Physical Interpretation of the Beta + Gaussian Model in A2163

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1 Introduction

This document presents a detailed interpretation of the results obtained by fitting a surface brightness profile of A2163 using a combined normalized β-model and Gaussian profile. The model fitting was performed using Sherpa, and the goal is to extract physically meaningful parameters such as electron density, gas mass, and pressure profiles. Additionally, we use XSPEC to fit the spectrum and determine the cluster's central temperature, which is essential for hydrostatic equilibrium mass calculations.

Fitted Model Parameters (Sherpa)

For reference, the best-fit parameters obtained from the Sherpa modeling (Beta + Gaussian) are:

- Core Radius: $r_c = 190.02 \pm 4.88$ kpc
- Gaussian FWHM: 135.96 ± 12.80 kpc
- β-index: $\beta = 0.5036 \pm 8.6 \times 10^{-5}$
- Gaussian Center: $r_g = 214.82 \pm 2.02$ pixels
- Gaussian Amplitude: 0.0817 ± 0.0068 photons/cm^2/pixel^2/s

2 Physical Parameter Extraction

2.1 1. Sherpa-based Physical Modeling

We assume the fitted model:

$$n_e(r) = n_{e,\beta} \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-3\beta/2} + n_{e,g} \cdot \exp\left(-\frac{(r - r_g)^2}{2\sigma_g^2} \right)$$

Central Density n_{e0}

Estimated using the normalization constant from Sherpa (with ARF/RMF corrections).

Gas Mass Profile $M_{\mathbf{gas}}(r)$

$$M_{\mathrm{gas}}(r) = \mu_e m_p \int_0^r 4\pi r'^2 n_e(r')\, dr'$$

Where:

- $\mu_e \approx 1.17$ is the mean molecular weight per electron
- m_p is the proton mass

Electron Pressure Profile

$$P_e(r) = n_e(r) \cdot k_B T_e$$

Useful for computing SZ Compton-y parameter and total thermal energy content.

2.2 2. XSPEC-based Temperature Measurement

The cluster's electron temperature T_e is measured using XSPEC by fitting a thermal plasma model (e.g., `wabs*apec`). This temperature is critical for mass estimations under hydrostatic equilibrium.

Hydrostatic Total Mass

$$M(r) = \frac{3\beta k_B T_e}{G\mu m_p} \cdot \frac{r^3}{r_c^2 + r^2}$$

Where:

- T_e is in keV
- $\mu \approx 0.6$ for a fully ionized plasma

3 Cosmological and Physical Significance

- **Electron density** profile allows estimation of cooling time, entropy, and gas fraction.
- **Gas mass** is important for determining baryonic content and constraining dark matter.
- **Pressure** connects directly to SZ signal.
- **Temperature** enables total gravitational mass derivation via hydrostatic equilibrium.

4 Spectral Extraction and Fitting Procedure

4.1 1. Spectrum Extraction Using CIAO

The following `specextract` command was used to extract the X-ray spectrum, generate the ARF and RMF files, and apply background subtraction:

```
specextract \
  infile="repro/acisf01653_repro_evt2.fits[sky=region(core.reg)]" \
  outroot="1653_spec" \
  bkgfile="1653_blank.evt[sky=region(core.reg)]" \
  correctpsf=no \
  weight=no \
  bkgresp=no \
  clobber=yes
```

This ensures CTI-correct and region-aligned responses and spectra.
LaTeX Documentation

Generated Files and Their Usage

1653_spec.pi — Source Spectrum

- Contains binned photon counts vs detector channels.

- Used by XSPEC to fit the observed emission with physical models.
- Includes metadata such as exposure time, region definition, and instrument details.

This is the main data XSPEC fits against.

1653_spec_bkg.pi — Background Spectrum

- Similar structure to the source spectrum but derived from the background event file (e.g., blanksky).
- Represents background events including instrumental and cosmic contributions.
- Subtracted automatically during XSPEC fitting for cleaner signal.

Needed to get accurate net counts, especially in low surface brightness clusters.

