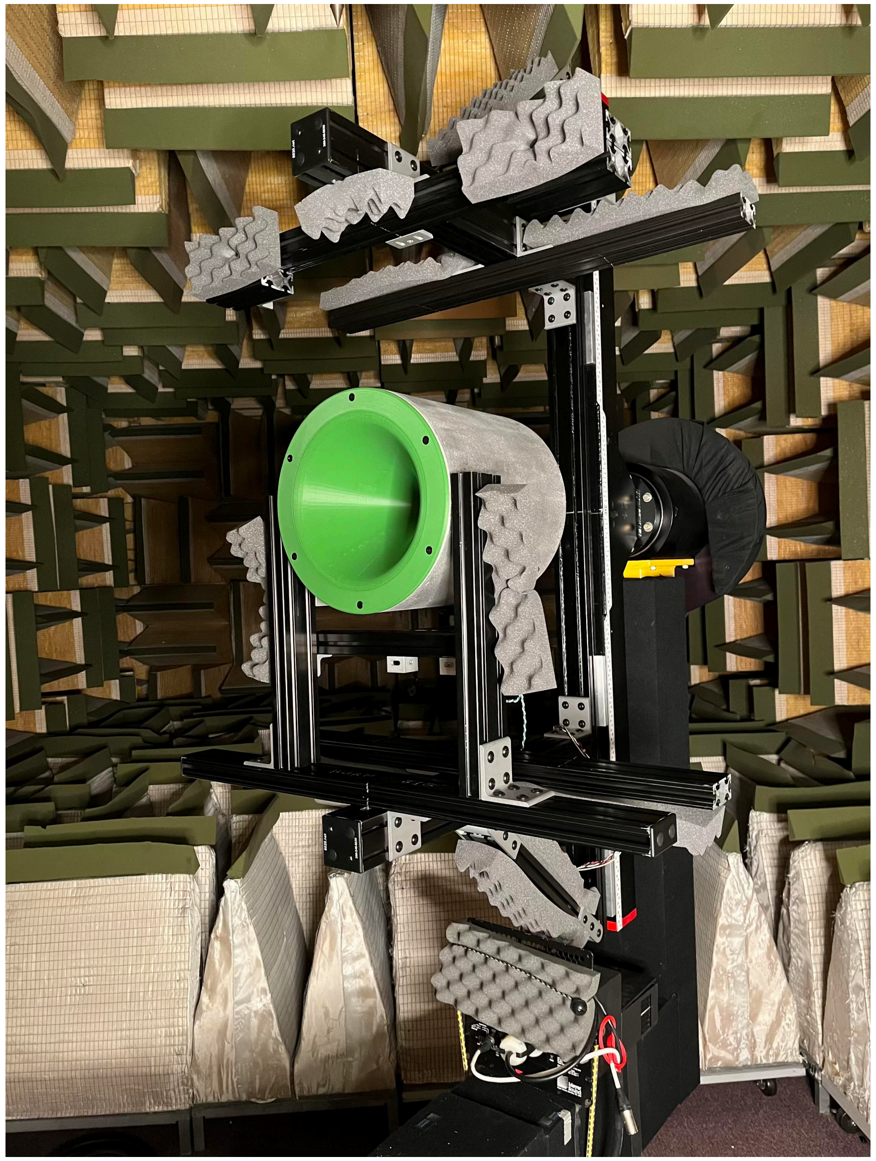
Ed Sheeran Makes Magic in Mumbai with Meyer Sound

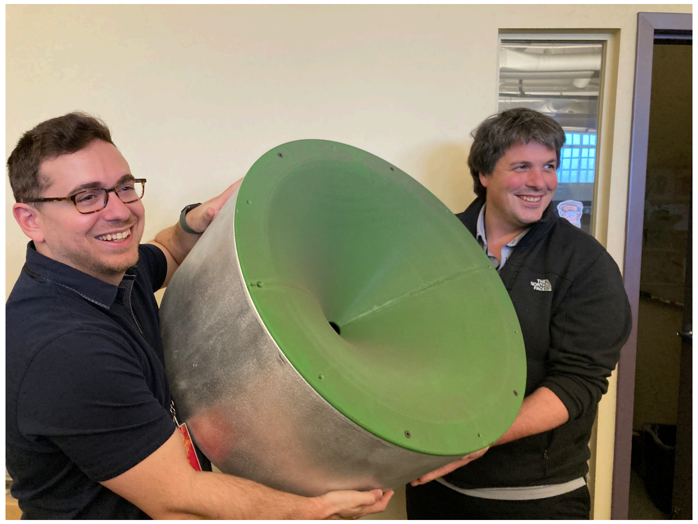
Asia Tour Finale Showcases Seamless Stadium Sound in the Round

