The `sdii` Users' Manual

`sdii` – computes significance of differences intervals (SDIs).[1]

Description

`sdii` computes SDIs to indicate whether two points estimates (typically the mean of distributions or of samples from the study population) are statistically or substantively distinct. The second `i` in the command name indicates that the command has "immediate" features, in the sense that for some specifications the command computes estimates not from the data stored in memory but from numbers typed as arguments.[2]

Quick start

Compute SDIs to compare two distributions with the following means and standard deviations: $\mu_1 = 10$*,* $\sigma_1 = 2$*, and* $\mu_2 = 5$*,* $\sigma_2 = 4$*. Further assume the distributions are correlated at the* 0.5 *level.*

```
    sdii, sample1(10 2) sample2(5 4) corr(.5)
```

Compute SDIs to compare two populations represented by samples x_1 *and* x_2*, with the following number of observations, means, and standard deviations:* $n_1 = 60$*,* $\bar{x}_1 = 10$*,* $sd_1 = 2$*, and* $n_2 = 40$*,* $\bar{x}_2 = 5$*,* $sd_2 = 4$.

```
    sdii, sample1(40 10 2) sample2(40 5 4)
```

As above, but assume unpaired samples and unequal population variances.

```
    sdii, sample1(60 10 2) sample2(40 5 4) unpaired unequal
```

Compute SDIs to compare the underlying populations for the two samples in variables v1 *and* v2.

```
    sdii v1 v2
```

As above, but compute percentile SDIs and report the median rather than mean as the summary statistic.

```
    sdii v1 v2, median percentile
```

Compute SDIs to compare the two group samples in variable v1 *defined by* groupvar.

```
    sdii v1, by(groupvar)
```

The `sdii` syntax

Compare two distributions or paired samples with set characteristics

`sdii,` sample1([$\#_{\text{obs1}}$] $\#_{\text{mean1}}$ $\#_{\text{sd1}}$) sample2([$\#_{\text{obs2}}$] $\#_{\text{mean2}}$ $\#_{\text{sd2}}$) [*options* *options*$_1$]

[1] The `sdii` program its associated manual come "as is" without warranty of any kind, either expressed or implied, including, but not limited to, the suitability and fitness for a particular purpose. Improvements and/or changes in the product and the program described in this manual may be made at any time and without notice.

[2] The description of the several syntax elements, options, and formulas borrow heavily, or reproduce excerpts ad litteram, from the official `pctile`, `power`, and `ttest` manuals (StataCorp 2017).

Compare two unpaired samples with set characteristics

`sdii,` sample1($\#_{\text{obs1}}$ $\#_{\text{mean1}}$ $\#_{\text{sd1}}$) sample2($\#_{\text{obs2}}$ $\#_{\text{mean2}}$ $\#_{\text{sd2}}$) unpaired [*options* *options*$_2$]

Compare two paired variable samples

`sdii` *varname1* *varname2* [*if*] [*in*] [*weight*] [, *options* *options*$_1$]

or

`sdii` *varname1* [*if*] [*in*] [*weight*] , variable2(*exp*) [*options* *options*$_1$]

Compare two unpaired variable samples

`sdii` *varname1* *varname2* [*if*] [*in*] [*weight*] , unpaired [*options* *options*$_2$]

or

`sdii` *varname1* [*if*] [*in*] [*weight*] , variable2(*exp*) unpaired [*options* *options*$_2$]

or

`sdii` *varname* [*if*] [*in*] [*weight*] , by(*groupvar*) [*options* *options*$_2$]

Note: Syntax elements within square brackets [ ] are optional. Underlining indicates minimal abbreviation.

options	Description
`difference`	report the difference in estimates
`level(#)`	set confidence level; default is `level(95)`
`mvalue(#)`	set meaningful value *m*; default is 0
`nolegend`	suppress legend of output items and explanatory notes
`power[(#)]`	power; default is `power(0.8)`
`precision(#)`	set level of precision for SDIs; default is 1
`reverse`	reverse order for the difference in estimates computation
`sample1`([$\#_{\text{obs1}}$] $\#_{\text{mean1}}$ $\#_{\text{sd1}}$)	input the number of observations, mean, and standard deviation of the first sample
`s1`([$\#_{\text{obs1}}$] $\#_{\text{mean1}}$ $\#_{\text{sd1}}$)	shorthand for `sample1`([$\#_{\text{obs1}}$] $\#_{\text{mean1}}$ $\#_{\text{sd1}}$)
`sample2`([$\#_{\text{obs2}}$] $\#_{\text{mean2}}$ $\#_{\text{sd2}}$)	input the number of observations, mean, and standard deviation of the second sample
`s2`([$\#_{\text{obs2}}$] $\#_{\text{mean2}}$ $\#_{\text{sd2}}$)	shorthand for `sample2`([$\#_{\text{obs2}}$] $\#_{\text{mean2}}$ $\#_{\text{sd2}}$)
`unpaired`	treat data as unpaired
`variable2(`*exp*`)`	generates the second comparison sample as defined by the expression *exp*
`var2(`*exp*`)`	shorthand for `variable2(`*exp*`)`

options$_1$ (with paired data)	Description
`correlation(#)`	set correlation level; default is 0
`distribution`	treat data as sampling distributions
`median`	report the median as the statistic value; default is mean
`percentile[(`*altdef*`)]`	compute percentile SDIs instead of the default, standard error-based SDIs; with suboption *altdef* an alternative formula is used to calculate percentiles
`pctile[(`*altdef*`)]`	shorthand for `percentile[(`*altdef*`)]`

Continued on next page

options$_2$ (with unpaired data)	Description
`by(`*groupvar*`)`	variable defining the groups
`unequal`	unpaired data have unequal variances
`welch`	use Welch's approximation

Note: Underlining indicates minimal abbreviation.

