A Simple Project

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System Model

- Two wireless nodes using Slotted ALLOHA.
- Packet Arrival probability: $p_i, i = 1, 2$.
- Packet Transmission probability: $q_i, i = 1, 2$.
- Node 2 is saturated $\rightarrow p_2 = 1$.
- Our goal is to design a policy for Node 1 that minimizes the average delay.

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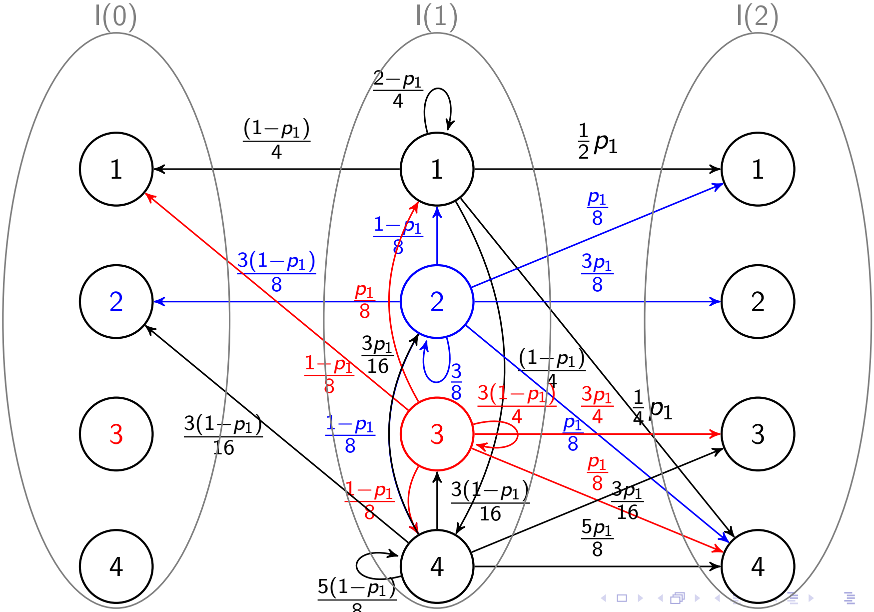
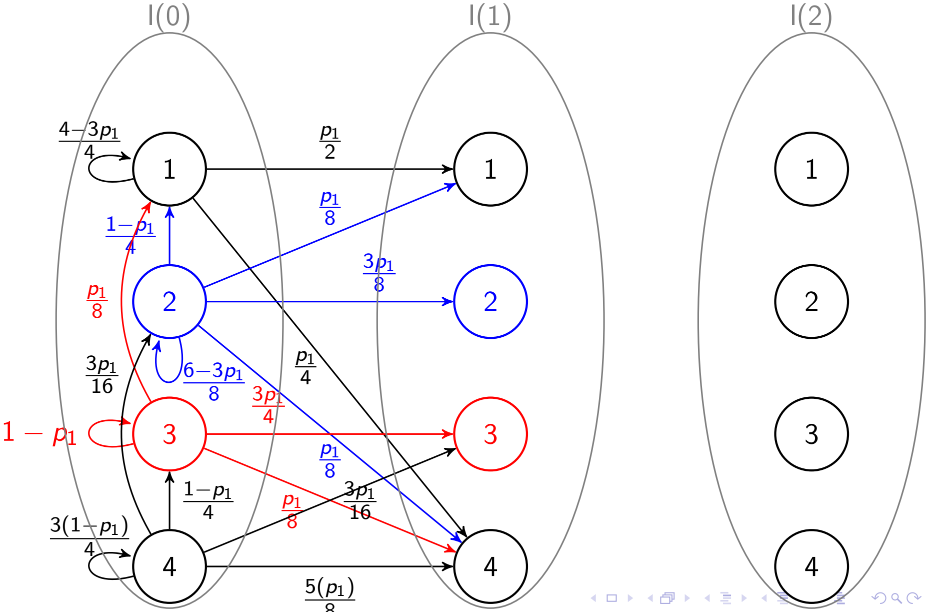
QBD