3 of 11

Horn loaded speakers



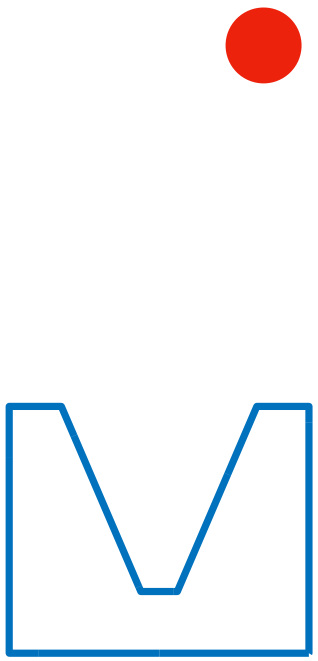


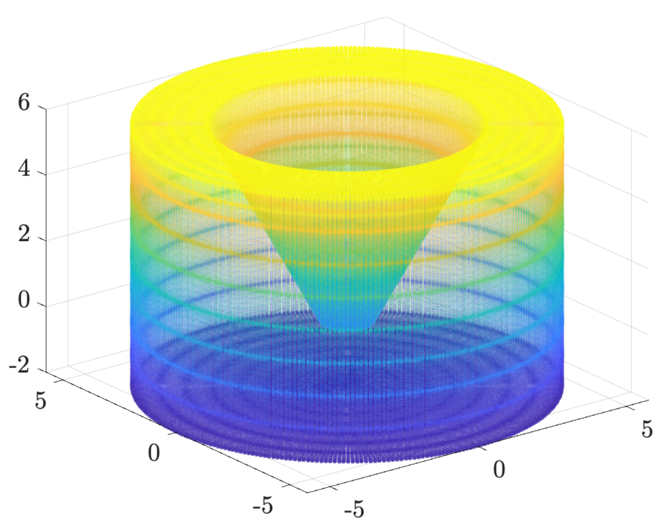


Horn loaded speakers

$$(\Delta + k^2)u = 0$$

$$n \cdot \nabla u = f$$