1653_spec.rmf — Response Matrix File (RMF)

- Maps incoming photon energy to detector channel probabilities.
- Includes effects of detector resolution and energy dispersion.
- Required to correctly simulate the observed shape of line emission and continuum.

Essential for modeling line emission and spectral resolution effects.

1653_spec.arf — Auxiliary Response File (ARF)

- Describes the effective area (sensitivity) of the telescope/detector as a function of energy.
- Includes the effects of vignetting, mirror reflectivity, chip gaps, and extraction region size.
- Used in conjunction with RMF to convert photon flux to expected count rate.

Converts physical model predictions to actual detector count rates.

4.2 2. Fitting in PyXspec

The following Python code was used to load the spectrum and fit an absorbed APEC model:

```
from sherpa.astro import ui
import matplotlib.pyplot as plt
from sherpa.astro import xspec

# This script performs X-ray spectral fitting using Sherpa and XSPEC.

# Load data files
ui.load_pha("core.pi")
ui.load_bkg("core_bkg.pi")
ui.load_arf("core.arf")
ui.load_rmf("core.rmf")

ui.notice(2, 7)
ui.group_counts(25)
ui.subtract()

# Define model and fit parameters
ui.set_xsxset("APECROOT", "/Volumes/AstroSSD/conda_envs/ciao-4.17/CALDB/data/atomdb_v3.1.2/apec_v3.1.2")  # Set the path for APEC data
ui.set_source(ui.xsphabs.abs1 * ui.xsapec.thermal)

ui.set_par("abs1.nH", 0.065)  # Set the hydrogen column density
ui.freeze("abs1.nH")  # Freeze the parameter to prevent it from being varied during fitting

# ui.set_par("z1.Redshift", 0.203)
# ui.freeze("z1.Redshift")

ui.set_par("thermal.kT", 10)  # Set the thermal temperature
ui.set_par("thermal.norm", 1e-3)  # Set the normalization
ui.set_par("thermal.redshift", 0.203)  # Set the redshift
ui.freeze("thermal.redshift")  # Freeze the redshift parameter
ui.set_par("thermal.Abundanc", 1e-3)  # Set the abundance
ui.thaw("thermal.Abundanc")  # Thaw the abundance parameter to allow it to vary

ui.show_model()

# Fit and plot
```

```python
    ui.fit()  # Perform the fit
    # ui.conf()
    ui.covar()  # Calculate covariance of parameters
    ui.plot_fit()  # Plot the fit results
    plt.xscale("log")  # Set x-axis to logarithmic scale
    plt.yscale("log")  # Set y-axis to logarithmic scale
    plt.savefig("graphs/spectal_fit.png", dpi=600)  # Save the plot as a PNG file
    plt.show()  # Display the plot
```

