Stabilising Volatility: A Case Study in Loss Development Smoothing

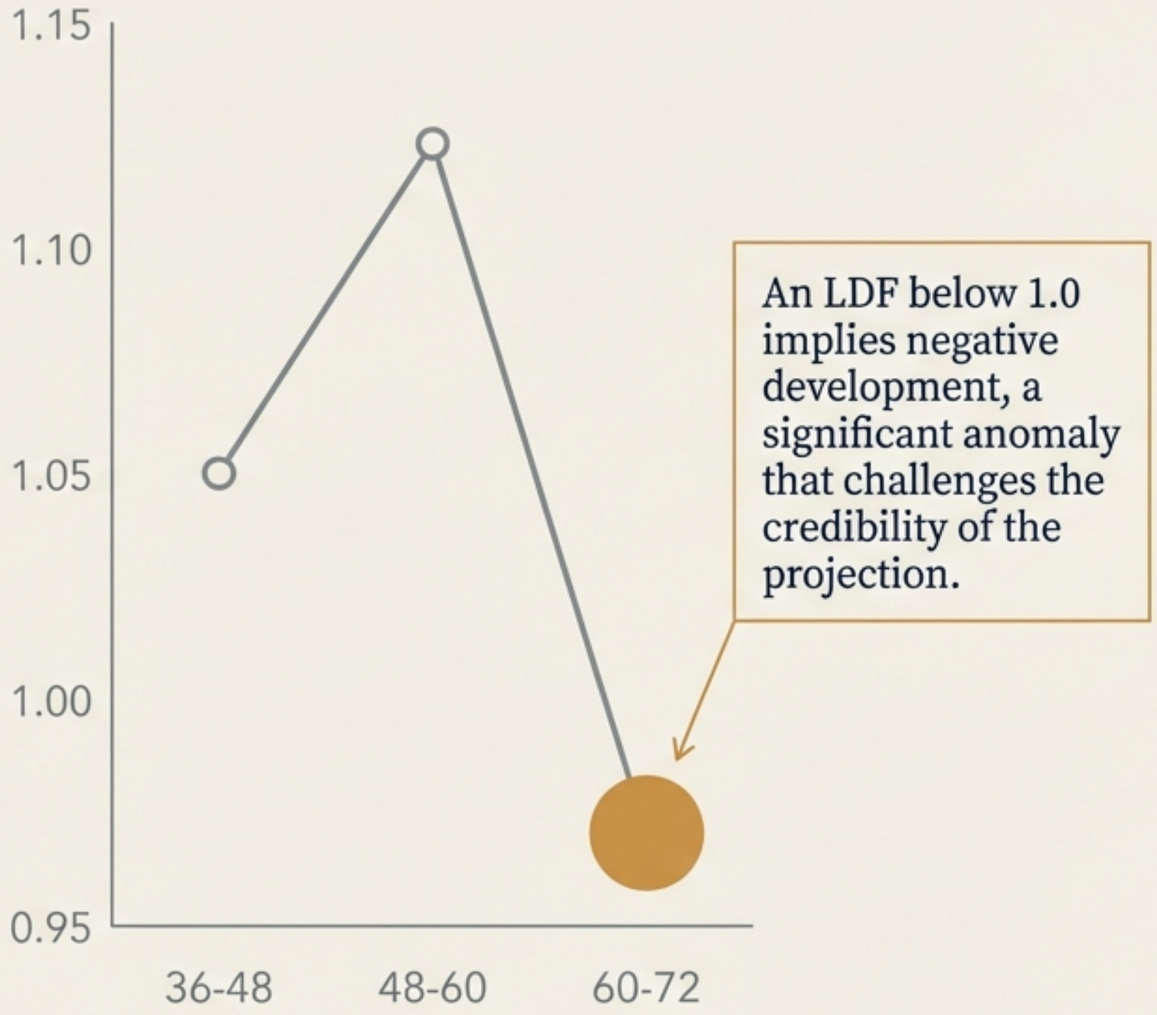
Applying Log-Relativity Benchmarking for a More Credible Curve

The Challenge: An Erratic Development Pattern

Actuarial judgment relies on logical data patterns. However, internal company data can often be volatile, especially in later development periods. This case study examines a common scenario where the selected Loss Development Factors (LDFs) show an inconsistent pattern that requires refinement.

