

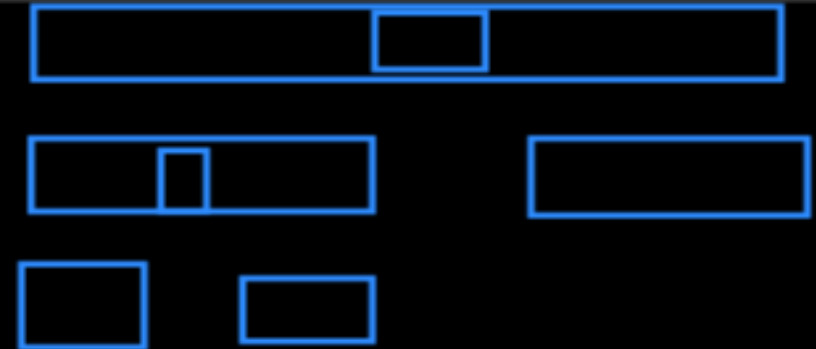
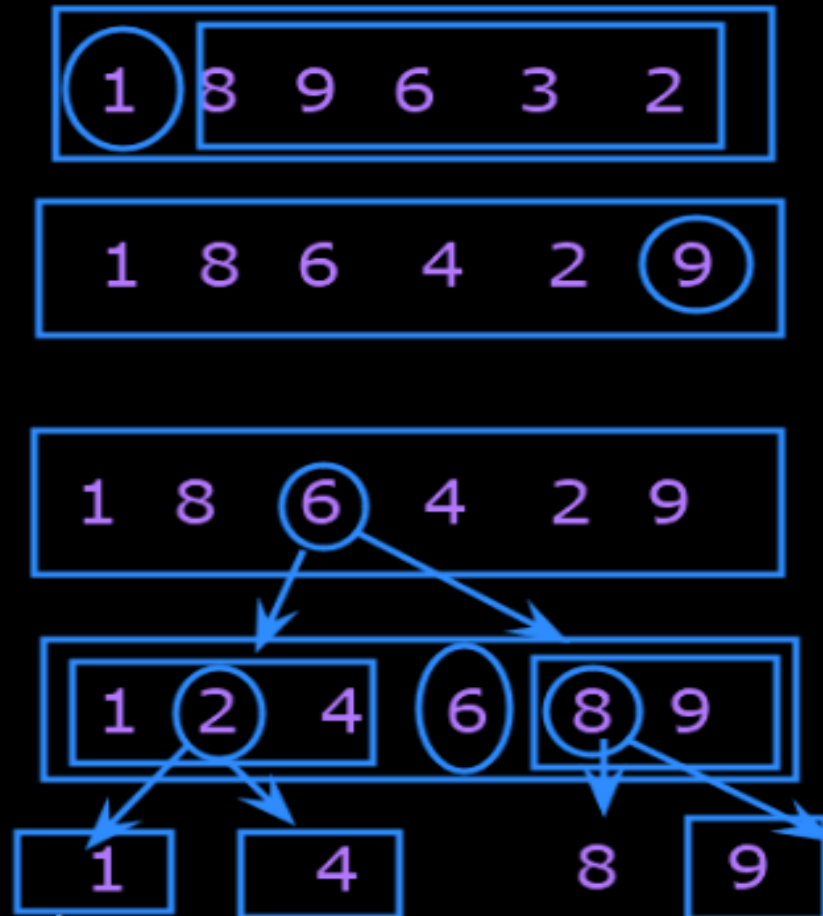
Algorithms & Data Structure

Kiran Waghmare

Date :16/06/2021

-Quick Sort

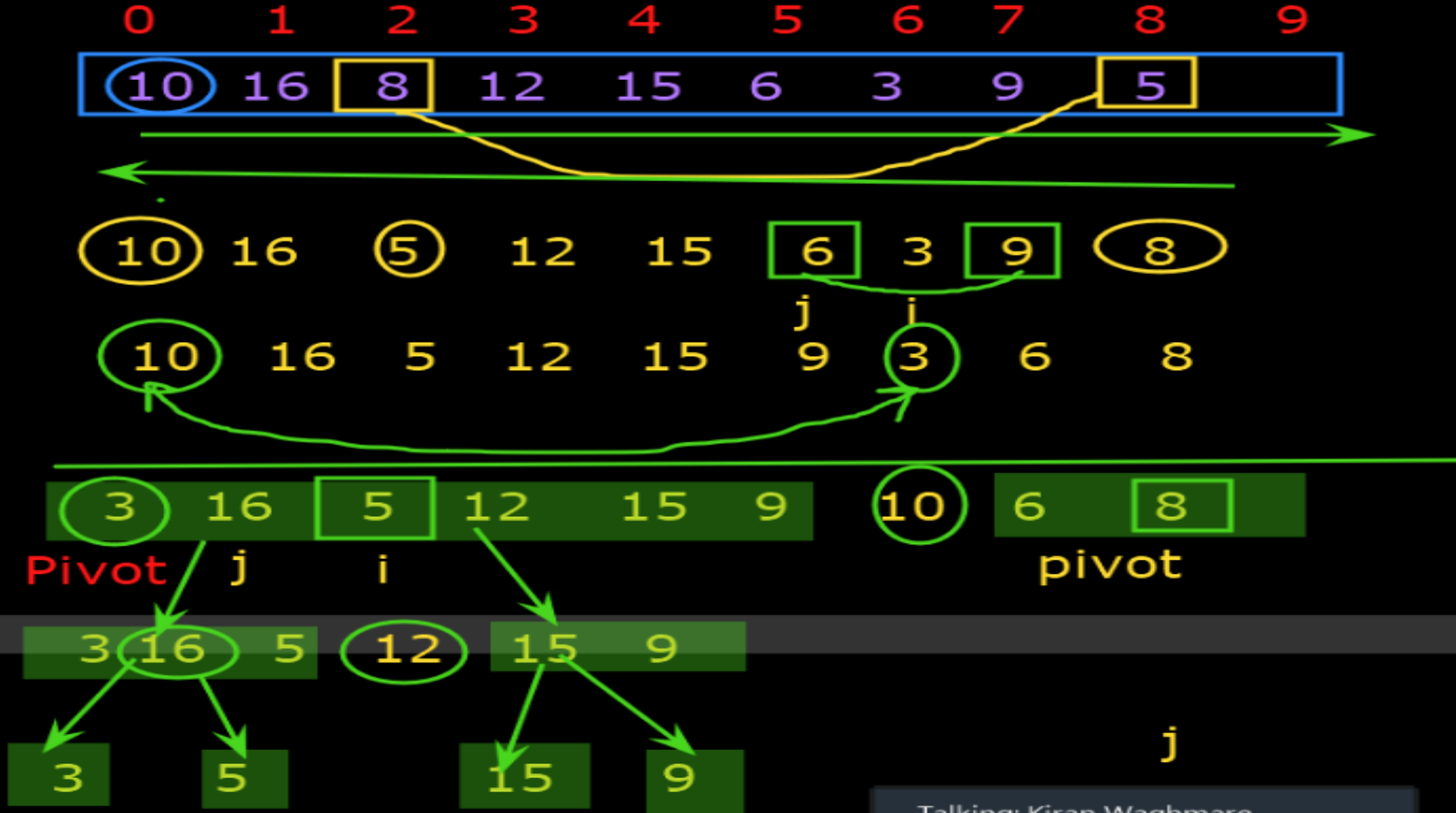
-pivot element



Quick Sort

-pivot element

Who can see what you share here?