- Initially, we assume node 1 has an infinite buffer.
- As each node after a collision has a backoff stage, our QBD has m=4 phases:
  - $q_1 = \frac{1}{2}\ q_2 = \frac{1}{2} \rightarrow$ (n,1)
  - $q_1 = \frac{1}{2}\ q_2 = \frac{1}{4} \rightarrow$ (n,2)
  - $q_1 = \frac{1}{4}\ q_2 = \frac{1}{2} \rightarrow$ (n,3)
  - $q_1 = \frac{1}{4}\ q_2 = \frac{1}{4} \rightarrow$ (n,4)
- The pair (l(n), i) represents the i-th phase of the n-th level.

QBD process representation:



QBD process representation:



QBD Transition Matrix

- $TransitionMatrix = \begin{bmatrix} B & A_0 & 0 & 0 & 0 & \dots \\ A_2 & A_1 & A_0 & 0 & 0 & \dots \\ 0 & & & & \dots & \end{bmatrix}$ where

$$B = \begin{bmatrix} \frac{4-3p_1}{4} & 0 & 0 & 0 \\ \frac{1-p_1}{4} & \frac{6-3p_1}{8} & 0 & 0 \\ \frac{p_1}{8} & 0 & 1-p_1 & 0 \\ \frac{3p_1}{16} & 0 & \frac{1-p_1}{4} & \frac{3(1-p_1)}{4} \end{bmatrix}, \quad A_0 = \begin{bmatrix} \frac{p_1}{2} & 0 & 0 & \frac{p_1}{4} \\ \frac{p_1}{8} & \frac{3p_1}{8} & 0 & \frac{p_1}{8} \\ 0 & 0 & \frac{3p_1}{4} & \frac{p_1}{8} \\ 0 & 0 & \frac{3p_1}{16} & \frac{5p_1}{8} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} \frac{2-p_1}{4} & 0 & 0 & \frac{1-p_1}{4} \\ \frac{1-p_1}{8} & \frac{3}{8} & 0 & \frac{1-p_1}{8} \\ \frac{p_1}{8} & 0 & \frac{3(1-p_1)}{4} & \frac{1-p_1}{8} \\ 0 & \frac{3p_1}{16} & \frac{3(1-p_1)}{16} & \frac{5(1-p_1)}{8} \end{bmatrix}, \text{ and}$$

$$A_2 = \begin{bmatrix} \frac{1-p_1}{4} & 0 & 0 & 0 \\ 0 & \frac{3(1-p_1)}{8} & 0 & 0 \\ \frac{1-p_1}{8} & 0 & 0 & 0 \\ 0 & \frac{3(1-p_1)}{16} & 0 & 0 \end{bmatrix}.$$

QBD Stability

if:

- QBD is irreducible
- $m < \infty$ $\rightarrow$ QBD is Possitive Reccurent [1]
- A is irreducible
- $\mu = \alpha A_0 \mathbf{1} - \alpha A_2 \mathbf{1} < 0$

where $A = A_0 + A_1 + A_2$,
$\alpha A = \alpha and, \alpha \mathbf{1} = \mathbf{1}$
in this case:

- $A = \begin{bmatrix} \frac{3}{4} & 0 & 0 & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{4} & 0 & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{3}{4} & \frac{1}{8} \\ 0 & \frac{3}{16} & \frac{3}{16} & \frac{5}{8} \end{bmatrix}$
- $\alpha = \begin{bmatrix} \frac{3}{13} & \frac{3}{13} & \frac{3}{13} & \frac{4}{13} \end{bmatrix}$
- $\mu = p_1 - \frac{3}{13} < 0 \rightarrow p_1 < \frac{3}{13} \rightarrow QBD : Stable$