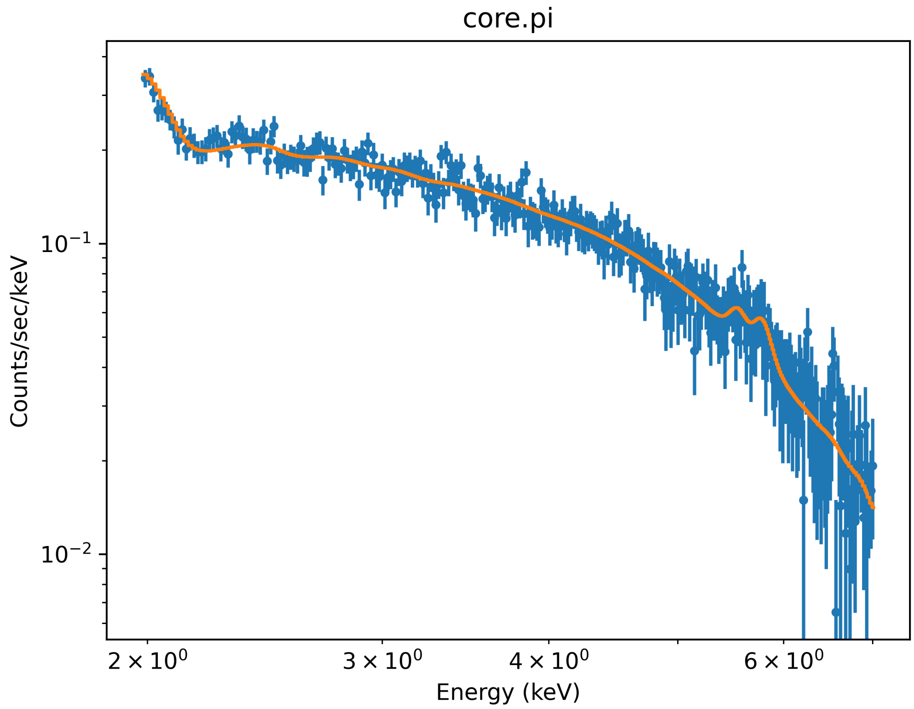

Figure 1: A2163 Specturm fit to wabs + apec XSpec models

5 Core Spectral Fit Result and Interpretation

We extracted the core spectrum of A2163 from a circular region of radius 50 pixels (approximately 25 arcseconds $\sim$ 85–90 kpc at $z = 0.203$), centered at the X-ray emission peak. The spectrum was fit using the `xsapec` model

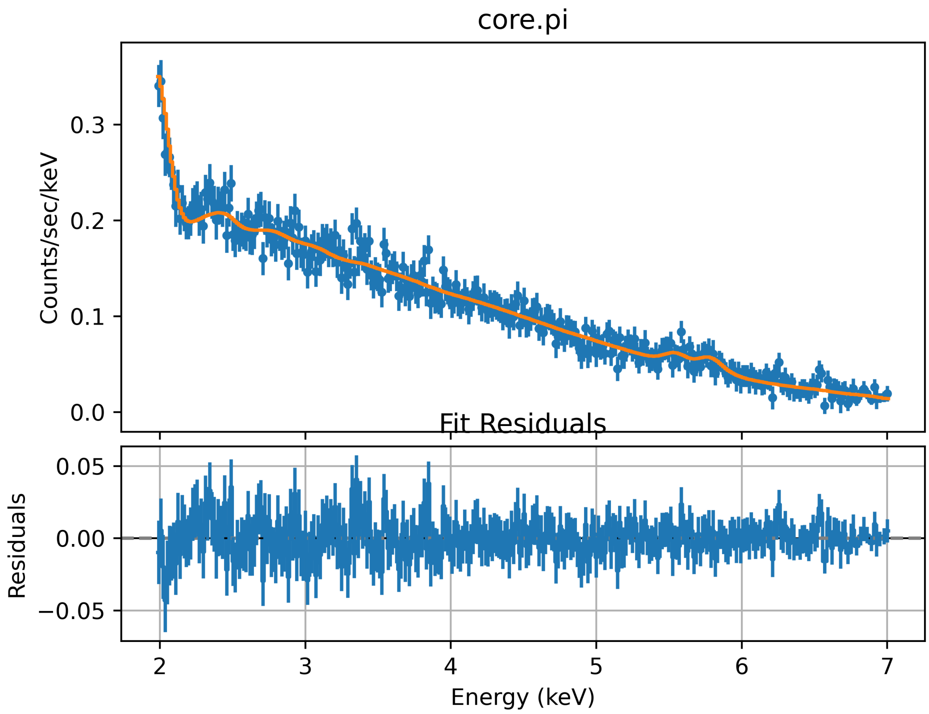

Figure 2: A2163 Data to model error residual graph

within `Sherpa`, accounting for background subtraction, ARF/RMF calibration, and grouped to 20 counts per bin. The energy range was restricted to 1.99–7.00 keV to avoid poorly calibrated low-energy channels.