Options:

Main

`difference` reports the point estimate for the difference in estimates (typically the difference in means) with its standard CI. The default is the 95% CI or as set by `set level`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`.

`mvalue(#)` computes SDIs that indicate significance of differences other than zero. By typing `mvalue(1.3)` we test whether the difference between the compared estimates is greater than 1.3 rather than 0. # can be either positive or negative; by default # = 0.

`nolegend` specifies that the legend detailing the output items and the explanatory notes be suppressed.

`power`[`(#)`] computes the minimum detectable value of the effect size δ (the difference in estimates), given power, significance level α, sample characteristics (size and standard deviation), and, for paired data only, the correlation level ρ. Power is technically defined as $(1-\beta)$, where β is the probability of type II error. # sets the power within the $(0,1)$ range; by default # = 0.8. The significance level is calculated from the specified level for confidence intervals, $\alpha = (1 - \texttt{level}/100)$.

`precision(#)` specifies the decimal precision of reported SDIs, ranging from no decimals to up to six decimals (# $\in$ {0,1,2,3,4,5,6}). The difference between typing `precision(0)` or `precision(3)` is that an 85-ish% SDI will be rounded to either 85% or 85.xxx%, respectively. The default is to report one digit precision SDIs, i.e., # = 1.

`reverse` reverses the order of the two samples when calculating the difference in estimates. By default `sdii` observes the order in which the samples are listed in *varlist* or `sample*()`. Specifically, *var2* or `sample2()` is subtracted from *var1* or `sample1()`, respectively. With `by()`, the group corresponding to the largest value in the variable in `by()` is subtracted from the group with the smallest value in `by()`. `reverse` reverses this behavior and the order in which variables appear in the table. `reverse` also reverses the sign of the meaningful value *m* (i.e., *m* = `mvalue(`$-$`#)`).

`sample1(`[$\#_{\text{obs1}}$] $\#_{\text{mean1}}$ $\#_{\text{sd1}}$`)` specifies the number of observations, mean, and standard deviation, respectively, of the first sample. When `sample1()` receives only two arguments, these numbers are assumed to represent the mean and standard deviation of a normal distribution rather than of a sample (e.g., `sample1(`$\#_{\text{mean1}}$ $\#_{\text{sd1}}$`))`.

`sample2(`[$\#_{\text{obs2}}$] $\#_{\text{mean2}}$ $\#_{\text{sd2}}$`)` same as above for the second sample. Specifying samples with different number of observations (i.e., $\#_{\text{obs1}} \neq \#_{\text{obs2}}$), implies `unpaired`.

`unpaired` specifies that the data be treated as unpaired.

`variable2(`*exp*`)` generates the second sample as defined by the expression in the suboption *exp*. *var2* can be a transformation of the compared sample *var1* (e.g., `variable2(var1^2)`), or of another exiting variable (e.g., `variable2(var3+1)`).

With paired data

`correlation(#)` indicates the correlation level, ρ, for paired data. It cannot be combined with *varlist* as in this case ρ is the observed correlation level between the two variables. By default $\rho = 0$, and its range is $[-1, 1]$.

`distribution` specifies that the compared variables are sampling distributions rather than sample groups. Sampling distributions contain a number of realizations of the quantity of interest (e.g., sample mean), and are typically computed via simulations using either canned Stata commands (e.g., `bootstrap`) or user-written programs (e.g., Clarify). The relevant difference between sampling distributions and samples from the population, is that the standard error for the latter is a function of the sample size, whereas for the former is simply the standard deviation. `distribution` does not affect percentile-based SDIs, since they are not a function of the standard error.

`median` reports the median as the summary statistic value, whereas mean is the default. `median` can be used only in combination with the `percentile` option.

`percentile`[`(`*altdef*`)`] specifies that percentile SDIs be calculated instead of standard error-based SDIs, which is the default. The default method for calculating percentiles is to invert the empirical distribution function by using averages, $(x_i + x_{i+1})/2$, where the function is flat. When the suboption *altdef* is specified, an alternative formula that uses an interpolation method is employed. Weights cannot be used when *altdef* is specified.

With unpaired data

`by(`*groupvar*`)` specifies the *groupvar* that defines the two group samples to be compared. Specifying `by()` implies `unpaired`.

`unequal` specifies that the unpaired data not to be assumed to have equal variances.

`welch` specifies that the approximate degrees of freedom for the test be obtained from Welch's (1947) formula rather than from the Satterthwaite's (1946) approximation formula, which is the default when `unequal` is specified. Specifying `welch` implies `unequal` and `unpaired`.