Company Selected LDFs (Age-to-Age)

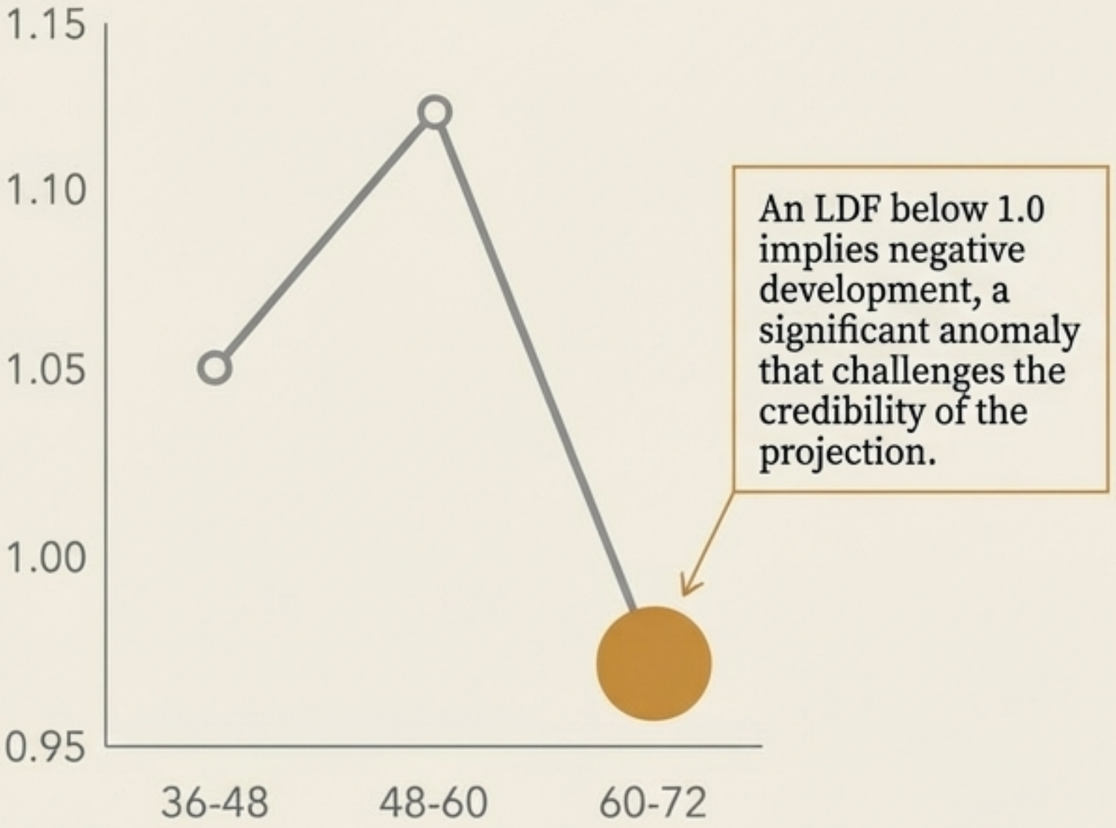
Age-to-Age	Company LDF
36 - 48	1.050
48 - 60	1.122
60 - 72	**0.970**



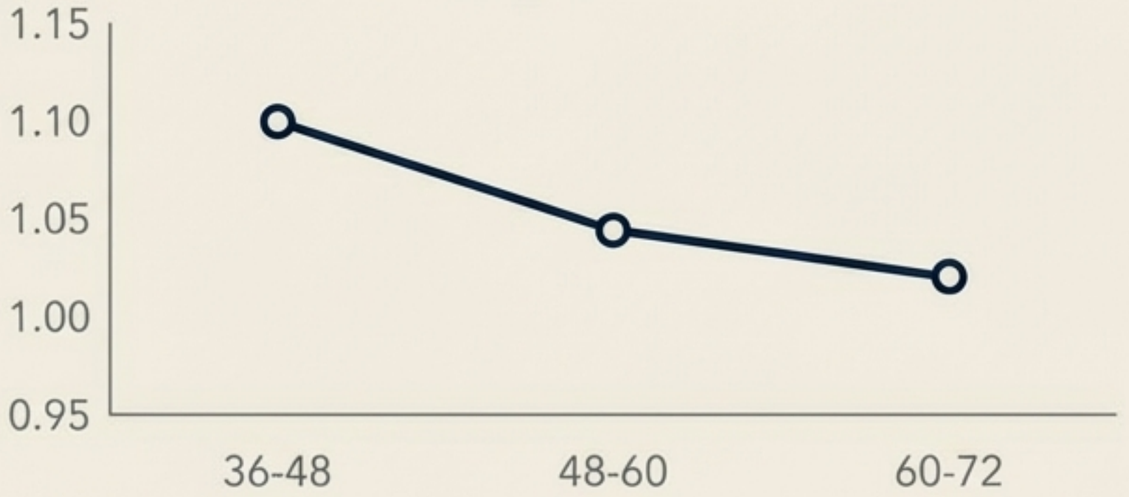
The Reference Point: Establishing a Stable Benchmark

When company-specific data is thin or noisy, the broader industry pattern provides a credible ‘shape’ for development. While the overall quantum of development should remain specific to the company, the benchmark can guide the distribution of that development over time.

