Setting up the environment

```
'''!pip uninstall -y pmdarima numpy
!pip install numpy==1.24.3
!pip install pmdarima --no-cache-dir'''
```

```
'!pip uninstall -y pmdarima numpy\n!pip install numpy==1.24.3\n!pip install pmdarima --no-cache-dir'
```

```
# Imports
import warnings
warnings.filterwarnings("ignore")
from statsmodels.tools.sm_exceptions import ConvergenceWarning, ValueWarning
warnings.simplefilter('ignore', ConvergenceWarning)
warnings.simplefilter('ignore', ValueWarning)
warnings.simplefilter('ignore', UserWarning)
import time
from itertools import product
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import statsmodels as ss
import seaborn as sns
from tqdm.notebook import tqdm
from scipy import stats
from statsmodels.tsa.arima.model import ARIMA
from statsmodels.graphics.tsaplots import month_plot, plot_acf, plot_pacf
from statsmodels.graphics.gofplots import qqplot
from statsmodels.tsa.stattools import adfuller, acf, pacf
from statsmodels.tsa.seasonal import STL, seasonal_decompose
from statsmodels.tsa.statespace.sarimax import SARIMAX # Corrected import
from statsmodels.tools.eval_measures import mse
from statsmodels.tsa.statespace.tools import diff
from sklearn.metrics import mean_squared_error
import pmdarima as pm
from scipy.fft import fft, fftfreq
np.random.seed(0)
```

```
def fft_analysis(signal):
  slope, intercept = np.polyfit(np.arange(len(signal)), signal, 1)
  trend = np.arange(len(signal))*slope + intercept
  detrended = signal - trend

  fft_values = fft(detrended)
  frequencies = np.fft.fftfreq(len(fft_values))

  positive_frequencies = frequencies[frequencies>0]
  magnitudes = np.abs(fft_values)[frequencies>0]

  dominant_frequency = positive_frequencies[np.argmax(magnitudes)]
  print(f"Dominant Frequency: {dominant_frequency:.3f}")

  dominant_period = 1/dominant_frequency
  print(f"Dominant Period: {dominant_period:.2f} time units")

  return dominant_period, positive_frequencies, magnitudes
```

```
data = pd.read_csv('time_data.csv')
data
```

	Id	Values
0	0	2.834420
1	1	4.279164
2	2	1.598206
3	3	0.069473
4	4	0.232714
...	...	...
796	796	286.592993
797	797	286.195649
798	798	285.752718
799	799	287.831402
800	800	287.942373

801 rows × 2 columns

```
data.set_index('Id', inplace=True)
data
```

	Values
Id	
0	2.834420
1	4.279164
2	1.598206
3	0.069473
4	0.232714
...	...
796	286.592993
797	286.195649
798	285.752718
799	287.831402
800	287.942373

801 rows × 1 columns

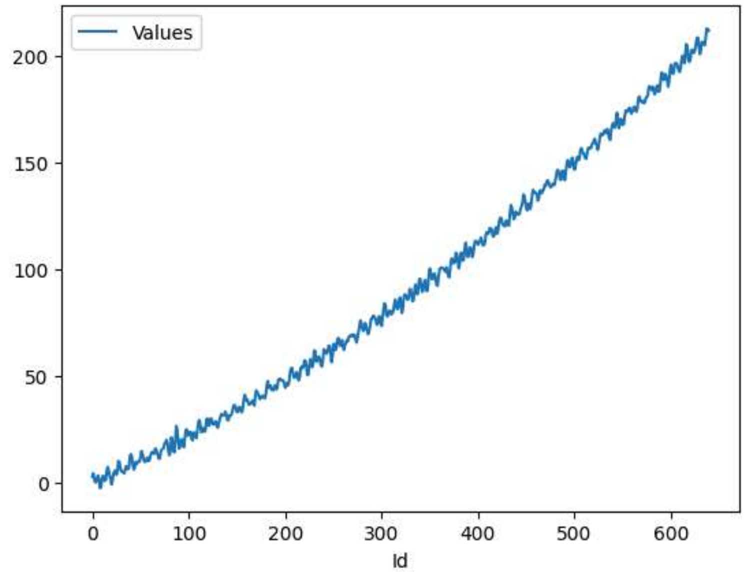
```
train_data = data.iloc[:int(len(data)*0.8)]
test_data = data.iloc[int(len(data)*0.8):]
```

```
out = fft_analysis(train_data.iloc[0:200]['Values'])
```

```
Dominant Frequency: 0.085
Dominant Period: 11.76 time units
```

```
train_data.plot()
```