Talking: Kiran Waghmare

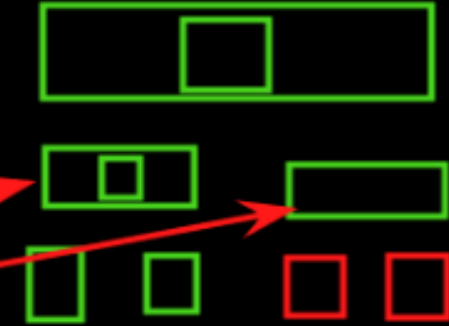
```
//partition
static int partition(int[] arr, int low, int high)
{
    int pivot = arr[high];

    int i = (low - 1);

    for(int j = low; j <= high - 1; j++)
    {
        if (arr[j] < pivot)
        {
            i++;
            swap(arr, i, j);
        }
    }
    swap(arr, i + 1, high);
    return (i + 1);
}
```

Talking: Kiran Waghmare

```
static void quickSort(int[] arr, int low, int high)
{
    if (low < high)
    {
        int pi = partition(arr, low, high);
        quickSort(arr, low, pi - 1);
        quickSort(arr, pi + 1, high);
    }
}
```



```
static void display(int[] arr, int size)
{
    for(int i = 0; i < size; i++)
        System.out.print(arr[i] + " ");
    System.out.println();
}
```

Hashing:

-It is a technique that determines an index or location for storage of an data structure.

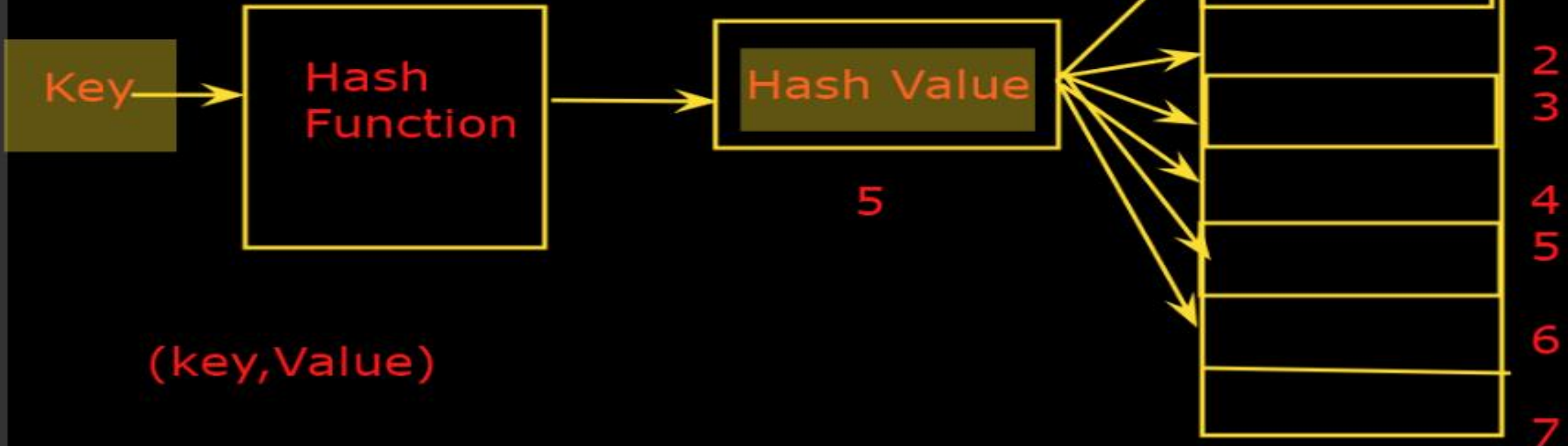
-search key - receive the information for any location

Hash Table

Linear Search: $O(n)$

Binary Search: $O(\log n)$

Hashing Search: $O(1) \Rightarrow$ Best (Fastest)



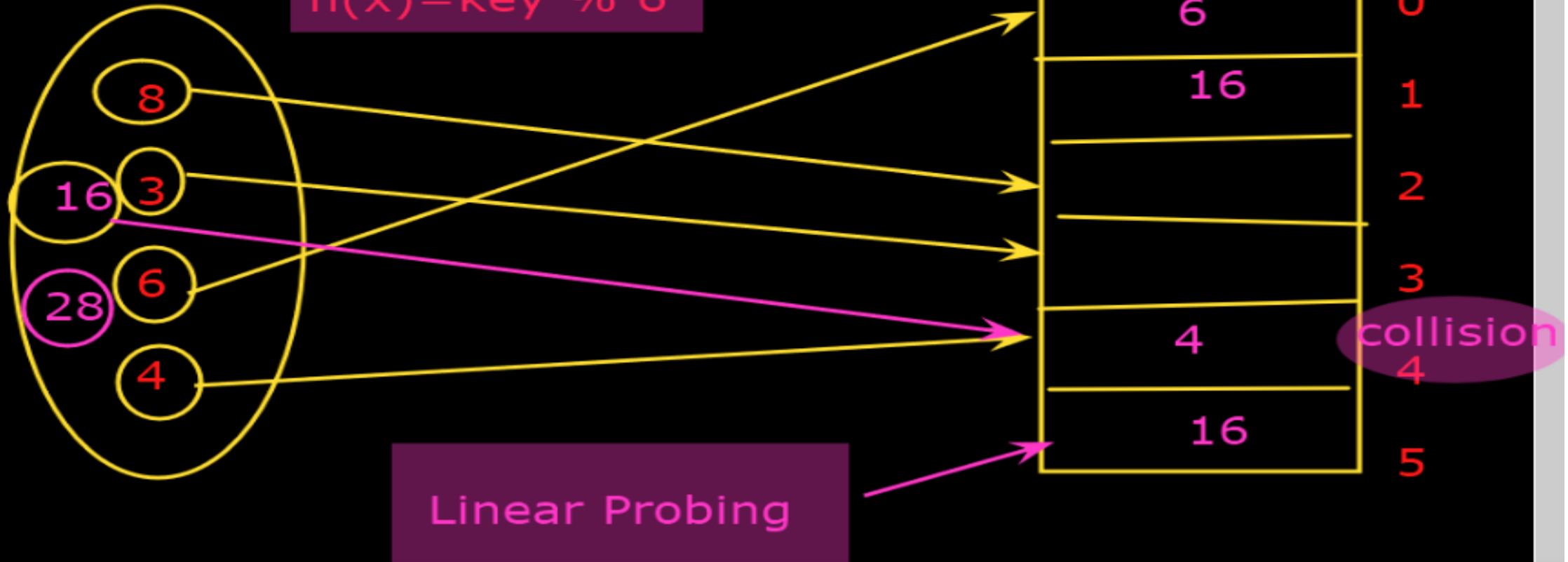
Hashing:

- It is a technique that determines an index or location for storage of an data structure.
- search key - receive the information for any location

Hash Function:

$$h(x) = \text{key} \% n \quad (\text{Hash table size})$$

$$h(x) = \text{key} \% 6$$

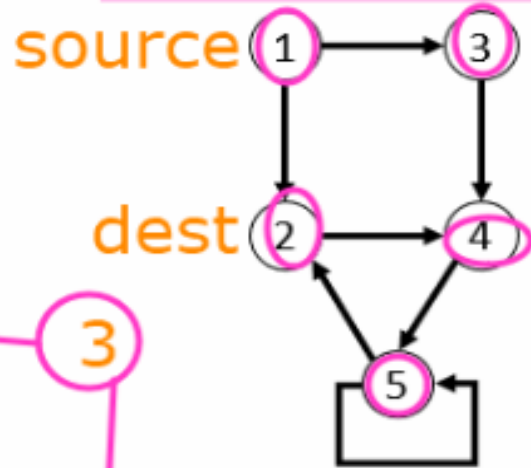


Graph definition

A graph is specified by a set of vertices (or nodes) V and a set of edges E .