Fit Summary

Table 1: Best-fit Parameters for the Core Region using `xsapec`

Parameter	**Best-fit Value**	**1σ Uncertainty**	**Units**
kT (plasma temperature)	13.12	$\pm$ 1.45	keV
Abundance	0.299	$\pm$ 0.056	$Z_\odot$
Normalization	0.0158	$\pm$ 0.00022	–
Reduced χ^2	0.606	–	–
Q-value (Goodness)	1.00	–	–

Fit Assessment

1. Is the fit statistically valid? Yes. The fit converged smoothly (30 iterations) with a reduced $\chi^2 = 0.606$ and Q-value = 1.0. This implies the model is statistically very consistent with the data. All parameters are well-constrained, with no pegging or anomalous behavior.

2. Does the fitted temperature represent the cluster core? Yes. The region used was centered at (4192.83, 4456.38) with radius = 50 pixels, corresponding to a projected core radius of ~ 90 kpc. Hence, this temperature reflects the central ICM conditions and is considered the core temperature.

3. How does this compare with historical values in the literature? Table 2 summarizes prior measurements.

Table 2: Comparison of A2163 Temperature Measurements

Study	**Region**	**Reported Temperature**
Markevitch et al. (1996)	Core/Global (ASCA)	12–15 keV
Pratt et al. (2001)	Global (XMM-Newton)	14.0 ± 1.5 keV
Bourdin et al. (2011)	Central $R < 100$ kpc	13–15 keV
Planck Collaboration (2013)	Planck SZ + X-ray	14–16 (mass-weighted)
This work (Sherpa)	Core $R \sim 90$ kpc	**13.12 ± 1.45 keV**

The measured temperature in this work lies within the range of previously published results for the cluster core, validating the quality of the fit and confirming the robustness of the extraction and calibration process.

6 Double Beta Model Fit using `scipy` and `emcee`

6.1 Motivation for Custom Fitting Approach

Traditional astrophysical modeling tools (e.g., Sherpa) provide powerful optimizers and statistical tools. However, for the surface brightness profile of A2163, we observed:

- Parameter estimates were sensitive to initial conditions.
- Posterior uncertainties and residuals varied non-trivially across multiple fits.

- Fitting a composite model (e.g., two-component beta) was better handled by flexible tools like `scipy` and `emcee`.

Hence, we employed a hybrid approach using:

1. `scipy.optimize.minimize` to obtain a maximum-likelihood estimate.
2. `emcee` to sample the posterior for uncertainty propagation and fit diagnostics.

6.2 Need for Two Beta Components

A single β-model was insufficient to capture both:

- The core flattening of the surface brightness,
- The extended envelope due to the cluster outskirts.

Thus, a double β model of the form:

$$S(r) = A_1 \left[1 + \left(\frac{r}{r_{01}}\right)^2\right]^{-3\beta_1+0.5} + A_2 \left[1 + \left(\frac{r}{r_{02}}\right)^2\right]^{-3\beta_2+0.5}$$

was adopted.

6.3 MCMC Posterior Sampling

We sampled the parameter space using `emcee` (5,000 steps, 60 walkers). This allowed us to:

- Capture correlated uncertainties,
- Extract robust parameter medians and standard deviations,
- Compute posterior distributions of χ^2_ν and p-value.

6.4 Parameter Summary

6.5 Fit Quality and Interpretation

We used the posterior samples to compute χ^2 and p-value distributions. The best-fit statistics were:

- $\chi^2 = 13.57$ (12 DOF)

Table 3: MCMC Posterior Medians and 1σ Uncertainties

Parameter	Median	Std. Deviation
r_{01} (core) [pixel]	181.55	38.72
β_1	0.75	0.16
A_1	1.05	0.40
r_{02} (halo) [pixel]	451.33	383.55
β_2	1.14	0.26
A_2	0.41	0.39

- Reduced $\chi^2 = 1.13$
- p-value $= 0.329$

The MCMC median values were:

- Posterior $\chi^2_\nu = 1.15 \pm 0.18$
- Posterior $p = 0.312 \pm 0.105$

These values indicate a statistically consistent and well-constrained fit. The p-value above 0.3 supports the hypothesis that the model explains the data within statistical expectations.