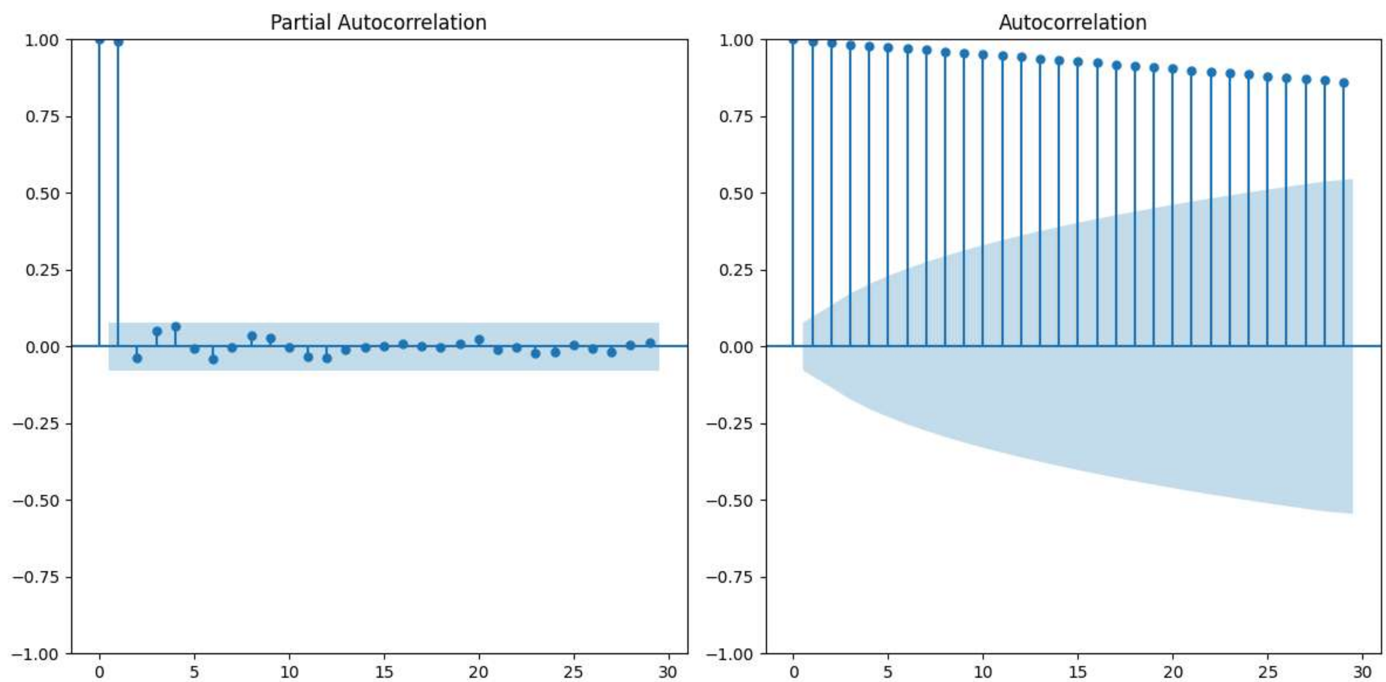
```
<Axes: xlabel='Id'>
```




```
out = adfuller(train_data['Values'])
print('ADF Statistic: %f' % out[0])
print('p-value: %f' % out[1])
for key, value in out[4].items():
  print('\t%s: %.3f' % (key, value))
```

```
ADF Statistic: 11.796410
p-value: 1.000000
        1%: -3.441
        5%: -2.866
        10%: -2.569
```

```
fig, ax = plt.subplots(1, 2, figsize=(12, 6))
plot_pacf(train_data['Values'], ax=ax[0])
plot_acf(train_data['Values'], ax=ax[1])
plt.tight_layout()
plt.show()
```