Horn loaded speakers

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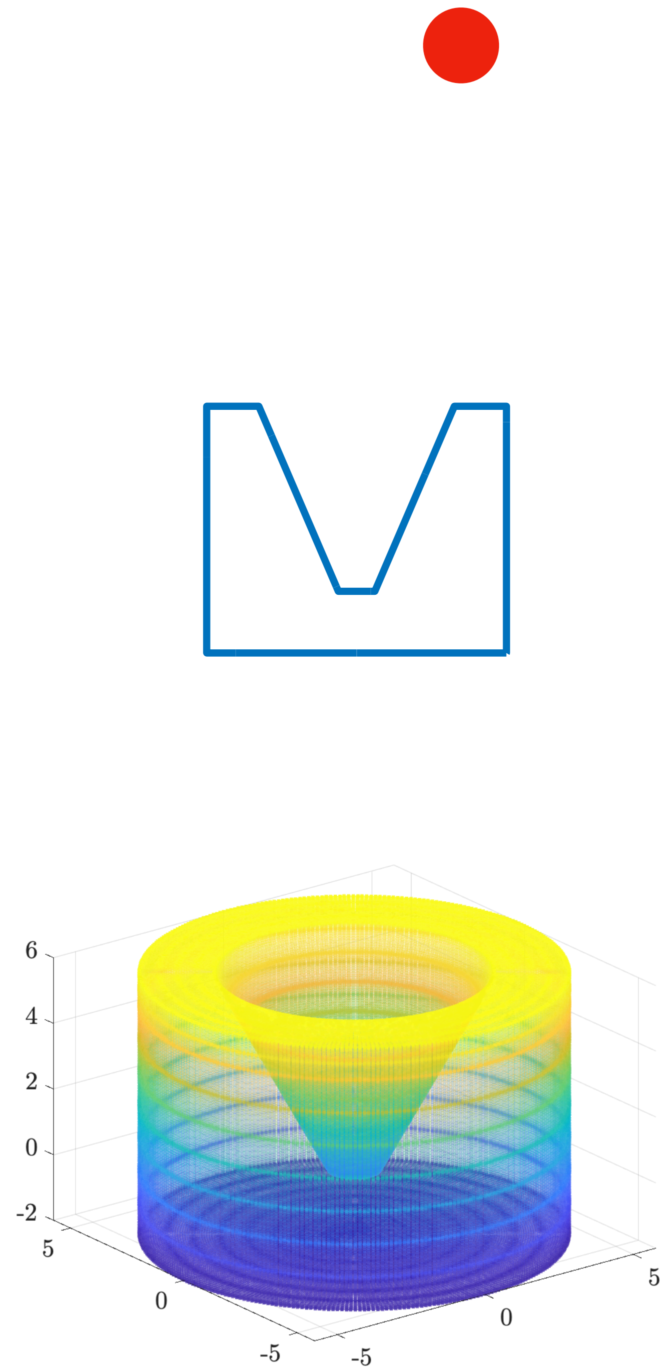
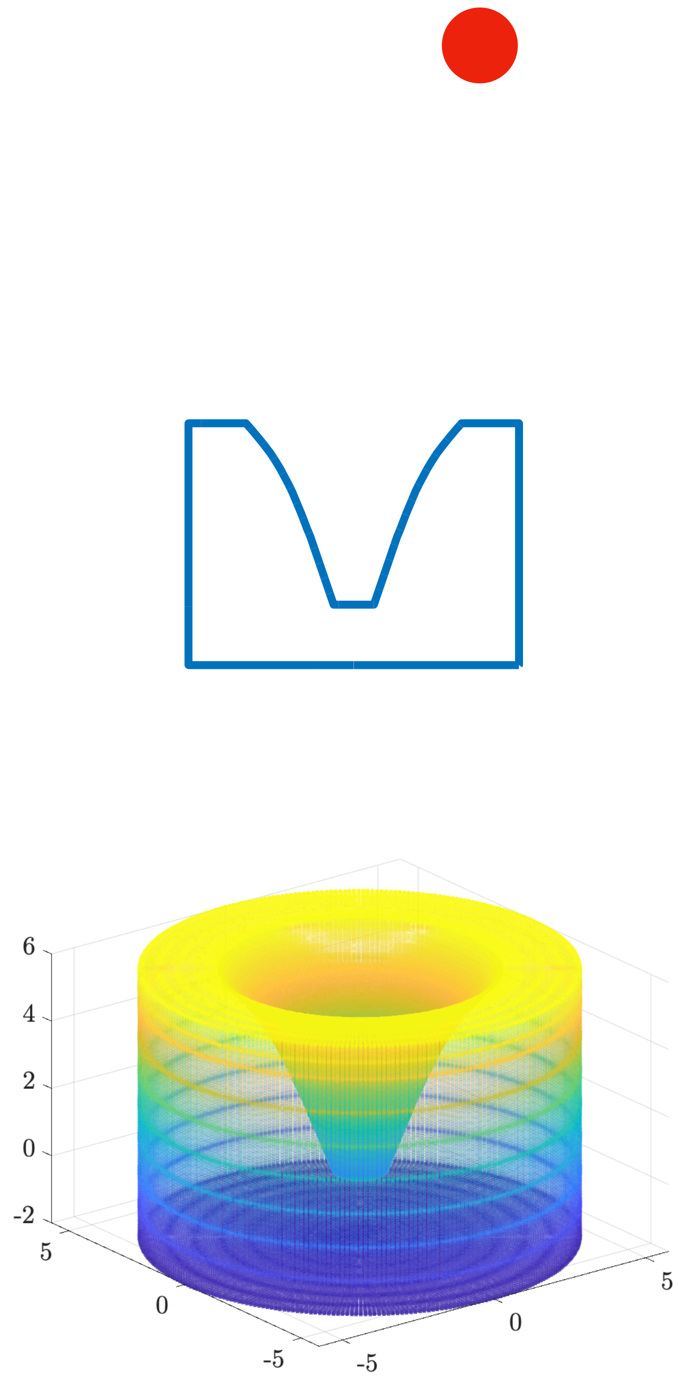
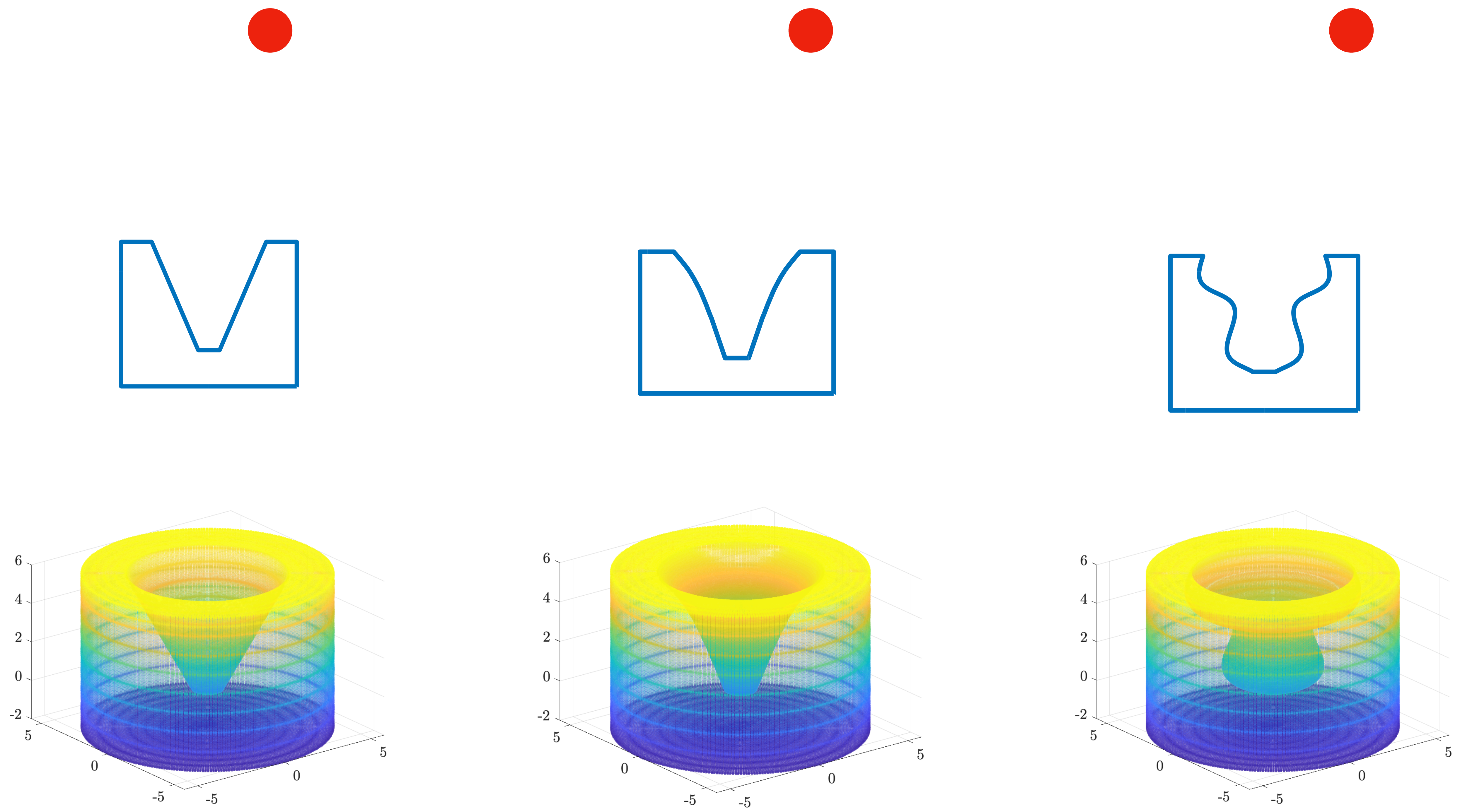