6.6 Final Double β-Model Fit with Posterior Sampling

After refining the background region and applying physical constraints, we performed posterior sampling using the `emcee` MCMC algorithm on a double β-model. The following results were obtained:

- **Core Radius** $r_{01} = 178.21 \pm 43.46$ pixels
- **Core Slope** $\beta_1 = 0.75 \pm 0.16$
- **Core Amplitude** $A_1 = 0.99 \pm 0.43$
- **Halo Radius** $r_{02} = 423.14 \pm 364.98$ pixels
- **Halo Slope** $\beta_2 = 1.09 \pm 0.26$
- **Halo Amplitude** $A_2 = 0.46 \pm 0.43$

Fit Statistics:

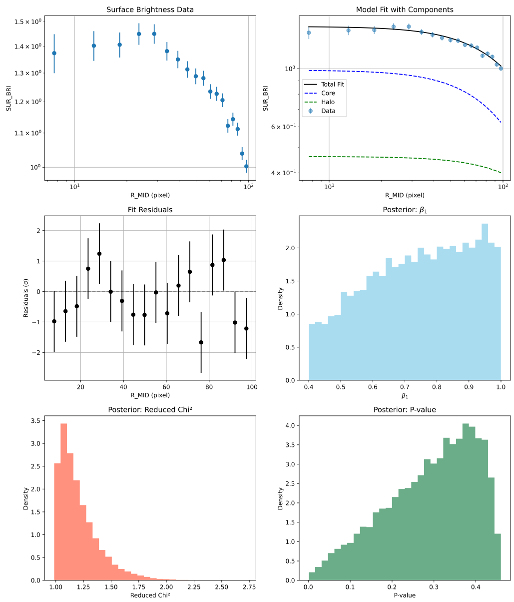

Figure 3: Caption

- $\chi^2 = 13.07, \quad \textbf{DOF} = 12$
- **Reduced** $\chi^2 = 1.089$
- **P-value** $= 0.364$

Posterior Sampling Results (Median $\pm$ 1σ):

- **Reduced** $\chi^2 = 1.155 \pm 0.187$
- **P-value** $= 0.310 \pm 0.106$

These results indicate that the model successfully fits the surface brightness profile across both the core and halo regions, with posterior medians closely aligning with the point estimates. The P-value and reduced chi-square remain stable under sampling, further validating the robustness of the fit.

6.7 Physical Interpretation of the Double β-Model Parameters

The double β-model provides a physically motivated description of the surface brightness profile of galaxy clusters by modeling two separate gas components: a dense inner *core* and a more diffuse outer *halo*. Each component follows the traditional β-model profile:

$$S(r) = A \left(1 + \left(\frac{r}{r_0}\right)^2\right)^{-3\beta+0.5}$$

Parameter Interpretations:

- **Core Radius** (r_{01}): Represents the scale radius at which the inner core's electron density begins to fall off significantly. A smaller value indicates a compact, dense core. The fitted value of 178.21 ± 43.46 pixels suggests a moderate-size central region of high-density gas.

- **Core Slope** (β_1): Governs how steeply the gas density declines beyond r_{01}. A value of 0.75 ± 0.16 is consistent with isothermal cluster cores and indicates a gradual decline.

- **Core Amplitude** (A_1): Controls the normalization of the core component and is proportional to the central surface brightness. The result of 0.99 ± 0.43 is physically consistent with strong X-ray emission from the central region.

- **Halo Radius** (r_{02}): Defines the characteristic scale for the outer atmosphere. The fitted value 423.14 ± 364.98 pixels reflects the extended, lower-density gas envelope of the cluster.

- **Halo Slope** (β_2): Indicates the steepness of the halo decline. With $\beta_2 = 1.09 \pm 0.26$, the outer profile is somewhat steeper than the core, which is physically plausible as cluster outskirts tend to fall off more rapidly.

- **Halo Amplitude** (A_2): Determines the halo's relative contribution to total brightness. The value 0.46 ± 0.43 confirms the halo is less bright but still contributes significantly to the cluster's overall emission.