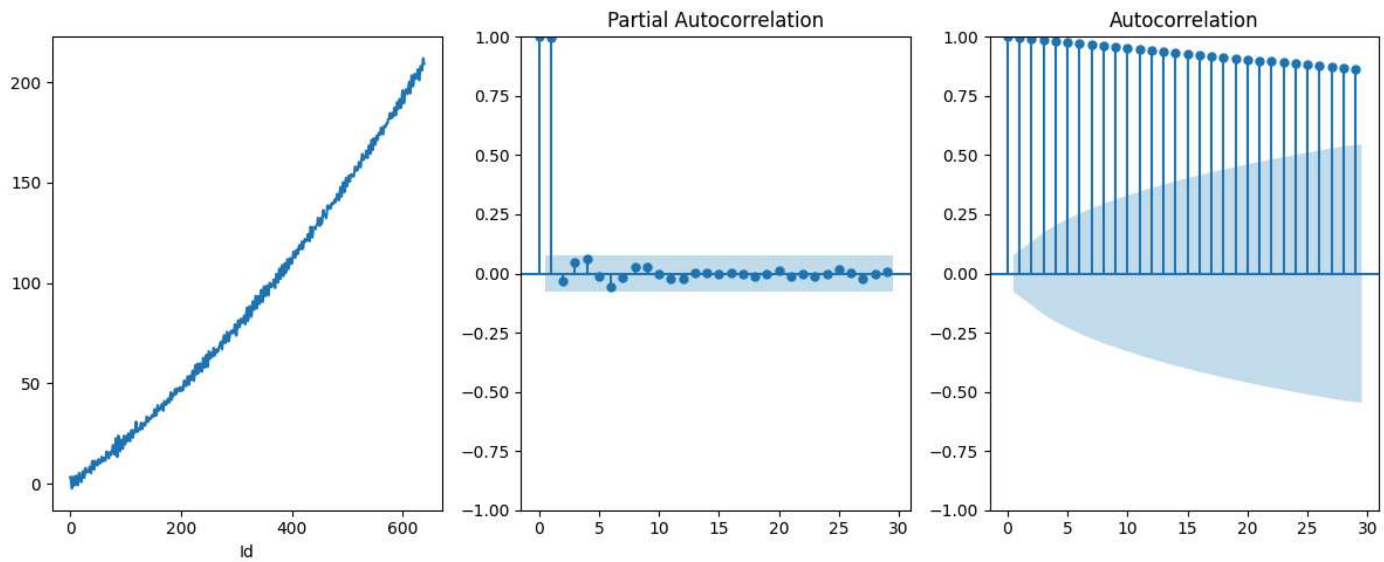


```
decomposition = seasonal_decompose(train_data['Values'], model='additive', period=12)
seasonal = decomposition.seasonal
tr = train_data['Values'] - seasonal
tr.head()
```

Id	0
0	3.222873
1	3.527865
2	-0.270101
3	-2.399960
4	-1.622402

dtype: float64

```
fig, ax = plt.subplots(1, 3, figsize=(12, 5))
tr.plot(ax=ax[0])
plot_pacf(tr, ax=ax[1])
plot_acf(tr, ax=ax[2])
plt.tight_layout()
plt.show()
```




```
process = train_data.copy()
```

```
out = adfuller(process['Values'])
print('ADF Statistic: %f' % out[0])
print('p-value: %f' % out[1])
for key, value in out[4].items():
  print('\t%s: %.3f' % (key, value))
```

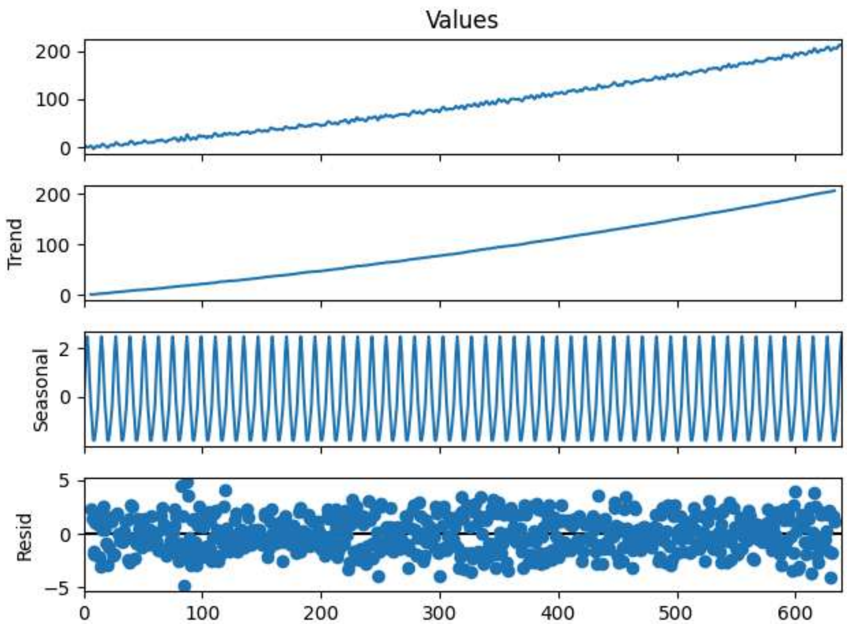
```
ADF Statistic: 11.796410
p-value: 1.000000
        1%: -3.441
        5%: -2.866
        10%: -2.569
```

```
decomposition = seasonal_decompose(process['Values'].dropna(), model='additive', period=12)
seasonal = decomposition.seasonal
tr = process['Values'] - seasonal
tr.dropna(inplace=True)
tr.head()
```

