Map Reduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer *i* and produces the list of pairs (*p*,*i*) such that *p* is a prime divisor of *i*. For example, map(12) = [(2,12),(3,12)].

The reduce function is addition. That is, reduce(p,[i_1,i_2,...,i_k]) is (p,i_1+i_2+...+i_k).

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Solution :-

prime number :- 2, 3, 5, 7, 11 . . . .

map [15] : [3, 15], [5, 15]

map (21) : [3, 21], [7, 21]

map [27) : [2, 27], [3, 24]

map [30) : [2, 30], [3, 30]

map (47) : [7, 49]

by combining all common parts i.e compare element & add right most to get the soln

reduce (2, 54)

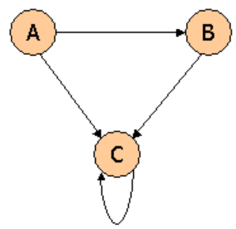
reduce (3, 90)

reduce (5, 45)

reduce (7, 70)

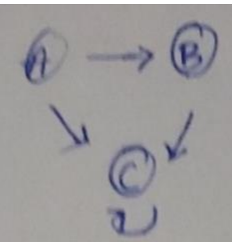
Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRank of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRank *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Solution :-

Q 

$$m = \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix} \end{array}$$

By rank Eq'n

$$r = \beta M \cdot r + (1-\beta)\left[1/N\right]_N$$

$$\beta M = 7/10 \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 7/20 & 0 & 0 \\ 7/20 & 7/10 & 7/10 \end{bmatrix}$$

$$\beta M \cdot r^0 = \begin{bmatrix} 0 & 0 & 0 \\ 7/20 & 0 & 0 \\ 7/20 & 7/10 & 7/10 \end{bmatrix}_{3\times3} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 0 \\ 0.1155 \\ 0.35 \times 0.35 + 0.7 \times 0.33 + 0.7 \times 0.33 \end{bmatrix}$$

$$= \begin{bmatrix} 0. \\ 0.1155 \\ 0.5775 \end{bmatrix},$$

$$(1-\beta)\left[\frac{1}{N}\right]_N = (1-0.7)\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$r^1 = \begin{bmatrix} 0 \\ 0.1155 \\ 0.5775 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.2155 \\ 0.6775 \end{bmatrix}$$

$$r^2 = \beta M r^1 + (1-\beta)\left[1/N\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.25 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2125 \\ 0.6775 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.025 \\ 0.6601 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7601 \end{bmatrix}$$

$$r^3 = \beta M r^2 + (1-\beta)\left[1/N\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.25 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7601 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0. \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.66157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix}$$

$$r^4 = \beta M r^3 + (1-\beta)\left[\frac{1}{N}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76157 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6625 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