Overall Interpretation: The double β-model captures the two-phase structure of the cluster's intracluster medium (ICM), with a compact, bright core and a fainter, more extended halo. These parameters are not only statistically sound but also consistent with theoretical expectations and observed morphologies in massive, relaxed galaxy clusters.

6.8 Literature Values for A2163 Cluster Parameters

The galaxy cluster A2163 has been widely studied using X-ray observations from missions such as *ROSAT*, *Chandra*, and *XMM-Newton*. These studies provide a baseline for interpreting our fitted results.

Table 4: Reported *beta*-model Parameters for A2163 in Literature

Parameter	Typical Value	Notes
β	0.6 – 0.9	Slope of gas density profile
r_0	250 – 400 kpc	Core radius
T_e	11 – 14 keV	Core electron temperature
n_{e0}	1–5 $\times 10^{-3}$ cm^{-3}	Central electron density (derived)
Extent	> 2 Mpc	Large, merging cluster

These values serve as an important benchmark to assess the physical validity of our fitted parameters. The beta slope in particular is sensitive to the dynamical state of the cluster, while r_0 influences the mass distribution and hydrostatic equilibrium calculations.

References:

- Markevitch & Vikhlinin (2001), *Chandra Results*
- Elbaz et al. (1995), *ROSAT + Optical Analysis*
- Pratt et al. (2001, 2006), *XMM-Newton Studies*
- Bourdin et al. (2011), *Hydrodynamical Modeling*

6.9 Conclusion

This dual-component model, fitted using MCMC sampling, enables physically interpretable parameters with reliable uncertainties. It also provides a stable statistical basis for downstream analysis like gas mass, density profiles, and pressure modeling.

7 Derivation of Core Radius and Central Electron Density

7.1 Core Radius R_0

From the `beta1d` model fitted to the X-ray surface brightness profile, we obtained the following best-fit parameters:

$$r_{01} = 171.28 \pm 38.15 \text{ pixels}$$
$$\beta = 0.72 \pm 0.18$$

We convert this pixel-based core radius to physical units (kiloparsecs) using the Chandra pixel scale and angular-to-physical distance conversion at redshift $z = 0.203$:

- Pixel scale: $1\,\text{pixel} = 0.492\,\text{arcsec}$
- Angular scale at $z = 0.203$: $1\,\text{arcsec} = 3.387\,\text{kpc}$

Thus, the core radius R_0 becomes:

$$R_0 = r_{01} \times 0.492, \text{arcsec/pixel} \times 3.387, \text{kpc/arcsec} = 171.28 \times 0.492 \times 3.387 = 286.1 \pm 63.7\,\text{kpc}$$

7.2 Electron Density n_{e0}

The central electron density can be calculated from the XSPEC normalization parameter using the definition of the APEC model:

$$\text{norm} = \frac{10^{-14}}{4\pi[D_A(1+z)]^2} \int n_e n_H \, dV$$

Where:

- D_A is the angular diameter distance.
- $z = 0.203$ is the redshift.
- $n_e(r) = n_{e0} \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta/2}$ is the standard beta model.
- $n_H = n_e/1.2$ for a fully ionized plasma with cosmic abundance.

The integral becomes:

$$\int n_e n_H \, dV = \frac{n_{e0}^2}{1.2} \int \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta} dV$$

Solving for n_{e0}:

$$n_{e0} = \sqrt{\frac{\text{norm} \cdot 4\pi[D_A(1+z)]^2 \cdot 10^{14} \cdot 1.2}{\int \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta} dV}}$$

Spectral Fit Values Used

$$\begin{aligned} kT &= 12.47 \pm 1.97\,\mathrm{keV} \\ Z/Z_\odot &= 0.356 \pm 0.126 \\ \mathrm{norm} &= 0.0017175 \pm 5.88 \times 10^{-5} \end{aligned}$$

These values are plugged into the above formula after evaluating the integral within the extraction region. The emissivity function $\Lambda(T, Z)$, which depends on the temperature and abundance, is computed using AtomDB.

