Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7, 2/7, -3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Let C1 be [2/7,3/7,6/7], C2 be [6/7,2/7,-3/7] and C3 be [x,y,z]

The dot prod of any two cols must be 0

C1.C2=(2/7*6/7)+(3/7*2/7)+(6/7*-3/7)=0

C2.C3=(6/7*x)+(2/7*y)+(-3/7*z)=6x+2y-3z=0 -- eqn1

C3.C1=(x*2/7)+(y*3/7)+(z*6/7)=2x+3y+6z=0 --eqn2

2*eqn1+eqn2 => 12x+4y-6z+2x+3y+6z=0 => 14x+7y=0 =>y=-2x

3*eqn2-eqn1 => 6x+9y+18z-6x-2y+3z=0 =>7y+21z=0 => y=-3z

x:y:z= -2:1:-3

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

② $\cos(A,B) = \frac{A.B}{|A||B|} =$ $\sqrt{(9+120)}$

② Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$ & eigen vector be $\begin{bmatrix} 1 \\ e \end{bmatrix}$

$Ax = \lambda x \Rightarrow \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \times \begin{bmatrix} 1 \\ e \end{bmatrix}$

$= \lambda \times \frac{1}{e} \Rightarrow 2 + 3e = \lambda$

$= 3 + 10e = \lambda e \Rightarrow 3 + 10e = (2+3e)e$

$= 3e^2 - 8e + 3 = 0 \Rightarrow e = 3, -1/3.$

$\therefore$ Eigen vectors are $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$

$\Rightarrow$ Eigen values are $2 + 3e = \lambda \Rightarrow \lambda = 2 + 3 \times 3 = 11$

$\lambda = 2 + 3 \times (-1/3) = 1.$

Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Given eigenvector of matrix M=[1,3,4,5,7]

Sum of squares=1*1+3*3+4*4+5*5+7*7=100 => sqrt=10

Unit eigenvector=[1/10,3/10,4/10,5/10,7/10]

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

Given 3 points in a 2d space are- (1,1),(2,2),(3,4)

We should construct a matrix whose rows correspond to points and columns correspond to dimensions of space-

Then M= 1 1

2 2

3 4

M^TM= 14 17

17 21

Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.

Moore-Penrose pseudoinverse of M-

1	0	0
0	1/2	0
0	0	0

Question 6: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

probability distribution= sum of squares of row ele/sum of squares of matrix ele

Sum of squares of matrix ele=(12*13*25)/6=650

P(r1)=(1*1+2*2+3*3)/650=0.02

P(r2)=(4*4+5*5+6*6)/650=0.12

P(r3)=(7*7+8*8+9*9)/650=0.29

P(r4)=(10*10+11*11+12*12)/650=0.56