Remarks and examples

Example 1: Compute SDIs to compare independent distributions

Let us say we want to compare two normal distributions. To do this we need to indicate the distributions' means (i.e., $\mu_1 = 10$ and $\mu_2 = 5$) and standard deviations (i.e., $\sigma_1 = 2$ and $\sigma_2 = 1$). Optionally, we can also specify the level of correlation. If not specified, as it is the case here, the distributions are assumed to be independent ($\rho = 0$). The last step is to check whether the SDIs around the respective means overlap, in which case the compared estimates are statistically indistinguishable. In this example there is no overlap as the lower SDI bound of the higher statistic (here the mean of distribution 1), is higher than the upper SDI bound of the lower statistic. Practically this means the two distributions are statistically different.

```
. sdii, sample1(10 2) sample2(5 1)

SDI Results

    Comparison      :          Two distributions with correlation ρ = 0
    (1)             :          sample1
    (2)             :          sample2
    Statistic       :          mean
    Interval type   :          Standard error-based SDIs

──────────┬───────────────────────────────────────────────────────────────────
          │      Statistic      Std. Err.      [Interval         Bounds]       (%) Type
──────────┼───────────────────────────────────────────────────────────────────
(1)       │             10              2      7.0782582       12.921742       85.6 SDI
(2)       │              5              1      3.5391291       6.4608709       85.6 SDI
──────────┴───────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 0.
```

Example 2: Compute SDIs to indicate substantive significance

Using the option `mvalue()`, next we test whether the difference between the same two distributions is statistically different from 1 rather than 0. Additionally, we also report the difference in estimates with its standard CI. In this case the associated SDIs overlap which means the difference in means is not statistically distinct from 1, even though it is distinct from 0 (see Example 1). Testing for significance of differences other than zero is a practical way to assess substantive significance.

```
. sdii, sample1(10 2) sample2(5 1) mvalue(1) difference

SDI Results

    Comparison      :          Two distributions with correlation ρ = 0
    (1)             :          sample1
    (2)             :          sample2
    Statistic       :          mean; (1-2) = mean(1) - mean(2)
    Interval type   :          Standard error-based SDIs & CI

──────────┬───────────────────────────────────────────────────────────────────
          │      Statistic      Std. Err.      [Interval         Bounds]       (%) Type
──────────┼───────────────────────────────────────────────────────────────────
(1)       │             10              2      6.4115915       13.588408       92.8 SDI
(2)       │              5              1      3.2057958       6.7942042       92.8 SDI
(1-2)     │              5       2.236068       .6173873       9.3826127         95  CI
```

```
──────────────┴──────────────────────────────────────────────────────────────────
Note: SDIs indicate significance of difference from 1.
```

Example 3: Compute SDIs to compare correlated distributions

Would the results change if the distributions were correlated at, let us say, the 0.5 level? As it turns out, in this case the SDIs do not overlap which means the two distributions are substantively different (given a meaning value $m = 1$). Since the only difference from Example 2 is the value of ρ, this example highlights the fact that the difference in estimates is a function of their correlation level.

```
. sdii, sample1(10 2) sample2(5 1) m(1) corr(.5) diff

SDI Results

    Comparison      :          Two distributions with correlation ρ = 0.50
    (1)             :          sample1
    (2)             :          sample2
    Statistic       :          mean; (1-2) = mean(1) - mean(2)
    Interval type   :          Standard error-based SDIs & CI

──────────────┬──────────────────────────────────────────────────────────────────
              │      Statistic      Std. Err.      [Interval        Bounds]        (%) Type
──────────────┼──────────────────────────────────────────────────────────────────
(1)           │             10              2      7.0701619      12.929838       85.8 SDI
(2)           │              5              1      3.5350809      6.4649191       85.8 SDI
(1-2)         │              5      1.7320508      1.6052428      8.3947572         95  CI
──────────────┴──────────────────────────────────────────────────────────────────
Note: SDIs indicate significance of difference from 1.
```

Example 4: Compute SDIs to compare paired samples

In many practical applications the researcher only has data on a subset of the population. As a result, we often end up comparing samples selected from the population. Below is a comparison between two paired samples with the respective number of observations, means, and standard deviations. Since the associated SDIs do not overlap, it means the two underlying populations are statistically distinct at the 10% level.

```
. sdii, sample1(40 10 2) sample2(40 5 4) level(90)

SDI Results

    Comparison      :          Two paired samples with correlation ρ = 0
    (1)             :          sample1
    (2)             :          sample2
    Number of obs   =          40
    Statistic       :          mean
    Interval type   :          Standard error-based SDIs

──────────────┬──────────────────────────────────────────────────────────────────
              │      Statistic      Std. Err.      [Interval        Bounds]        (%) Type
──────────────┼──────────────────────────────────────────────────────────────────
(1)           │             10      .31622777      9.6028711      10.397129       78.4 SDI
(2)           │              5      .63245553      4.2057423      5.7942577       78.4 SDI
──────────────┴──────────────────────────────────────────────────────────────────
Note: SDIs indicate significance of difference from 0.
```

Example 5: Compute SDIs to compare unpaired samples

While in some cases the researcher has multiple readings on the same subjects (e.g., patient data over time), in other cases the subjects are distinct (e.g., different survey waves, treated and untreated experimental groups, etc.). The latter type of data are unpaired and to compare sample means requires specific adjustments. For such cases we need to use the `unpaired` option. Unlike paired data, unpaired samples can potentially have different number of observations. In this example the underlying populations represented by the two unpaired samples are statistically distinct, as indicated by the lack of SDI overlap.

```
. sdii, sample1(60 10 2) sample2(40 5 4) unpaired

SDI Results

    Comparison          :          Two unpaired samples with equal population variances
    (1)                 :          sample1
    Number of obs (1)   =          60
    (2)                 :          sample2
    Number of obs (2)   =          40
    Statistic           :          mean
    Interval type       :          Standard error-based SDIs

──────────┬──────────────────────────────────────────────────────────────────────
          │     Statistic      Std. Err.      [Interval         Bounds]       (%) Type
──────────┼──────────────────────────────────────────────────────────────────────
(1)       │            10      .25819889      9.6536289       10.346371       81.6 SDI
(2)       │             5      .63245553      4.1463896       5.8536104       81.6 SDI
──────────┴──────────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 0.
```

