Lecture 3

Divide and Conquer, Merge Sort, Solving Recurrences

Divide-and-conquer Method

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Combine: Use solutions to subproblems to create solution for the problem.

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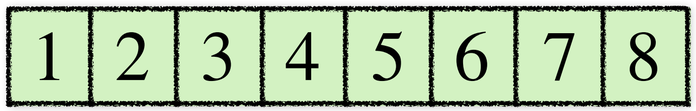

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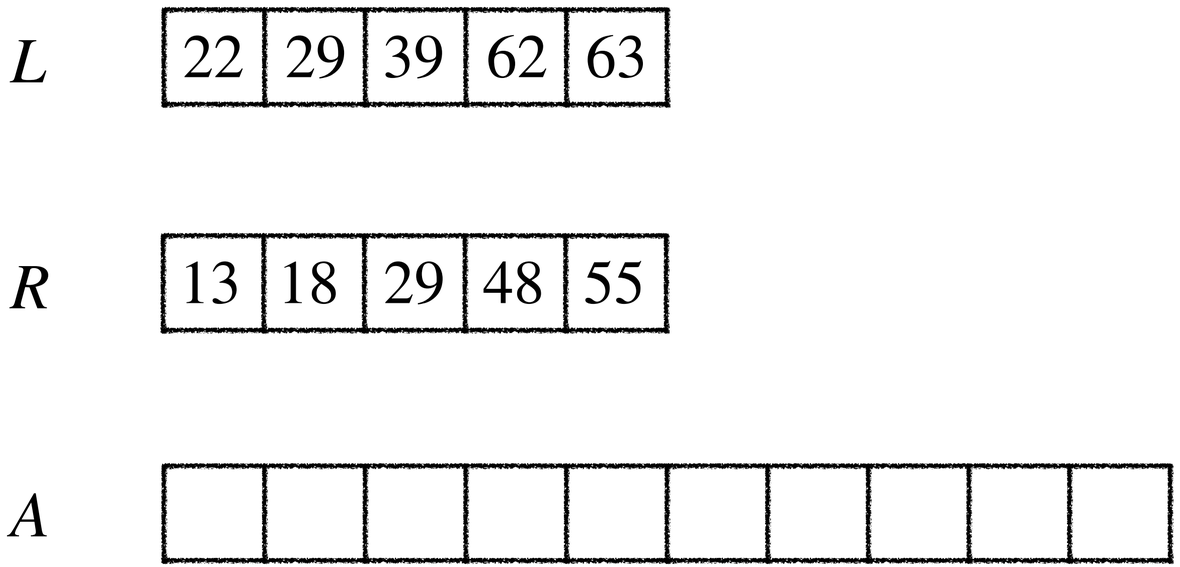

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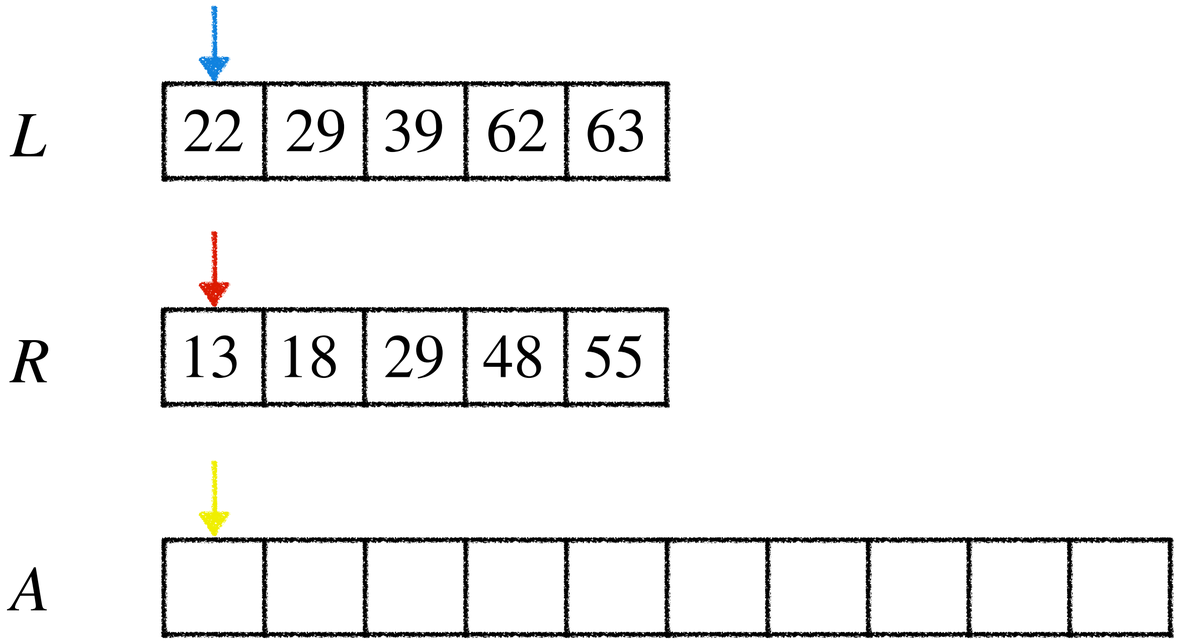

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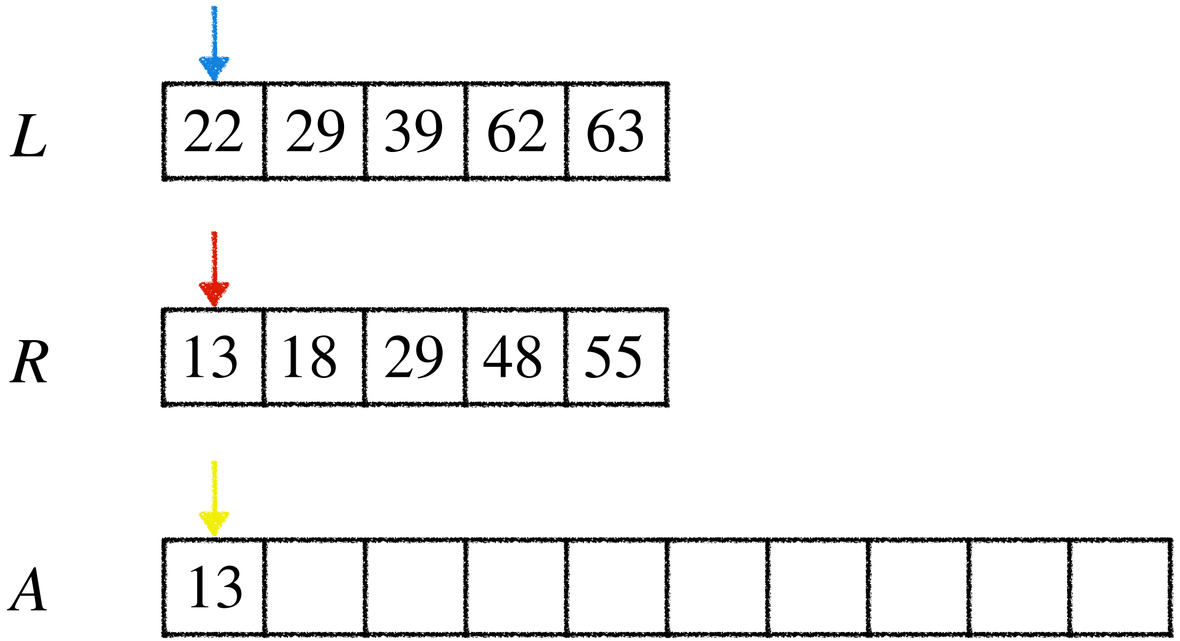

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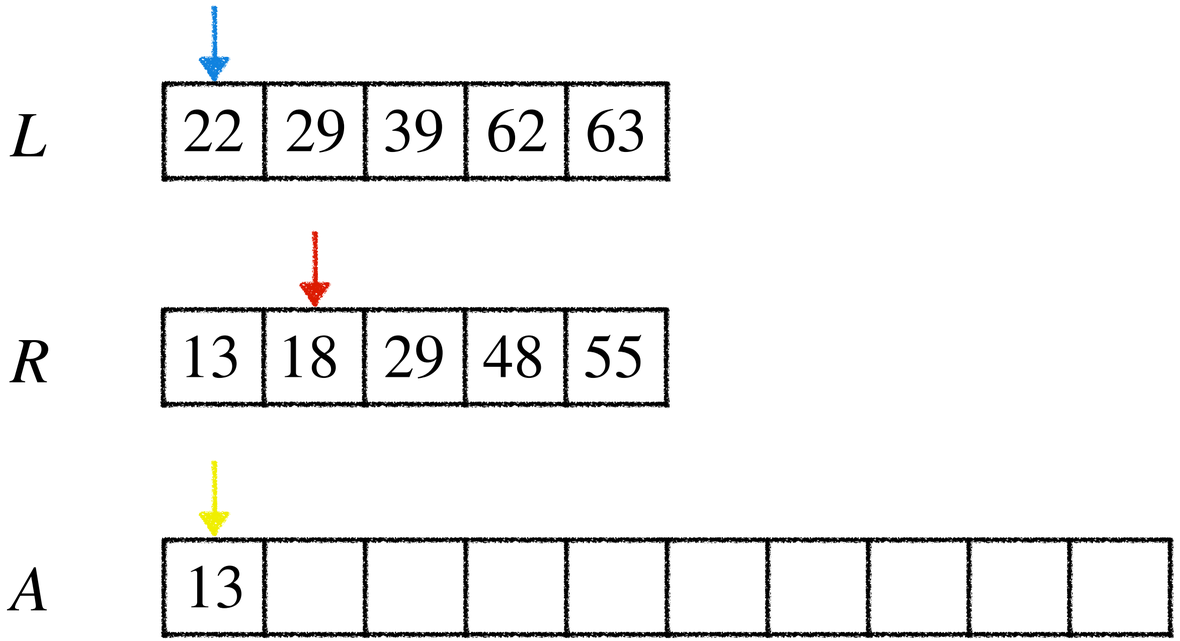

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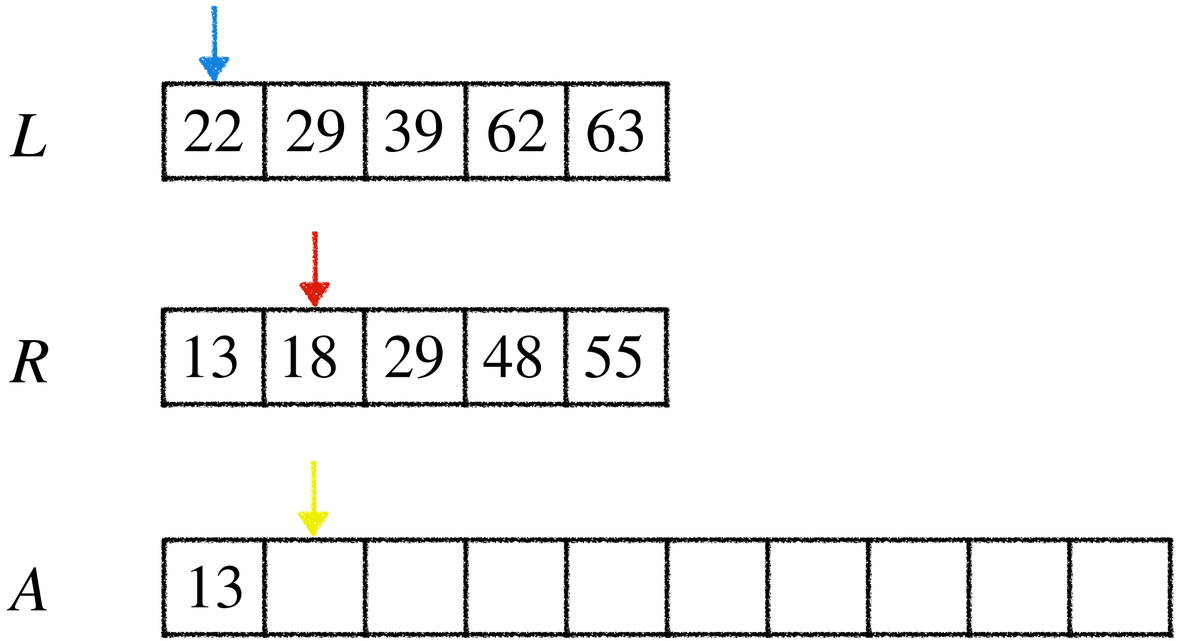

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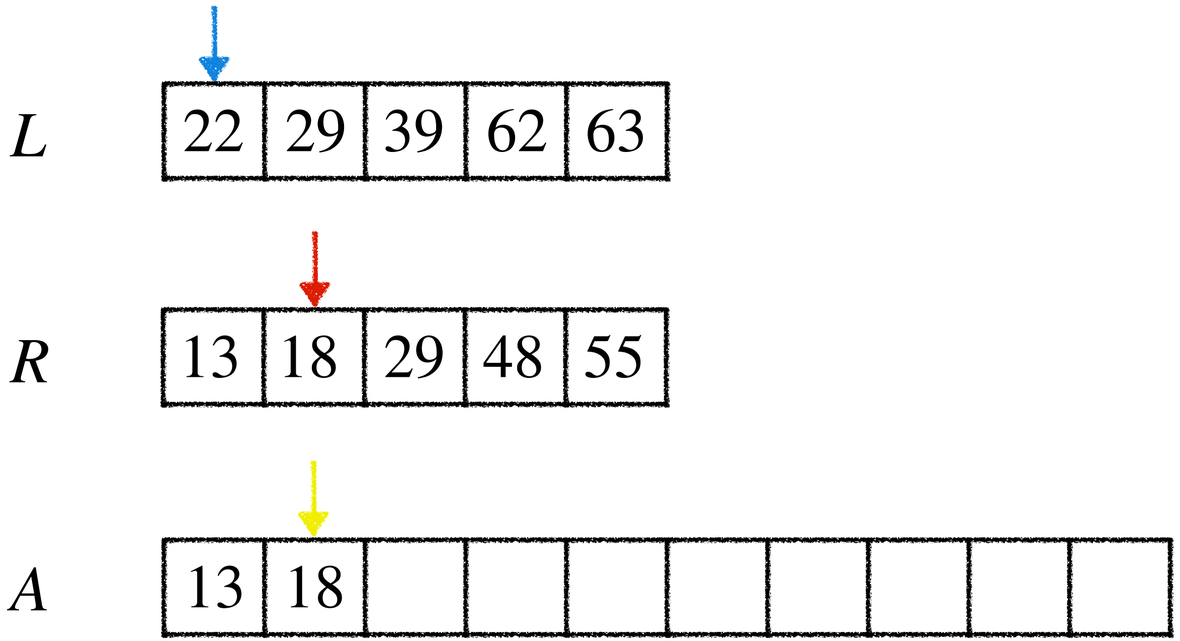

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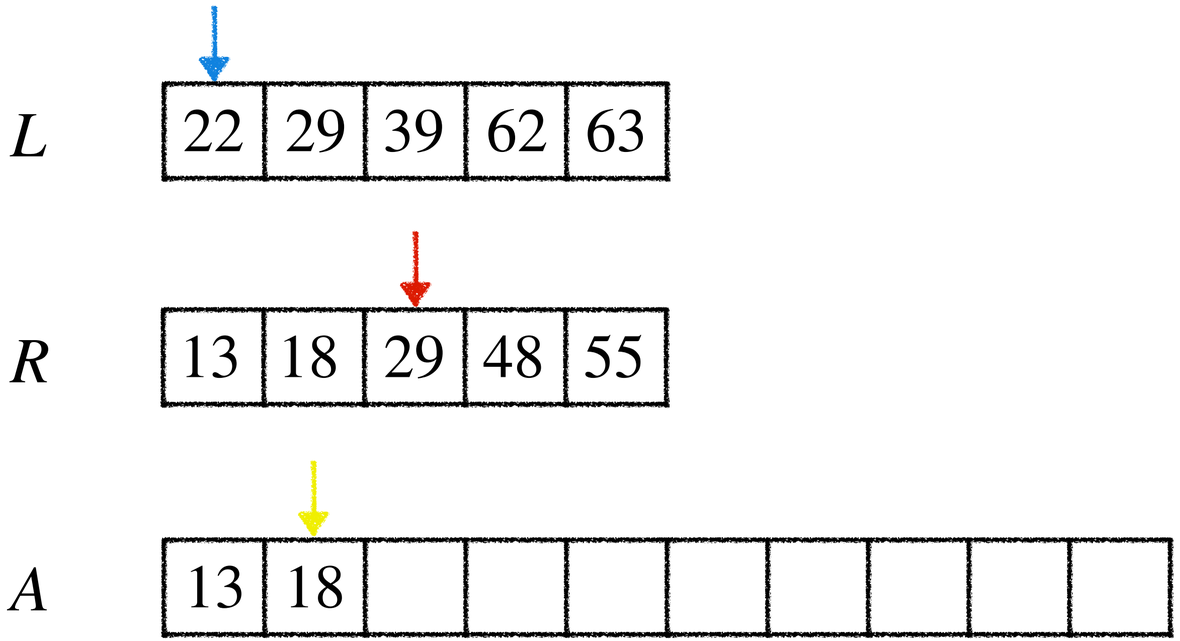

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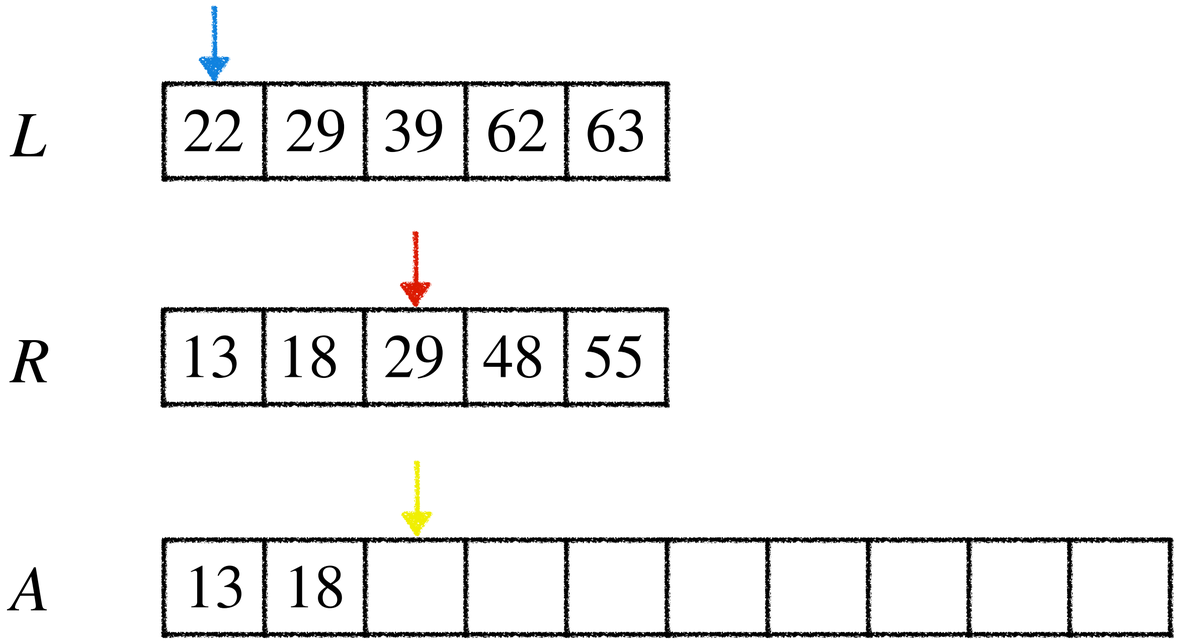

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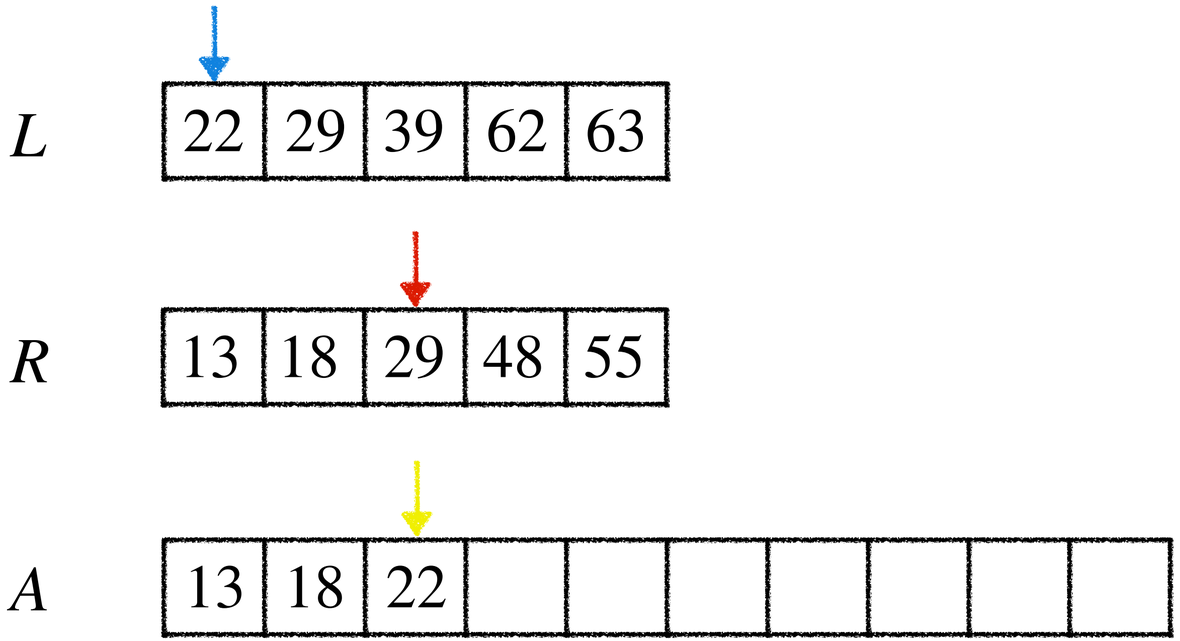

