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Compute $\mathbf{u} \cdot \mathbf{u}$, $\mathbf{v} \cdot \mathbf{u}$, and $\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}$ using the vectors $\mathbf{u} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

If $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix}$ are vectors in \mathbb{R}^n , then the inner product of \mathbf{u} and \mathbf{v} is as shown below.

$$\begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix} = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$

By this definition, $\mathbf{u} \cdot \mathbf{u}$ is given by $\begin{bmatrix} -7 & 2 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

Compute $\mathbf{u} \cdot \mathbf{u}$.

$$\begin{bmatrix} -7 & 2 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \end{bmatrix} = (-7)(-7) + (2)(2) = 53$$

Thus, $\mathbf{u} \cdot \mathbf{u} = 53$.

By the definition stated previously, $\mathbf{v} \cdot \mathbf{u}$ is given by $\begin{bmatrix} 4 & 5 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

Compute $\mathbf{v} \cdot \mathbf{u}$.

$$\begin{bmatrix} 4 & 5 \end{bmatrix} \begin{bmatrix} -7 \\ 2 \end{bmatrix} = (4)(-7) + (5)(2) = -18$$

Thus, $\mathbf{v} \cdot \mathbf{u} = -18$.

As found previously, $\mathbf{u} \cdot \mathbf{u} = 53$. Use these values to compute $\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}$.

$$\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} = \frac{-18}{53}$$

Thus, $\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} = -\frac{18}{53}$.