Example 6: Compute SDIs to compare unpaired samples with unequal population variances

In this example we use the same data but relax the assumption that the underlying populations have equal variances. We also use Welch's formula (rather than Satterthwaite's) to calculate the approximate degrees of freedom. Compared to the results from Example 5 the SDI level is slightly higher, but the substantive results remain largely unchanged.

```
. sdii, sample1(60 10 2) sample2(40 5 4) unpaired unequal welch

SDI Results

    Comparison          :          Two unpaired samples with unequal population variances
    (1)                 :          sample1
    Number of obs (1)   =          60
    (2)                 :          sample2
    Number of obs (2)   =          40
    Statistic           :          mean
    Interval type       :          Standard error-based SDIs

──────────┬──────────────────────────────────────────────────────────────────────
          │     Statistic      Std. Err.      [Interval         Bounds]       (%) Type
──────────┼──────────────────────────────────────────────────────────────────────
(1)       │            10      .25819889      9.6047904        10.39521       86.9 SDI
(2)       │             5      .63245553      4.0248925       5.9751075       86.9 SDI
──────────┴──────────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 0.
```

Example 7: Compute SDIs to compare paired variable samples

Let us say we want to compare two samples saved in distinct variables. For this example, as well as the next two, we use the `fuel` data available from the StataCorp website. The data contains two variables, *mpg1* and *mpg2*, that record the miles per gallon consumption of cars with and without fuel treatment. To determine whether the fuel treatment is effective, we compare the sample means by computing their associated SDIs. Since the SDIs do not overlap, we can conclude that the cars that were treated (*mpg1*) have on average a lower fuel consumption than their counterparts from *mpg2*.

```
. use https://www.stata-press.com/data/r17/fuel, clear

. sdii mpg1 mpg2

SDI Results

    Comparison      :          Two paired samples
    (1)             :          mpg1 (variable)
    (2)             :          mpg2 (variable)
    Number of obs   =          12
    Statistic       :          mean
    Interval type   :          Standard error-based SDIs

----------+-------------------------------------------------------------------
          |    Statistic      Std. Err.      [Interval        Bounds]       (%) Type
----------+-------------------------------------------------------------------
(1)       |           21      .78817011      20.216612      21.783388      65.9 SDI
(2)       |        22.75      .93844649      21.817248      23.682752      65.9 SDI
----------+-------------------------------------------------------------------
Note: SDIs indicate significance of difference from 0.
```

Example 8: Compute percentile SDIs

For variable samples, `sdii` can alternatively compute percentile- rather than standard error-based SDIs. Percentile intervals are often used with skewed data and in such cases the mean of the distribution may not be the relevant summary statistic. With `percentile`, researchers have the option to request the median rather than mean as the point estimate. In this example the percentile SDIs overlap which suggests that the two car samples have similar fuel consumptions. The low number of observations, however, should make us cautious of these results since the accuracy of percentile intervals increases with sample size.

```
. sdii mpg1 mpg2, unpaired

SDI Results

    Comparison          :          Two unpaired samples with equal population variances
    (1)                 :          mpg1 (variable)
    Number of obs (1)   =          12
    (2)                 :          mpg2 (variable)
    Number of obs (2)   =          12
    Statistic           :          mean
    Interval type       :          Standard error-based SDIs

----------+-------------------------------------------------------------------
          |    Statistic      Std. Err.      [Interval        Bounds]       (%) Type
```

```
                ─────────────────────────────────────────────────────────────────────────────
(1)          │            21      .78817011      19.839819      22.160181       83.1 SDI
(2)          │         22.75      .93844649      21.368613      24.131387       83.1 SDI
─────────────────────────────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 0.
```

Example 9: Compute SDIs to compare unpaired variable samples

Just because the compared samples are in adjacent variables with equal number of observations, does not necessarily mean the data are paired. When this is not the case, we need to specify `unpaired`. As our example illustrates, it matters whether the data is paired or unpaired. Compared to the results from Example 7, in this analysis the SDI level is substantively higher and the SDIs overlap. This in turn suggests that treated and untreated cars have similar fuel consumptions.

```
. sdii, sample1(10 2) sample2(5 1) mvalue(1) difference

SDI Results

    Comparison      :          Two distributions with correlation ρ = 0
    (1)             :          sample1
    (2)             :          sample2
    Statistic       :          mean; (1-2) = mean(1) - mean(2)
    Interval type   :          Standard error-based SDIs & CI

─────────────────────────────────────────────────────────────────────────────────────────
             │      Statistic      Std. Err.      [Interval         Bounds]        (%) Type
─────────────┼───────────────────────────────────────────────────────────────────────────
(1)          │            10              2      6.4115915      13.588408       92.8 SDI
(2)          │             5              1      3.2057958      6.7942042       92.8 SDI
(1-2)        │             5       2.236068       .6173873      9.3826127         95  CI
─────────────────────────────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 1.
```