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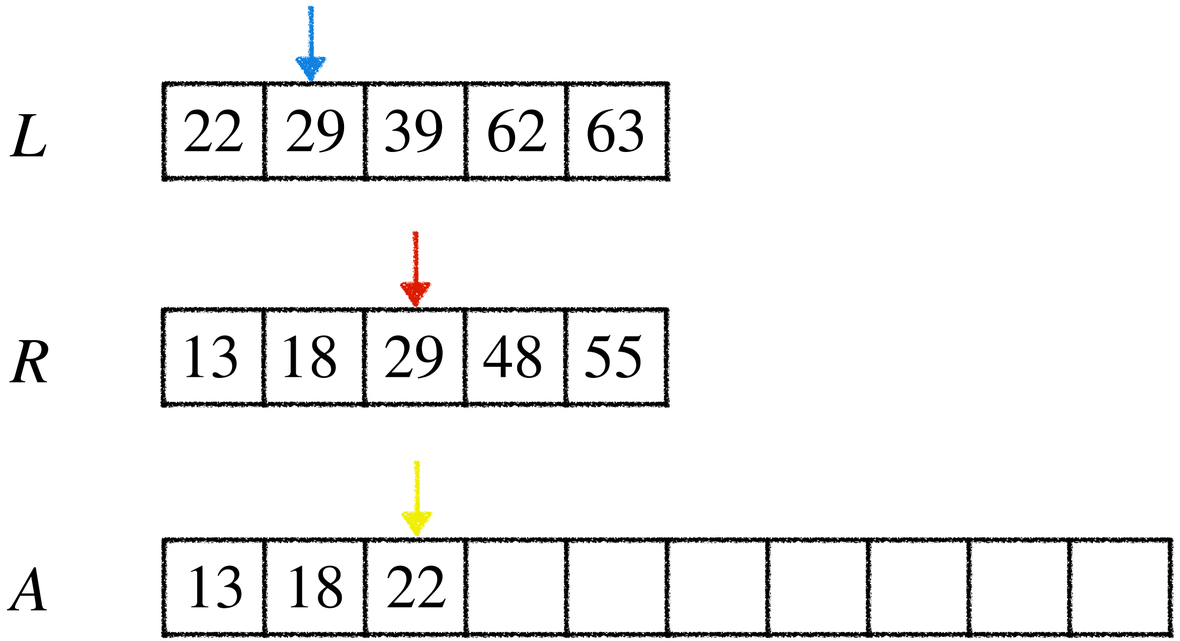

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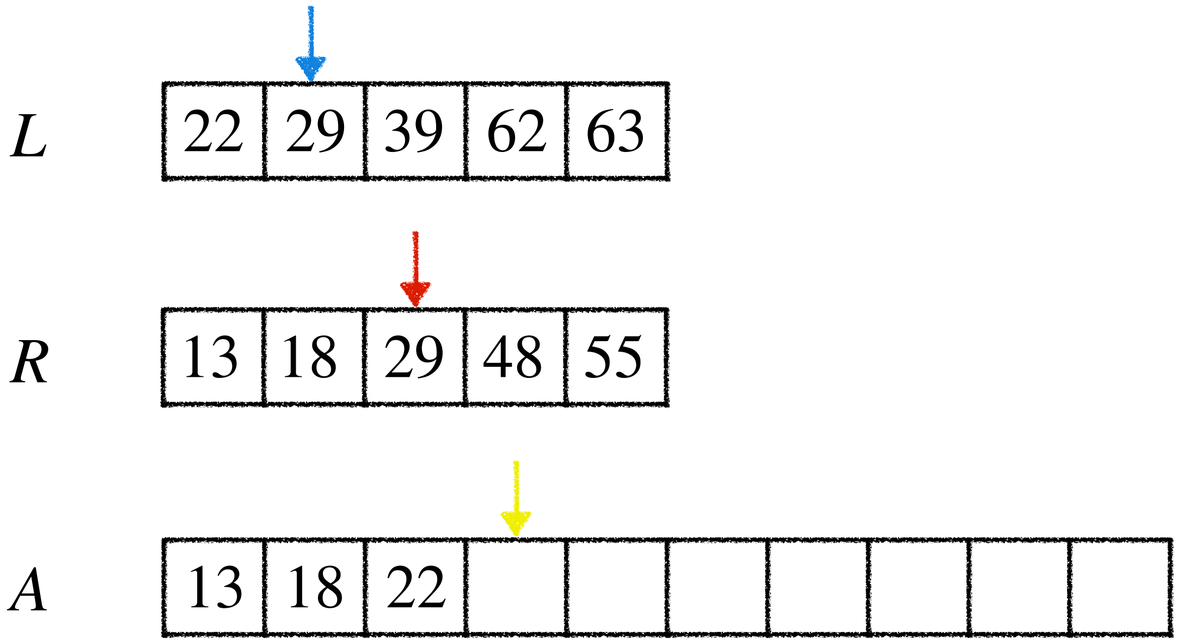

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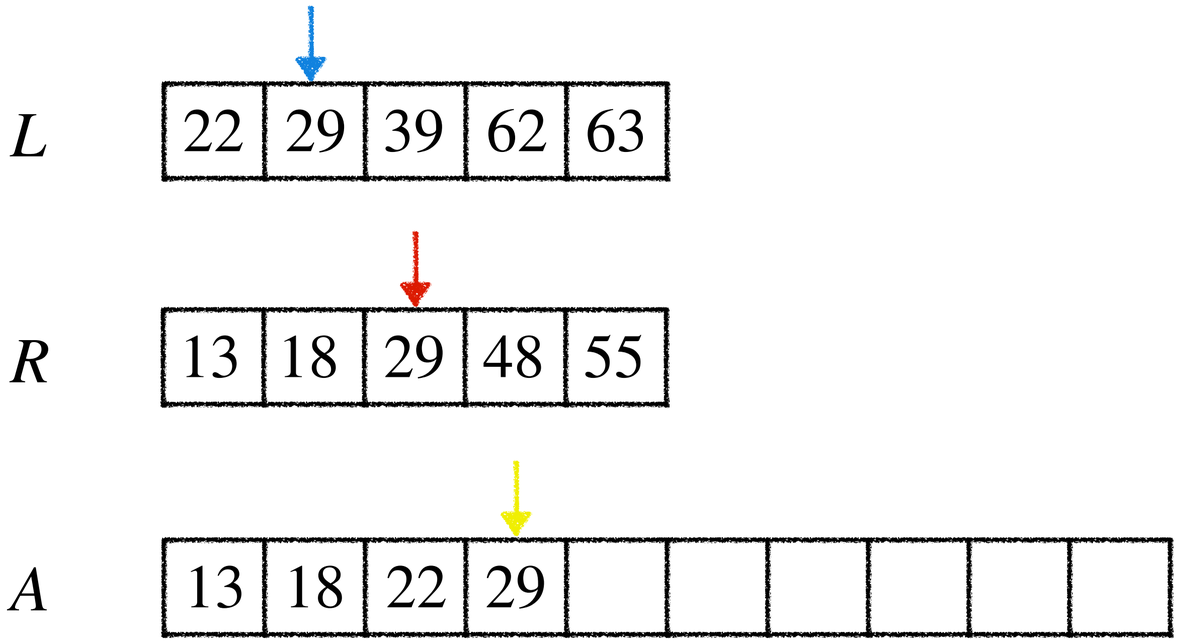

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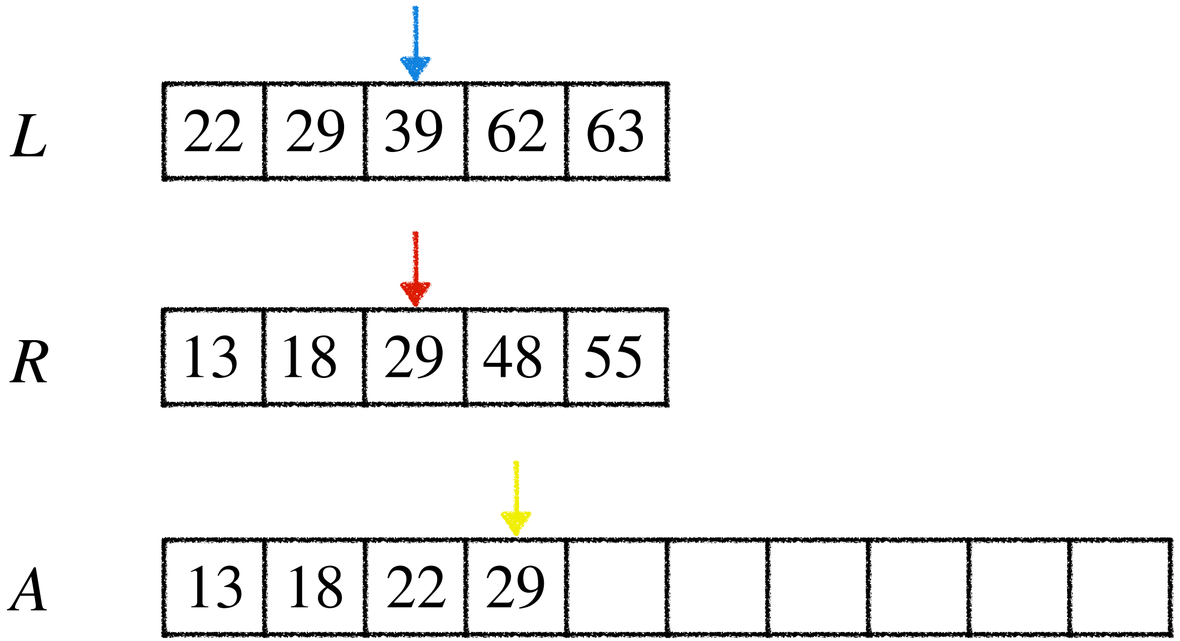

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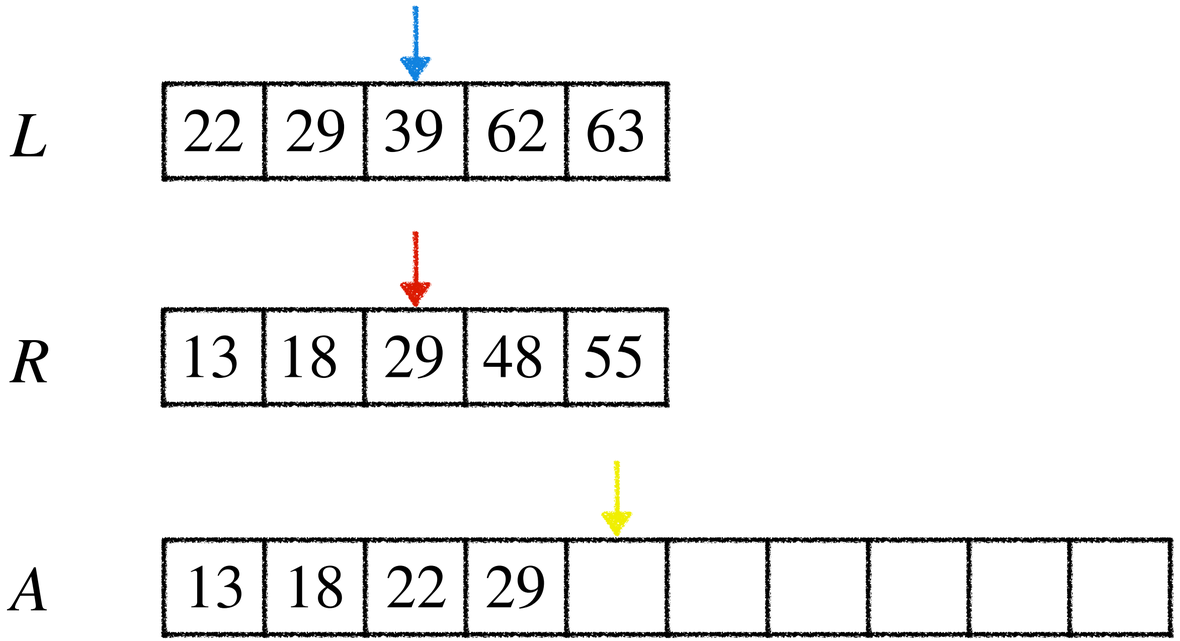

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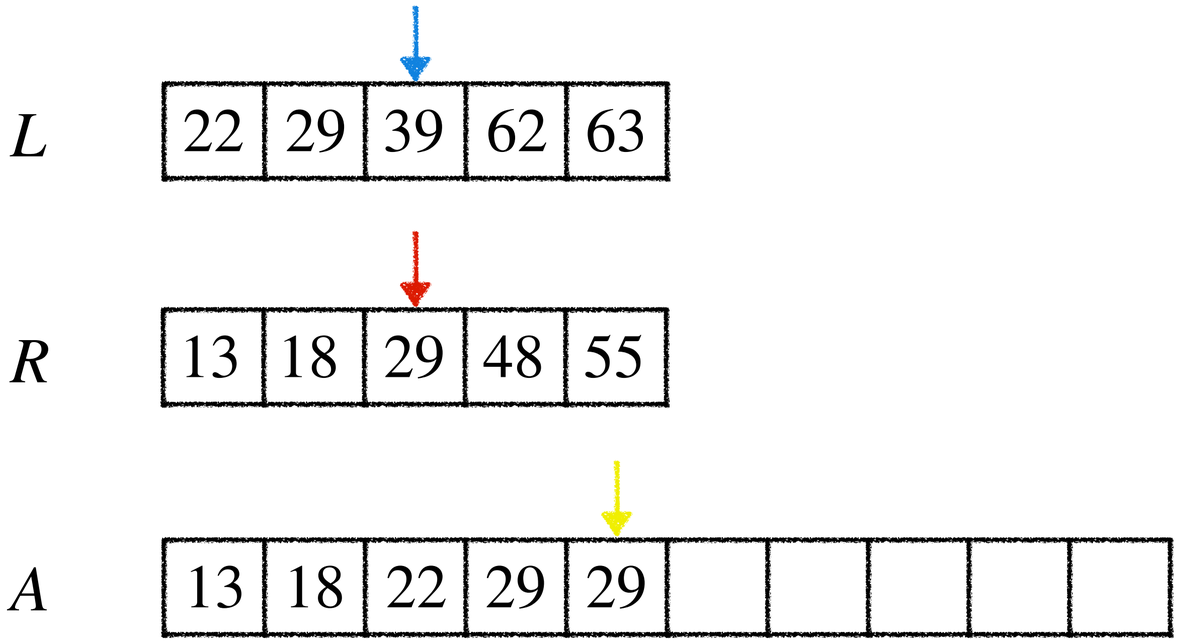

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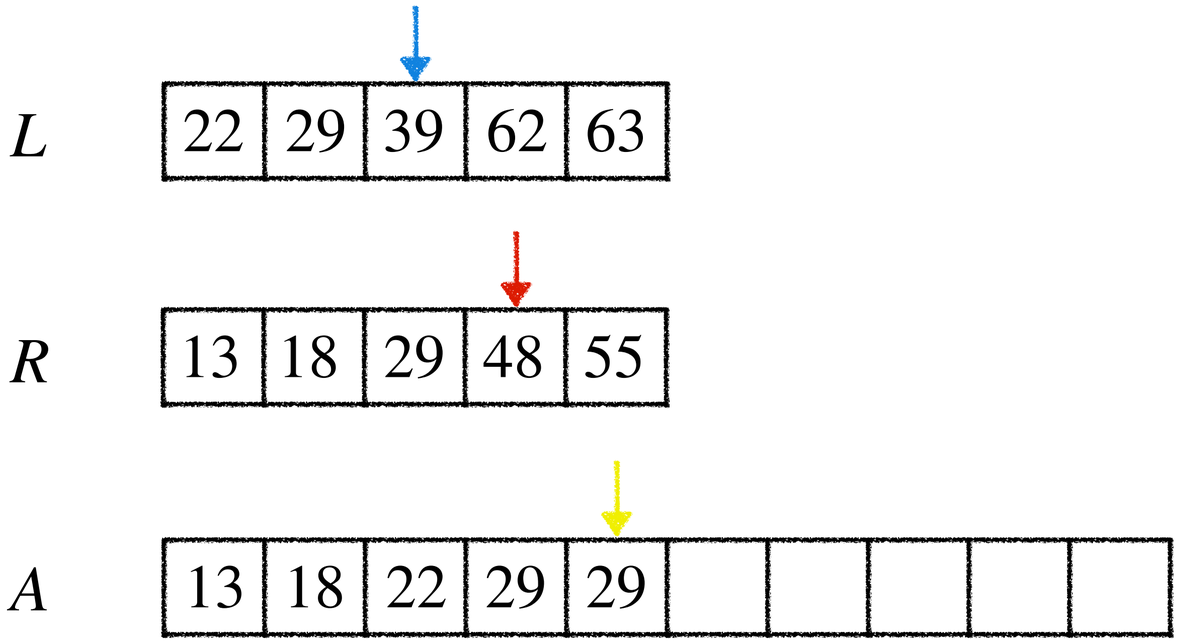

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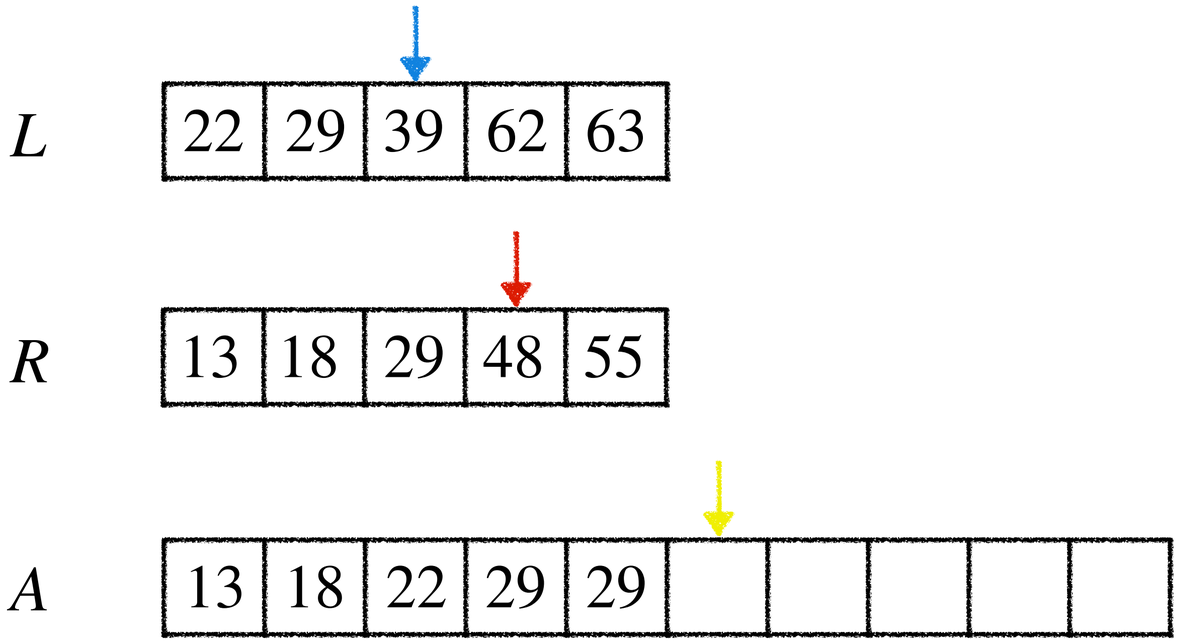

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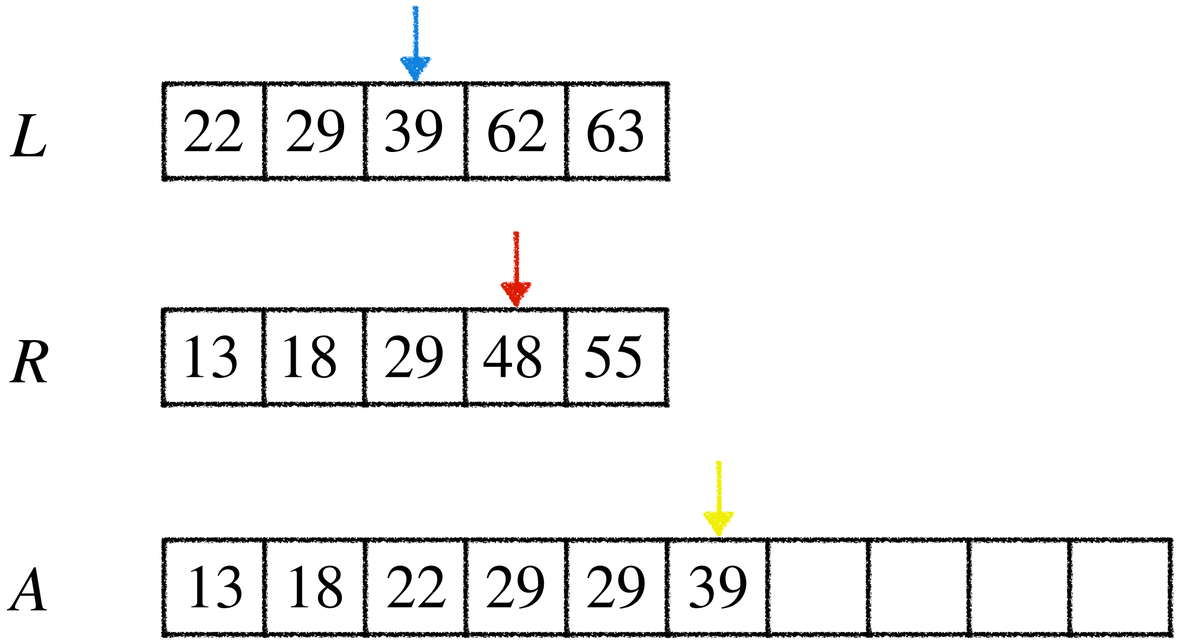