Id	0
0	3.222873
1	3.527865
2	-0.270101
3	-2.399960
4	-1.622402

dtype: float64

```
decomposition.plot();
```




```
out = adfuller(tr)
print('ADF Statistic: %f' % out[0])
print('p-value: %f' % out[1])
for key, value in out[4].items():
  print('\t%s: %.3f' % (key, value))
```

```
ADF Statistic: 8.929851
p-value: 1.000000
        1%: -3.441
        5%: -2.866
        10%: -2.569
```

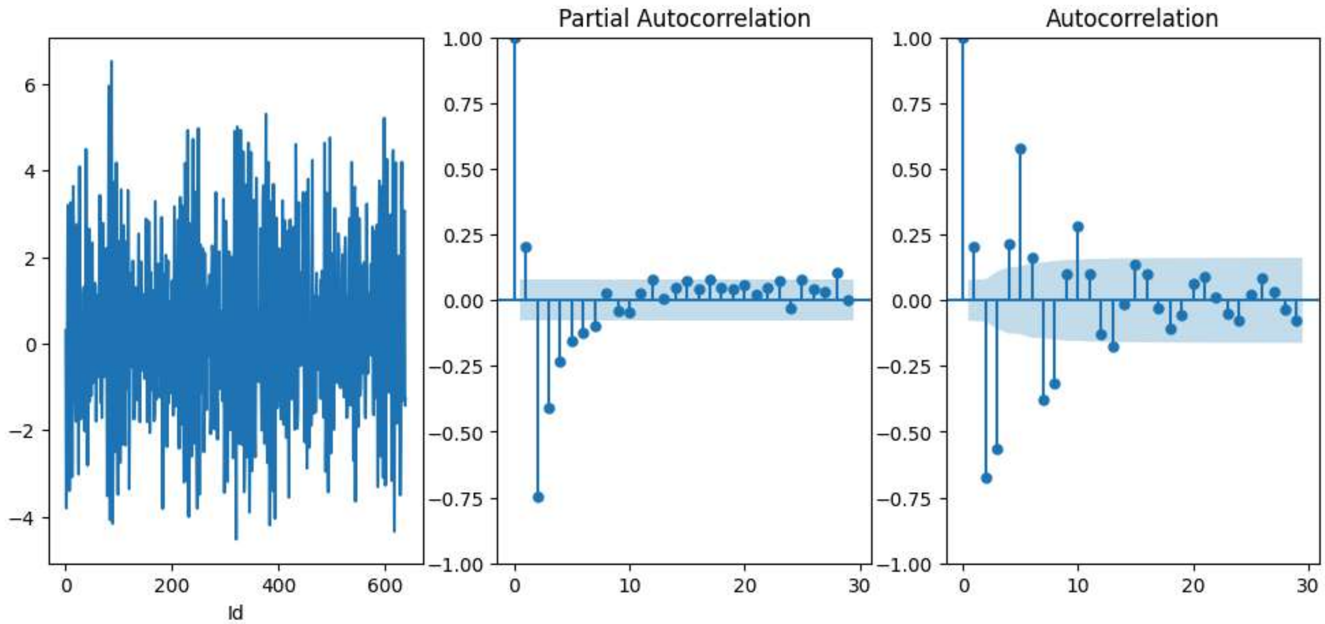
```
tr = tr.diff().dropna()
```

```
out = adfuller(tr.dropna())
print('ADF Statistic: %f' % out[0])
print('p-value: %f' % out[1])
for key, value in out[4].items():
  print('\t%s: %.3f' % (key, value))
```