Horn loaded speakers



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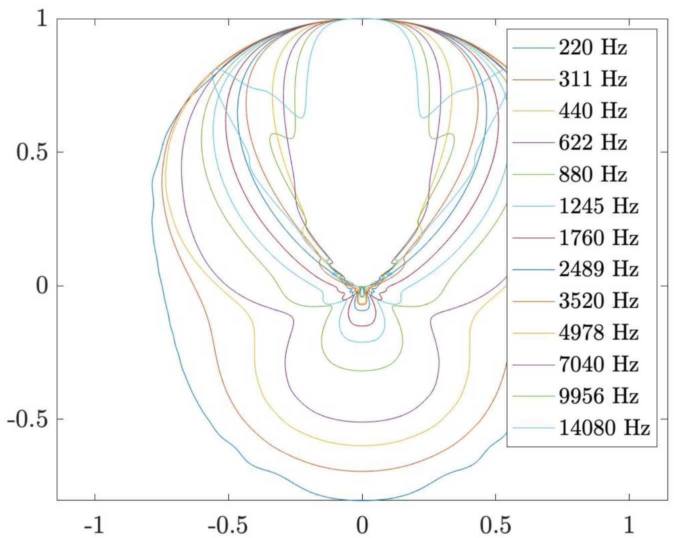
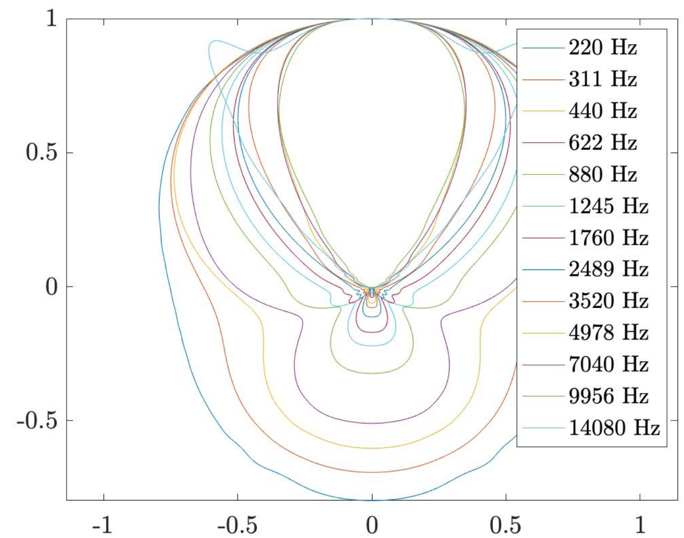