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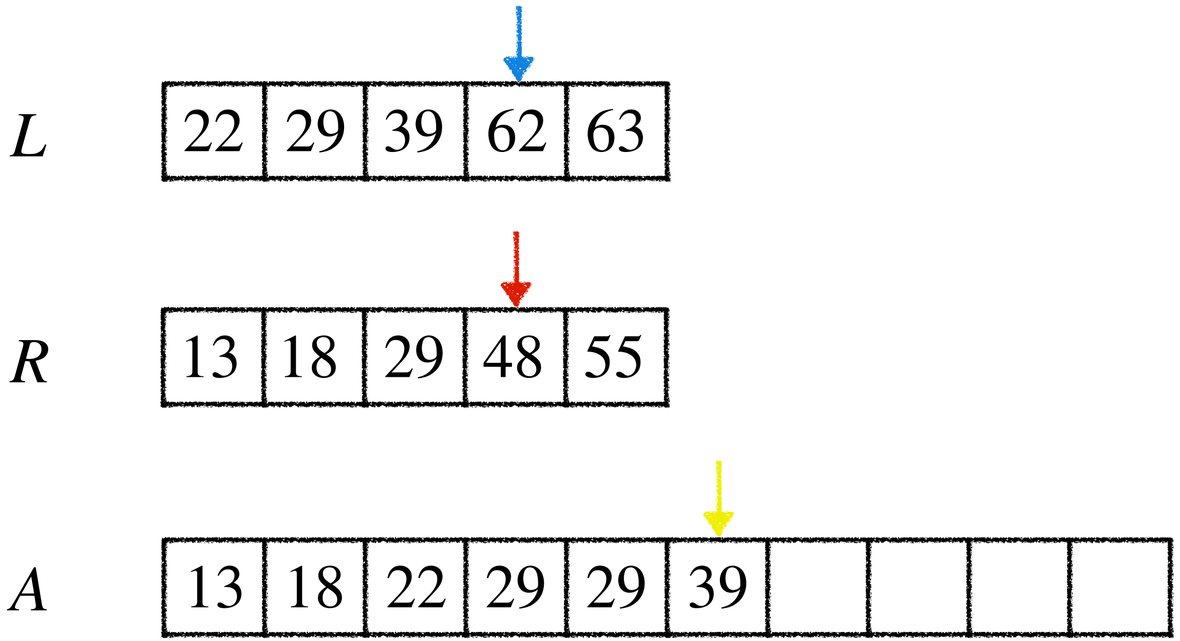

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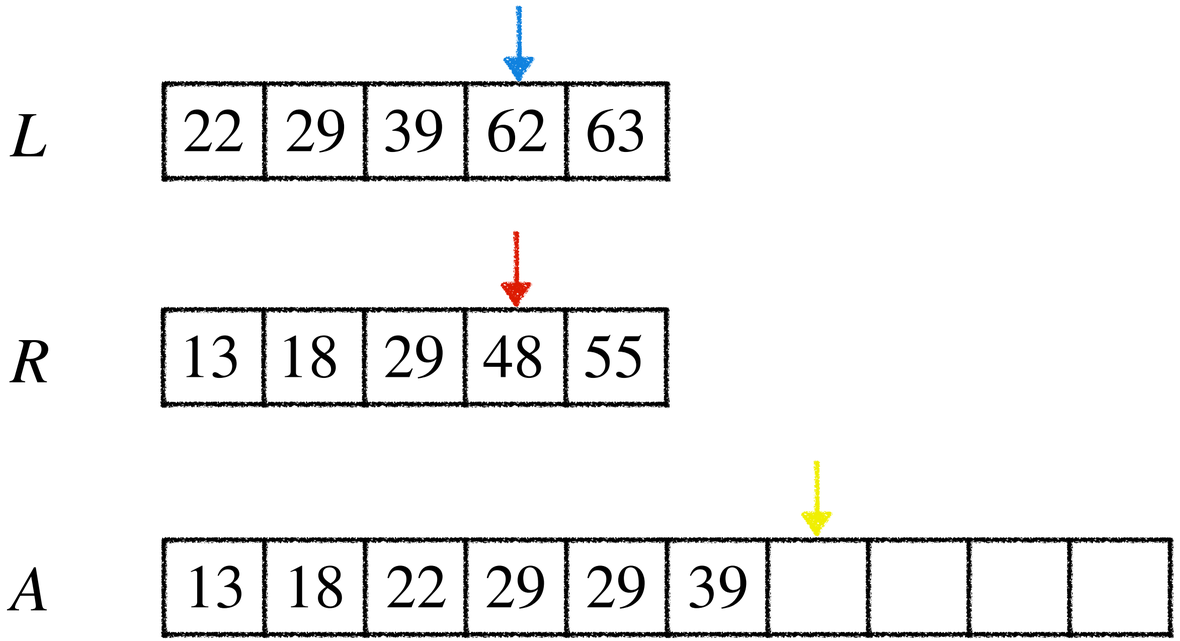

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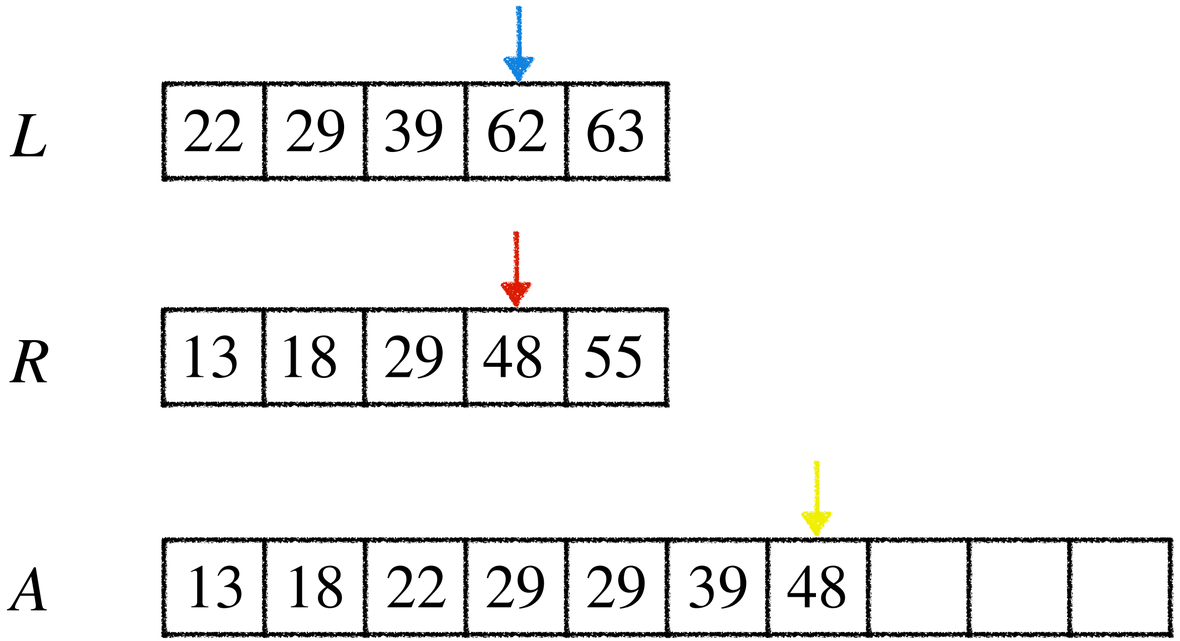

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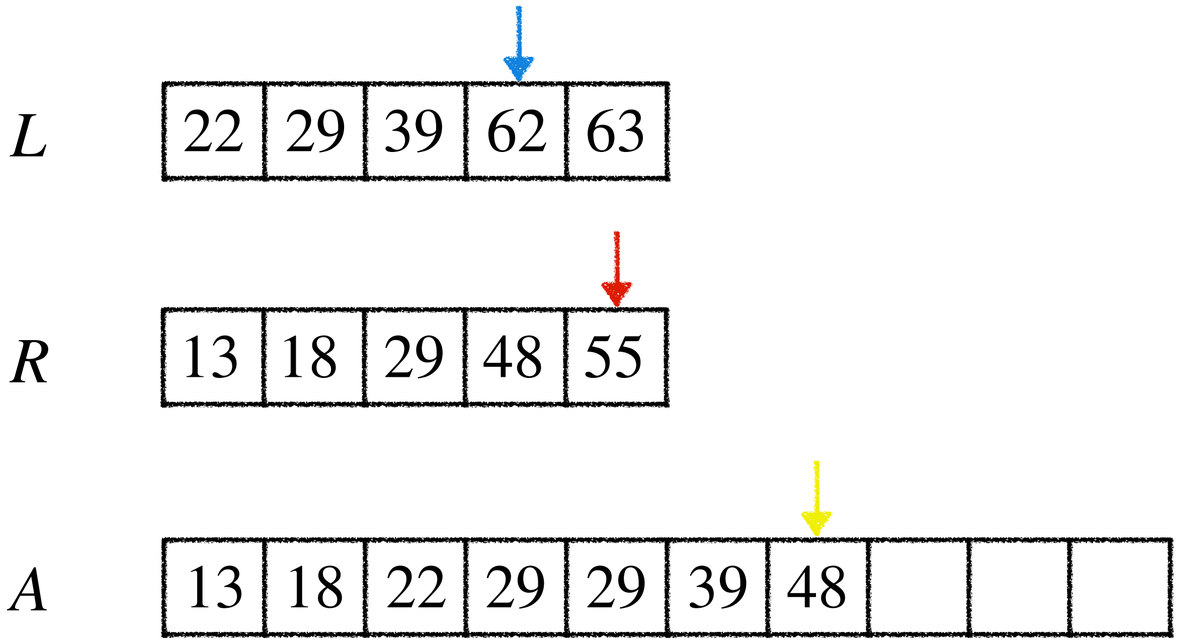

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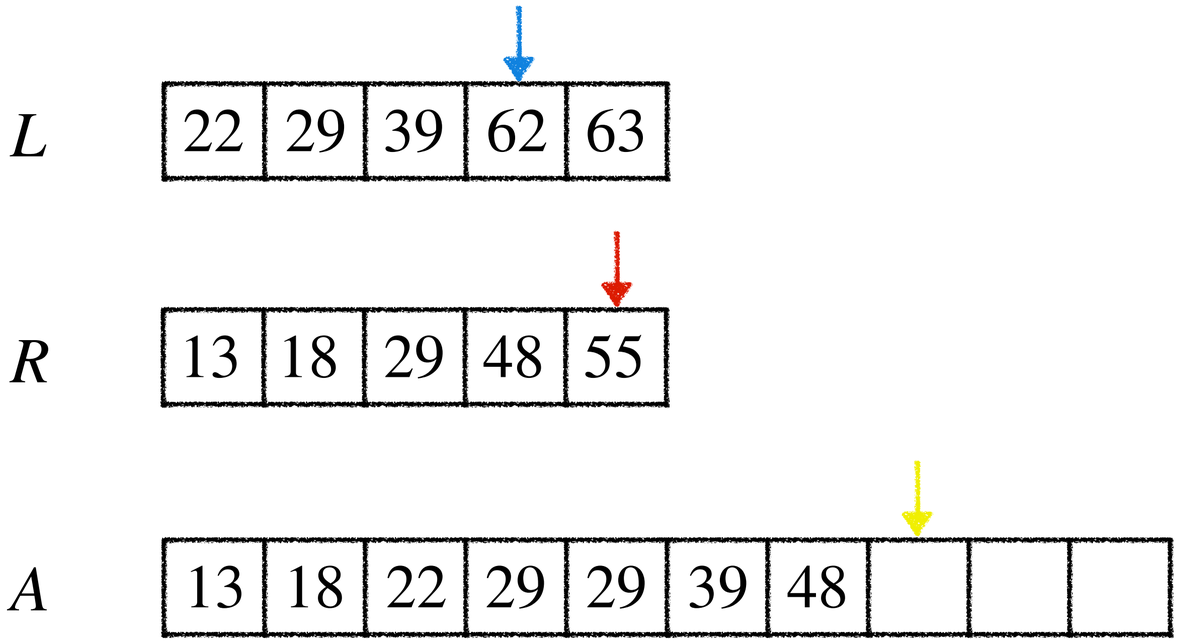

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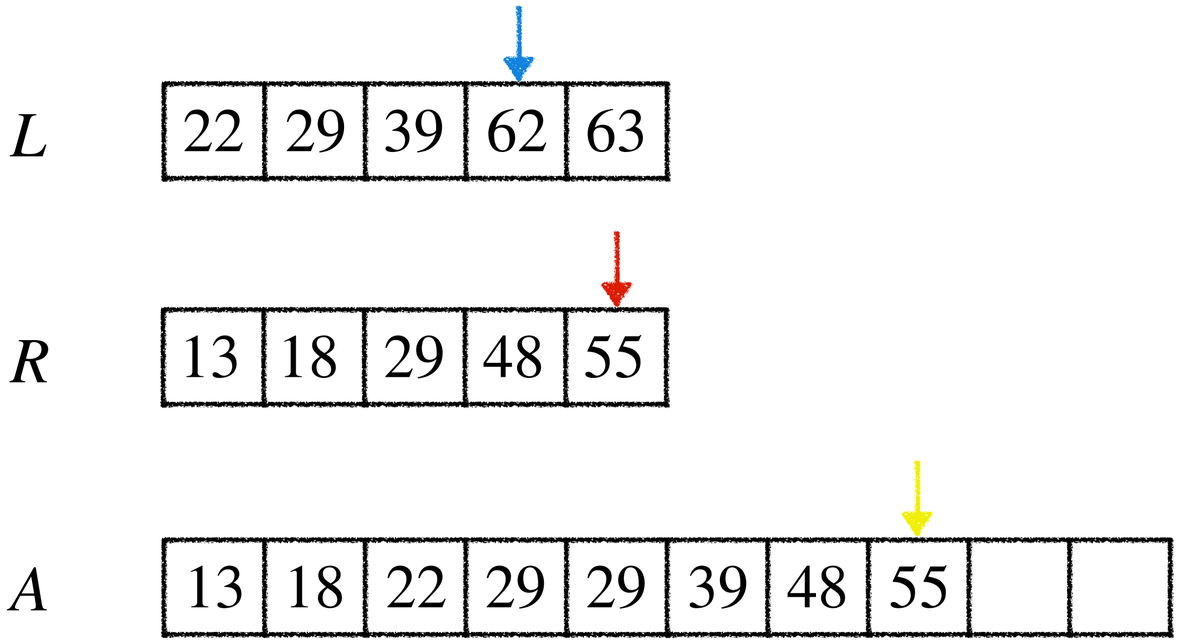

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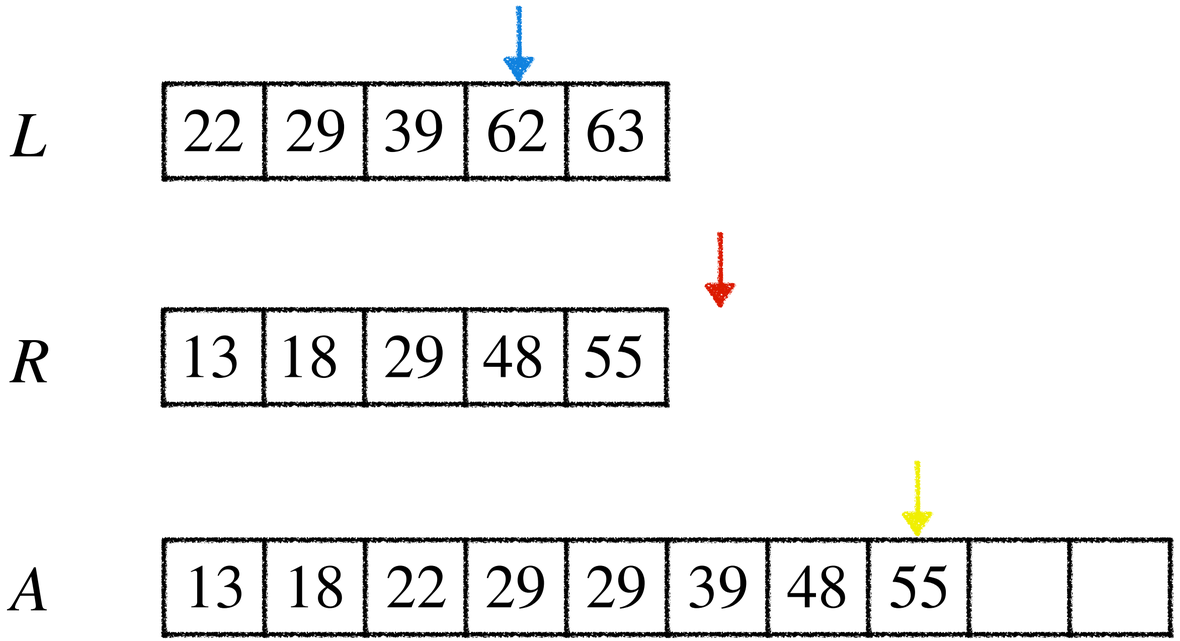

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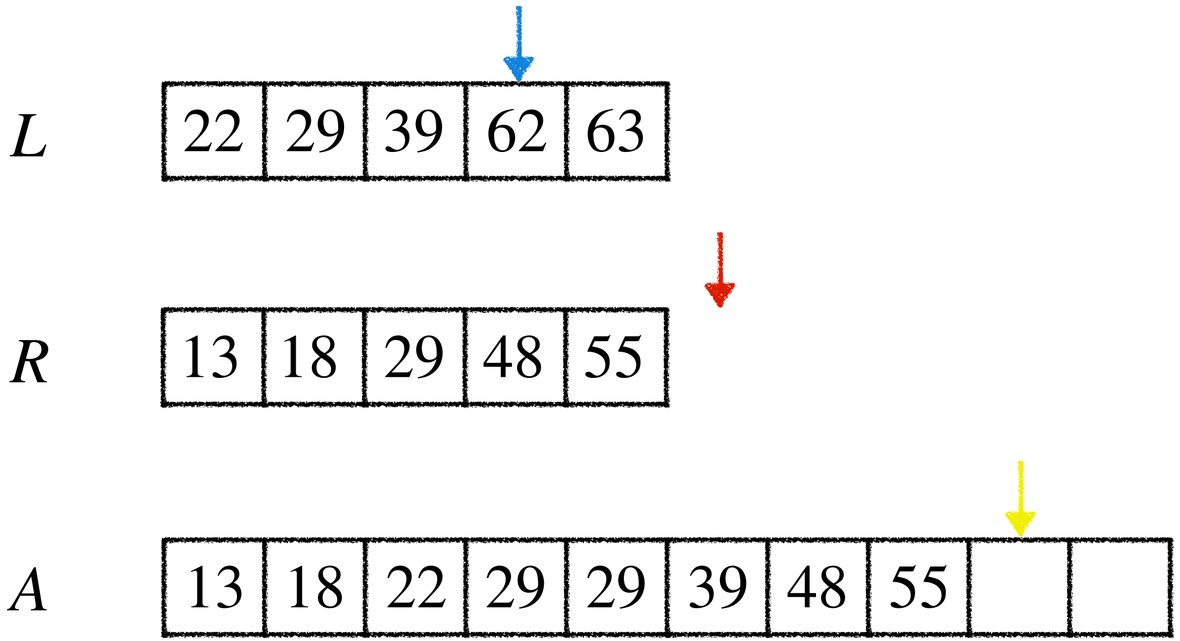

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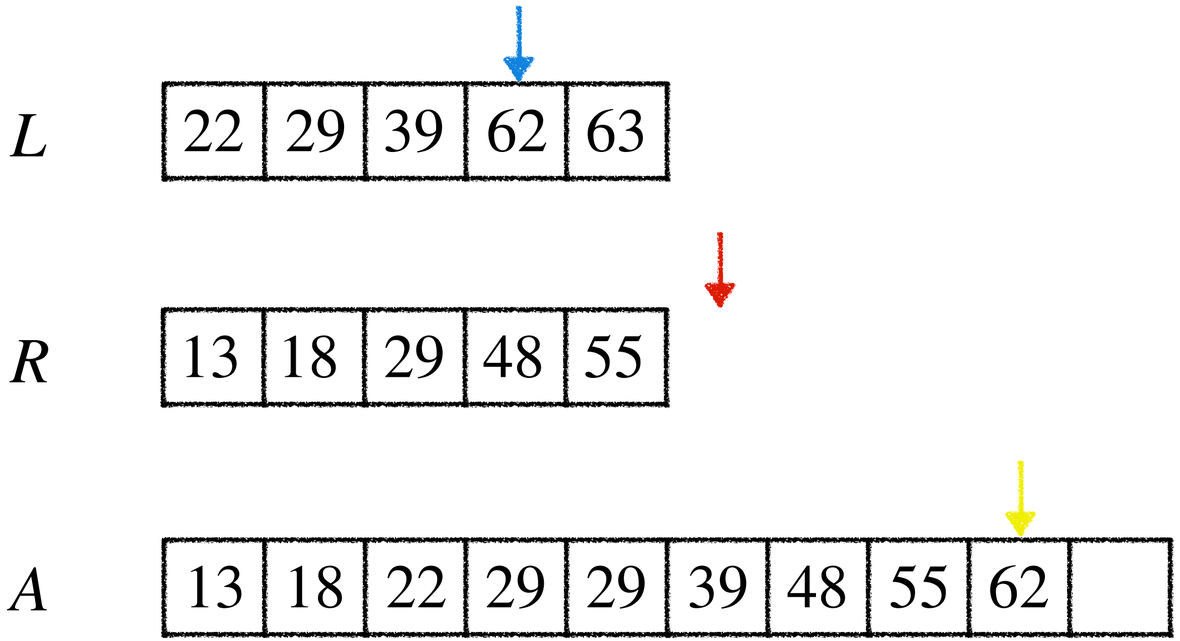