$G(V, E)$

Directed Graph



$V = \{1, 2, 3, 4, 5\}$

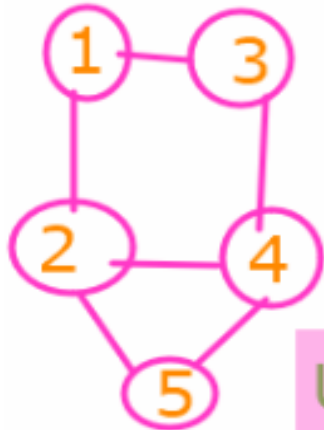
=> set of no. of nodes

$E = \{(1, 2), (1, 3), (2, 4), (3, 4), (4, 5), (5, 2), (5, 5)\}$

$E(\text{Source}, \text{dest})$

Graphs can be directed or undirected.

Undirected graphs have a symmetric relation.



Undirected Graph

KW:CDAC Mumbai

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Graph definition

A graph is specified by a set of vertices (or nodes) V and a set of edges E .

Path : sequence of nodes that are followed in order

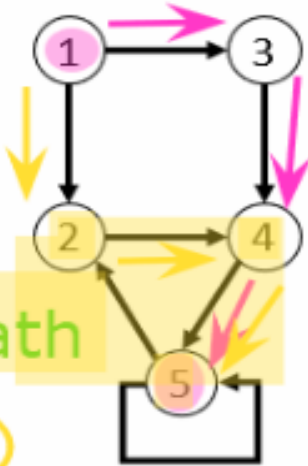
$$V = \{1, 2, 3, 4, 5\}$$

$$V = \{1, 2, 3, 4, 1, 3\}$$

Cycle:

$$E = \{(1, 2), (1, 3), (2, 4), (3, 4), (4, 5), (5, 2), (5, 5)\}$$

Closed Path

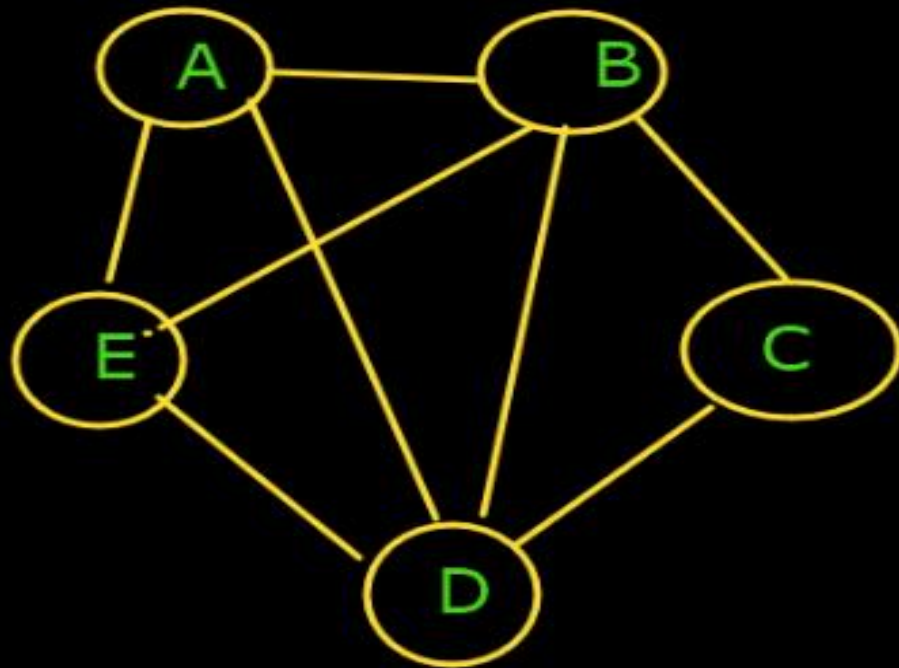


Connected Graph

KW:CDAC Mumbai

Complete Graph

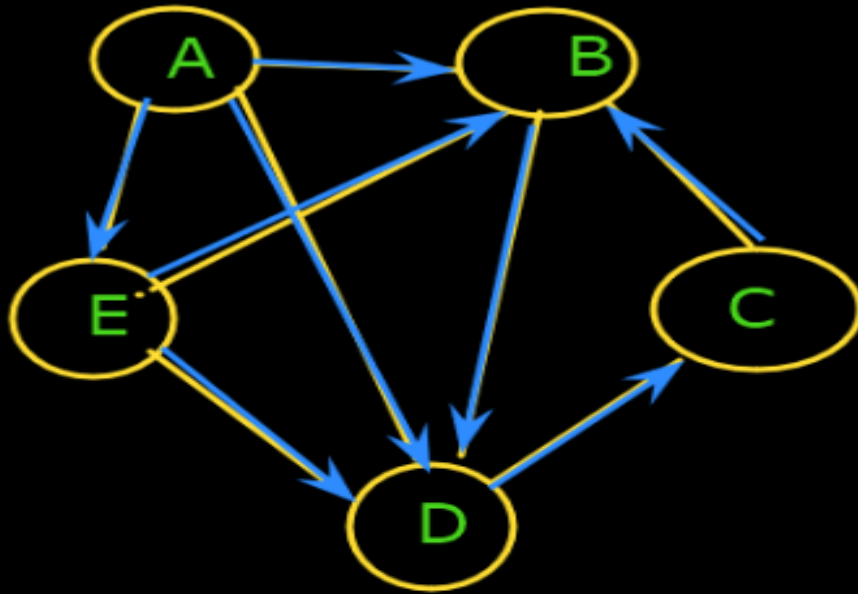
$$n(n-1)/2 = \text{No. of edges}$$



UNDIRECTED GRAPH

Adjacency Matrix

	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	1	1
C	0	1	0	1	0
D	1	1	1	0	1
E	1	1	0	1	0