QBD

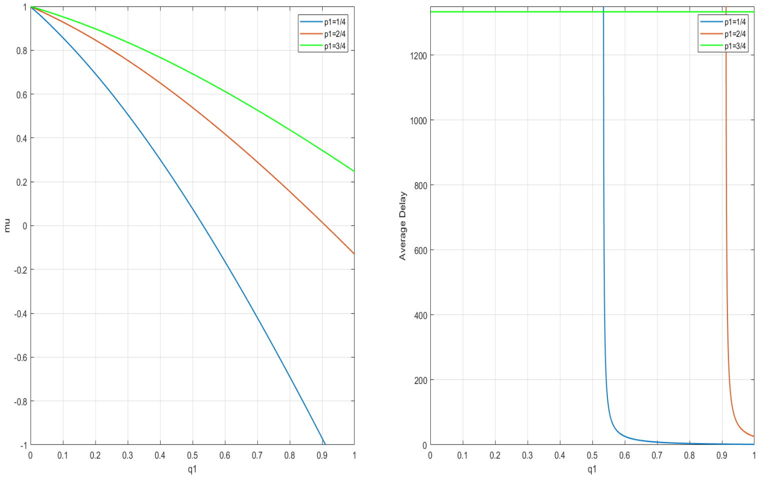


Figure: $q_2 = \frac{1}{2}$

QBD

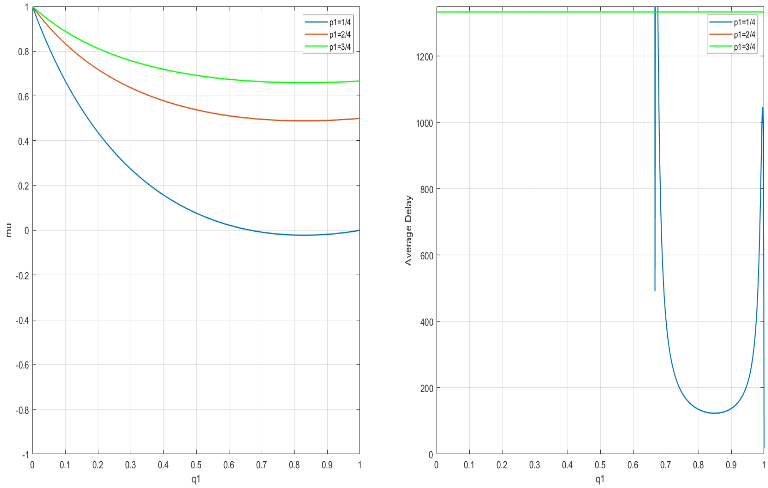


Figure: $q_1 = q_2$

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MDP-Transmission Control

- We assume that node 1 has a buffer of size M($Mislimited$).
- To model the system with an MDP we need to define State and Action Space and reward function for node 1:
  - $S = \{s_1, s_2, s_3, ..., s_{2M+2}\}$
  - $A = \{1, 0\}, \quad 1 = send \quad and \quad 0 = don't \quad send$
  - $r(s, a) = -E\{Number \quad of \quad Packets \quad in \quad Buffer\}$
- For Node 1 we want to design a policy that minimizes the average delay with less than 1% blocking probability.

MDP-Transmission Control

- To find the optimal policy we need to solve this linear programming [2]:

$$
\begin{aligned}
\max_{x,y} \quad & \sum_{s\in S}\sum_{a\in A_s} r(s,a)x(s,a) && (1)\\
s.t. \quad & \sum_{a\in A_j} x(j,a) - \sum_{s\in S}\sum_{a\in A_s} p(j|s,a)x(s,a) = 0, \forall j \in S\\
& \sum_{a\in A_j} x(j,a) + \sum_{a\in A_j} y(j,a) - \sum_{s\in S}\sum_{a\in A_s} p(j|s,a)y(s,a) = \alpha_j, \forall j \in S\\
& \sum_{s\in S}\sum_{a\in A_s} c(s,a)x(s,a) = 0.01,\\
& x(s,a) > 0 \forall (s,a) \in S \times A\\
& y(s,a) > 0 \forall (s,a) \in S \times A
\end{aligned}
$$

MDP-Admission Control

- We assume that node 1 has a buffer of size M($Mislimited$) and $q_1 = q_2 = \frac{1}{2}$.
- To model the system with an MDP we need to define State and Action Space and reward function for node 1:
  - $S = \{s_1, s_2, s_3, ..., s_{2M+2}\}$
  - $A = \{1, 0\}, \quad 1 = Accept \quad and \quad 0 = don't \quad Accept$
  - $r(s, a) = -E\{Number \quad of \quad Packets \quad in \quad Buffer\}$
- For Node 1 we want to design a policy that maximizes the throughput with less than 1% blocking probability.