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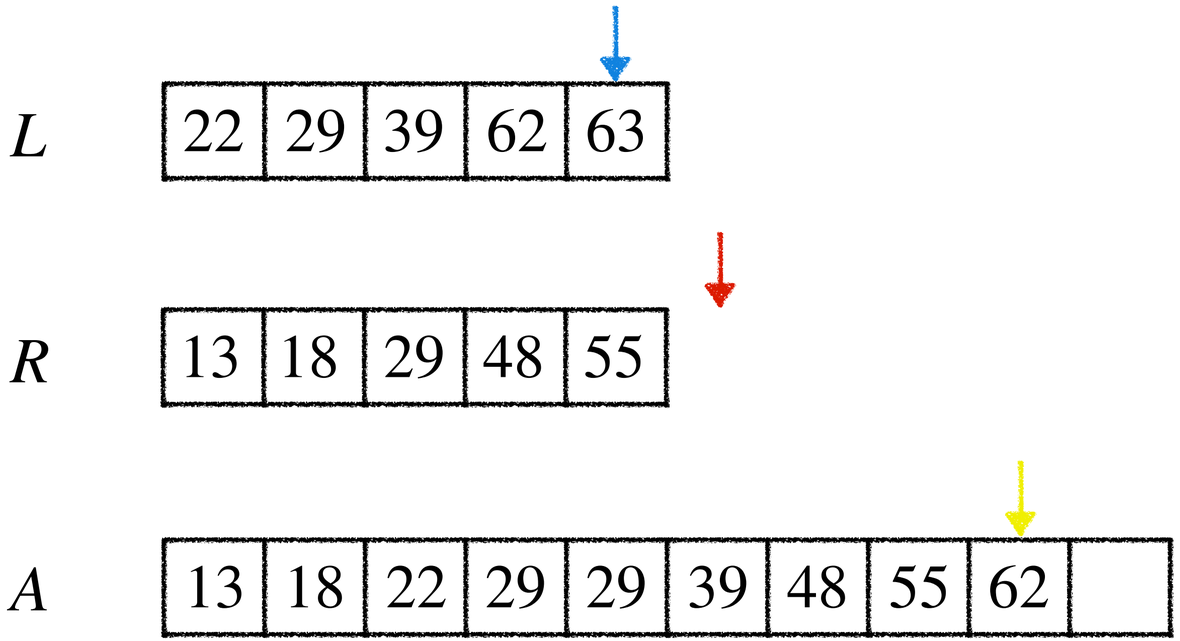

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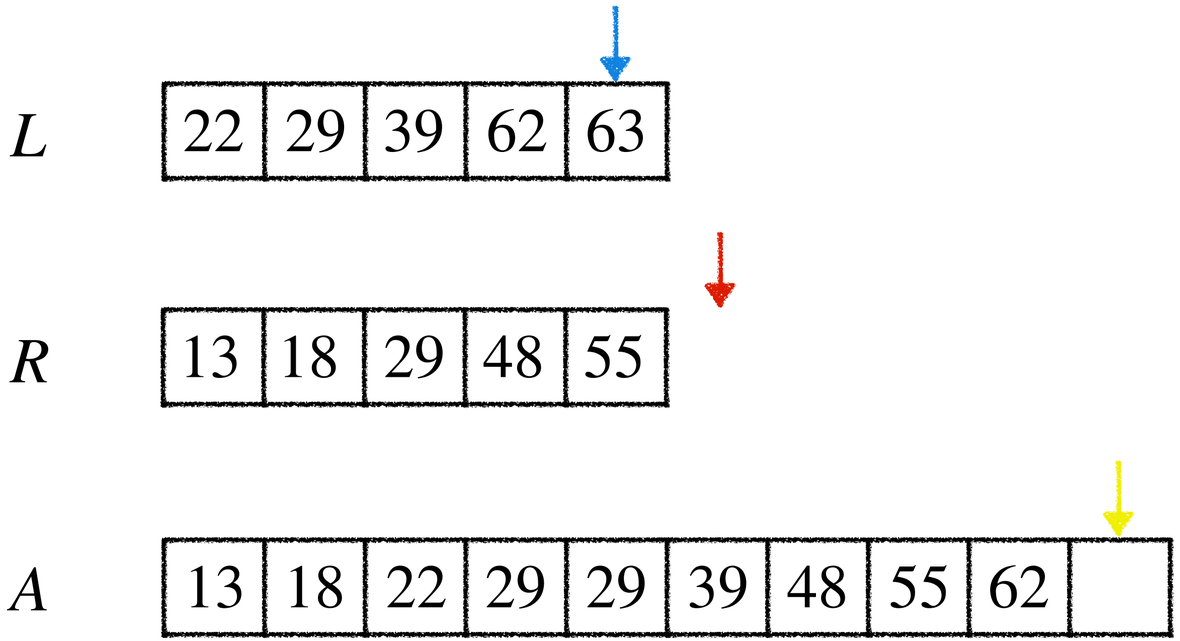

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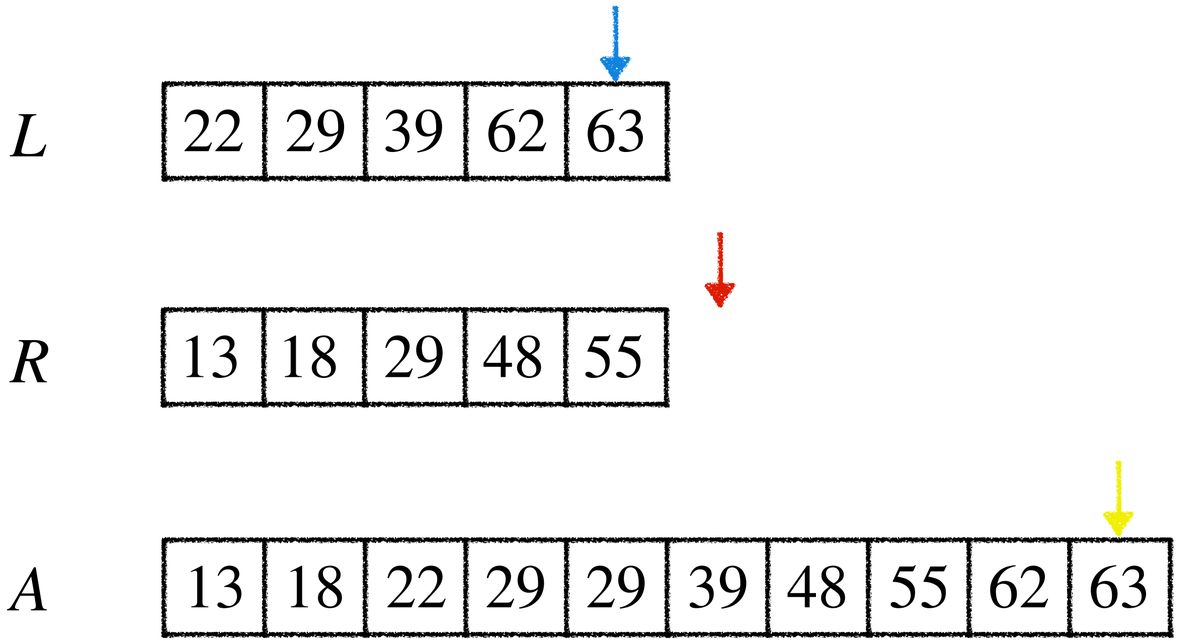

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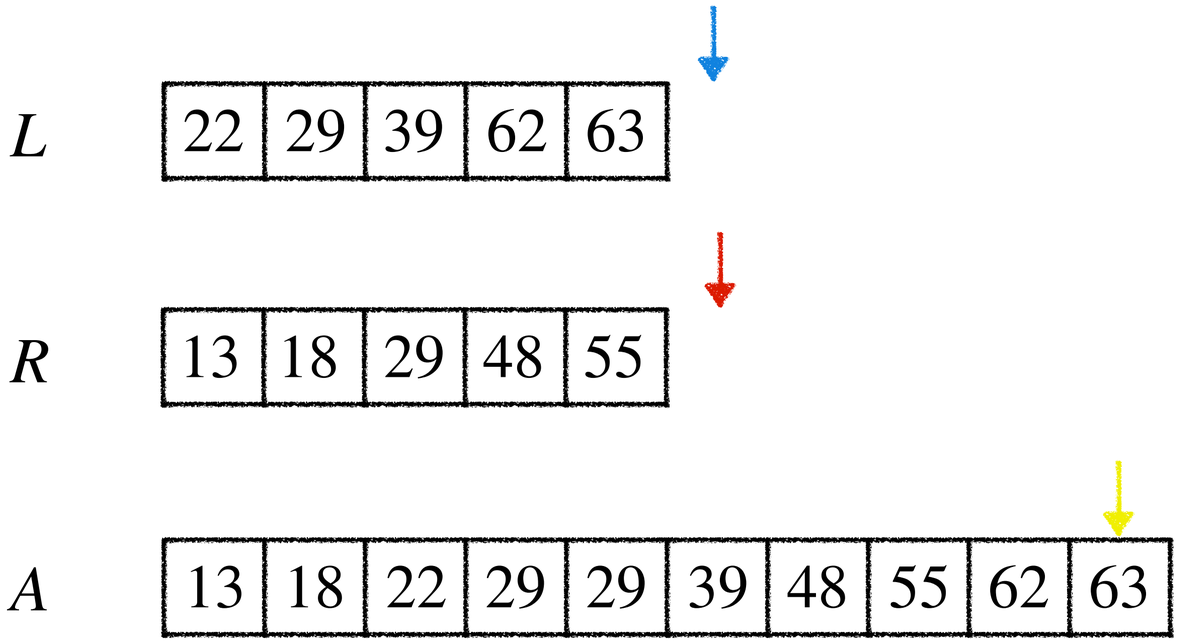

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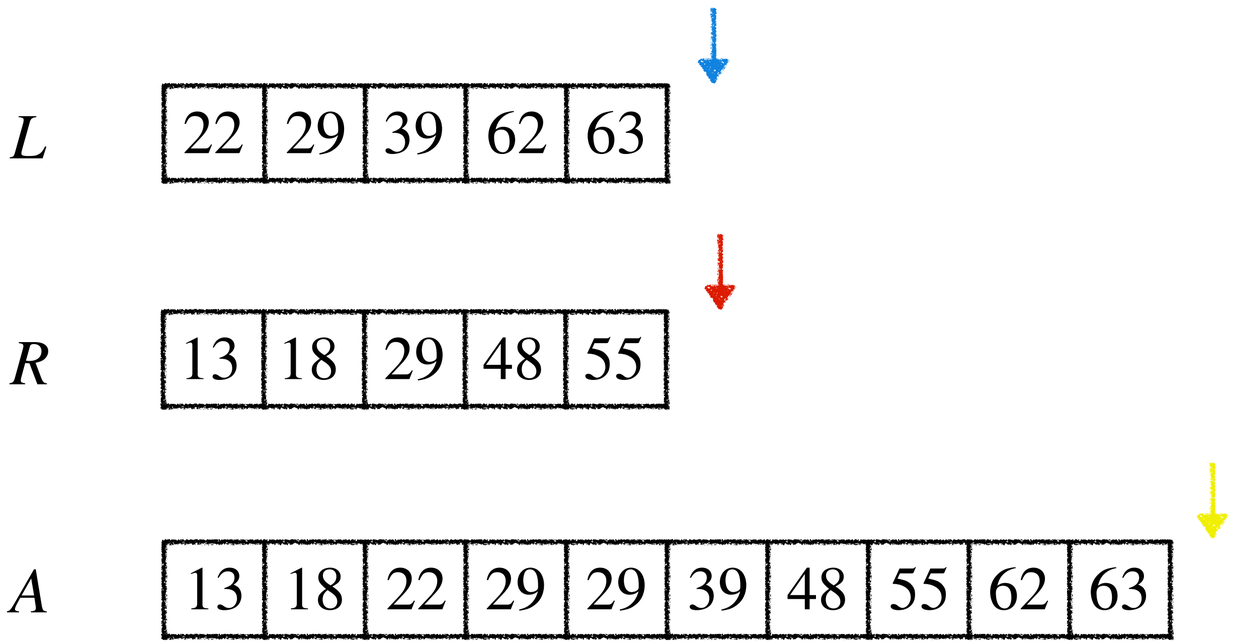

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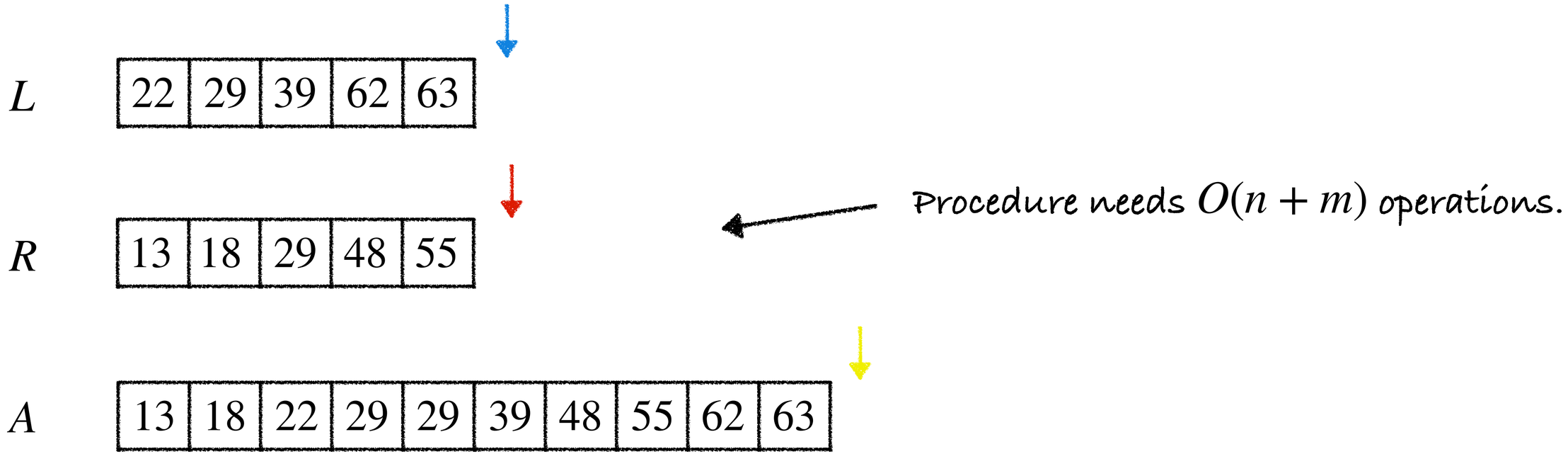

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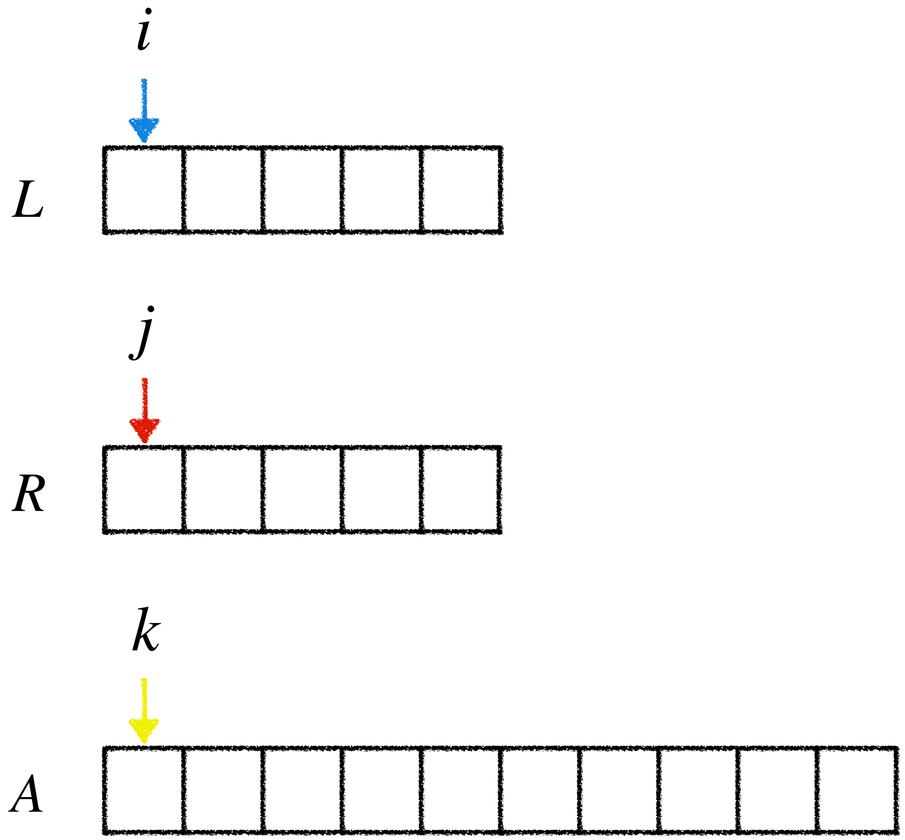

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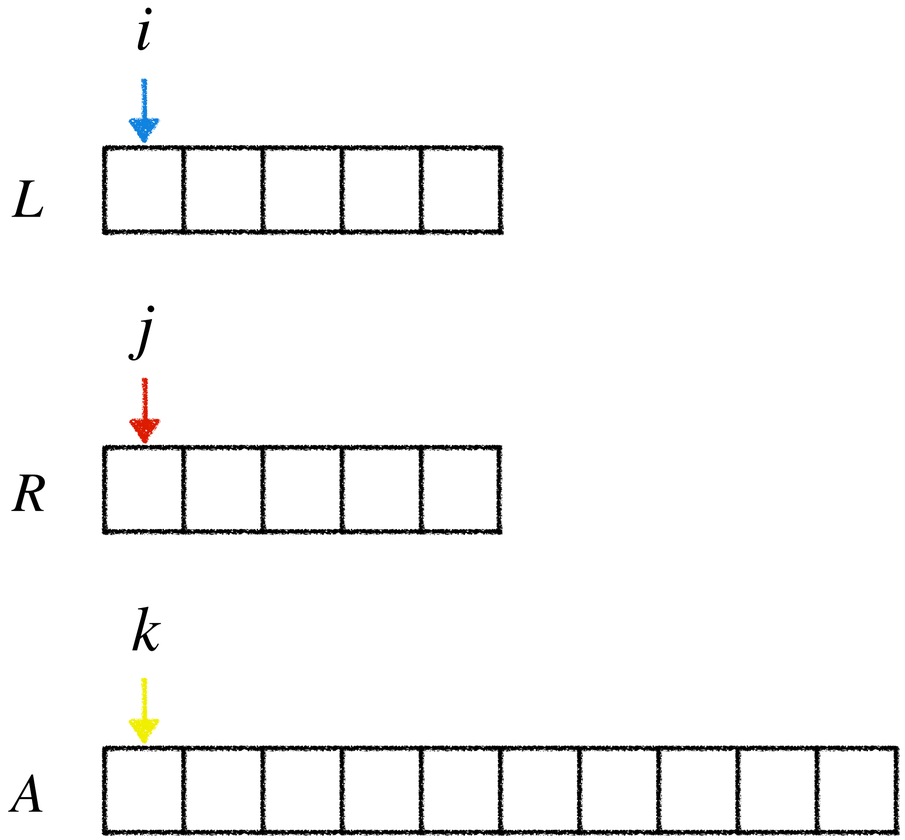

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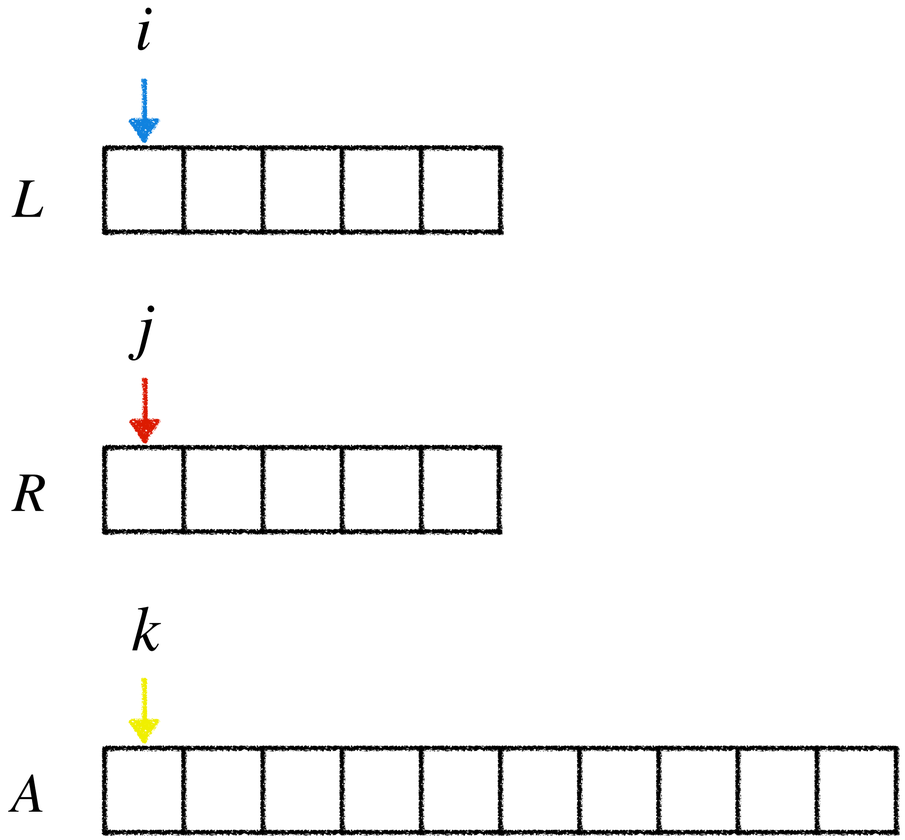