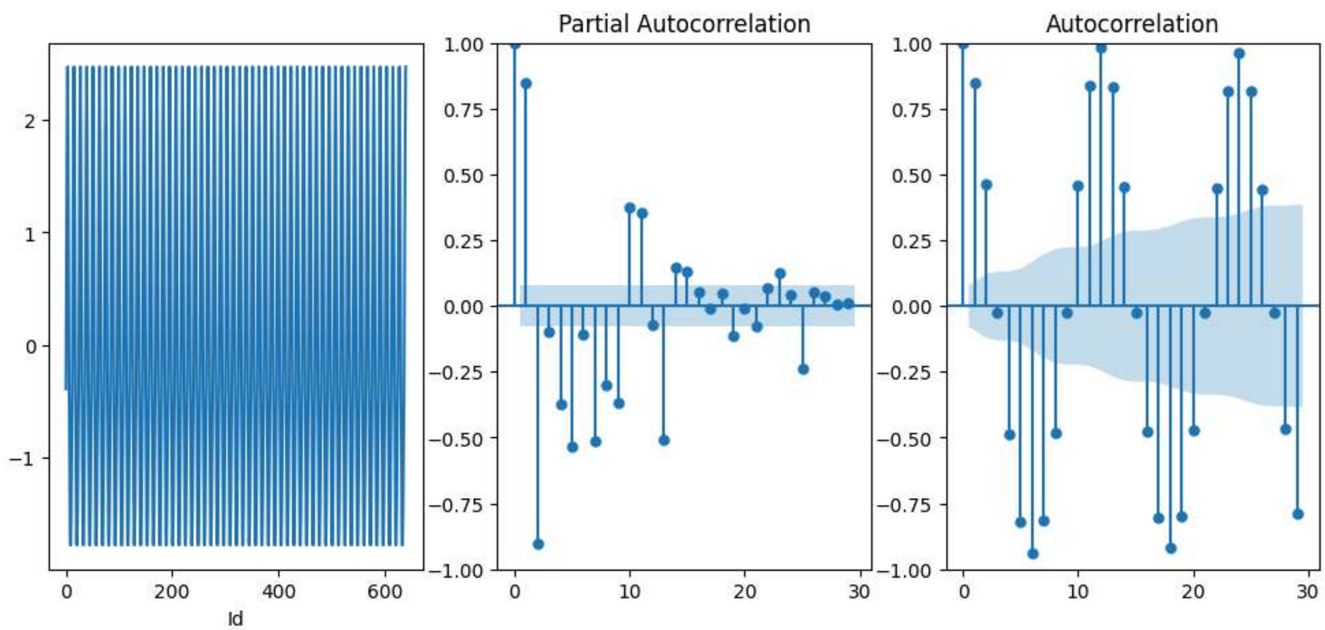
```
ADF Statistic: -4.140160
p-value: 0.000831
        1%: -3.441
        5%: -2.866
        10%: -2.569
```

```
fig, ax = plt.subplots(1, 3, figsize=(12, 5))
tr.plot(ax=ax[0])
plot_pacf(tr, ax=ax[1])
```

```
plot_acf(tr, ax=ax[2])
plt.show();
```




```
fig, ax = plt.subplots(1, 3, figsize=(12, 5))
seasonal.plot(ax=ax[0])
plot_pacf(seasonal, ax=ax[1])
plot_acf(seasonal, ax=ax[2])
plt.show();
```




```
out = adfuller(seasonal.dropna())
print('ADF Statistic: %f' % out[0])
print('p-value: %f' % out[1])
for key, value in out[4].items():
  print('\t%s: %.3f' % (key, value))
```

```
ADF Statistic: -312950858164238.750000
p-value: 0.000000
        1%: -3.441
        5%: -2.866
        10%: -2.569
```