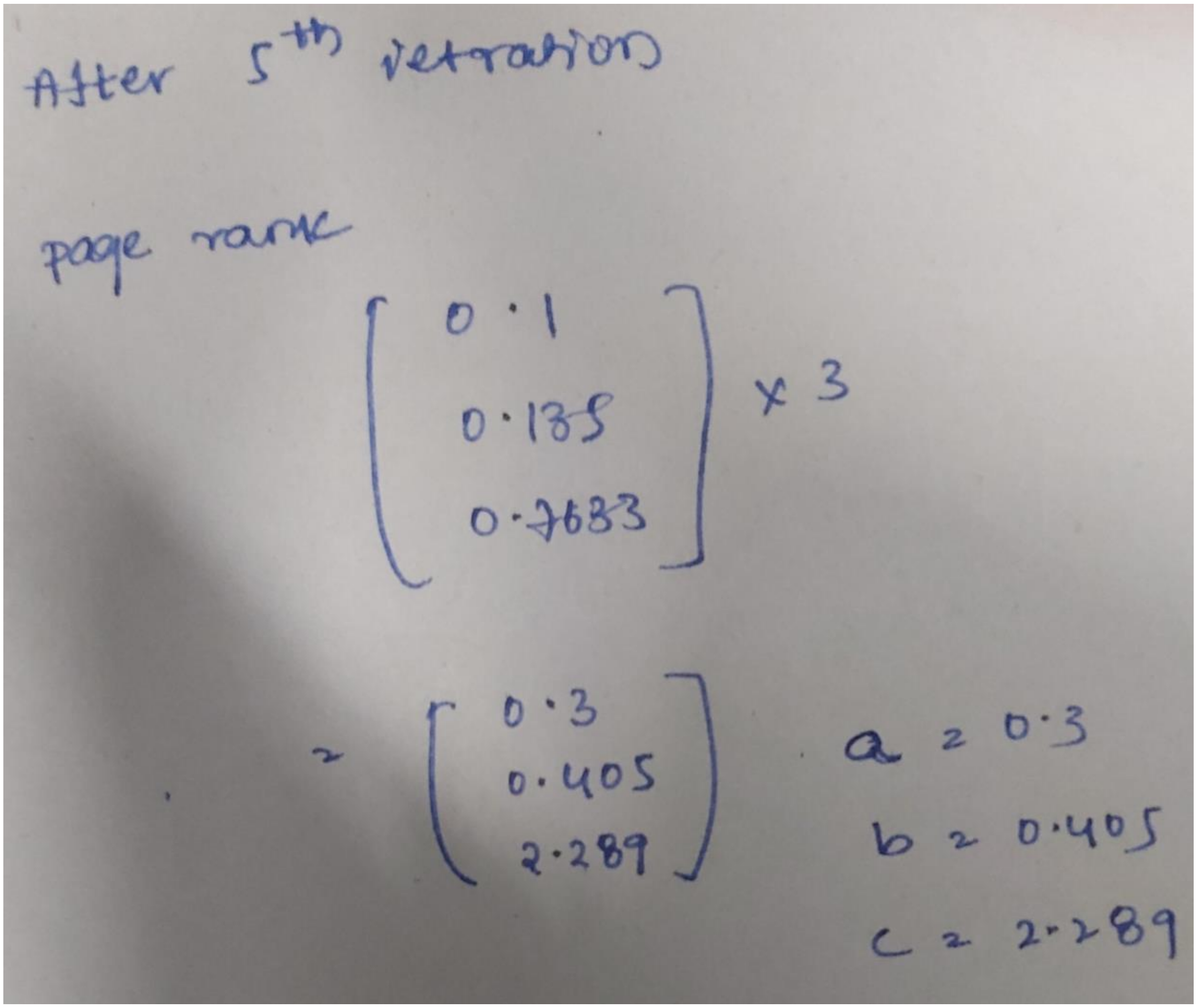
$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7625 \end{bmatrix}$$

$$r^5 = \beta M r^4 + (1-\beta)\left[\frac{1}{N}\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.35 & 0 & 0 \\ 0.35 & 0.7 & 0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.135 \\ 0.7625 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.035 \\ 0.6633 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 \\ 0.125 \\ 0.7633 \end{bmatrix}$$



After 5^{th} iteration

page rank

$$\begin{bmatrix} 0.1 \\ 0.135 \\ 0.7683 \end{bmatrix} \times 3$$

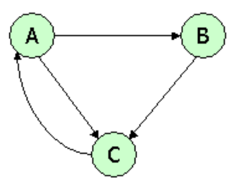
$$= \begin{bmatrix} 0.3 \\ 0.405 \\ 2.289 \end{bmatrix}$$

$a = 0.3$

$b = 0.405$

$c = 2.289$

Question 3:



Suppose we compute PageRank with β=0.85. Write the equations for the PageRanks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Solution :-

3) Formula

$$A = \beta \cdot C + (1-\beta)\, 1/3$$

$$B = \beta * A/2 + (1-\beta)\, 1/3$$

$$C = \beta * [A/2 * B] + (1-\beta)\, 1/3$$

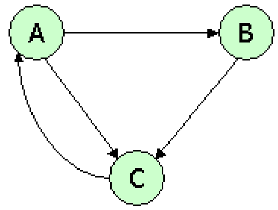
since $\beta = 0.85$

$$A = 0.85C + (1-0.85)\, 1/3 \qquad \boxed{A = 0.85C + 0.05}$$

$$B = 0.85 \times 0.5A\, (1-0.85)\, 1/3 \qquad \boxed{B = 0.425A + 0.05}$$

$$C = 0.85\, [0.5A + 0.85] + 0.05 \qquad \boxed{C = 0.425A + 0.85B + 0.0}$$

Question 4:



Assuming no "taxation," compute the PageRank *a*, *b*, and *c* of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank *a* = *b* = *c* = 1. Compute as far as the 5th iteration, and also determine what the PageRank are in the limit.

Solution:-

4) Formula $A = C$ $\quad B = A/2$ $\quad C = A/2 + B$

At 0^{th} iteration

$A = 1 \quad B = 1 \quad C = 1$

1^{st} iteration

$A = 1 \quad B = 1/2 \quad C = 3/2$

2^{nd} iteration

$A = 3/2 \quad B = 1/2 \quad C = 1/2 + 1/2 = 1$

3^{rd} iteration

$A = 1 \quad B = 3/2 \times 1/2 = 3/4 \quad C = 3/4 + 1/2$

$C = 5/4$