Example 10: Compute SDIs to compare group samples within the same variable

In the previous example the two samples were recorded in different variables. Alternatively, they can be recorded in the same variable, with an additional variable identifying the groups. In such cases we need to use the `by()` option. We use the `stack` command to stack variables *mpg1* and *mpg2*, a command which conveniently creates the `by()` variable, that is, *_stack*. The results are identical to those from Example 9, the one with two variable samples.

```
. stack mpg1 mpg2, into(mpg) clear

. sdii mpg, by(_stack) unpaired

SDI Results

    Comparison          :          Two unpaired samples with equal population variances
    (1)                 :          mpg if _stack==1 (variable)
    Number of obs (1)   =          12
    (2)                 :          mpg if _stack==2 (variable)
    Number of obs (2)   =          12
    Statistic           :          mean
    Interval type       :          Standard error-based SDIs
```

```
             |     Statistic     Std. Err.      [Interval        Bounds]       (%) Type
-------------+---------------------------------------------------------------------
(1)          |            21     .78817011      19.839819      22.160181       84.5 SDI
(2)          |         22.75     .93844649      21.368613      24.131387       84.5 SDI
-----------------------------------------------------------------------------------

Note: SDIs indicate significance of difference from 0.
```

Using `sdii` *interactively with estimation and postestimation commands*

Example 11: Compute SDIs to compare estimated statistics

The next two examples use the data from the Second National Health and Nutrition Examination Survey, available from the StataCorp website (`nhanes2f.dta`). While the dataset is comprehensive and has many variables, we estimate a simple logit model with two covariates. The dependent variable, *diabetes*, is a dummy variable coded one if the respondent has diabetes. *race*, one of the two covariates, is a three category variable (1 = White, 2 = Black, and 3 = Other), whereas the second covariate, *age*, is continuous. Looking at the *race* coefficients, the negative and statistically significant coefficient on *Black* indicates that blacks are more likely to have diabetes than whites (the reference category). Yet there is no statistical difference between other racial minorities and whites.

How do blacks fare compared to other minorities? Notably, it is not warranted to conclude that blacks are more likely to have diabetes simply because the *Other* coefficient is statistically indistinguishable from the base category, whereas *Black* is not. Since the associated 95% CIs overlap, we cannot tell from the available information whether the two coefficients are statistically distinct. One approach to answer this question is to compute the difference in coefficients via `nlcom`. If the standard CI of the difference contains zero, the estimates are statistically indistinguishable. Since this is the case in our analysis, there is no statistical difference between blacks and other minorities in terms of their probability of having diabetes.

```
. webuse nhanes2f, clear

. logit diabetes age i.race, nolog

Logistic regression                             Number of obs     =     10,335
                                                LR chi2(3)        =     371.75
                                                Prob > chi2       =     0.0000
Log likelihood = -1813.1895                     Pseudo R2         =     0.0930

------------------------------------------------------------------------------
    diabetes |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0595519   .0037348    15.95   0.000     .0522318     .066872
             |
        race |
       Black |   .7221263   .1270265     5.68   0.000     .4731588    .9710937
       Other |   .1930256   .3517485     0.55   0.583    -.4963888      .88244
             |
       _cons |  -6.330175   .2318176   -27.31   0.000    -6.784529    -5.87582
------------------------------------------------------------------------------

. nlcom _b[2.race]-_b[3.race]
```

```
       _nl_1:  _b[2.race]-_b[3.race]

    ------------------------------------------------------------------------------
        diabetes |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
    -------------+----------------------------------------------------------------
           _nl_1 |   .5291007   .3666378     1.44   0.149    -.1894962    1.247698
    ------------------------------------------------------------------------------
```

How can we outline this finding using SDIs around the compared coefficients? We can do this by using `sdii` interactively with the results saved by the logit estimation. For our example we are interested in the *2.race* and *3.race* coefficients, which are saved in the **e(b)** matrix. The coefficients' standard error is the square root of their variance (var), which can be retrieved from the variance-covariance matrix **e(V)**. Finally, the correlation level is a function of the respective variances and covariance (cov): $\rho(\textit{2.race, 3.race}) = \mathrm{cov}(\textit{2.race, 3.race})/(\sqrt{\mathrm{var}(\textit{2.race})} \times \sqrt{\mathrm{var}(\textit{3.race})})$. After making copies of the two matrices of interest, we need to select the matrix elements that correspond to the arguments of `sample`$*$`(`μ_* σ_*`)` from the `sdii` syntax. To do this we use the `el()` matrix function, and then save each required element in a scalar. In matrix notation, the coefficients of interest are in the first row, columns three and four (i.e., **eb**[1,3] and **eb**[1,4]). In the variance-covariance matrix, $\mathrm{cov}(\textit{2.race, 3.race})$ corresponds to element **eV**[4,3], and the respective variances to **eV**[3,3] and **eV**[4,4]. Now we have everything we need to issue the `sdii` command. Since the associated SDIs overlap, the two coefficients are statistically indistinguishable. As an accuracy check we also report the difference in estimates. The point estimate, standard error, and confidence interval of the difference in coefficients (i.e., the $(1-2)$ estimate) match perfectly the `nlcom` results.

```
. matrix eb = e(b)

. matrix eV = e(V)

. mat list eb

eb[1,5]
      diabetes:   diabetes:   diabetes:   diabetes:   diabetes:
                        1b.          2.          3.            
            age        race        race        race       _cons
y1   .05955192           0   .72212626   .19302558  -6.3301745

. mat list eV

symmetric eV[5,5]
                    diabetes:   diabetes:   diabetes:   diabetes:   diabetes:
                                      1b.          2.          3.            
                          age        race        race        race       _cons
    diabetes:age    .00001395
diabetes:1b.race            0           0
 diabetes:2.race    .00002225           0   .01613574
 diabetes:3.race    .00003014           0   .00271974   .12372701
  diabetes:_cons    -.000844            0  -.00401792  -.00449537   .05373939

. tempname m1 s1 m2 s2 cor12

. scalar `m1´ = el(eb,1,3)

. scalar `s1´ = sqrt(el(eV,3,3))

. scalar `m2´ = el(eb,1,4)
```

~~~
. scalar `s2´ = sqrt(el(eV,4,4))

. scalar `cor12´ = el(eV,4,3)/(sqrt(el(eV,3,3))*sqrt(el(eV,4,4)))

. sdii, sample1(`=`m1´´ `=`s1´´) sample2(`=`m2´´ `=`s2´´) corr(`=`cor12´´) diff

SDI Results

    Comparison      :         Two distributions with correlation ρ = 0.06
    (1)             :         sample1
    (2)             :         sample2
    Statistic       :         mean; (1-2) = mean(1) - mean(2)
    Interval type   :         Standard error-based SDIs & CI

------------------------------------------------------------------------------
             |       Statistic      Std. Err.      [Interval         Bounds]        (%) Type
-------------+----------------------------------------------------------------
(1)          |       .72212626      .12702654      .53147122       .91278129       86.7 SDI
(2)          |       .19302558      .35174851     -.33491625        .7209674       86.7 SDI
(1-2)        |       .52910068      .36663779     -.18949618       1.2476975         95  CI
------------------------------------------------------------------------------

Note: SDIs indicate significance of difference from 0.
~~~