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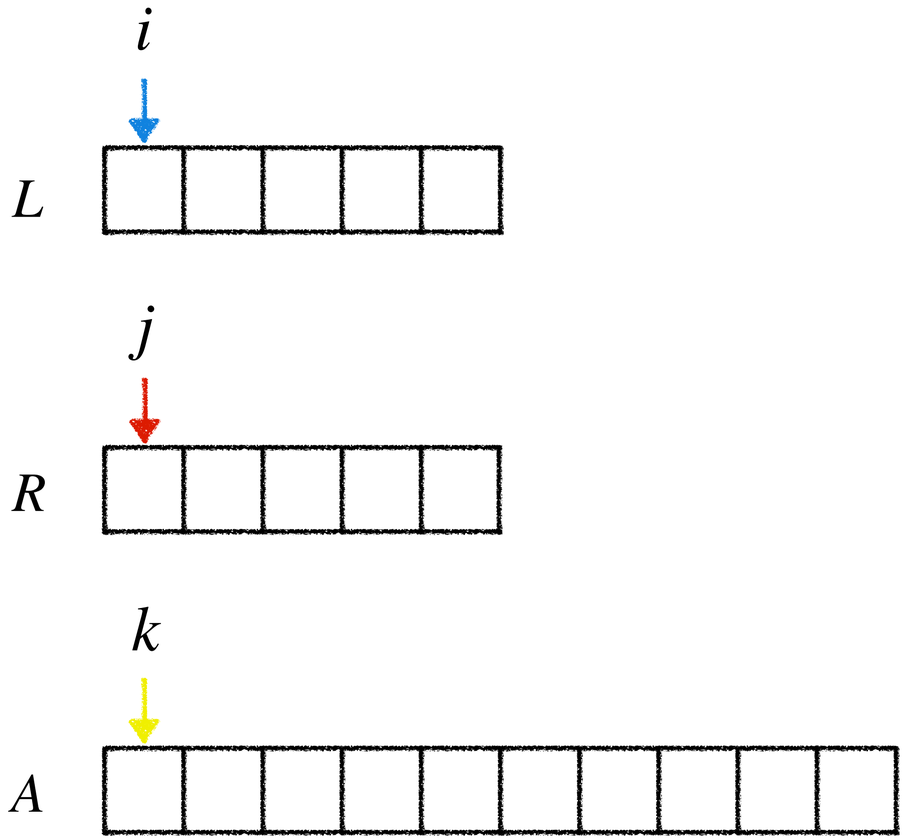

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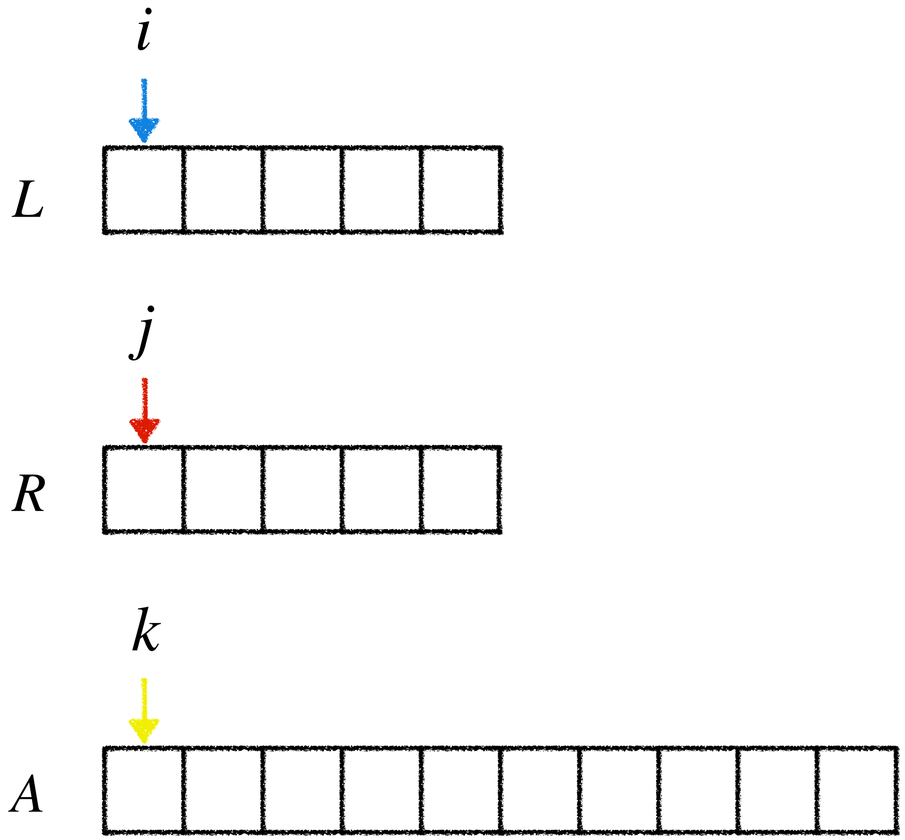

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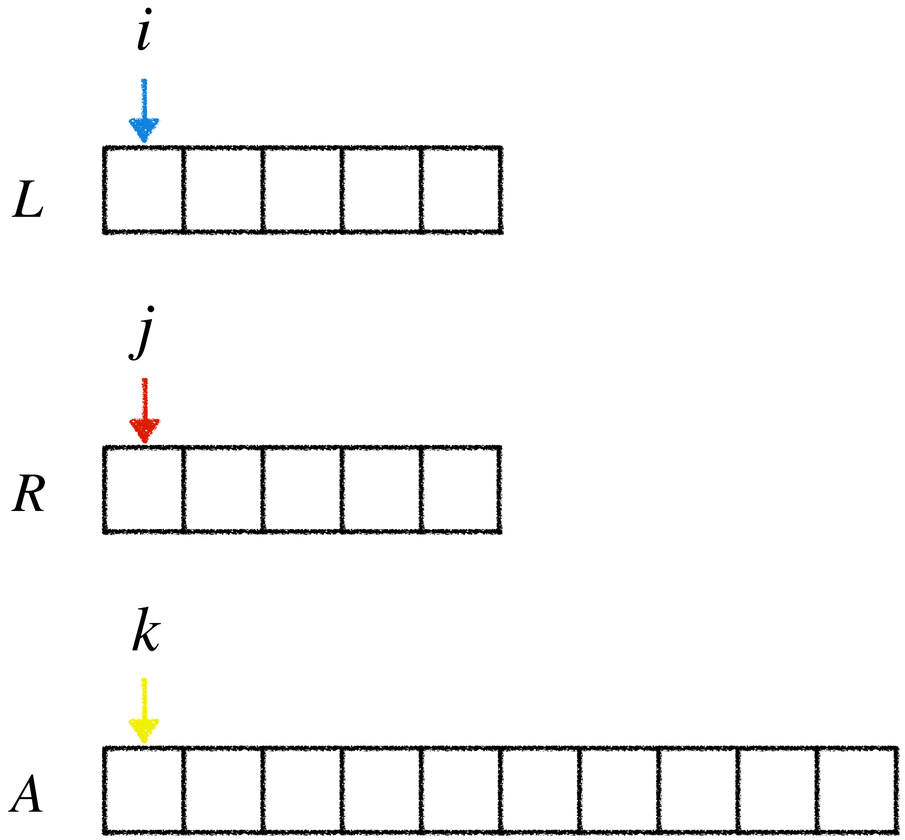

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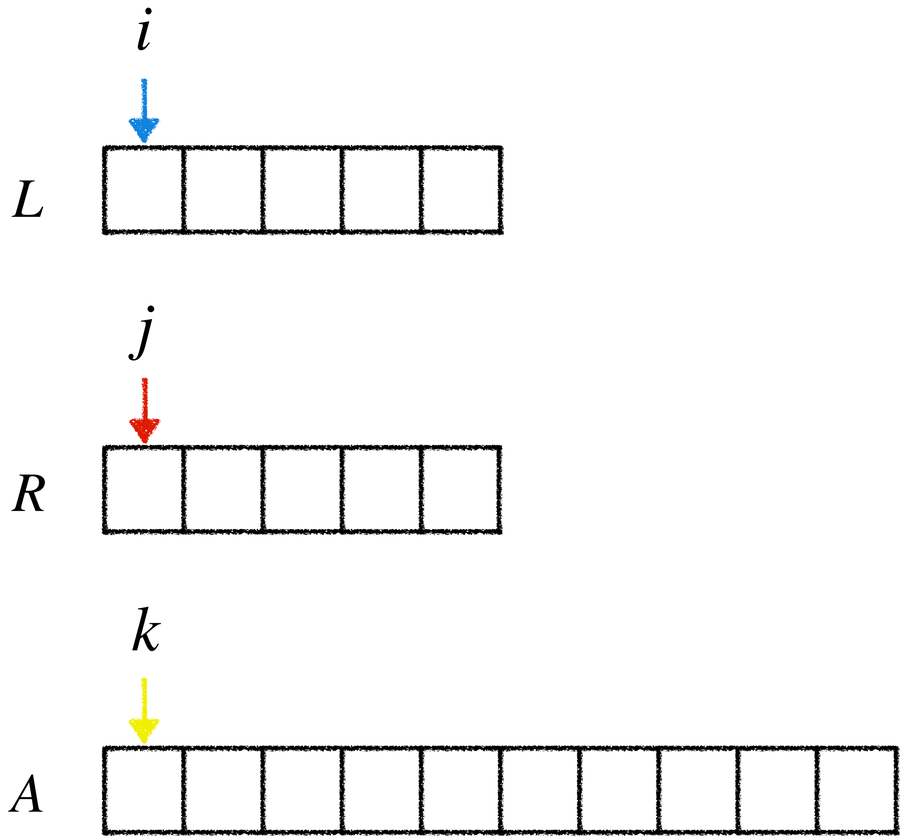

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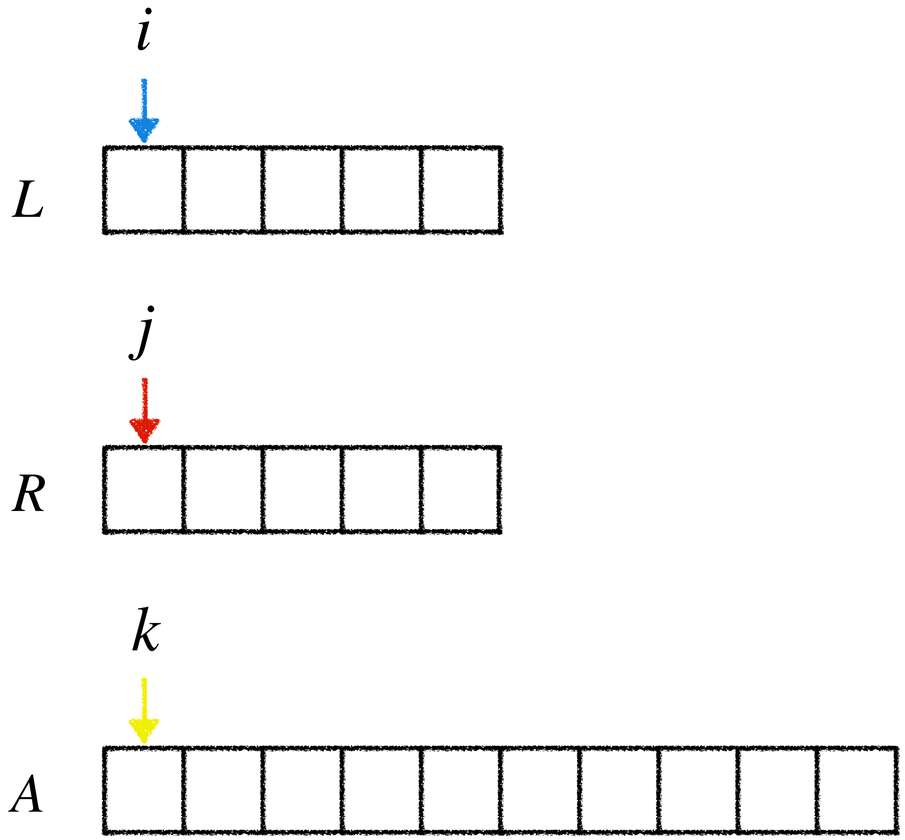

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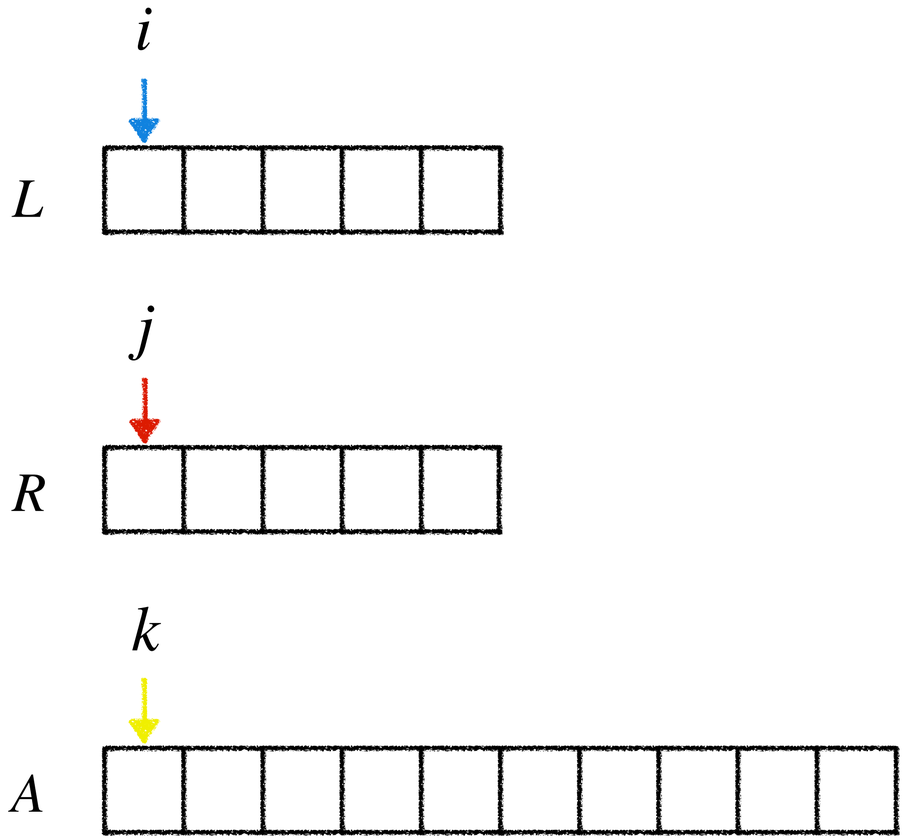

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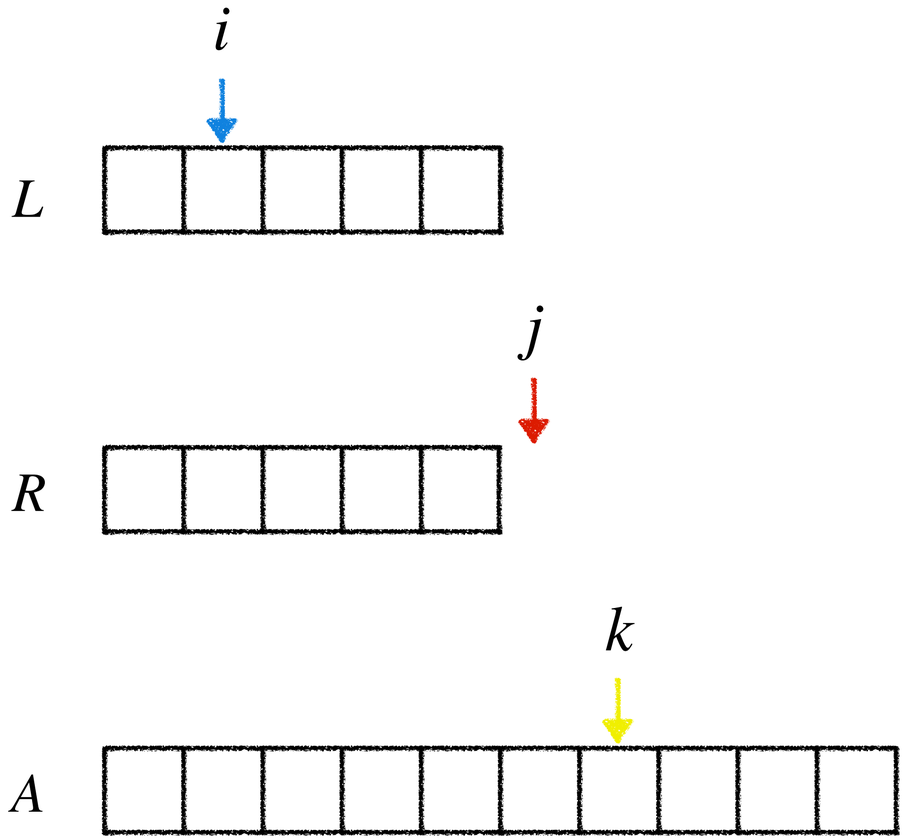

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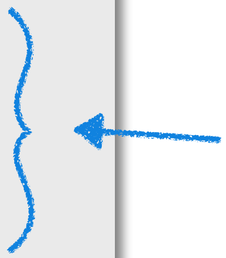

Fill the remaining elements of R in A

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Fill the remaining elements of R in A

Merging

```
MERGE(L, R):
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16.   while j ≤ n_r
17.       A[k] = R[j]
18.       j = j + 1, k = k + 1
20.   return A
```

Fill the remaining elements of R in A

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Return if $A[p : r]$ contains just one element

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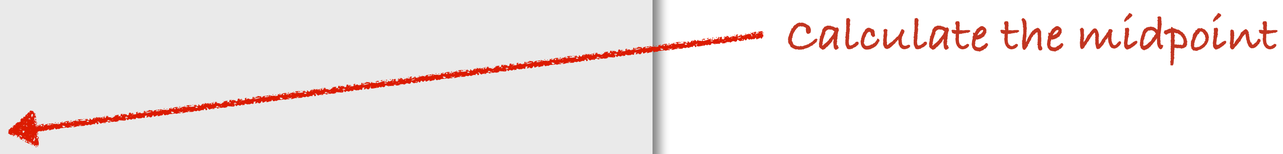

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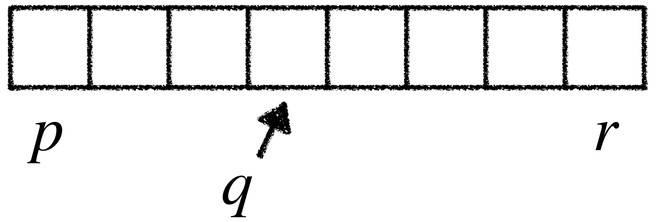

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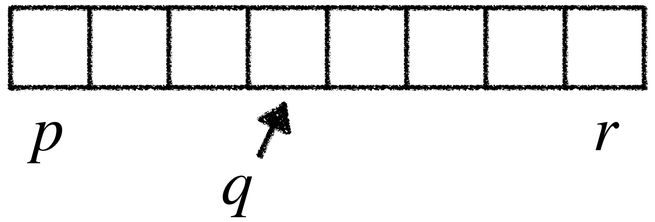

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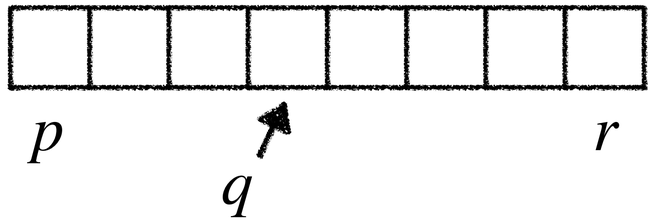

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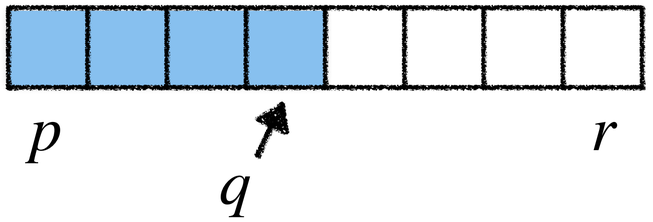

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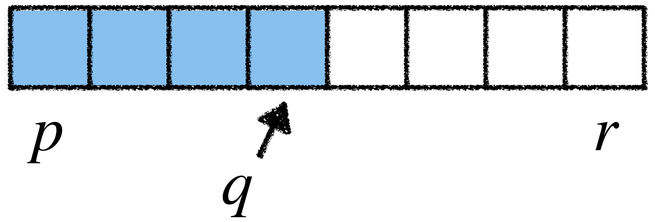

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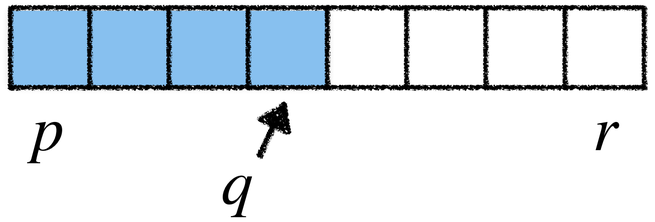