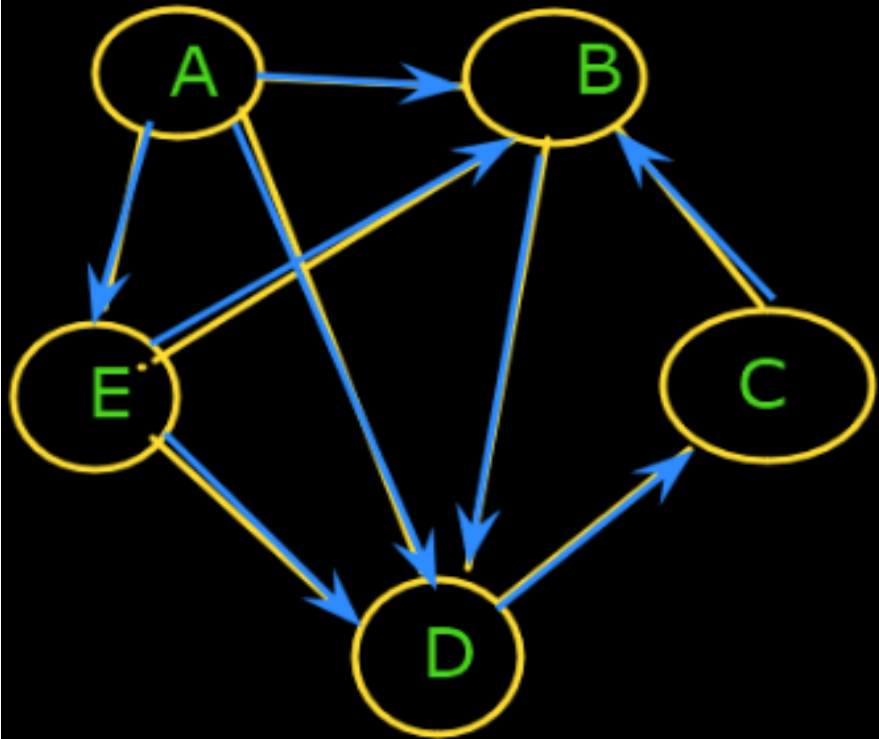


DIRECTED GRAPH

Adjacency Matrix

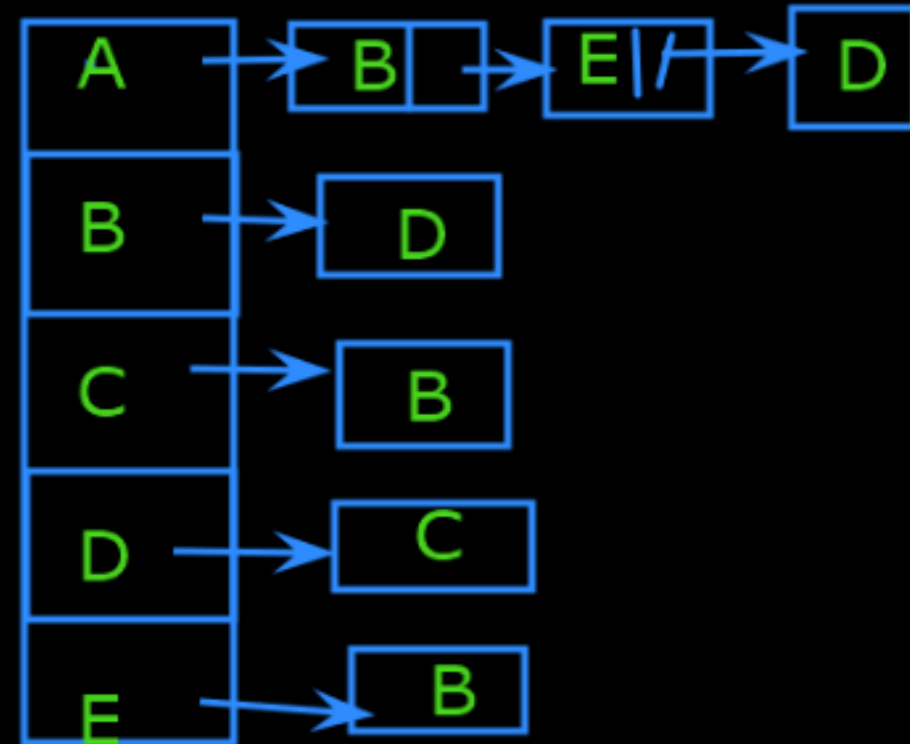
	A	B	C	D	E
A	0	1	0	1	1
B	0	0	0	1	0
C	0	1	0	0	0
D	0	0	1	0	0
E	0	1	0	1	0

COMPLEXITY: $O(n^2)$



DIRECTED GRAPH

Adjacency List



Finding the nodes reachable from another node

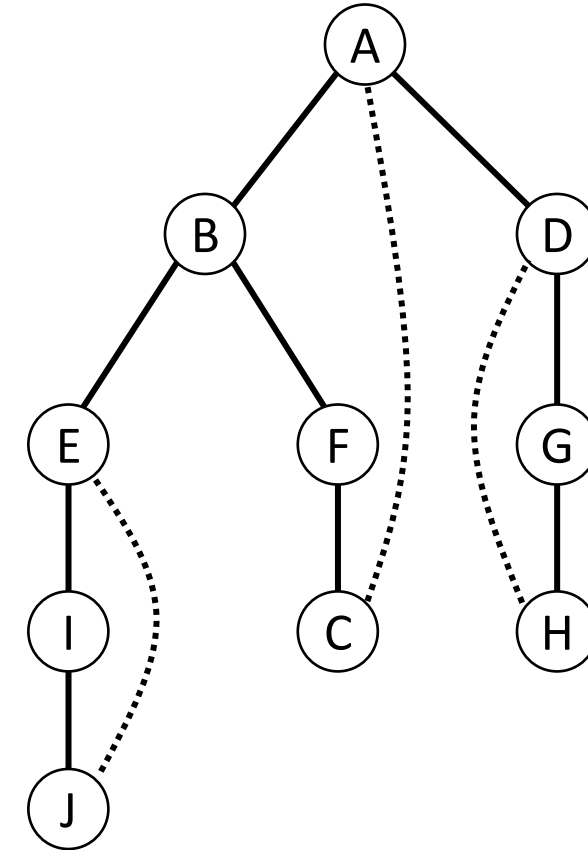
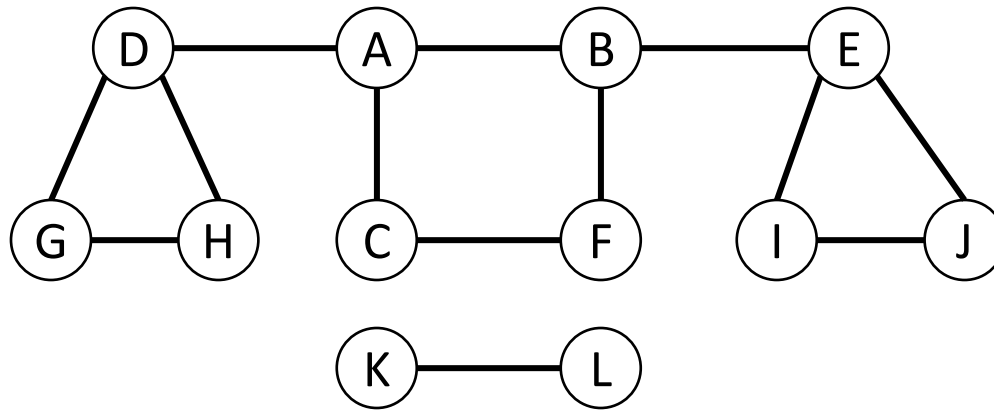
- **function explore(G, v):**
 - // Input: $G = (V, E)$ is a graph**
 - // Output: visited(u) is true for all the**
 - // nodes reachable from v**
- **visited(v) = true**
- **previsit(v)**
- **for each edge $(v, u) \in E$:**
- **if not visited(u): explore(G, u)**
- **postvisit(v)**

Notes:

- Initially, visited(v) is assumed to be *false* for every $v \in V$.
- pre/postvisit functions are not required now.

Finding the nodes reachable from another node

```
• function explore( $G, v$ ):  
  visited( $v$ ) = true  
  for each edge ( $v, u$ )  $\in E$ :  
    if not visited( $u$ ): explore( $G, u$ )
```



Dotted edges are ignored (*back edges*): they lead to previously visited vertices.
The solid edges (*tree edges*) form a tree.