Instead of using numerical tables, some researches prefer to report their findings using graphs like the one in Figure 1. In particular, Figure 1a1 plots the `nlcom` result, that is, the difference between the *Blacks* and *Other Minorities* coefficients. Since the CI of the difference contains zero, it means the two coefficients are not statistically different. Notably, the difference does not reveal the values used to create it, which is necessary to assess the rate of change. Moreover, while both our coefficients are positive, a 0.5 point estimate could also be the difference between two negative coefficients (e.g., $(-0.5-(-1))$). To put this finding in context, we need to graph the compared coefficients as well (i.e., the logit estimates), Figure 1a2. Taken together, these two sets of results illustrate the counterintuitive scenario where the compared estimates are statistically indistinguishable, even though only one (not both) of them is significant.

Figure 1b plots the compared coefficients with SDIs that can be used for direct comparisons. Since the associated 86.7% SDIs overlap, the two coefficients are not statistically different. By presenting a single set of results the SDI method is more parsimonious. Conversely, having two sets of results creates redundancy and may lead to confusion. By using different line patterns, SDI graphs can also outline whether the individual estimates are statistically significant. Specifically, in Figure 1b we use solid lines for the SDI if a particular coefficient is significant at the 0.05 level, and dashed lines otherwise. Thus, the SDI *level* indicates whether the estimates are statistically different from each other, whereas the SDI *pattern* indicates whether they are different from zero. In this case both the CI and the SDI associated with the *Other Minorities* coefficient cross zero. However, since SDIs are narrower than the standard CI, there are cases where only the CI (but not the SDI) contains zero.

Example 12: Compute SDIs to compare `margins` results

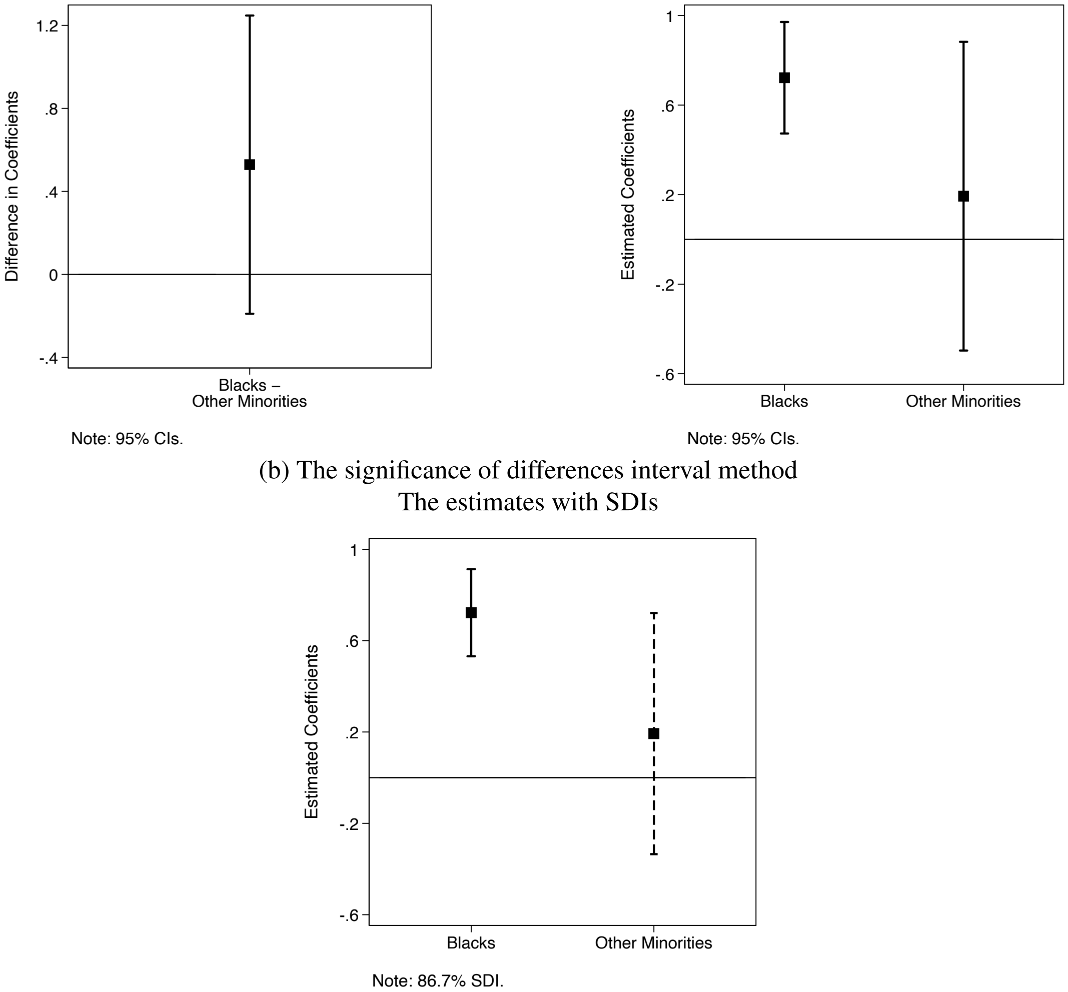
In our last example we show how to use `sdii` to compare predicted probabilities computed via `margins`. Let us say we want to examine whether older people are more likely to have diabetes than younger people. Specifically, we are going to compare the probability of being diabetic when *age* is fixed at its 25th and