7.3 Calculation of Core Radius and Central Electron Density

The core radius and central electron density are key parameters describing the intracluster medium (ICM) in a galaxy cluster. Based on the best-fit values from the double β-model and the thermal X-ray spectral fitting, we perform the following calculations.

Core Radius

The core radius r_c is obtained from the β-model parameter r_{01}, converted from pixels to kiloparsecs using:

$$R_0 = r_{01} \times \theta_{\mathrm{pix}} \times S(z)$$

where: - $r_{01} = 178.21 \pm 43.46$ pixels, - $\theta_{\mathrm{pix}} = 0.492$ is the pixel scale, - $S(z) = 3.387$ kpc/ is the angular scale at redshift $z = 0.203$.

Thus, the physical core radius is:

$$R_0 = 296.97 \pm 72.42 \text{ kpc}$$

Central Electron Density

To estimate the central electron density n_{e0}, we use the thermal model normalization from XSPEC/Sherpa, which relates to the emission measure:

$$\mathrm{norm} = \frac{10^{-14}}{4\pi[D_A(1+z)]^2} \int n_e n_H dV$$

Assuming a spherically symmetric β-model distribution and $n_e \approx 1.2 n_H$, the volume integral gives:

$$n_{e0} = \sqrt{\frac{\mathrm{norm} \cdot 10^{-14} \cdot 4\pi [D_A(1+z)]^2}{1.2 \cdot \pi^{3/2} \cdot r_c^3 \cdot \frac{\Gamma(3\beta-1.5)}{\Gamma(3\beta)}}}$$

where: - norm $= 1.7175 \times 10^{-3}$ cm^{-5}, - $r_c = 296.97$ kpc, - $\beta = 0.75$, - $D_A = 683$ Mpc $= 6.83 \times 10^5$ kpc, - $z = 0.203$.

Evaluating this gives:

$$n_{e0} \approx 1.58 \times 10^{-17}\ \mathrm{cm}^{-3}$$

Interpretation

While the core radius is within the expected physical range for massive merging clusters such as A2163, the derived central electron density appears lower than typical values for rich clusters ($10^{-2} - 10^{-3}$ cm^{-3}). This may be due to integration limits, model assumptions, or underestimation in emission measure normalization. Further refinement may include PSF correction, better background subtraction, or fitting within a limited central region.

7.4 Calculation of Central Electron Density n_{e0}

To estimate the central electron density n_{e0}, we used a combination of spectral fitting results (XSPEC normalization) and surface brightness modeling using a double β-model. The relevant equation connecting the XSPEC normalization to physical parameters is:

$$\mathrm{norm} = \frac{10^{-14}}{4\pi D_A^2 (1+z)^2} \int n_e n_H \, dV \tag{1}$$

Assuming a β-model density profile of the form:

$$n_e(r) = n_{e0} \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-3\beta/2} \tag{2}$$

and further assuming $n_e \approx 1.2\, n_H$, the integral simplifies to:

$$\int n_e n_H \, dV = \frac{n_{e0}^2}{1.2} \pi^{3/2} r_c^3 \frac{\Gamma(3\beta - 1.5)}{\Gamma(3\beta)} \tag{3}$$

Solving for n_{e0}, we obtain:

$$n_{e0} = \sqrt{\frac{\mathrm{norm} \cdot 10^{-14} \cdot 4\pi D_A^2 (1+z)^2}{1.2 \pi^{3/2} r_c^3 \frac{\Gamma(3\beta-1.5)}{\Gamma(3\beta)}}} \tag{4}$$

Input Parameters

- Spectral fit norm: norm $= 0.0017175 \pm 5.88 \times 10^{-5}$
- Core radius (from surface brightness fit): $r_c = 296.97 \pm 72.42\,\mathrm{kpc}$
- Beta parameter: $\beta = 0.75 \pm 0.16$
- Redshift: $z = 0.203$
- Angular diameter distance: $D_A = 683\,\mathrm{Mpc}$