MDP-Admission Control

- To find the optimal policy we need to solve this linear programming [2]:

$$
\begin{aligned}
\max_{x,y} \quad & \sum_{s\in S}\sum_{a\in A_s} r(s,a)x(s,a) && (2)\\
s.t. \quad & \sum_{a\in A_j} x(j,a) - \sum_{s\in S}\sum_{a\in A_s} p(j|s,a)x(s,a) = 0, \forall j \in S\\
& \sum_{a\in A_j} x(j,a) + \sum_{a\in A_j} y(j,a) - \sum_{s\in S}\sum_{a\in A_s} p(j|s,a)y(s,a) = \alpha_j, \forall j \in S\\
& \sum_{s\in S}\sum_{a\in A_s} c(s,a)x(s,a) = 0.01,\\
& x(s,a) > 0 \forall (s,a) \in S \times A\\
& y(s,a) > 0 \forall (s,a) \in S \times A
\end{aligned}
$$

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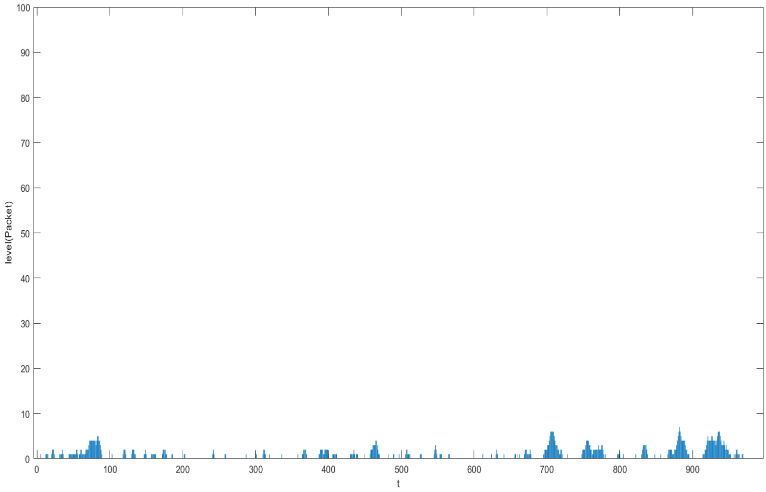

Figure: $q_1 = 1, \quad q_2 = \frac{1}{2}, p_1 = \frac{1}{4}, AverageDelay = 0.7$

Simulation(QBD)

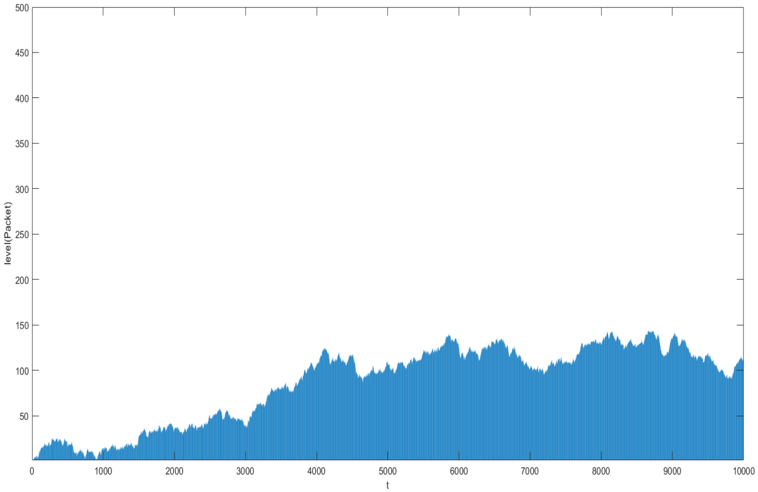

Figure: $q_1 = q_2 = 0.6, p_1 = \frac{1}{4}, AverageDelay = 86$

SIMULATION(QBD)