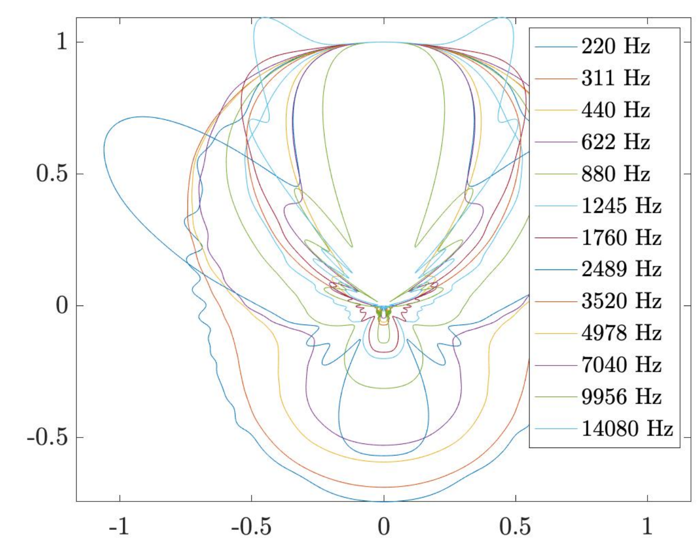
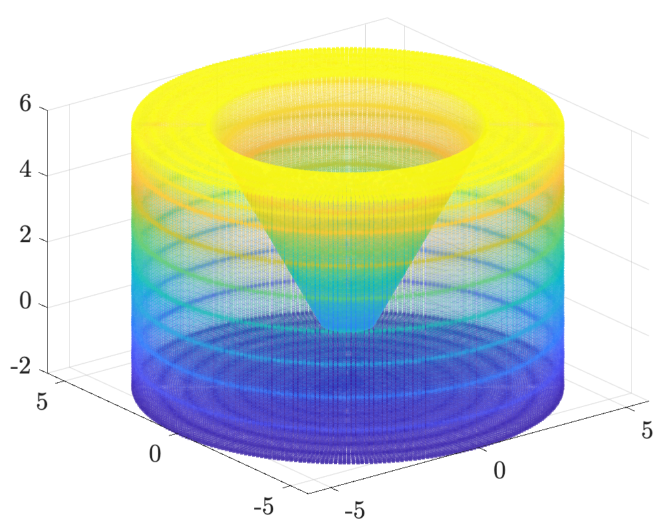
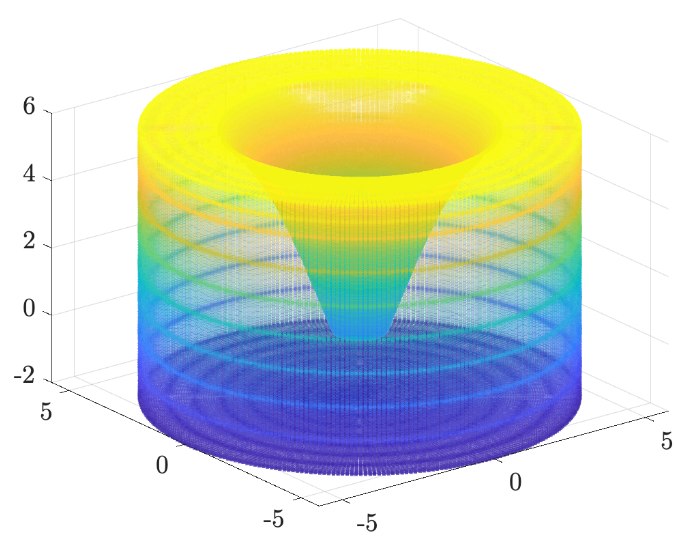
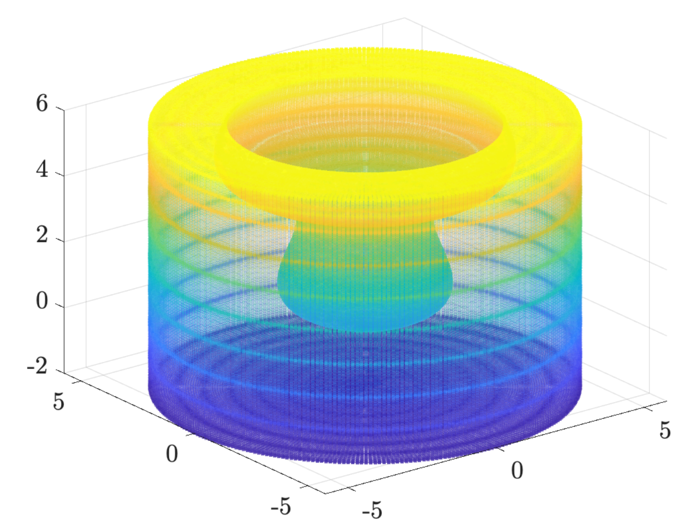