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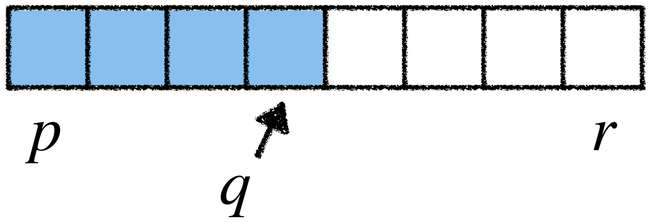

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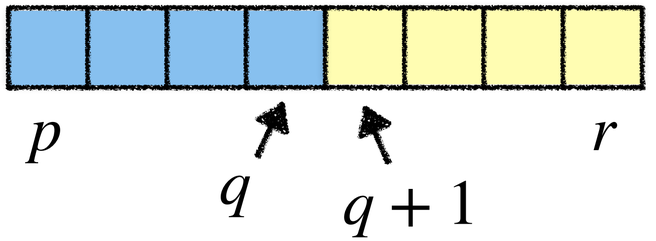

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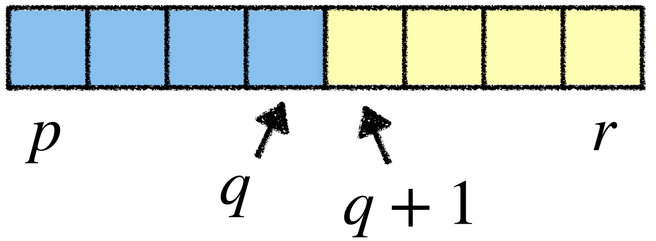

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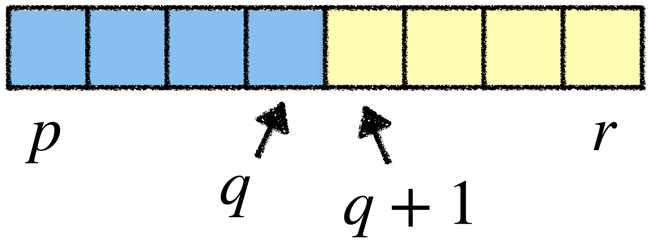

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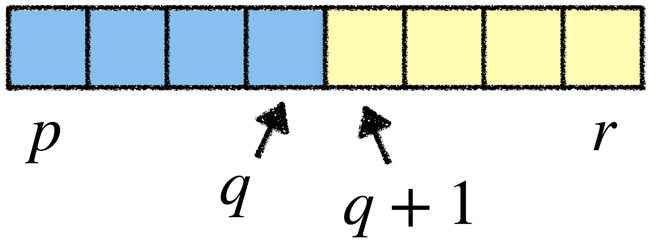

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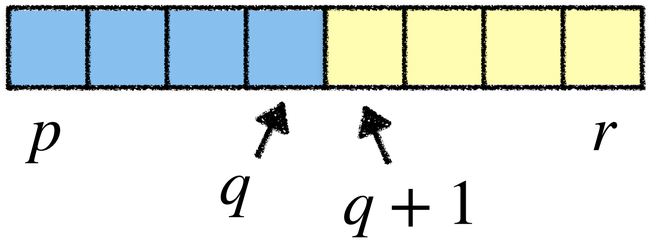

Time Complexity: $T(n) = c + 2.T(n/2) + c'n$

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MERGE-SORT(A, p, r)**:**

1. **if** $p == r$
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3. $q = \lfloor (p + r)/2 \rfloor$
4. **MERGE-SORT**(A, p, q)
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6. $A[p : r] =$ **MERGE**$(A[p : q], A[q + 1 : r])$
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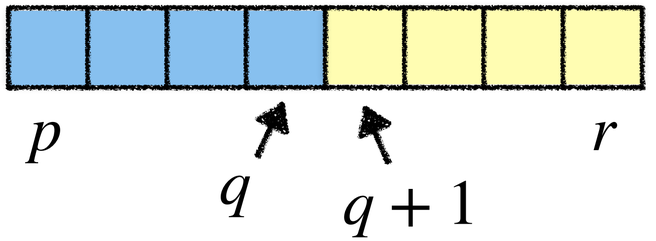

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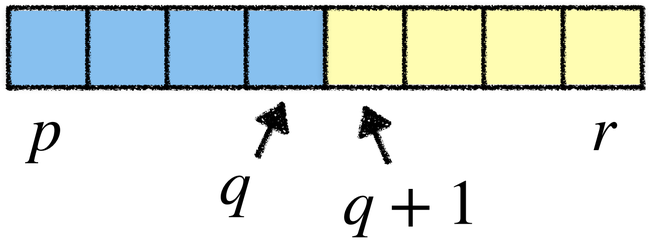

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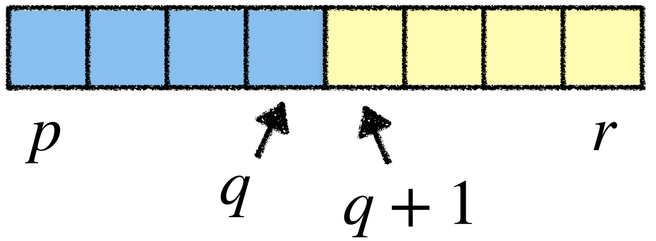

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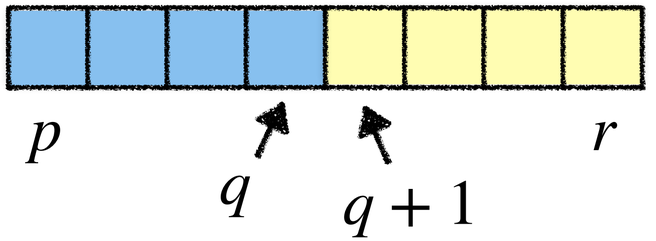

Time Complexity: $T(n) = c + 2.T(n/2) + c'n$ if $n > 1$, else c''.

Time to divide — Time to conquer — Time to combine

Merge Sort: Illustration

Division:

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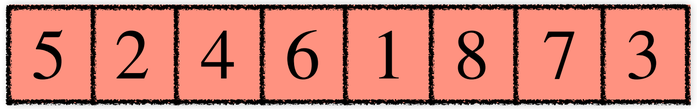

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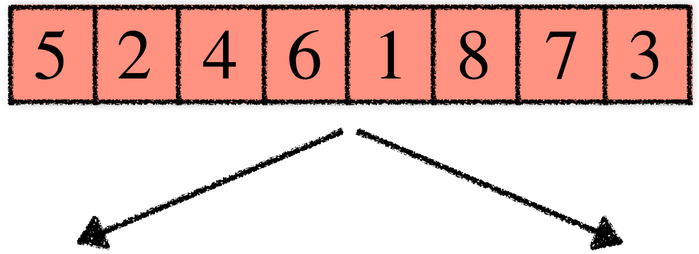

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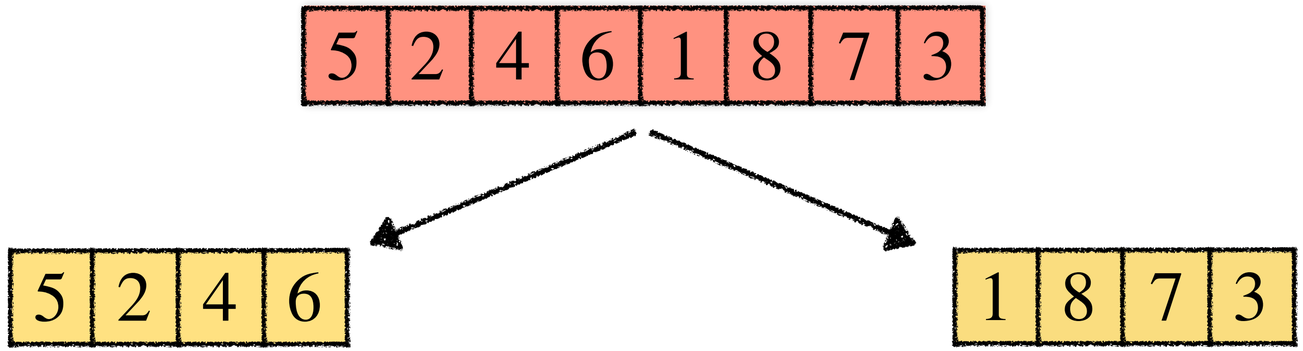

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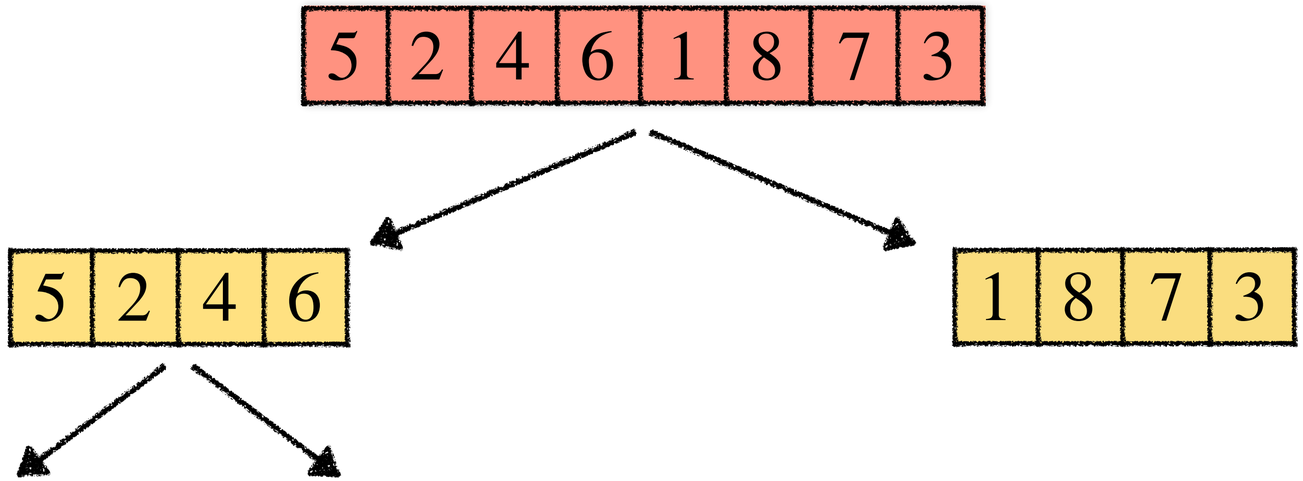

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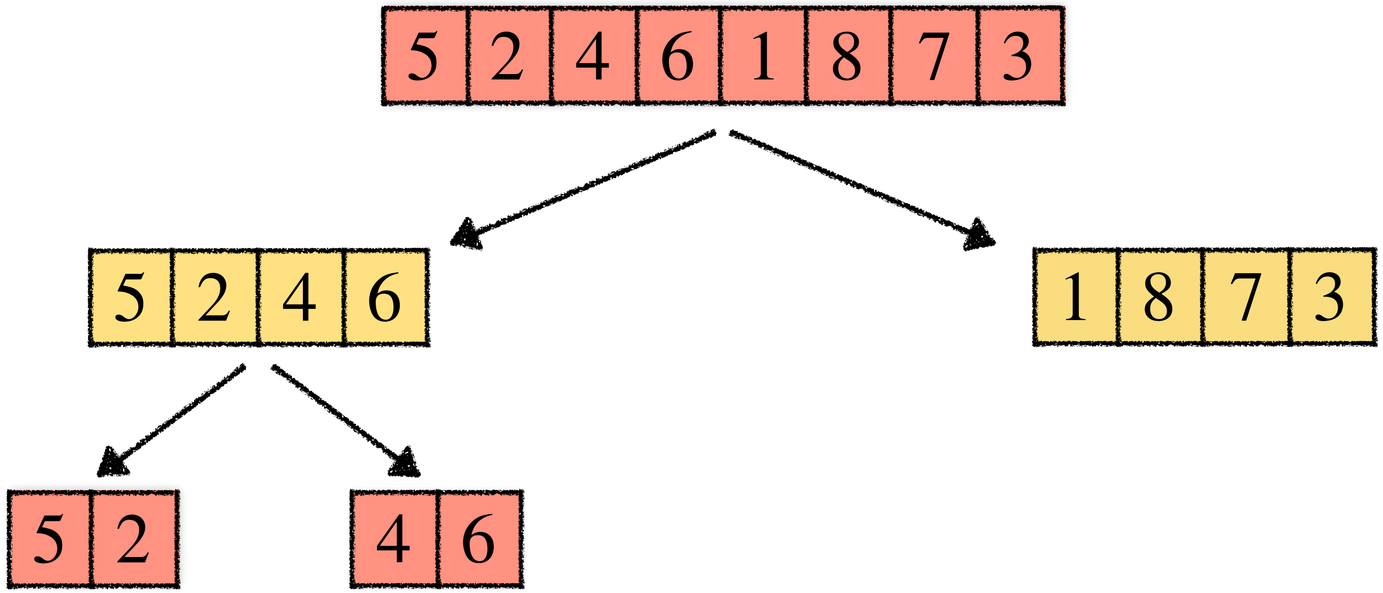

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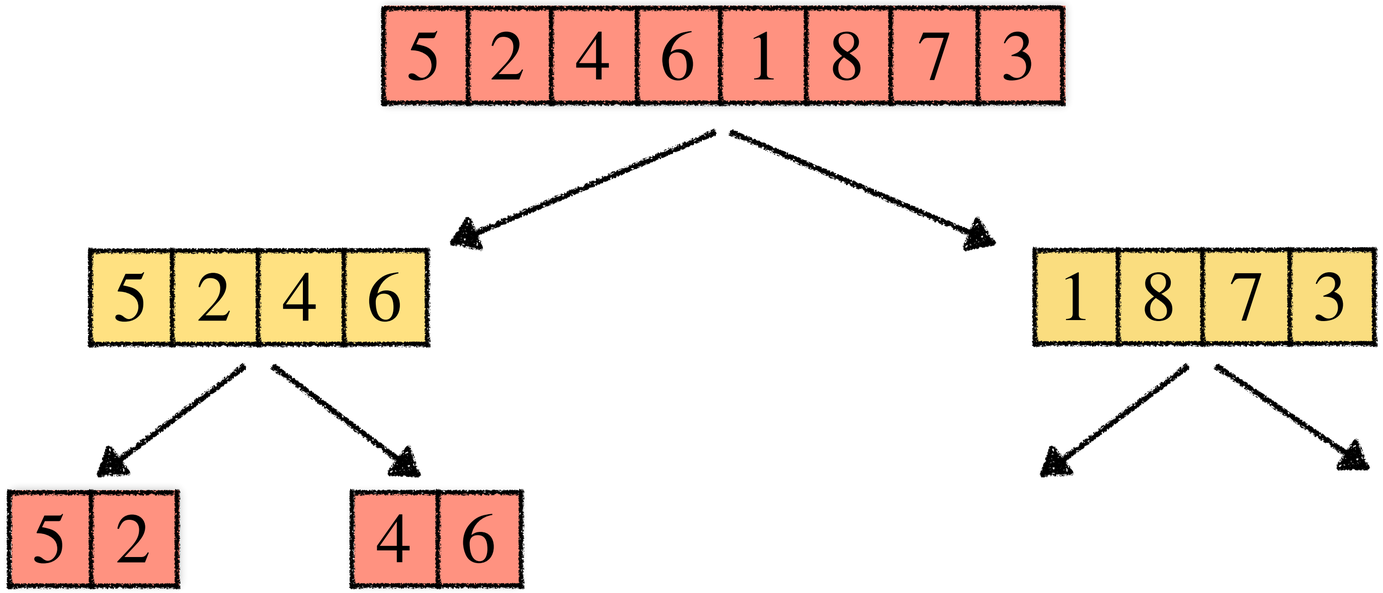

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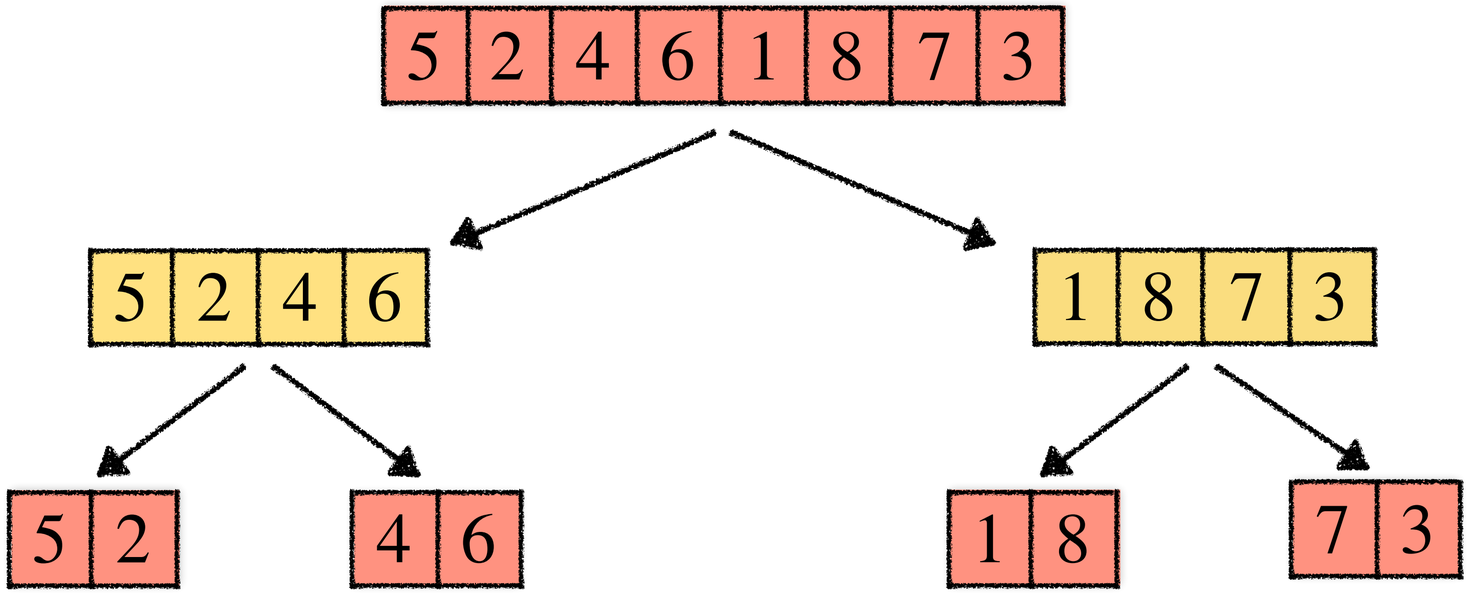

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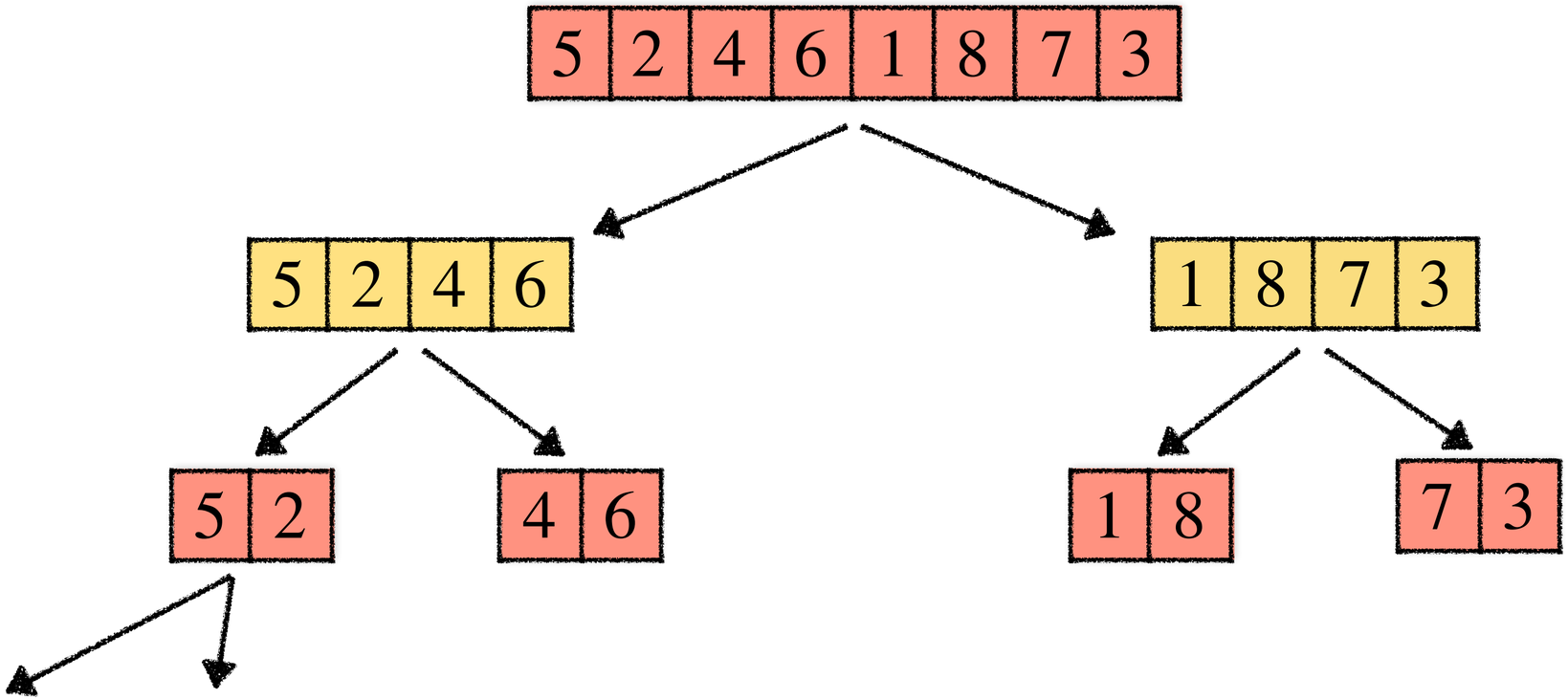