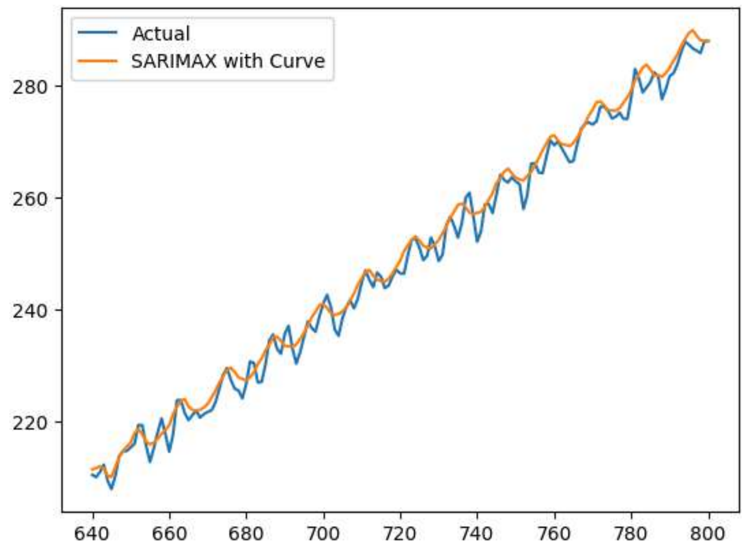
```
'''model = pm.auto_arima(train_data,
                       start_p=2, start_q=1,
                       test='adf',
                       max_p=4, max_q=1, m=12,
                       start_P=0, start_Q=0,
                       max_P=1, max_Q=1,
                       seasonal=True,
                       d=1,
                       D=1,
                       trace=True,
                       error_action='ignore',
                       trend='ct',
                       suppress_warnings=True,
                       stepwise=True)'''
```

```
'model = pm.auto_arima(train_data,\n                       start_p=2, start_q=1,\n                       test='adf',\n
max_p=4, max_q=1, m=12,\n                       start_P=0, start_Q=0,\n                       max_P=1, max_Q=1,\n
seasonal=True,\n                       d=1,\n                       D=1,\n                       trace=True,\n
error_action='ignore',\n                       trend='ct',\n                       suppress_warnings=True,\n                       s
```

```
from statsmodels.tsa.statespace.sarimax import SARIMAX
from sklearn.metrics import mean_squared_error

n = len(train_data)
x = np.arange(n)
exog_train = np.column_stack([x, x**2])



model = SARIMAX(train_data,
                order=(2,1,1),
                seasonal_order=(0,1,1,12),
                exog=exog_train)

model_fit = model.fit(disp=False)


h = len(test_data)
future_x = np.arange(n, n + h)
exog_test = np.column_stack([future_x, future_x**2])


forecast = model_fit.predict(start=n, end=n+h-1, exog=exog_test)
noise = np.random.normal(0, 0.1, len(forecast))
forecast_noisy = forecast+noise


mse = mean_squared_error(test_data, forecast_noisy)
print(f"Forecast MSE: {mse:.3f}")
```

```
Forecast MSE: 4.763
```

```
plt.plot(test_data.index, test_data, label='Actual')
plt.plot(test_data.index, forecast_noisy, label='SARIMAX with Curve')
plt.legend()
plt.show()
```




```
model_fit.summary()
```