Depth-first search

- **function DFS(G):**
- **for all $v \in V$:**
 visited(v) = false
- **for all $v \in V$:**
 if not visited(v): explore(G, v)

DFS traverses the entire graph.

Complexity:

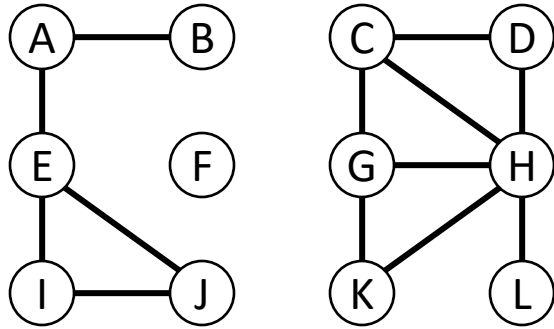
- Each vertex is visited only once (thanks to the chalk marks)
- For each vertex:
 - A fixed amount of work (pre/postvisit)
 - All adjacent edges are scanned

Running time is $O(|V| + |E|)$.

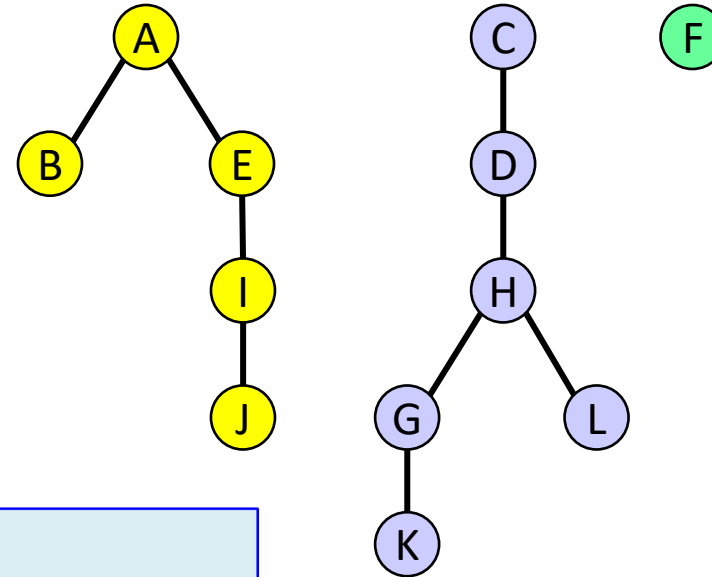
Difficult to improve: reading a graph already takes $O(|V| + |E|)$.

DFS example

Graph

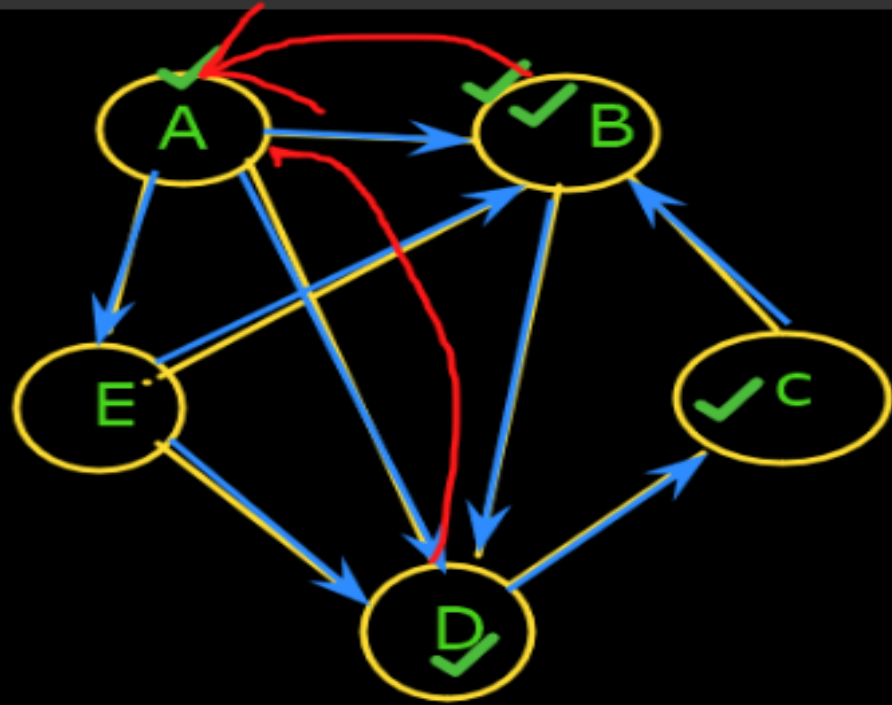


DFS forest



```
function DFS( $G$ ):  
  for all  $v \in V$ :  
    visited( $v$ ) = false  
  for all  $v \in V$ :  
    if not visited( $v$ ): explore( $G, v$ )
```

- The outer loop of DFS calls **explore** three times (for A, C and F)
- Three trees are generated. They constitute a *forest*.



DFS: A, B, D, C, E

A: ~~B~~, ~~D~~, E

B: ~~D~~

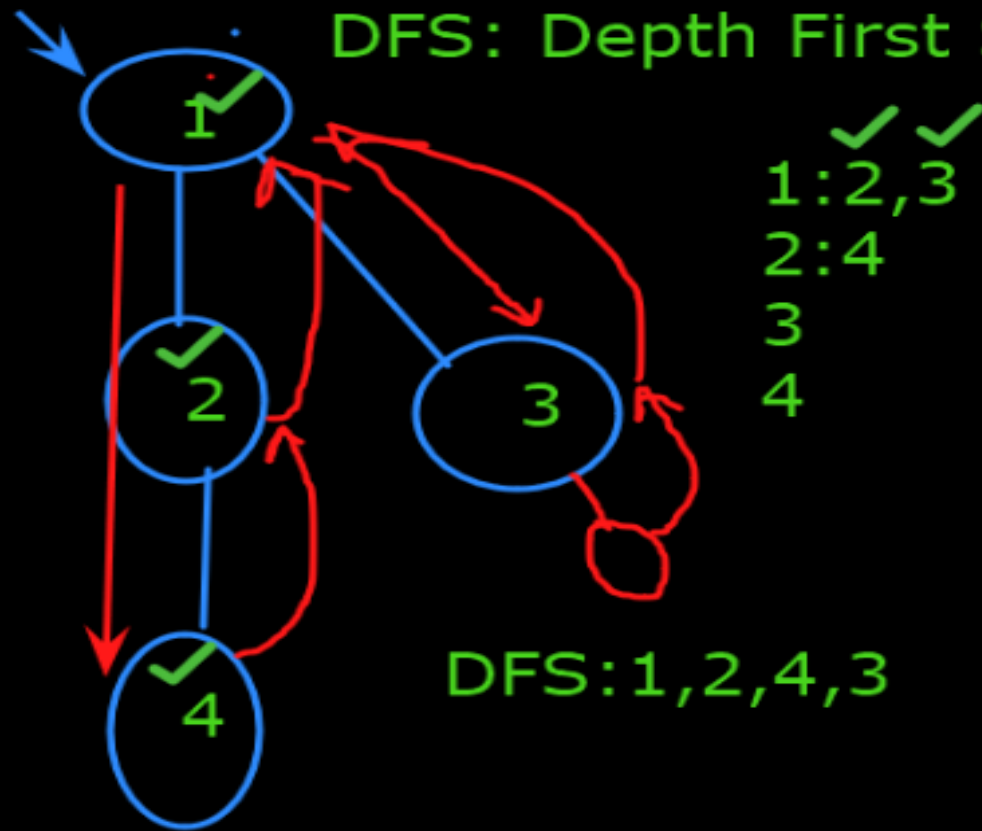
C: ~~B~~

D: ~~C~~

E: B, D

GRAPH TRAVERSAL

DFS: Depth First Search



1: ~~2~~, ~~3~~
2: 4
3
4

DFS: 1, 2, 4, 3

BFS algorithm

- BFS visits vertices layer by layer: $0, 1, 2, \dots, d$.
- Once the vertices at layer d have been visited, start visiting vertices at layer $d + 1$.
- Algorithm with two active layers:
 - Vertices at layer d (currently being visited).
 - Vertices at layer $d + 1$ (to be visited next).
- Central data structure: a queue.