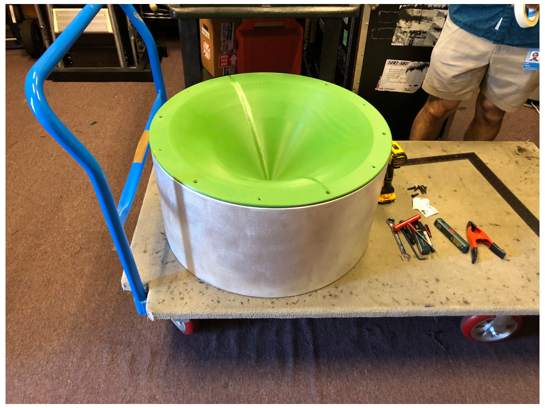


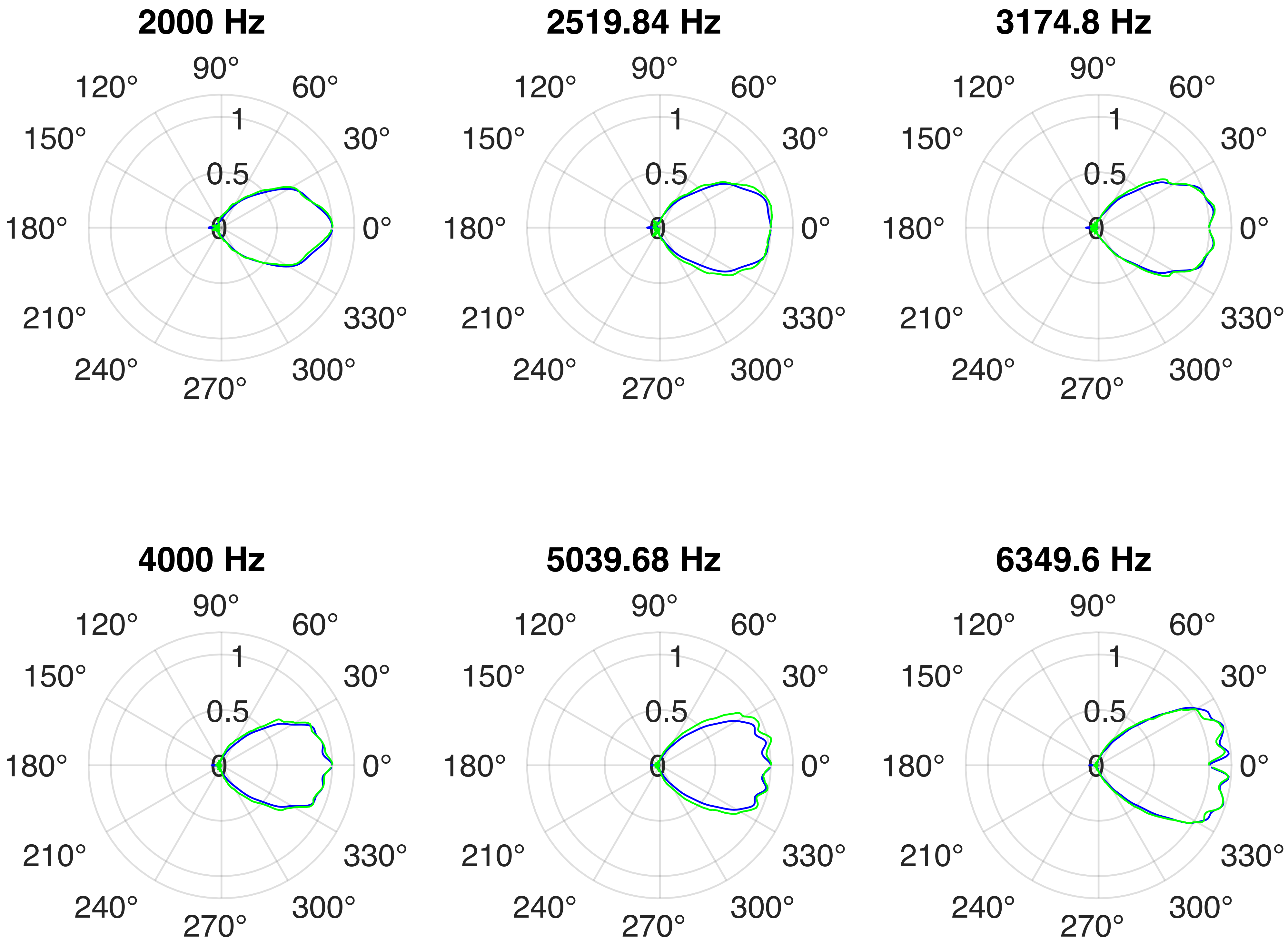






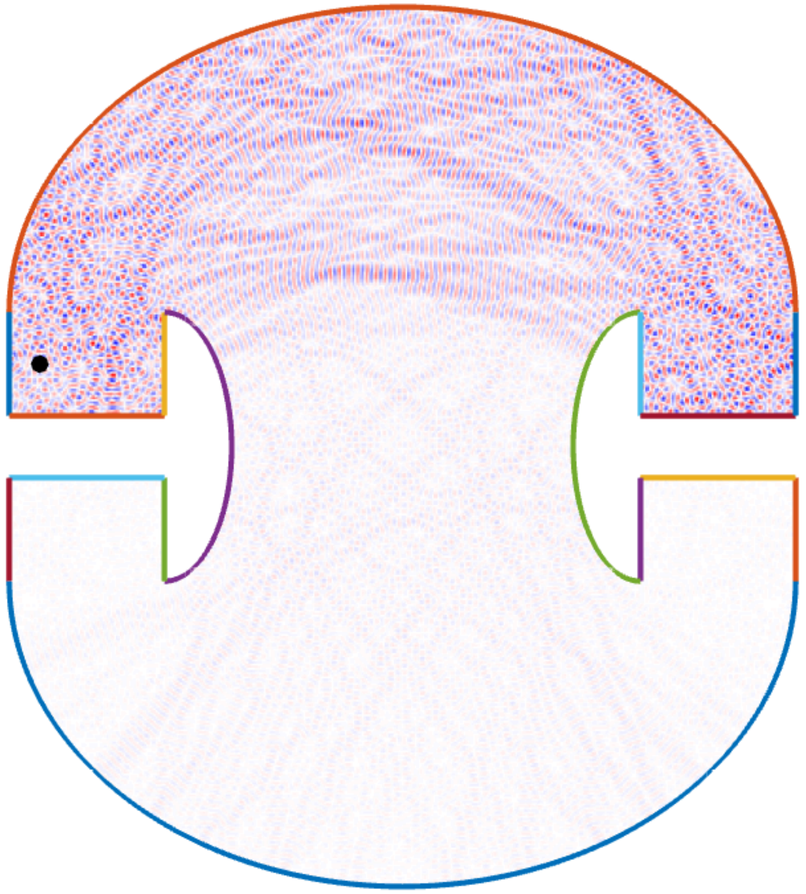
Speaker design and modeling





6 step to live the good life through integral equations

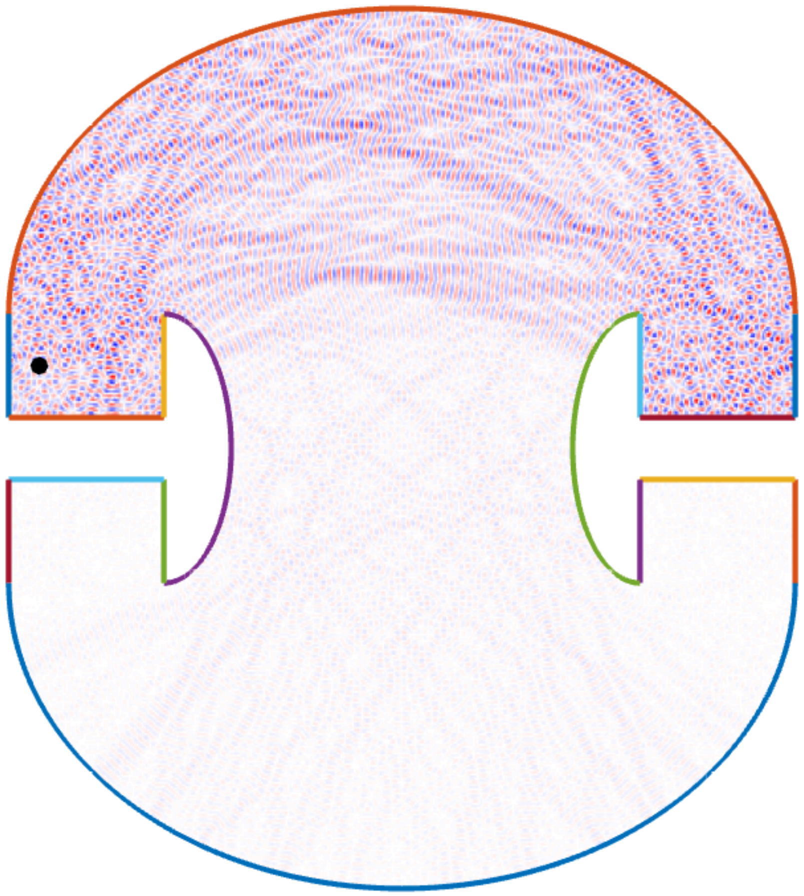
1. Identify a PDE and boundary conditions
2. Convert it into an integral equation (Lecture 2)
3. Discretize geometry (Lecture 1)
4. Discretize integral operators (Lecture 3)
5. Solve the integral equation (Lecture 4+5)