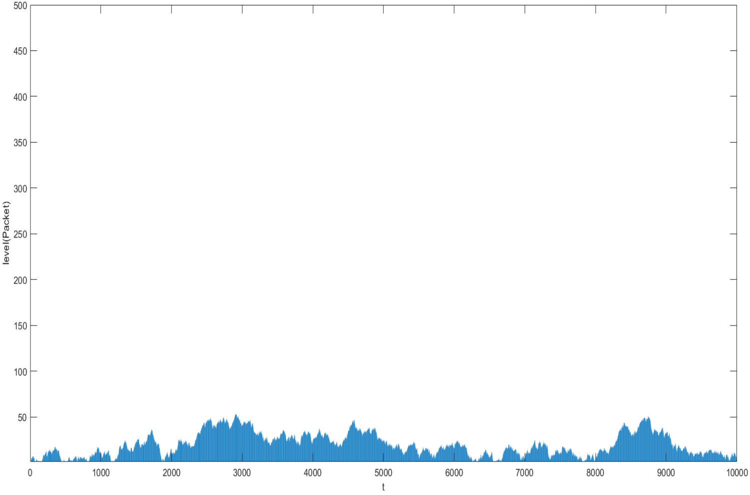

Figure: $q_1 = q_2 = 0.85, p_1 = \frac{1}{4}, AverageDelay = 19.61$

Simulation(QBD)

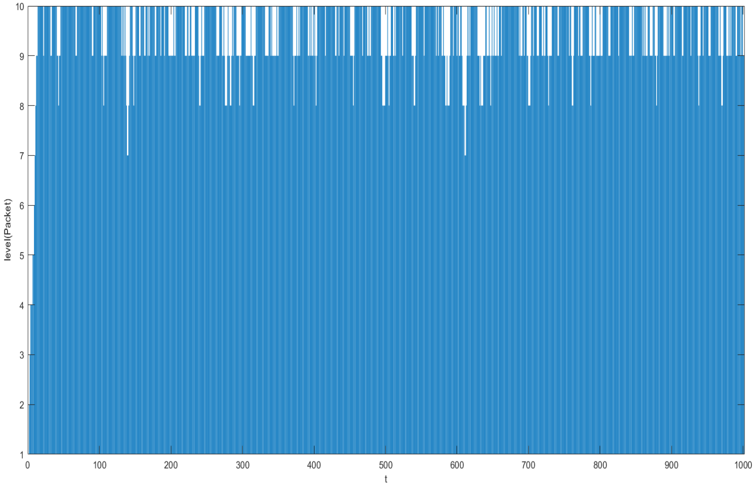

Figure: $q_1 = q_2 = 0.85, p_1 = \frac{3}{4}, AverageDelay = 89.51$

Simulation(MDP-Transmission Control)

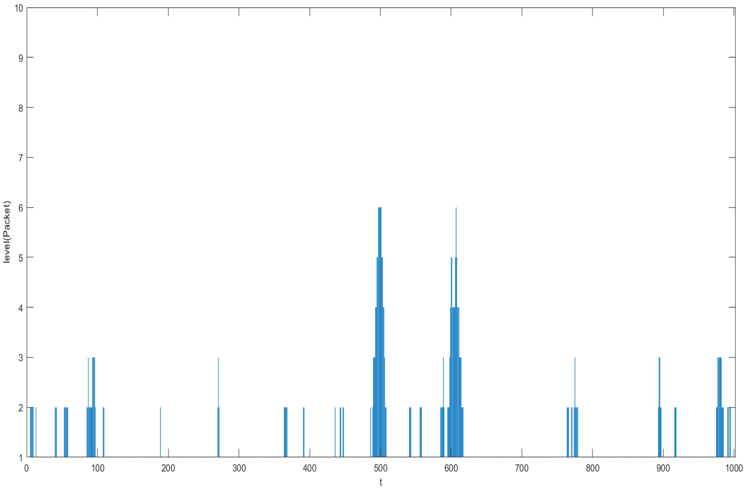

Figure: $p_1 = \frac{1}{4}, \quad q_1 = 1, \quad q2 = \frac{1}{2}, \quad AverageDelay = 0.56$