Company Data



Industry Benchmark

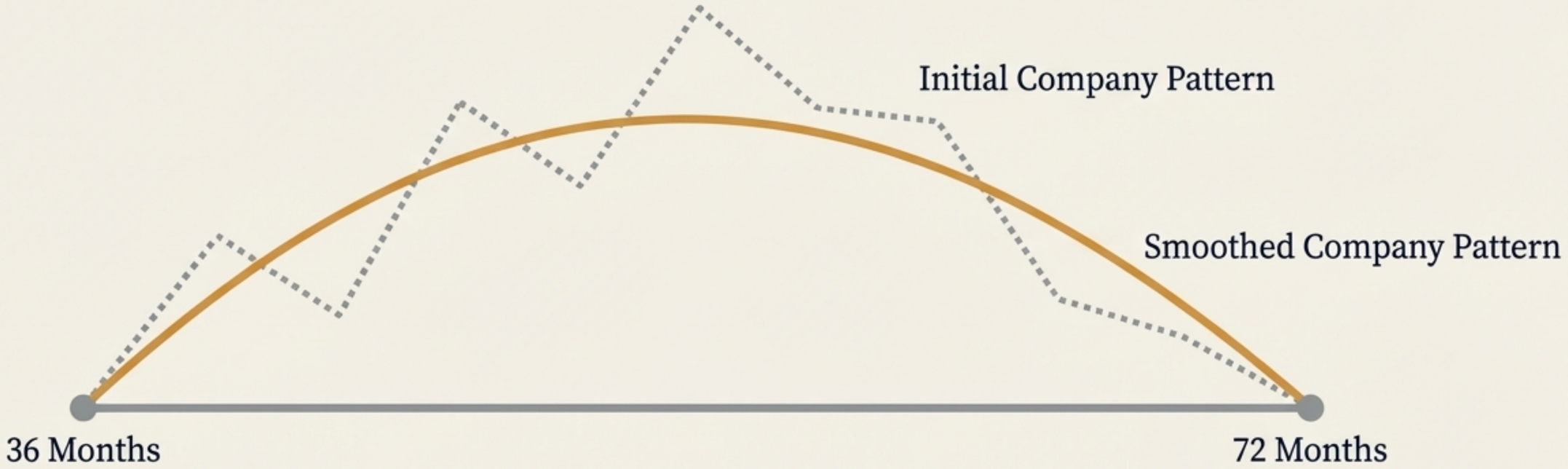


Age-to-Age	Company LDF	Industry Benchmark
36 - 48	1.050	1.100
48 - 60	1.122	1.045
60 - 72	**0.970**	1.020

The Objective: Reshape the Pattern, Preserve the Ultimate

The goal is not to replace company experience with the industry's. Instead, we will use the industry pattern's 'shape' to redistribute the company's own total development for the 36-72 month period.

We will change *when* the development is expected to emerge, not the total *amount* of development.



The total cumulative factor remains constant. We are only smoothing the path between the start and end points.

The Method: Log-Relativity Smoothing

The Log-Relativity approach is a robust technique to quantify the ‘shape’ of the benchmark pattern and apply it to our company data. It works by converting multiplicative LDFs into an additive, proportional scale using natural logarithms. This allows us to isolate the pattern of development from the absolute quantum.

[Shape from Benchmark] × [Quantum from Company] = [Smoothed Company Factors]

1. **Calculate Total Development**
   Determine the cumulative factor for both Company and Industry.

2. **Quantify the Benchmark Shape**
   Calculate the ‘log relativities’ for the industry benchmark.

3. **Apply to Company Data**
   Redistribute the company’s total development using these relativities.

Step 1: Calculate the Cumulative Development Factor

First, we establish the total development over the 36 to 72-month period for both the benchmark and the company by multiplying their respective age-to-age LDFs. This gives us the total "quantum" of development to be redistributed for the company.

Industry Benchmark (36-72)

$$1.100 \times 1.045 \times 1.020 = 1.17249$$

Industry Cumulative Development Factor (CDF)

Company Data (36-72)

$$1.050 \times 1.122 \times 0.970 = \mathbf{1.142757}$$

Company Cumulative Development Factor (CDF)

The Company's total development factor of **1.142757** is the figure we must preserve.