Figure 1: Comparing estimated coefficients

(a) The difference in estimates approach

(a1) The difference with the standard CI

(a2) The estimates with the standard CI



(b) The significance of differences interval method
The estimates with SDIs

Note: Figure 1a illustrates the difference in estimates approach, whereas Figure 1b the SDI method.

75th percentile, respectively. All other covariates are held at their observed values. To obtain the respective percentiles of *age*, we use `summarize, detail`; to keep things concise, we use `quietly` to suppress the output. As we will use `nlcom` to calculate the difference in probabilities, we also need to post the `margins` results. In this analysis it happens that the 95% CI of the individual probabilities do not overlap and, as a result, we can conclude that younger people are significantly less likely to have diabetes. However, the proper test (one that also covers cases where there is overlap) is to check whether the CI of the difference in probabilities contains zero. Since the two probabilities are distinct, the CI of the `nlcom` estimate does not.

```
. quietly sum age, detail

. margins, at(age=(`r(p25)´ `r(p75)´)) post
```

```
Predictive margins                              Number of obs     =     10,335
Model VCE    : OIM

Expression   : Pr(diabetes), predict()

1._at        : age            =          31

2._at        : age            =          63

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   .0124204    .001453     8.55   0.000     .0095726    .0152683
          2  |   .0775952   .0034146    22.72   0.000     .0709027    .0842877
------------------------------------------------------------------------------

. nlcom _b[1._at]-_b[2._at]

       _nl_1:  _b[1._at]-_b[2._at]

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _nl_1 |  -.0651748   .0034555   -18.86   0.000    -.0719474   -.0584022
------------------------------------------------------------------------------
```

Can we consolidate all this information (i.e., the individual probabilities (the `margins` results) and the significance of difference test (the `nlcom` finding)), into a single set of results? This can be achieved by computing SDIs around the probability of having diabetes for young and old respondents, as shown below. As before, we use the information that `margins` saves in **r(b)** and **r(V)**. Since the associated SDIs do not overlap, we can conclude that the two probabilities are distinct. The `sdii` results pass again the accuracy test, that is, the information on the difference in probabilities matches perfectly the `nlcom` results. Since we posted the results from the previous `margins` command, we first have to rerun the logit model and reissue the `margins` command.

```
. quietly logit diabetes age i.race

. quietly sum age, detail

. quietly margins, at(age=(`r(p25)´ `r(p75)´))

. tempname m1 s1 m2 s2 cor12

. scalar `m1´ = el(r(b),1,1)

. scalar `s1´ = sqrt(el(r(V),1,1))

. scalar `m2´ = el(r(b),1,2)

. scalar `s2´ = sqrt(el(r(V),2,2))

. scalar `cor12´ = el(r(V),2,1)/(sqrt(el(r(V),1,1))*sqrt(el(r(V),2,2)))

. sdii, sample1(`=`m1´´ `=`s1´´) sample2(`=`m2´´ `=`s2´´) corr(`=`cor12´´) diff

SDI Results

    Comparison      :          Two distributions with correlation ρ = 0.18
    (1)             :          sample1
```

```
   (2)             :          sample2
   Statistic       :          mean; (1-2) = mean(1) - mean(2)
   Interval type   :          Standard error-based SDIs & CI

──────────┬───────────────────────────────────────────────────────────────────
          │     Statistic      Std. Err.      [Interval         Bounds]       (%) Type
──────────┼───────────────────────────────────────────────────────────────────
(1)       │     .01242043        .001453      .01039877      .01444209       83.6 SDI
(2)       │     .07759522       .0034146      .07284426      .08234617       83.6 SDI
(1-2)     │    -.06517479      .00345548      -.0719474     -.05840218         95  CI
──────────┴───────────────────────────────────────────────────────────────────

Note: SDIs indicate significance of difference from 0.
```

Stored results

`sdii` stores the following in `r()`:

Scalars

`r(level)`	confidence level
`r(mvalue)`	meaningful value *m*
`r(sd_1)`	standard deviation for distribution or sample 1
`r(sd_2)`	standard deviation for distribution or sample 2
`r(sd_d)`	with `difference`; (combined) standard deviation for the difference in estimates

For samples

`r(alpha)`	with `power`; significance level
`r(delta)`	with `power`; minimum effect size
`r(N_1)`	sample size for sample 1
`r(N_2)`	sample size for sample 2
`r(N_d)`	with `difference`; (combined) sample size for the difference in estimates
`r(power)`	with `power`; power