BFS algorithm

```
• function BFS( $G, s$ )  
  // Input: Graph  $G(V, E)$ , source vertex  $s$ .  
  // Output: For each vertex  $u$ ,  $\text{dist}[u]$  is  
  //         the distance from  $s$  to  $u$ .  
  
  for all  $u \in V$ :  $\text{dist}[u] = \infty$   
  
   $\text{dist}[s] = 0$   
   $Q = \{s\}$  // Queue containing just  $s$   
  while not  $Q.\text{empty}()$ :  
     $u = Q.\text{pop\_front}()$   
    for all  $(u, v) \in E$ :  
      if  $\text{dist}[v] = \infty$ :  
         $\text{dist}[v] = \text{dist}[u] + 1$   
         $Q.\text{push\_back}(v)$ 
```

Runtime $O(|V| + |E|)$: Each vertex is visited once, each edge is visited once (for directed graphs) or twice (for undirected graphs).

Reachability: BFS vs. DFS

Input: A graph G and a source node s .

Output: $\forall u \in V: \text{reached}[u] \Leftrightarrow u$ is reachable from s .

- **function** BFS(G, s)

for all $u \in V$:
 reached[u] = **false**

$Q = \square$ // **Empty queue**

$Q.\text{push_back}(s)$

reached[s] = **true**

while not $Q.\text{empty}()$:

$u = Q.\text{pop_front}()$

for all $(u, v) \in E$:

if not **reached**[v]:

reached[v] = **true**

$Q.\text{push_back}(v)$

function DFS(G, s)

for all $u \in V$:
 reached[u] = **false**

$S = \square$ // **Empty stack**

$S.\text{push}(s)$

while not $S.\text{empty}()$:

$u = S.\text{pop}()$

if not **reached**[u]:

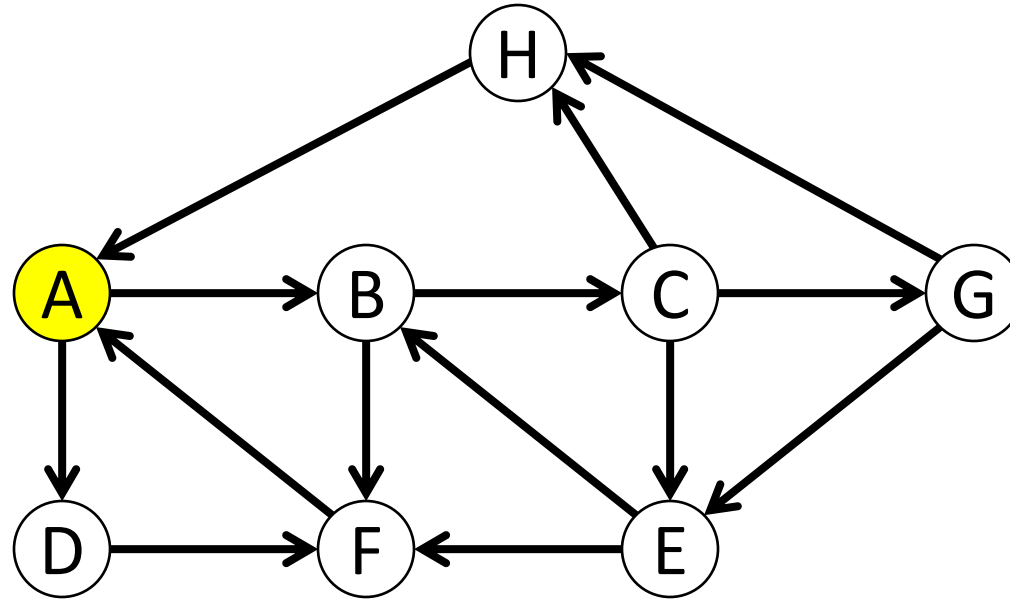
reached[u] = **true**

for all $(u, v) \in E$:

if not **reached**[v]:

$S.\text{push}(v)$

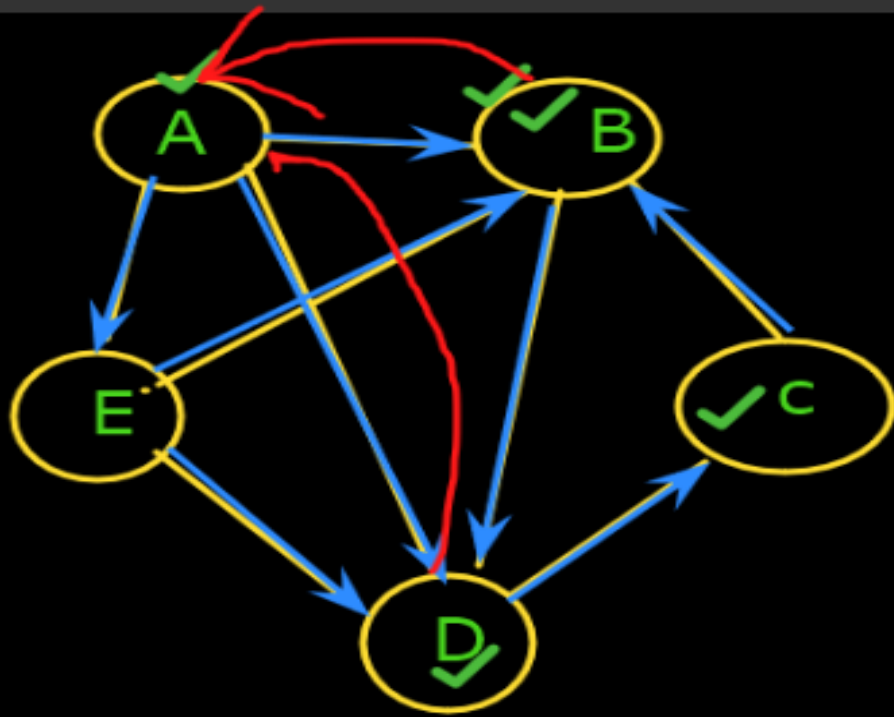
Reachability: BFS vs. DFS



DFS order: A B C E F G H D

BFS order: A B D C F E G H

Distance: 0 1 1 2 2 3 3 3



BFS: A, B, D, E, C

DFS: A, B, D, C, E

A: ~~B~~, ~~D~~, E

B: ~~D~~

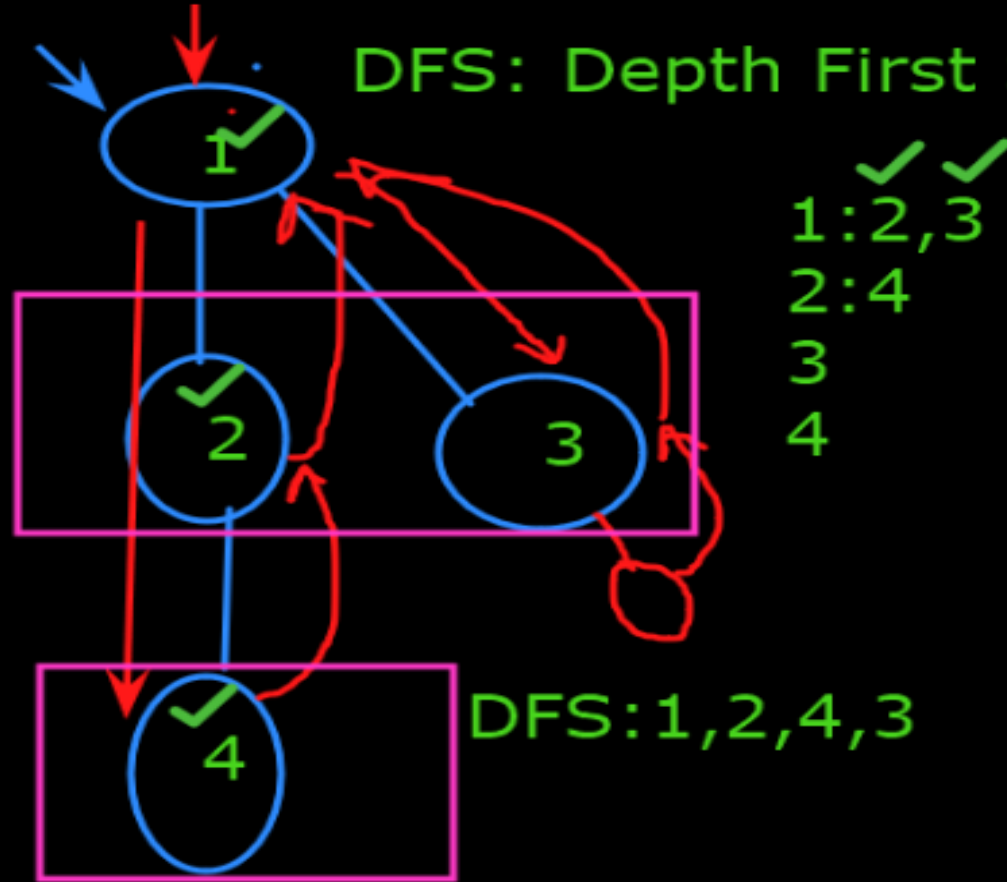
C: ~~B~~

D: ~~C~~

E: B, D

GRAPH TRAVERSAL

DFS: Depth First Search



1: 2, 3
2: 4
3
4

DFS: 1, 2, 4, 3

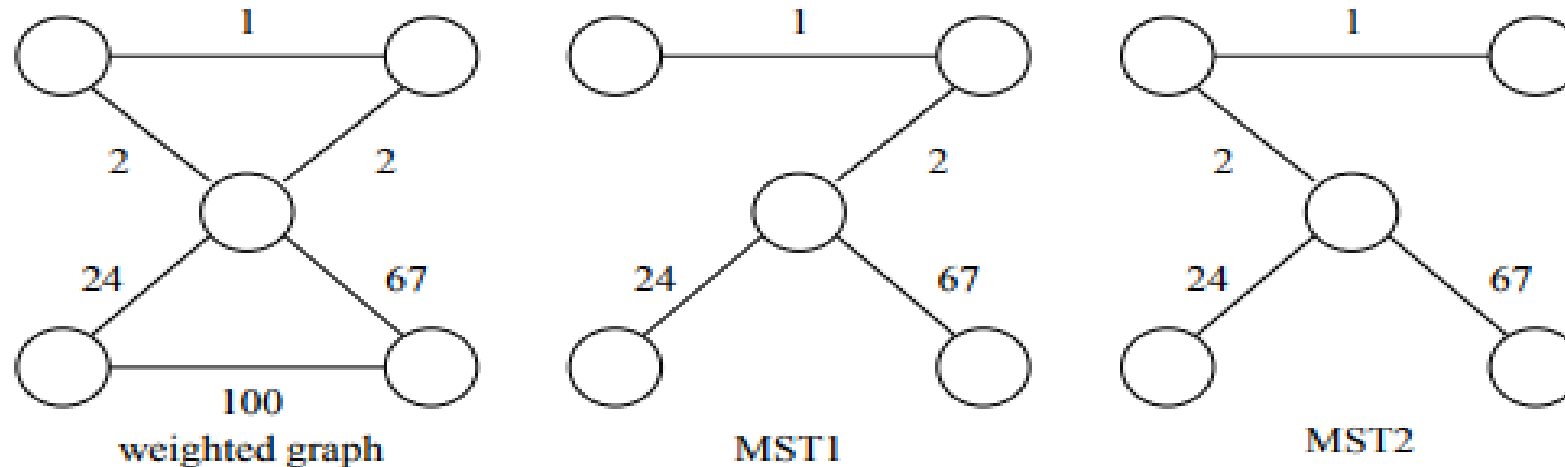
BFS: Breadth First Search

BFS: 1, 2, 3, 4

Minimum Spanning Trees

Remark: The minimum spanning tree may not be unique. However, if the weights of all the edges are pairwise distinct, it is indeed unique (we won't prove this now).