Step 2: Determine the Industry 'Log Relativities'

We now quantify the benchmark's shape by determining what proportion of the total logarithmic growth occurs in each period. This 'relativity' value represents each period's contribution to the total development curve.

$$\text{Relativity} = \frac{\ln(\text{Industry LDF}_{\text{period}})}{\ln(\text{Industry CDF}_{36-72})}$$

$$\frac{\ln(\mathbf{1.100})}{\ln(\mathbf{1.17249})} = \frac{0.09531}{0.15915} \approx \mathbf{0.5989}$$

This shows that the 36-48 month period accounts for approximately 59.9% of the total logarithmic development in the benchmark.

Calculated Log Relativities

Age-to-Age	Industry LDF	Log Relativity
36 - 48	1.100	**0.5989**
48 - 60	1.045	**0.2764**
60 - 72	1.020	**0.1247**
Total		**1.0000**

Step 3: Apply Relativities to Reshape Company Factors

Finally, we apply the industry-derived relativities to the company's own cumulative development factor. This redistributes the company's total development (1.142757) according to the stable pattern of the benchmark.

Formula:

Smoothed LDF = (Company $CDF_{36\text{-}72}$) ^ Relativity

Example Calculation (36-48 months):

1.142757 ^ **0.5989** = **1.0832**

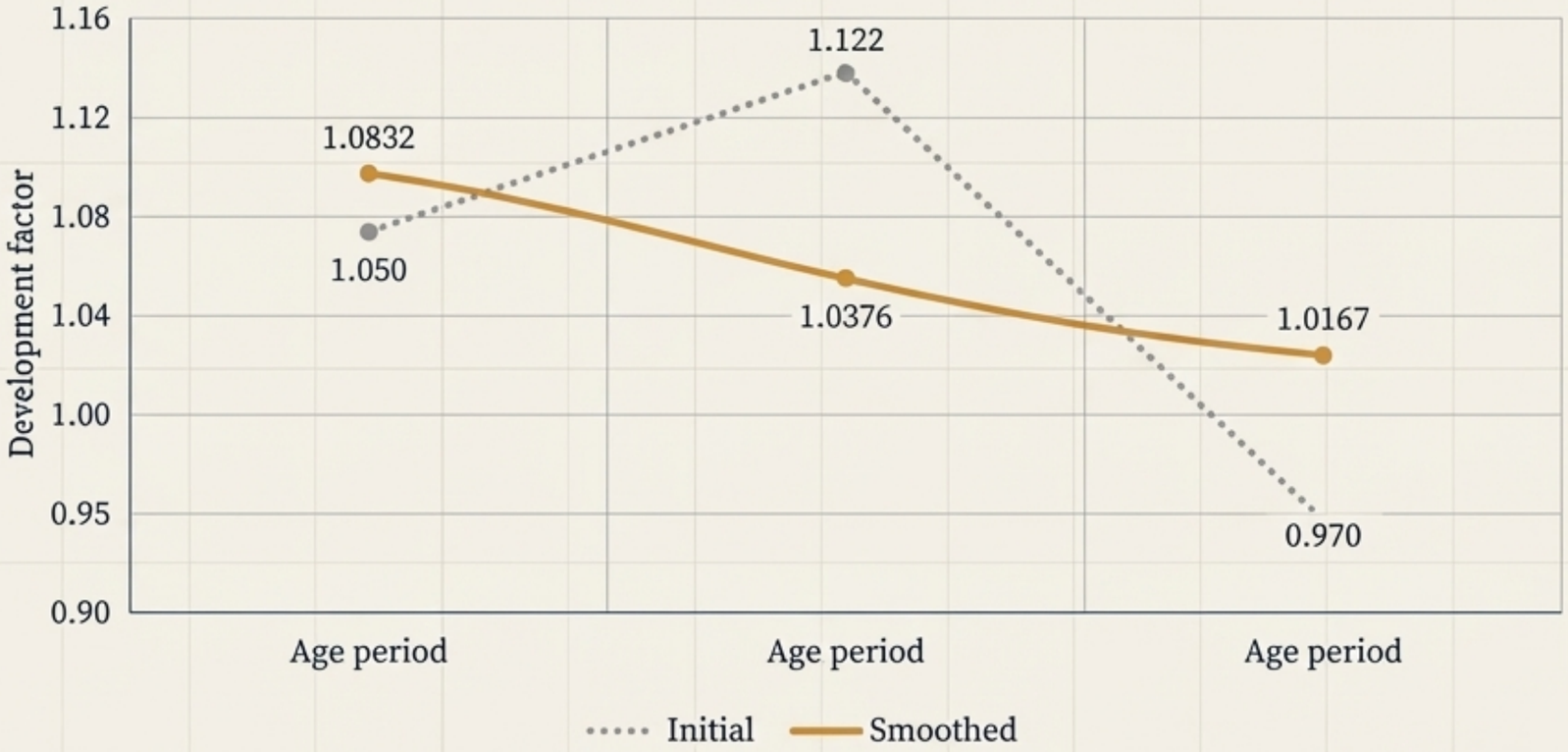
Final Smoothed LDF Calculation

Age-to-Age	Company CDF	Relativity	Calculation	Smoothed LDF
36 - 48	1.142757	**0.5989**	$1.142757^{0.5989}$	**1.0832**
48 - 60	1.142757	**0.2764**	$1.142757^{0.2764}$	**1.0376**
60 - 72	1.142757	**0.1247**	$1.142757^{0.1247}$	**1.0167**

The Result: From Volatile Data to a Credible Curve

Initial Company Selected
1.050
1.122
0.970

Smoothed Company
1.0832
1.0376
1.0167



The log-relativity method has successfully smoothed the erratic pattern, producing a set of LDFs that are mathematically consistent and professionally defensible.