SARIMAX Results

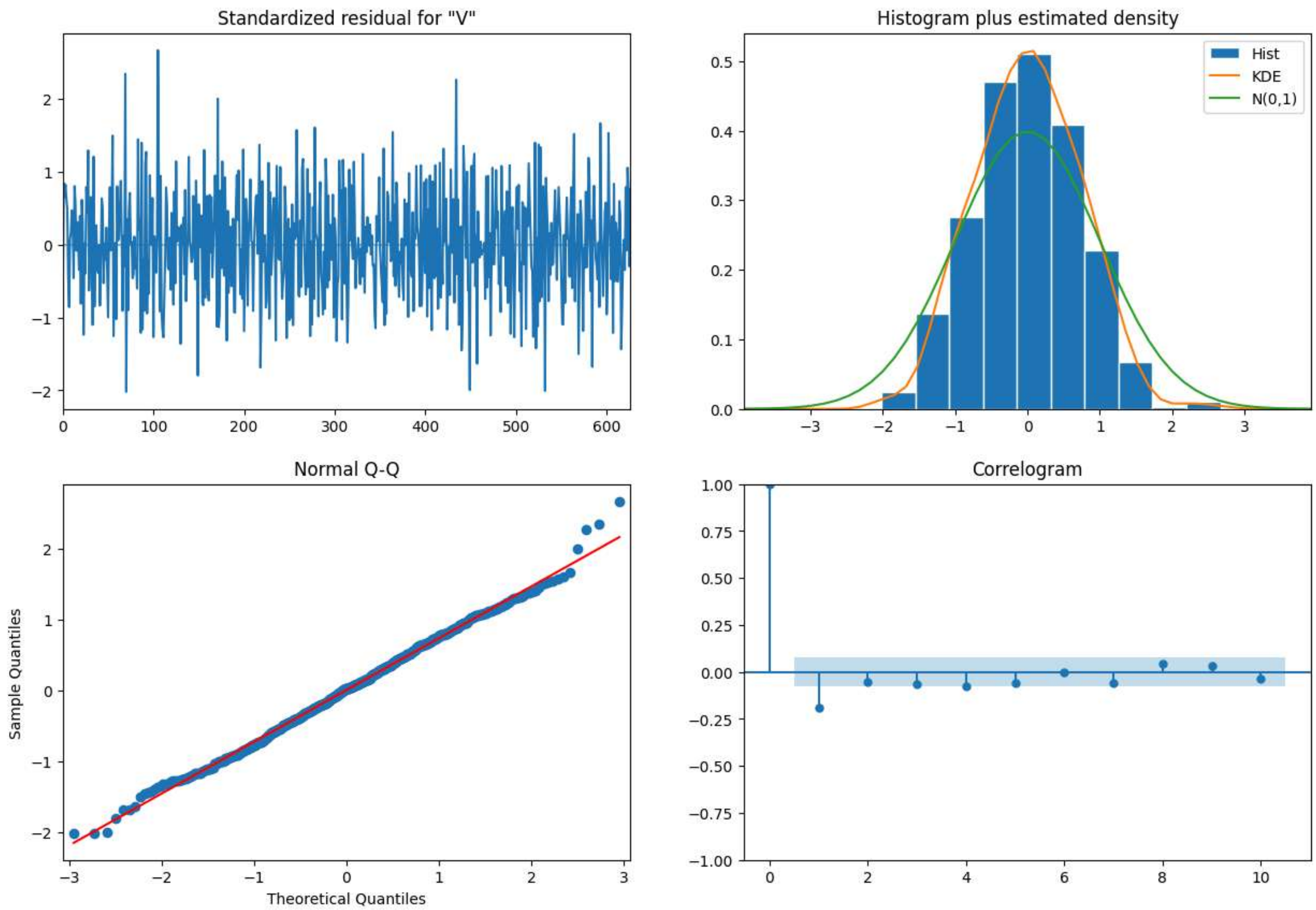
Dep. Variable:	Values	**No. Observations:**	640
Model:	SARIMAX(2, 1, 1)x(0, 1, 1, 12)	**Log Likelihood**	-1044.362
Date:	Sun, 20 Jul 2025	**AIC**	2102.724
Time:	13:14:21	**BIC**	2133.811
Sample:	0 - 640	**HQIC**	2114.802
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
x1	5.357e-14	3.13e-09	1.71e-05	1.000	-6.14e-09	6.14e-09
x2	0.0002	0.000	1.647	0.100	-3.92e-05	0.000
ar.L1	0.5941	0.049	12.058	0.000	0.498	0.691
ar.L2	-0.8414	0.044	-18.922	0.000	-0.929	-0.754
ma.L1	-0.5982	0.079	-7.605	0.000	-0.752	-0.444
ma.S.L12	-0.9198	0.048	-19.087	0.000	-1.014	-0.825
sigma2	2.5075	0.236	10.623	0.000	2.045	2.970

Ljung-Box (L1) (Q):	22.96	**Jarque-Bera (JB):**	0.68
Prob(Q):	0.00	**Prob(JB):**	0.71
Heteroskedasticity (H):	1.03	**Skew:**	0.07
Prob(H) (two-sided):	0.83	**Kurtosis:**	2.92

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
[2] Covariance matrix is singular or near-singular, with condition number 2.65e+23. Standard errors may be unstable.

```
model_fit.plot_diagnostics(figsize=(15, 10));
```




```
n_steps = len(test_data)
start_index = len(train_data)
end_index = start_index + n_steps - 1
forecast = model_fit.predict(start=start_index, end=end_index, exog=exog_test)
forecast
```

	predicted_mean
640	211.347815
641	211.684269
642	211.974666
643	211.445199
644	210.203148
...	...
796	289.735604
797	288.958890
798	288.158189
799	287.997108
800	287.972016