Simulation(MDP-Transmission Control)

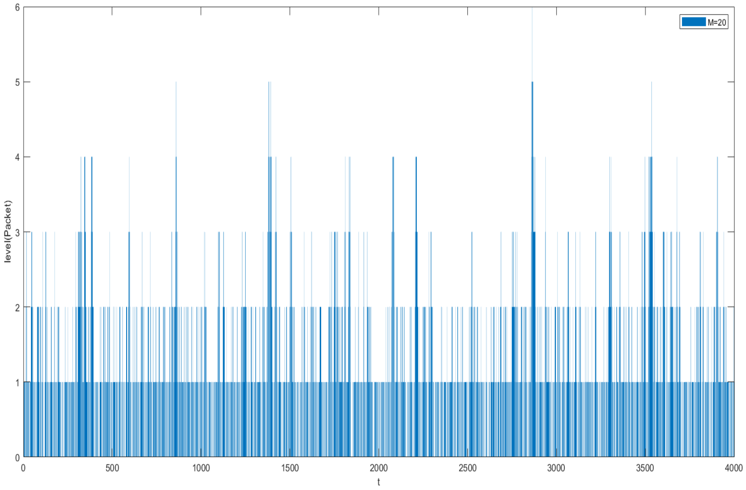

Figure:

$p_1 = \frac{2}{4}, q_1 = 1, q_2 = \frac{1}{2}, M = 20, AverageDelay = 1.094, BlockingProbability = 0$

Simulation(MDP-Transmission Control)

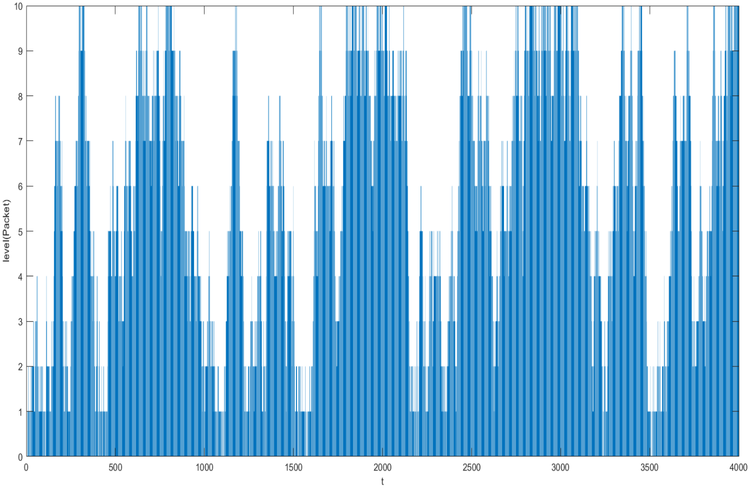

Figure:
$p_1 = \frac{3}{4}, q_1 = 1, q_2 = \frac{1}{2}, M = 10, AverageDelay = 8.31, BlockingProbability = 0.3$

Simulation(MDP-Admission Control)