6. Post-process/recover PDE solution (Lecture 4+5)



6 step to live the good life through integral equations

1. Identify a PDE and boundary conditions

$$(\Delta + k^2)u = 0 \quad \text{in } \Omega$$
$$u = f \quad \text{on } \partial\Omega$$



$$(\Delta + k^2)u = 0 \quad \text{in } \Omega$$
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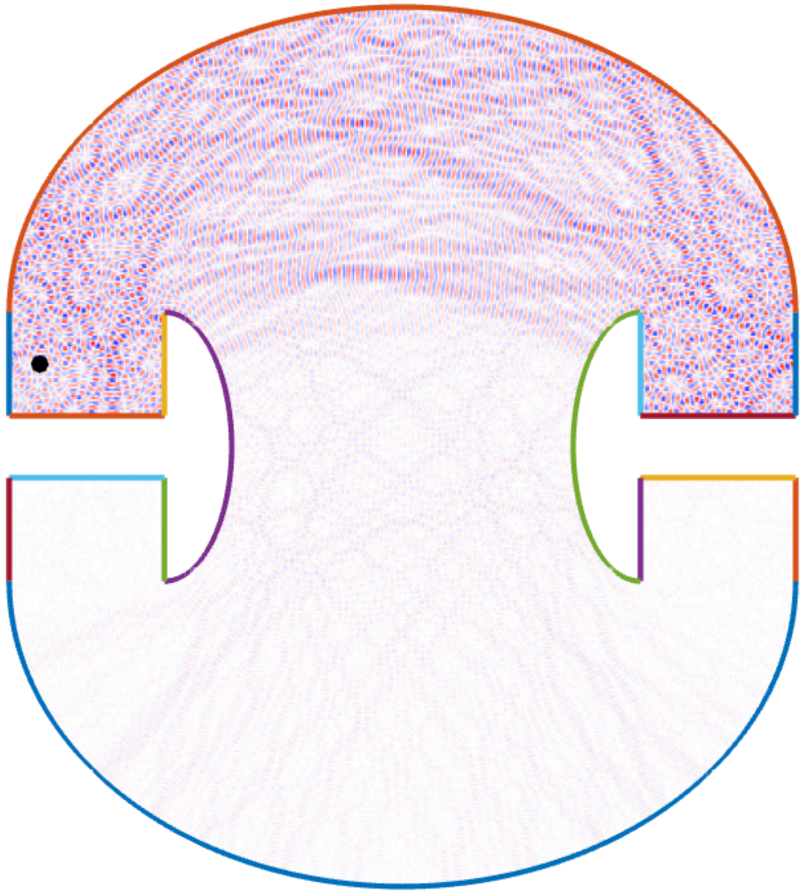
6 step to live the good life through integral equations