For paired data

`r(rho)`	correlation level

Macros

`r(cmd)`	`sdii`
`r(cmdline)`	command as typed
`r(type)`	type of the uncertainty interval

Matrices

`r(sdii)` contains the output table results plus some additional information:

`b`	estimated statistic
`se`	standard errors
`t/z`	test statistics for `b`
`pvalue`	observed significance level for `t/z`
`df`	degrees of freedom associated with `b`
`crit`	critical value associated with `t/z`
`ll`	lower limit of confidence interval
`ul`	upper limit of confidence interval
`level`	confidence level of the specific uncertainty interval

Methods and formulas

Formulas for significance of differences critical scores

To compute SDIs to indicate whether two distributions, let us say Q_1 and Q_2, are statistically or substantively distinct we need to use a significance of differences $z_{d|m}$-score that satisfies the following equality

$$z_{d|m} = \frac{z\sqrt{SE_1^2 + SE_2^2 - 2\rho SE_1 SE_2} + m}{SE_1 + SE_2}$$

where SE_1 and SE_2 are Q_1's and Q_2's standard errors, z is the standard score, ρ is the correlation level, and m is the value of a meaningful effect. When checking for statistical significance only, $m = 0$.

The analogous $t_{d|m}$-score for paired samples is

$$t_{d|m} = \frac{T\left(df, \tfrac{1}{2}\alpha\right)\sqrt{SE_1^2 + SE_2^2 - 2\rho SE_1 SE_2} + m}{SE_1 + SE_2}$$

where $T\left(df, \tfrac{1}{2}\alpha\right)$ is the function for the survivor Student's t distribution, df is the degrees of freedom, and α is the significance level (Gosset [Student, pseud.], 1908).

The formula for the $t_{d*|m}$-scores for unpaired samples with unequal number of observations is

$$t_{d1|m}SE_1 + t_{d2|m}SE_2 = t_{\mathit{diff}}SE_{\mathit{diff}} + m$$

$$T\left(df_1, \frac{1}{2}\alpha_d\right)SE_1 + T\left(df_2, \frac{1}{2}\alpha_d\right)SE_2 = T\left(df_{\mathit{diff}}, \frac{1}{2}\alpha\right)SE_{\mathit{diff}} + m$$

This equation does not have an analytical solution.

Formulas for paired and unpaired sample data

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown but $\sigma_x = \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left\{\frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2}\right\}^{1/2}\left(\frac{1}{n_x} + \frac{1}{n_y}\right)^{1/2}}$$

The result is distributed as Student's t with $n_x + n_y - 2$ degrees of freedom.

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown and $\sigma_x \neq \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left(s_x^2/n_x + s_y^2/n_y\right)^{1/2}}$$

The result is distributed as Student's t with v degrees of freedom, where v is given by (with Satterthwaite's (1946) formula)

$$\frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\frac{\left(s_x^2/n_x\right)^2}{n_x-1} + \frac{\left(s_y^2/n_y\right)^2}{n_y-1}}$$

With Welch's formula (1947), the number of degrees of freedom is given by

$$-2 + \frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\frac{\left(s_x^2/n_x\right)^2}{n_x+1} + \frac{\left(s_y^2/n_y\right)^2}{n_y+1}}$$

Formulas for percentiles

The default formula for percentiles is as follows. Let $x_{(j)}$ refer to the x in ascending order for $j = 1, 2, \ldots, n$. Let $w_{(j)}$ refer to the corresponding weights of $x_{(j)}$; if there are no weights, $w_{(j)} = 1$. Let $N = \sum_{j=1}^{n} w_{(j)}$.

To obtain the pth percentile, denoted as $x_{[p]}$, let $P = Np/100$, and let

$$W_{(i)} = \sum_{j=1}^{i} w_{(j)}$$

Find the first index, i, such that $W_{(i)} > P$. The pth percentile is then

$$x_{[p]} = \begin{cases} \frac{x_{(i-1)} + x_{(i)}}{2} & \text{if} \quad W_{(i-1)} = P \\ x_{(i)} & \text{otherwise} \end{cases}$$

When the *altdef* suboption is specified, the following alternative definition is used. Here weights are not allowed. Let i be the integer floor of $(n+1)p/100$; that is, i is the largest integer $i \leq (n+1)p/100$. Let h be the remainder $h = (n+1)p/100 - i$. The pth percentile is then

$$x_{[p]} = (1-h)x_{(i)} + hx_{(i+1)}$$

where $x_{(0)}$ is taken to be $x_{(1)}$ and $x_{(n+1)}$ is taken to be $x_{(n)}$.

References

Gosset [Student, pseud.], W. S. 1908. "The probable error of a mean." *Biometrika* 6: 1–25. `https://doi.org/10.2307/2331554`.

Satterthwaite, F. E. 1946. "An approximate distribution of estimates of variance components." *Biometrics Bulletin* 2: 110–114. `https://doi.org/10.2307/3002019`.

StataCorp. 2017. "Stata 15 Base Reference Manual." Stata Press: College Station, TX.

Welch, B. L. 1947. "The generalization of 'student's' problem when several different population variances are involved." *Biometrika* 34: 28–35. `https://doi.org/10.2307/2332510`.

Tomz, M., Wittenberg, J., & King, G. 2003. "Clarify: Software for Interpreting and Presenting Statistical Results." *Journal of Statistical Software* 8(1): 1–30. `https://doi.org/10.18637/jss.v008.i01`.