161 rows × 1 columns

dtype: float64

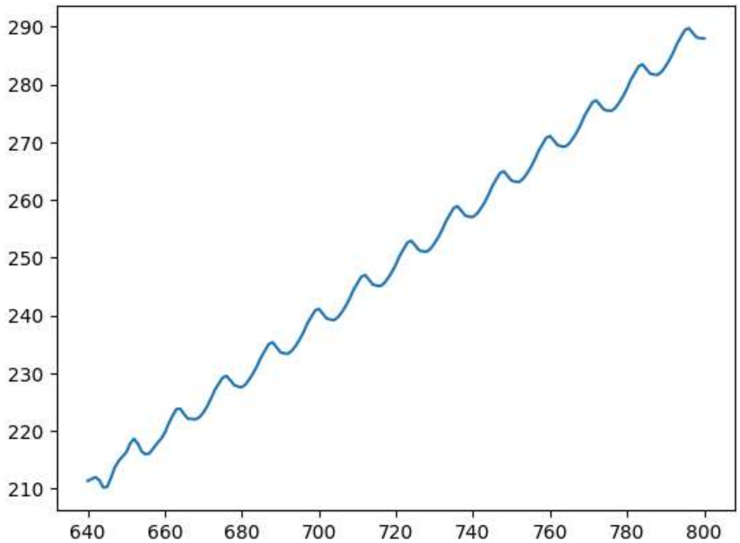
```
from sklearn.metrics import mean_squared_error
```

```
mse = mean_squared_error(forecast, test_data['Values'])
print(f"Mean Squared Error: {mse}")
```

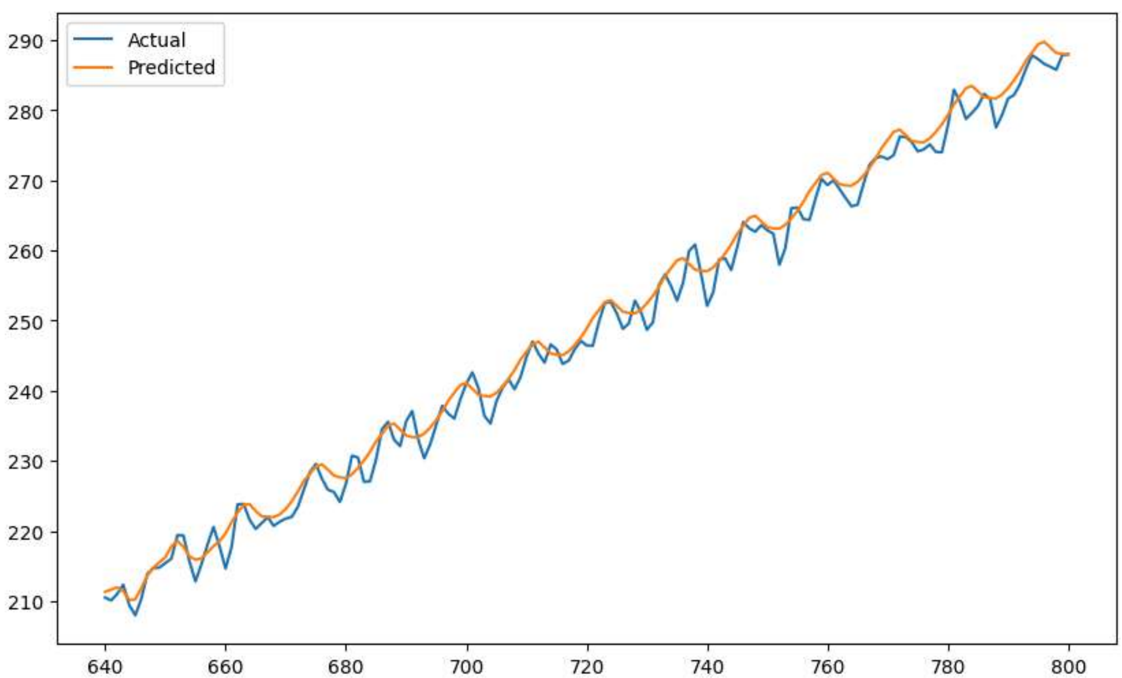
```
Mean Squared Error: 4.732093016900805
```

```
residuals = test_data - forecast
residuals.dropna(inplace=True)
forecast.plot()
```

```
<Axes: >
```



```
fig, ax = plt.subplots(1, 1, figsize=(10,6))
plt.plot(test_data['Values'])
plt.plot(forecast)
plt.legend(['Actual', 'Predicted'])
plt.show()
```




```
n = len(data)
x = np.arange(n)
exog = np.column_stack([x, x**2])
model = SARIMAX(data, order=(2,1,1), seasonal_order=(0,1,1,12), exog=exog
result = model.fit(disp=False)
```

```
h = 199
```

```
future_x = np.arange(n, n + h)
exog_future = np.column_stack([future_x, future_x**2])
forecast = result.forecast(steps=h, exog=exog_future)
noise = np.random.normal(0, 0.25, len(forecast))
forecast = forecast+noise
```

	predicted_mean
801	287.285540
802	286.521671
803	287.695603
804	290.252651
805	293.094809
...	...
995	395.875473