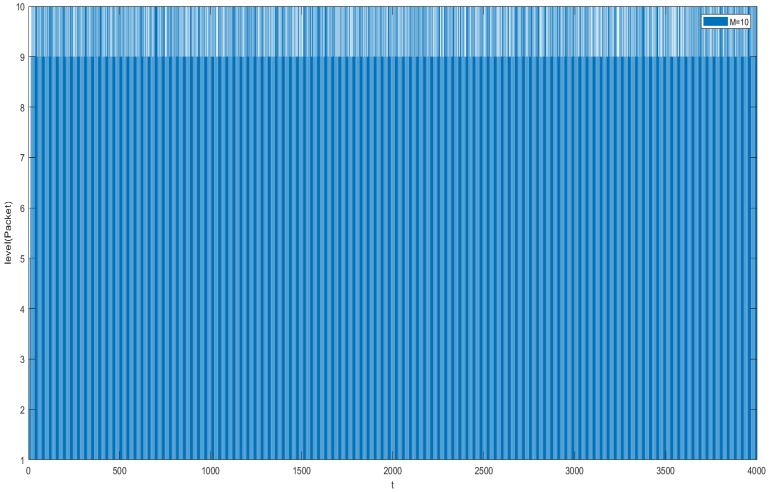

Figure: $p_1 = 1(except\ buffer\ is\ full)$, $q_1 = q_2 = \frac{1}{2}, M = 10,$ $Throughput = 0.33, AverageDelay = 9.71, BlockingProbability = 0$

Simulation(MDP-Admission Control)

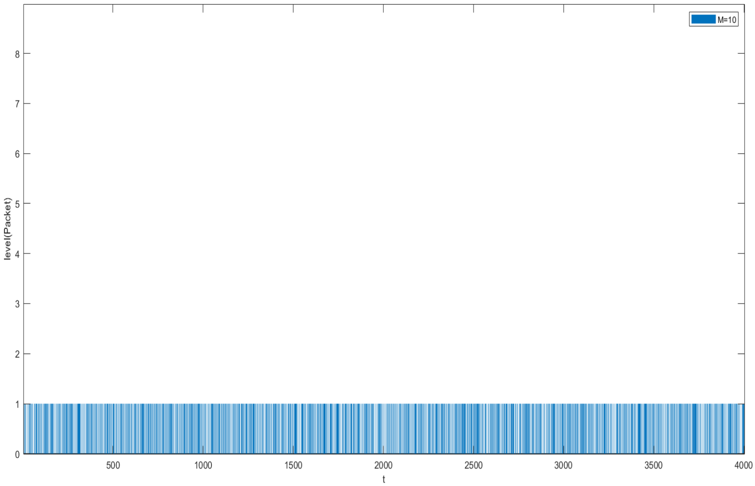

Figure: $p_1 = Optimal, \quad q_1 = q_2 = \frac{1}{2}, M = 10, \quad Throughput = 0.32, \quad AverageDelay = 0.65, \quad BlockingProbability = 0$

SIMULATION(MDP-Admission Control)

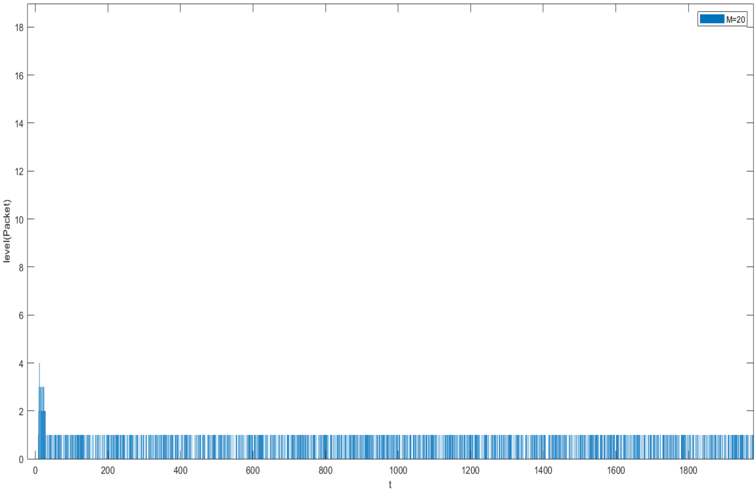

Figure: $p_1 = Optimal, \quad q_1 = q_2 = \frac{1}{2}, M = 20, \quad Throughput = 0.33, \quad AverageDelay = 0.66, \quad BlockingProbability = 0$

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References

[1] G. Latouche and V. Ramaswami. *Introduction to Matrix Analytic Methods in Stochastic Modeling*. Society for Industrial and Applied Mathematics, 1999. DOI: 10.1137/1.9780898719734.

[2] Martin L Puterman. *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons, 2014.

Thank You!