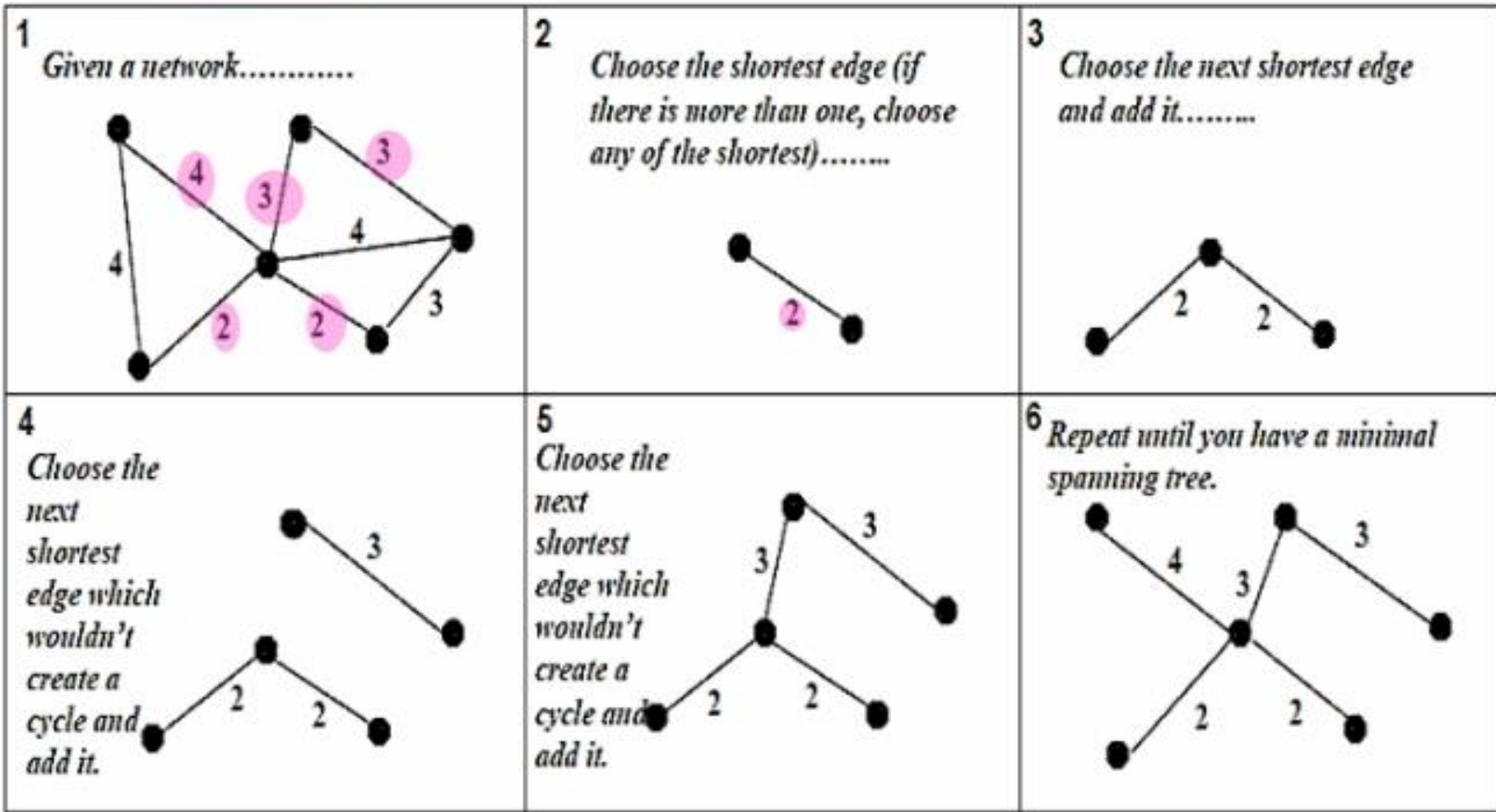
Example:



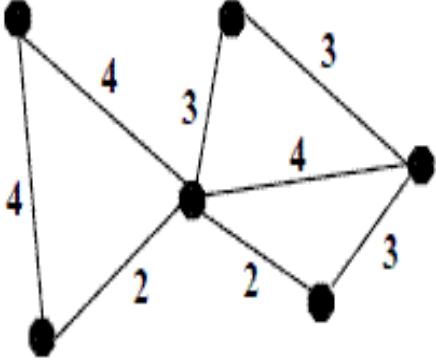

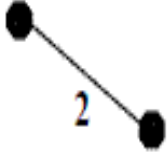
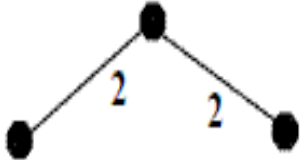
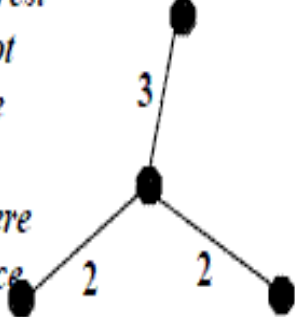
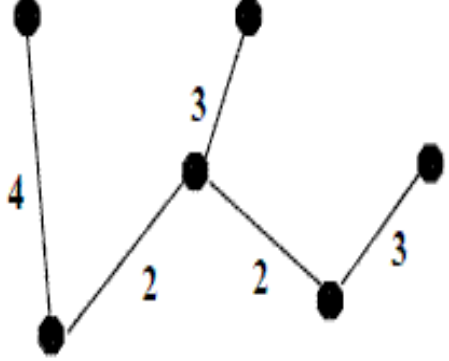
Finding Minimum Spanning Trees

1. Kruskal's algorithm,
2. Prim's algorithm

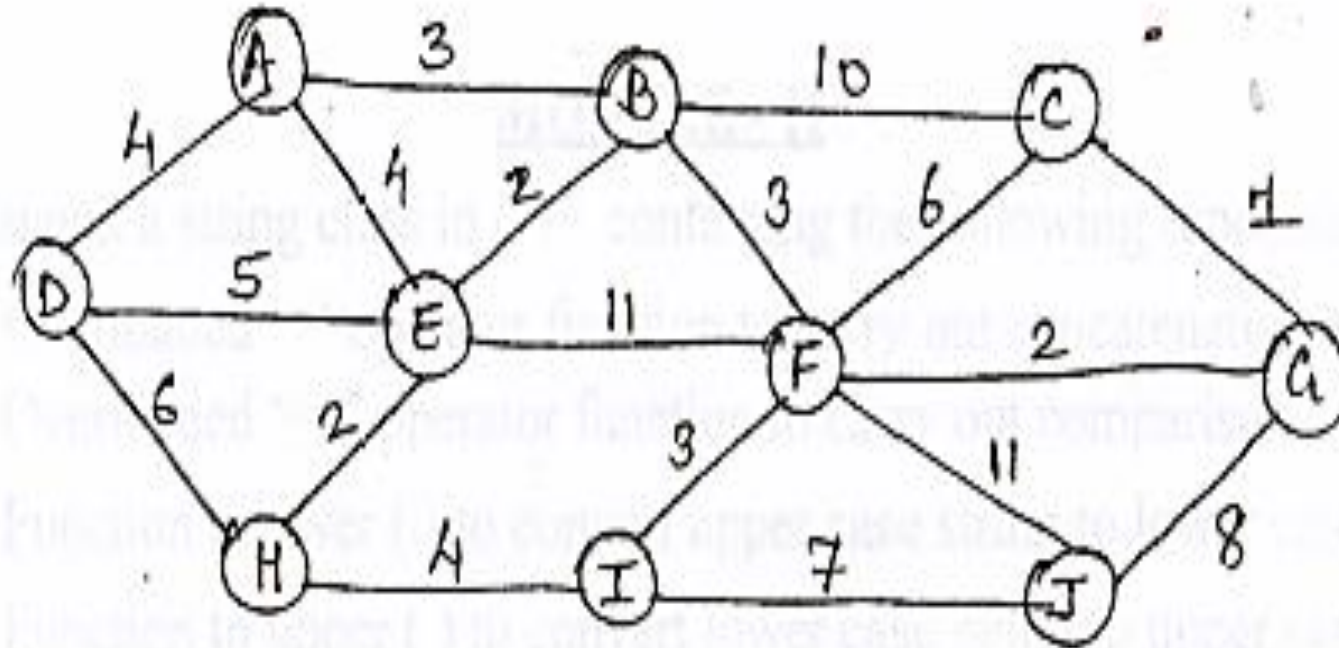
Kruskal's Algorithm



Prim's Algorithm

<p>1 <i>Given a network.....</i></p> 	<p>2 <i>Choose a vertex</i></p> 	<p>3 <i>Choose the shortest edge from this vertex.</i></p> 
<p>4 <i>Choose the nearest vertex not yet in the solution.</i></p> 	<p>5 <i>Choose the next nearest vertex not yet in the solution, when there is a choice choose either.</i></p> 	<p>6 <i>Repeat until you have a minimal spanning tree.</i></p> 

Obtain minimum cost spanning tree for the following graph using Kruskal's algorithm.



Obtain minimum cost spanning tree for the following graph using Kruskal's algorithm.

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