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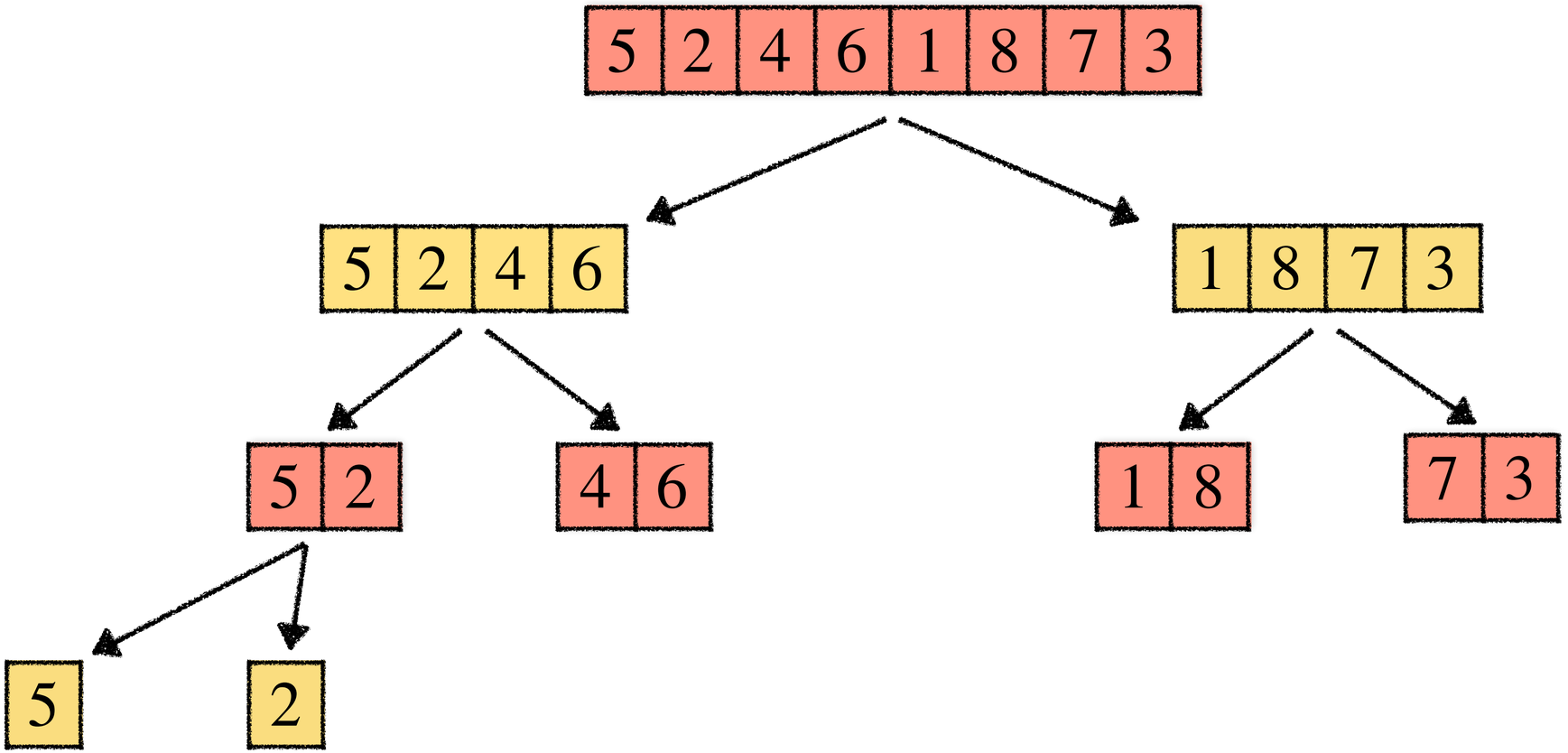

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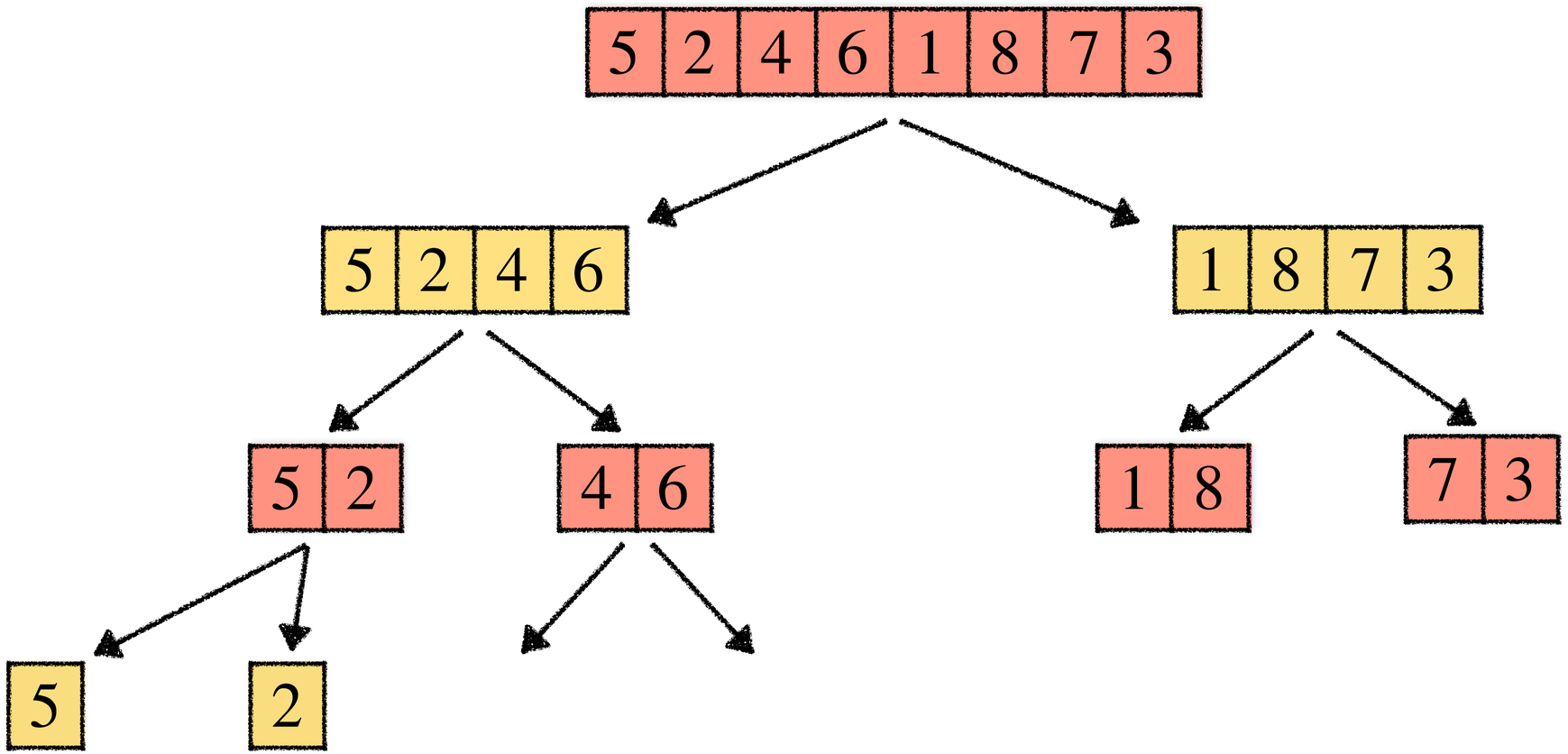

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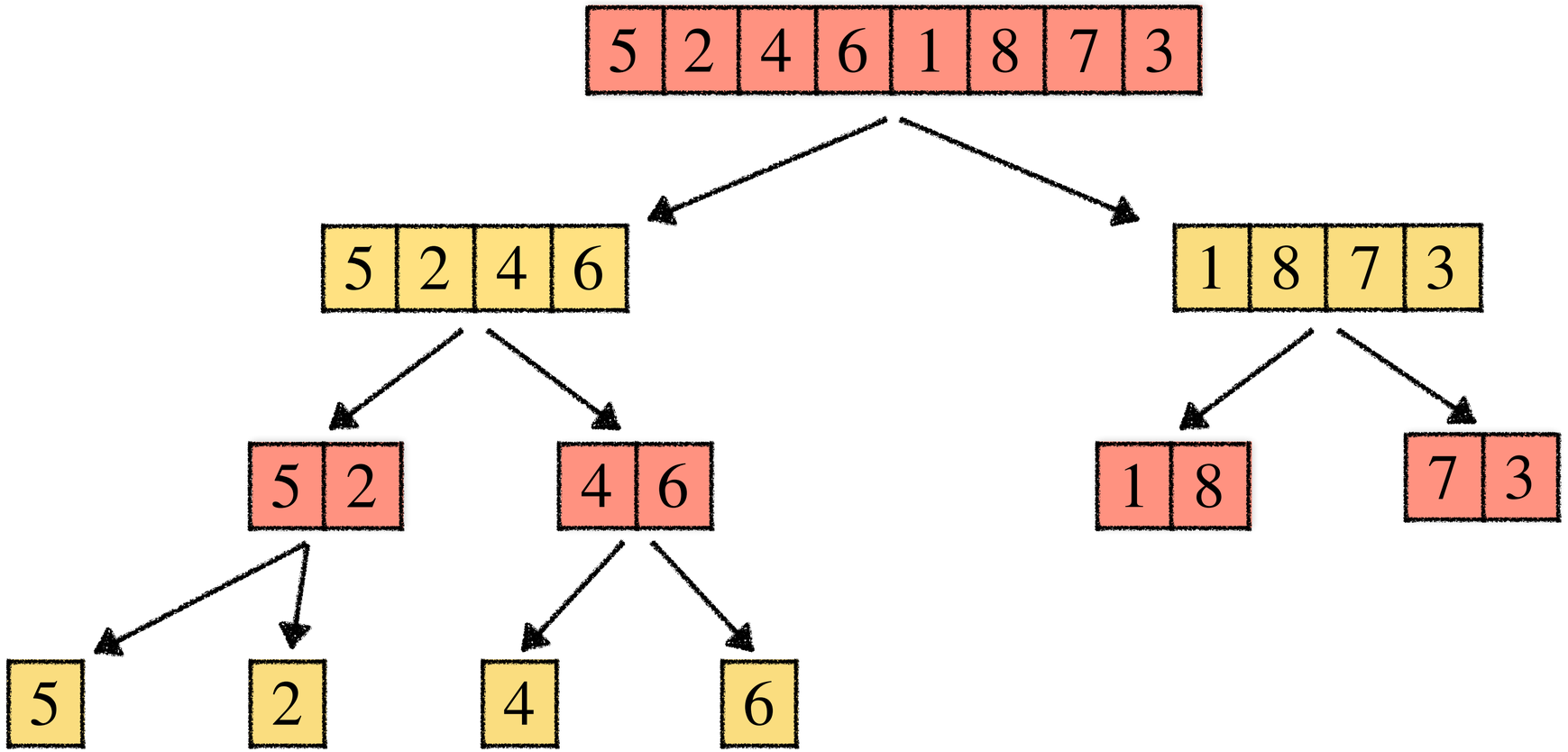

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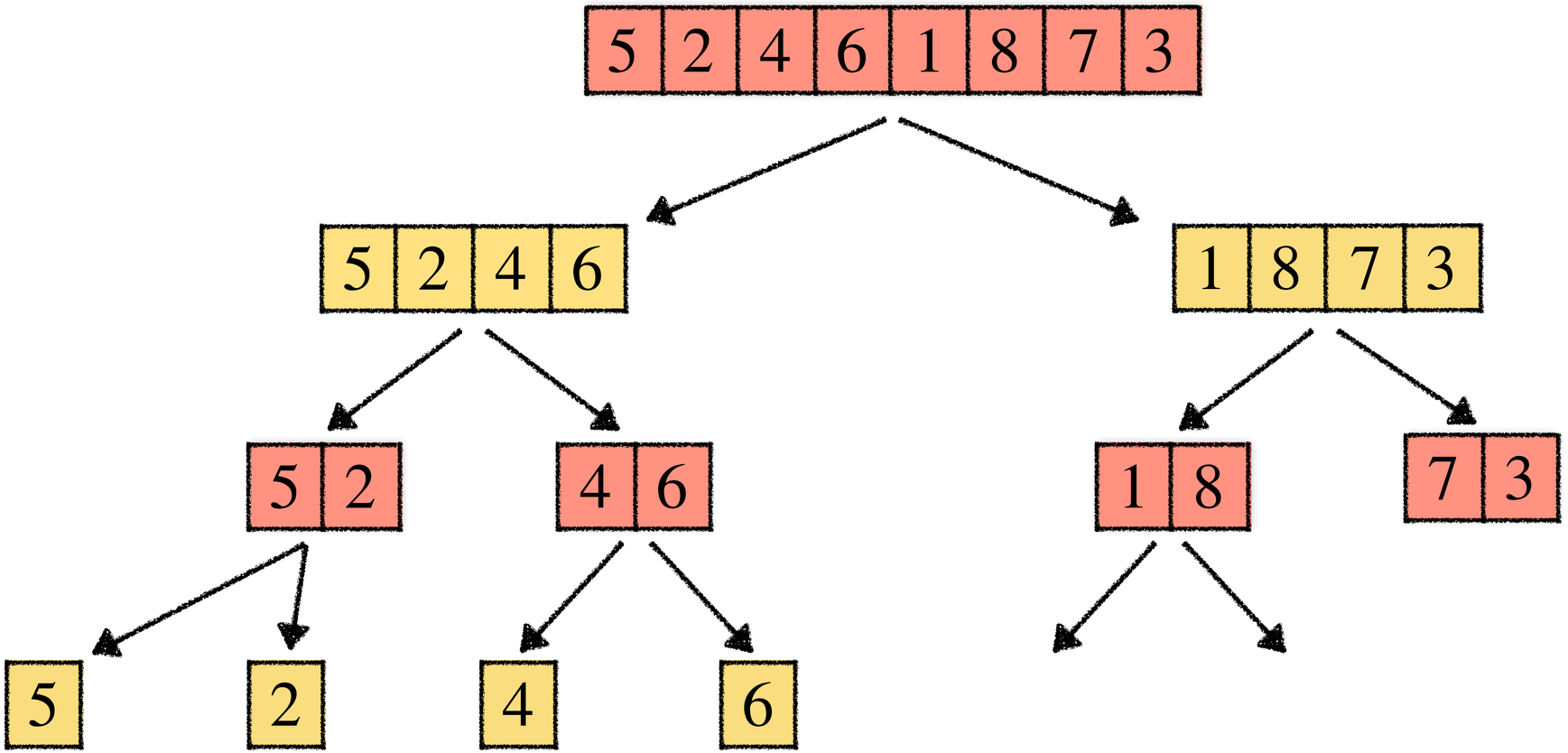

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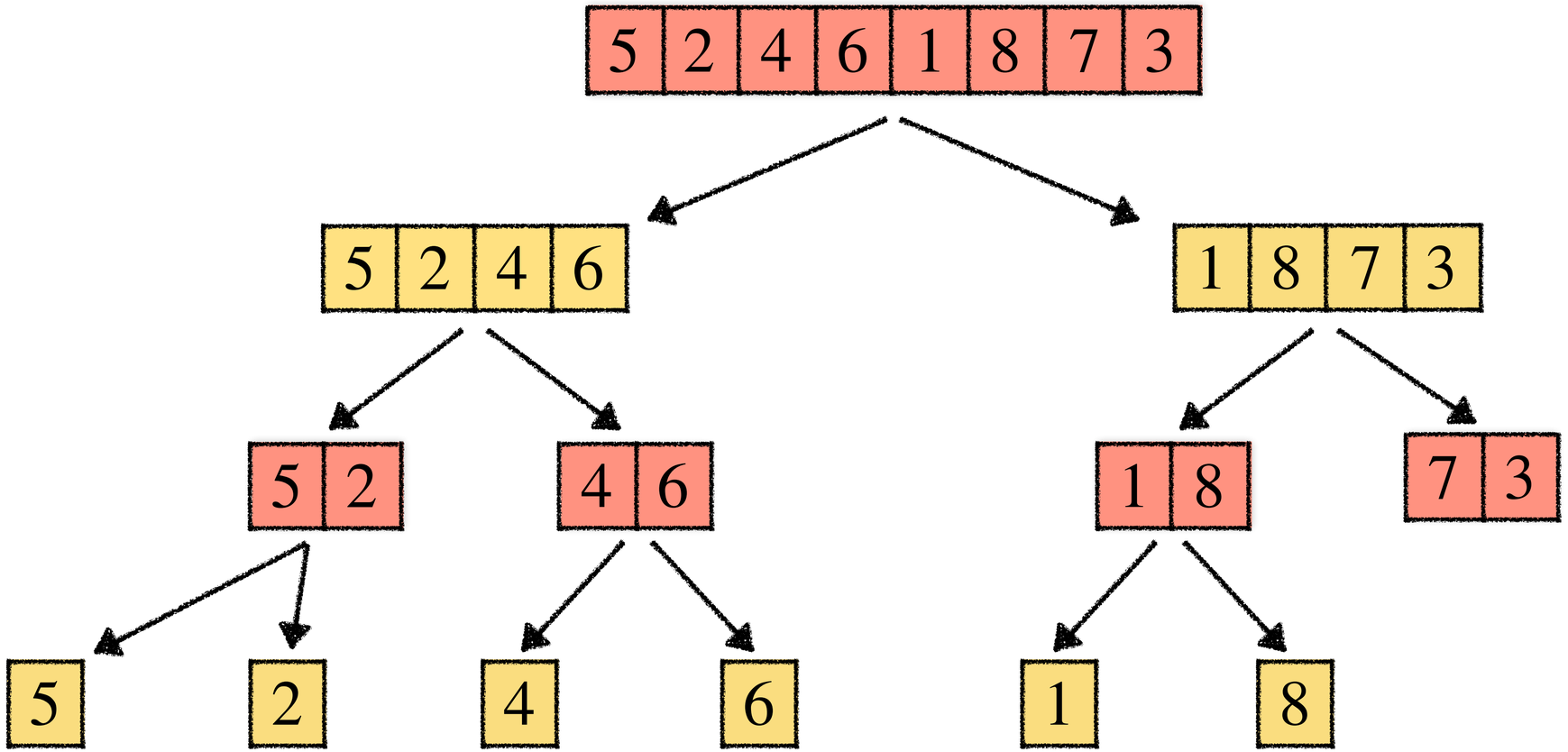

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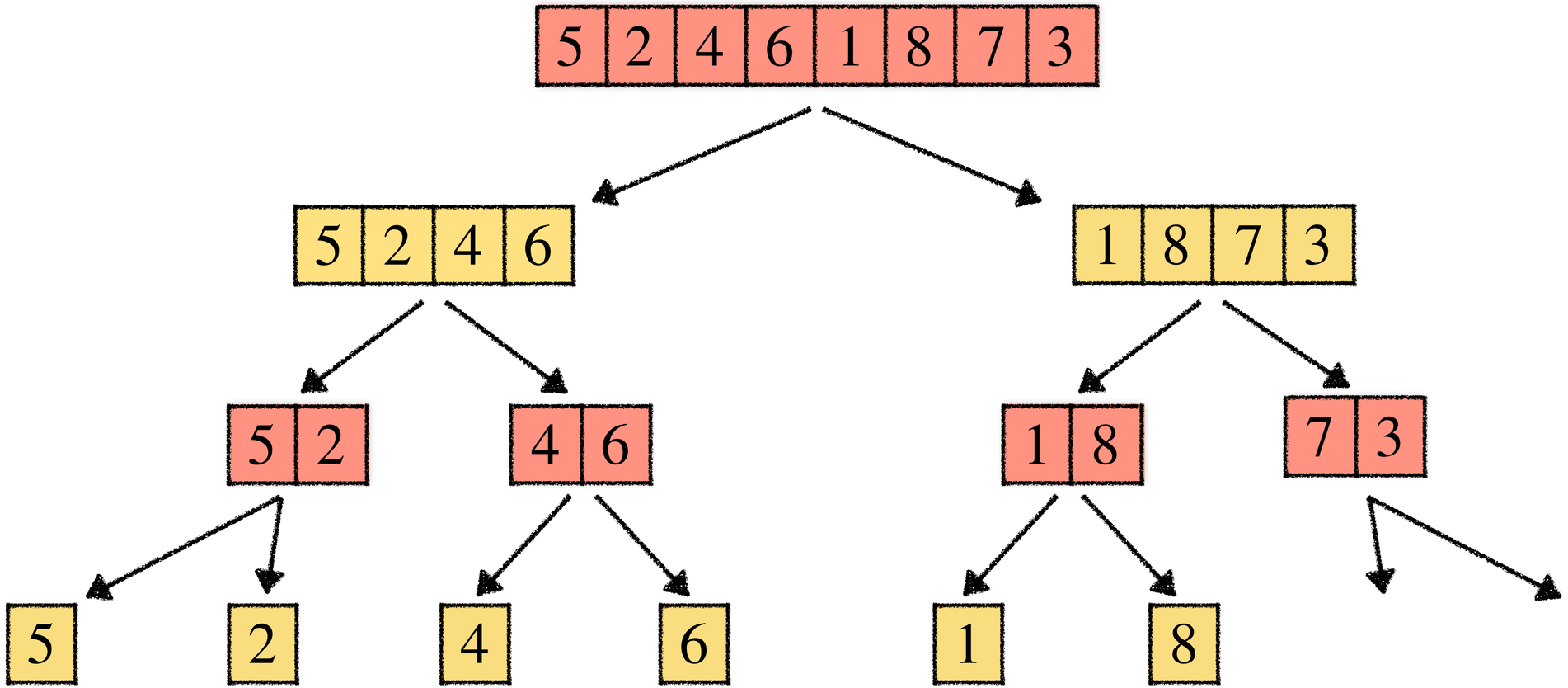