Validation: The Company's Ultimate Factor is Preserved

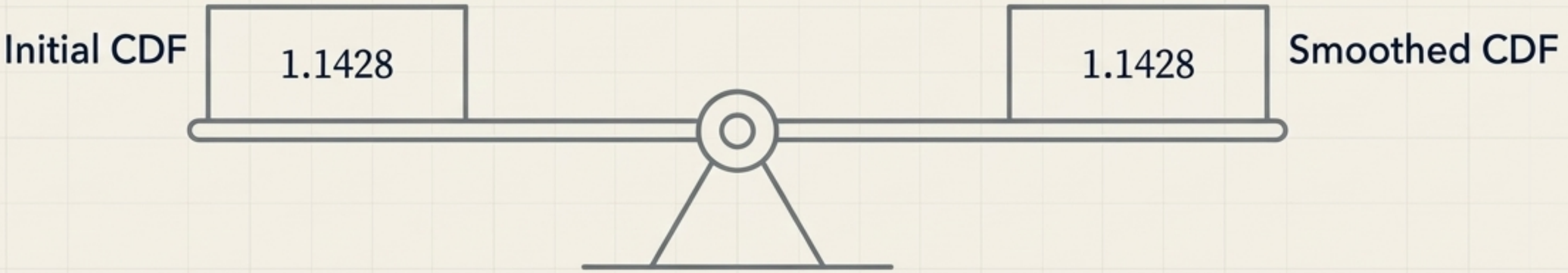
The most critical check is to ensure that our smoothing process has only redistributed the development pattern, not altered the total cumulative amount. The product of the new smoothed factors must equal the product of the original factors.

Initial Factors

1.050 × 1.122 × 0.970 = **1.142757**

Smoothed Factors

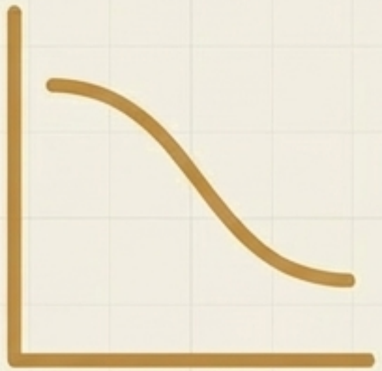
1.0832 × 1.0376 × 1.0167 = **1.142757**



The Actuary's Toolkit: The Benefits of a Smoothed Approach

Credibility

Lends stability to projections when company data is thin or noisy, which is common in long-tail or commercial lines. It provides a robust and defensible basis for selections.



Consistency

Ensures LDFs follow a rational, monotonically decreasing pattern. It eliminates illogical "dips" that imply negative development and complicate stakeholder communication.

Validation

Provides a data-driven method that preserves the company's unique total development quantum. It blends internal experience with the stability of external benchmarks, creating a well-justified final selection.

Blending Art and Science for a Defensible Reserve

Effective reserving is not about choosing between company data and industry benchmarks. It is **about the intelligent synthesis of both.**

The log-relativity smoothing technique is a prime example of this synthesis. It respects the unique experience captured in a company's data while shaping it with the stability and logic of the wider market, leading to projections that are not only mathematically sound but also professionally credible.