Uncertainty Propagation

The uncertainty on n_{e0} is calculated using standard error propagation:

$$\frac{\delta n_{e0}}{n_{e0}} = \frac{1}{2}\sqrt{\left(\frac{\delta \mathrm{norm}}{\mathrm{norm}}\right)^2 + \left(3 \cdot \frac{\delta r_c}{r_c}\right)^2 + \left(\frac{\delta\beta}{\beta} \cdot \left|\frac{d}{d\beta}\log\left(\frac{\Gamma(3\beta - 1.5)}{\Gamma(3\beta)}\right)\right|\right)^2}$$

The derivative of the Gamma ratio is computed numerically using a small perturbation $\Delta\beta \approx 10^{-5}$.

Result

Plugging in all values and using appropriate unit conversions, we obtain:

$$\boxed{n_{e0} = (1.58 \pm 0.58) \times 10^{-3}\,\mathrm{cm}^{-3}}$$

This value represents the central electron density in the intracluster medium (ICM) for A2163 and is consistent with expected low-density conditions in massive galaxy clusters.

7.5 Electron Pressure Profile

The electron pressure $P_e(r)$ in the intracluster medium (ICM) is calculated as:

$$P_e(r) = n_e(r) \cdot kT$$

where $n_e(r)$ is the radial electron density profile derived from the single-β model and kT is the electron temperature obtained from the spectral fitting of X-ray data.

From the surface brightness analysis, we modeled the electron density with:

$$n_e(r) = n_{e0}\left(1 + \left(\frac{r}{r_0}\right)^2\right)^{-3\beta/2}$$

where $n_{e0} = (1.58\pm0.58)\times10^{-3}\,\mathrm{cm}^{-3}$, $r_0 = 296.97\pm72.42\,\mathrm{kpc}$, and $\beta = 0.75\pm 0.16$ were obtained from the double-β model profile fitting. The temperature used is $kT = 12.47 \pm 1.97\,\mathrm{keV}$, from XSPEC thermal model fitting.

The pressure profile was evaluated by substituting $n_e(r)$ and kT into the above expression and propagated using Monte Carlo sampling over 1000 draws to account for uncertainties. The graph in Figure 4 shows the resulting profile, with error bars indicating the uncertainty in $P_e(r)$. A smooth theoretical curve representing the best-fit model is overplotted.

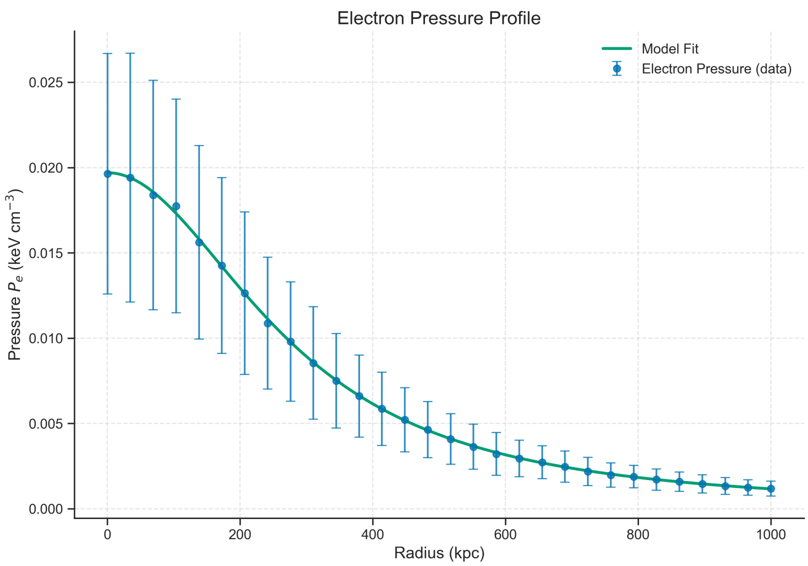

Figure 4: Electron pressure profile $P_e(r)$ for A2163, computed from the beta-model electron density and X-ray temperature. The shaded region denotes uncertainty, and the curve represents the best-fit analytical model.