1. Identify a PDE and boundary conditions
2. **Convert it into an integral equation**
   - Integral representations use Green's functions/ fundamental solutions to represent the solution

$$(\Delta_x + k^2)G(x,y) = -\delta_{x=y}$$

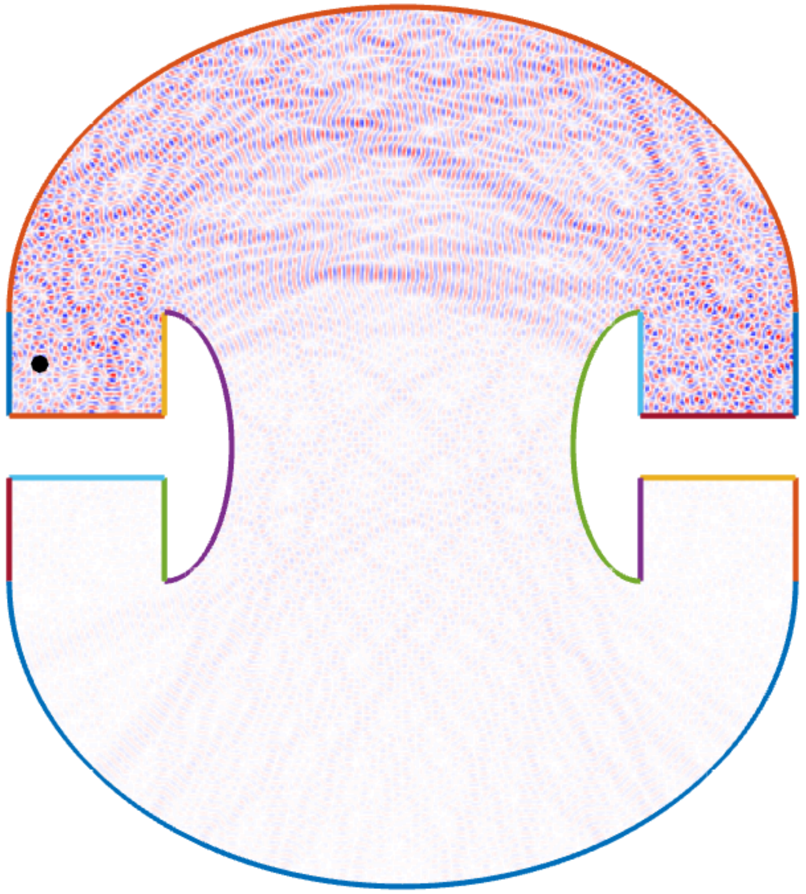
$$G(x,y) = \frac{i}{4}H_0^{(1)}(k|x-y|)$$



   - Information lives on the boundary alone, construct solution from bunch of Green's functions located on the boundary, i.e. set $u(x) = \int_{\partial\Omega} G(x,y)\sigma(y)ds_y$ for $x \in \Omega$
   - Need to solve for σ such that $\int_{\partial\Omega} G(x,y)\sigma(y) = f(x)$ for $x \in \partial\Omega$

6 step to live the good life through integral equations

1. Identify a PDE and boundary conditions
2. Convert it into an integral equation (Lecture 2)
3. **Discretize geometry**
    - Need a way to represent functions and integral equations on the computer, thus need to be able to interpolate functions from finite number of samples, compute their integrals…



What is function interpolation/representation?

Given:

- Discretization/sample points $\{x_j\}_{j=1}^{N}$
- Basis functions $\{\phi_j(x)\}_{j=1}^{M}$
- Exact samples of a function f at discretization points $\{f(x_j)\}_{j=1}^{N}$

Construct:

$$p(x) = \sum_{j=1}^{M} c_j \phi_j(x) \quad \text{with} \quad p(x_j) = f(x_j)$$

Function types

1. Nice and periodic
2. Nice and aperiodic
3. Not so nice

- Structure unknown: Adaptive
- Not niceness known: Generalized chebyshev methods