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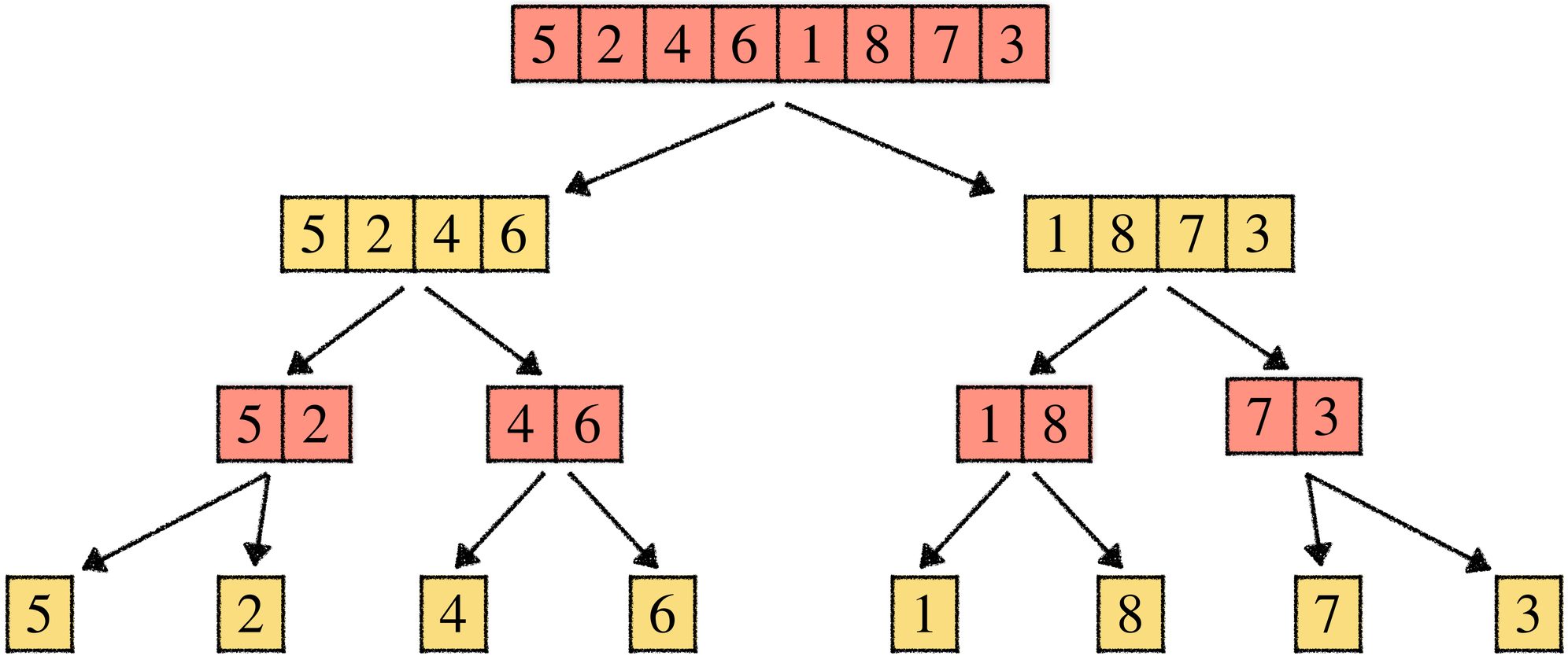

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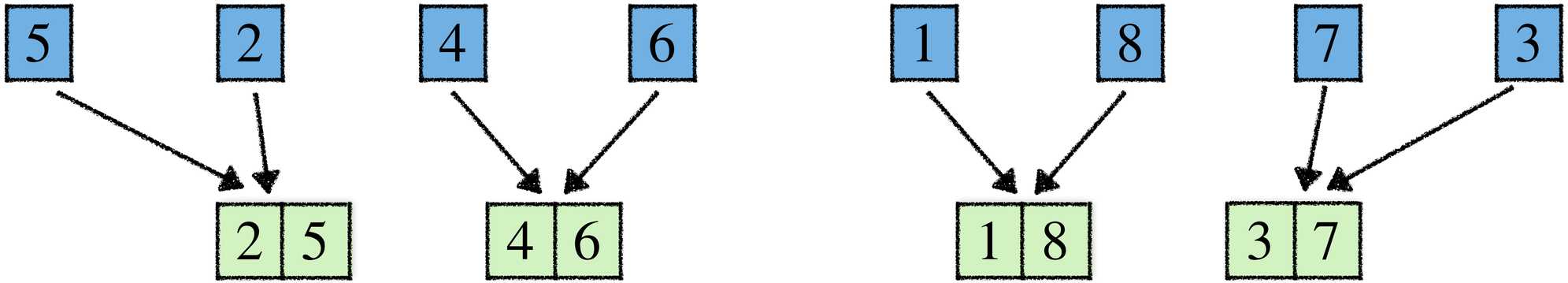

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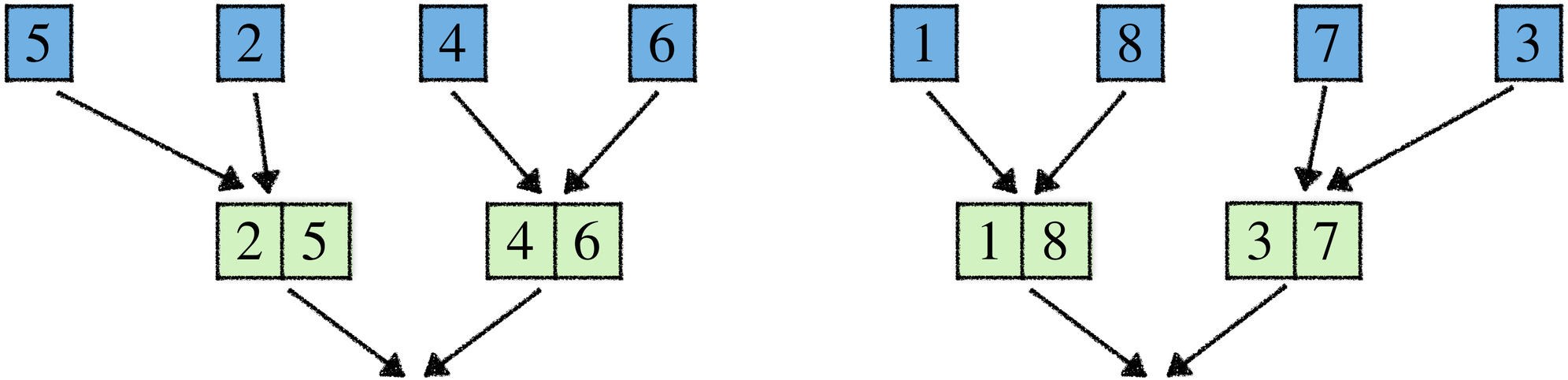

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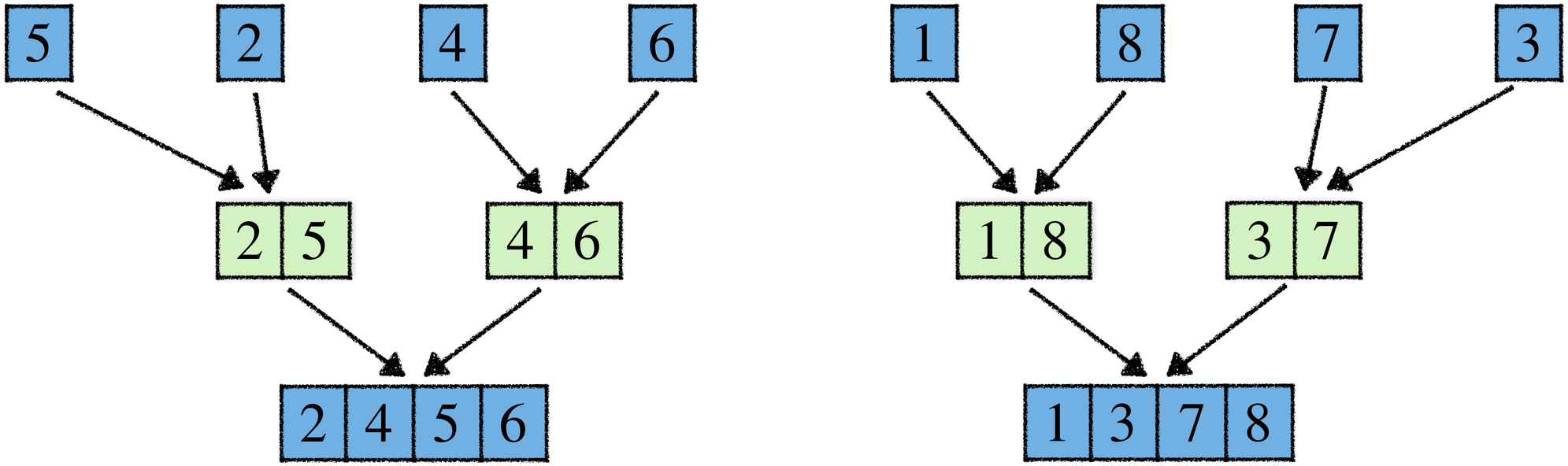

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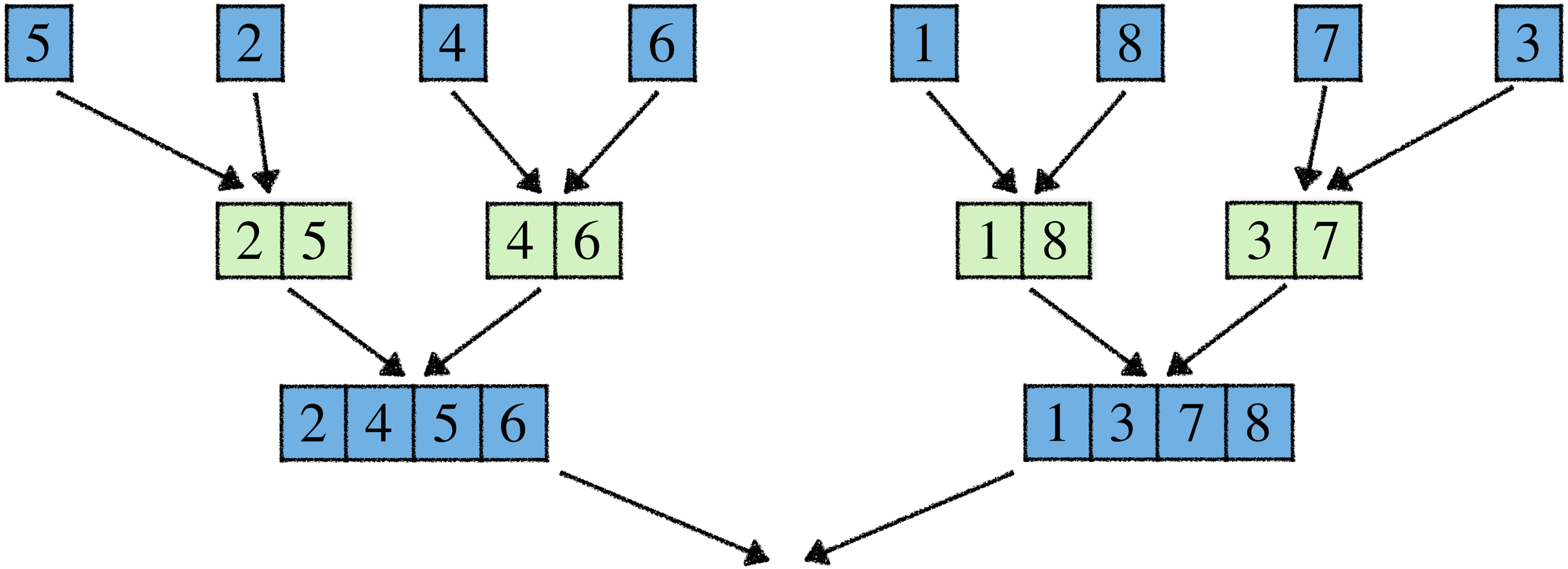

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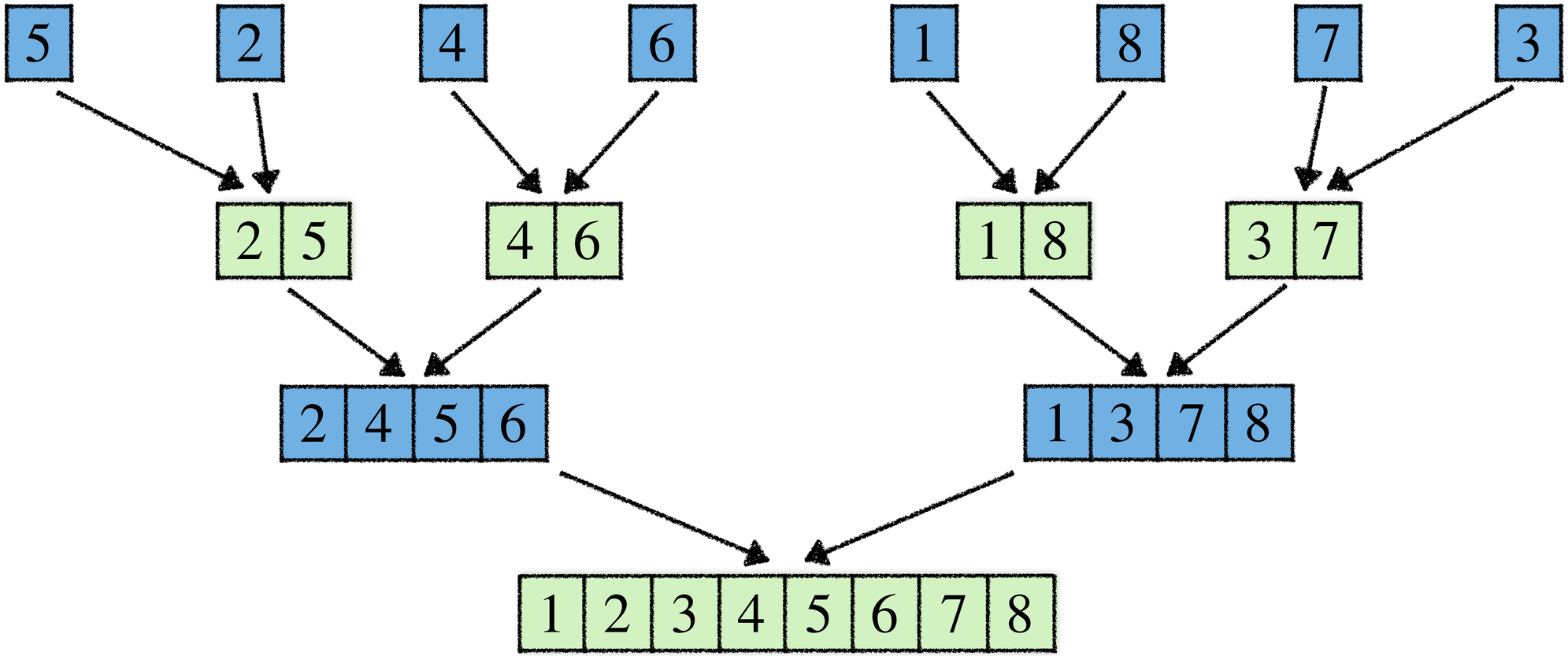

Recurrence Relation

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Idea: Expand the run time into a tree.

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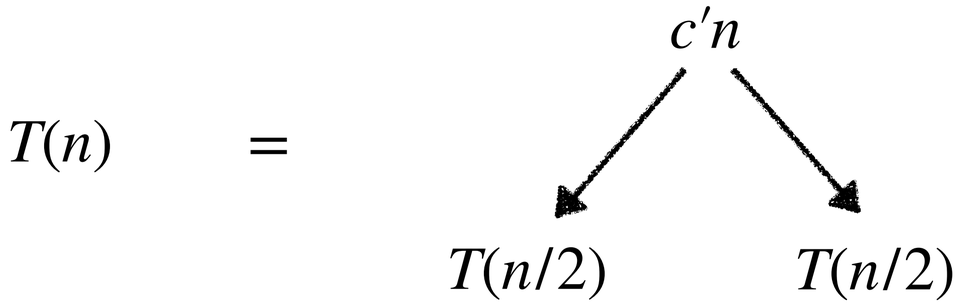

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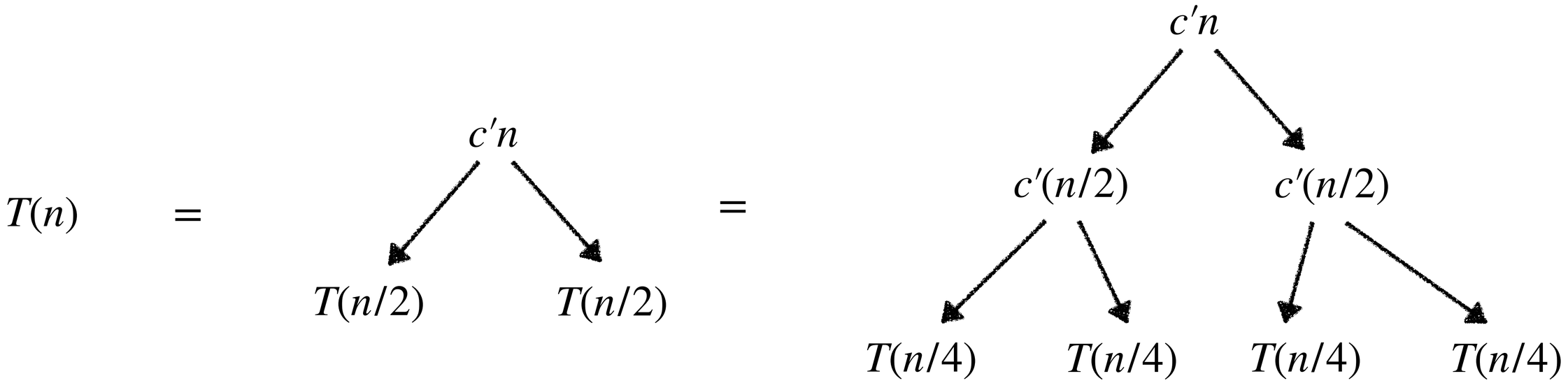

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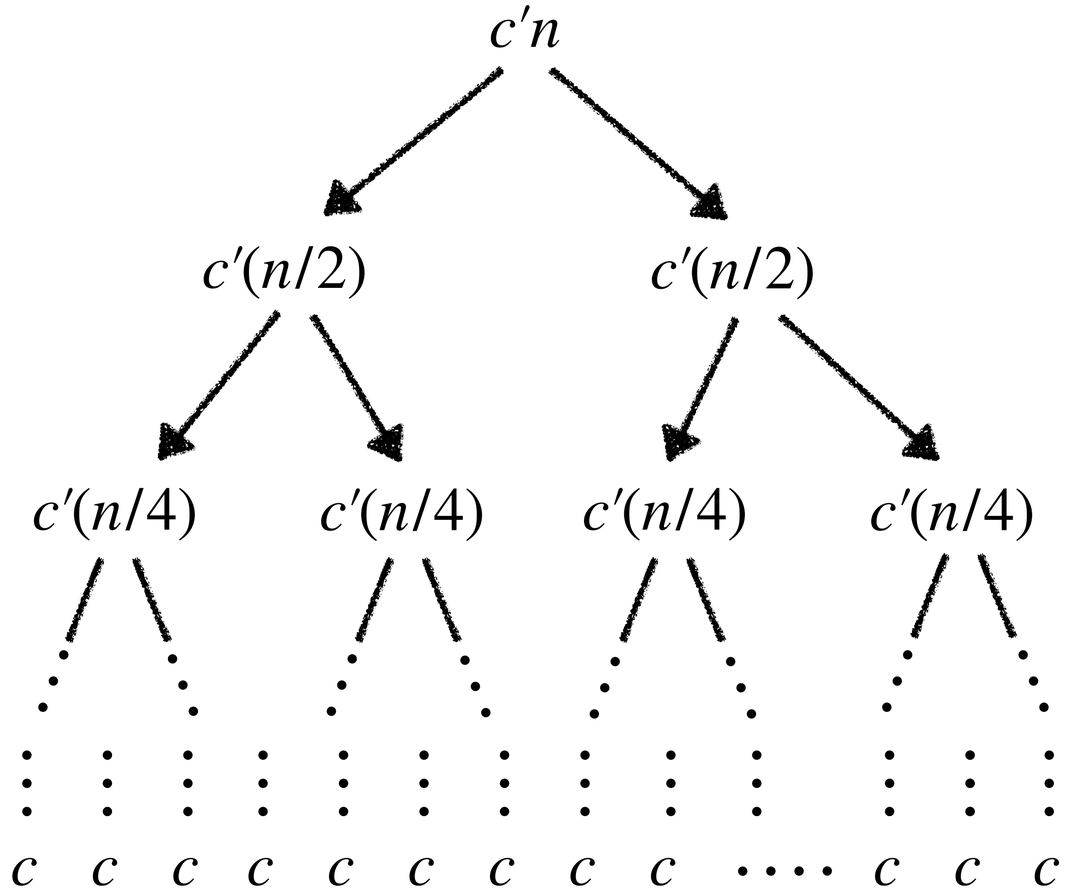

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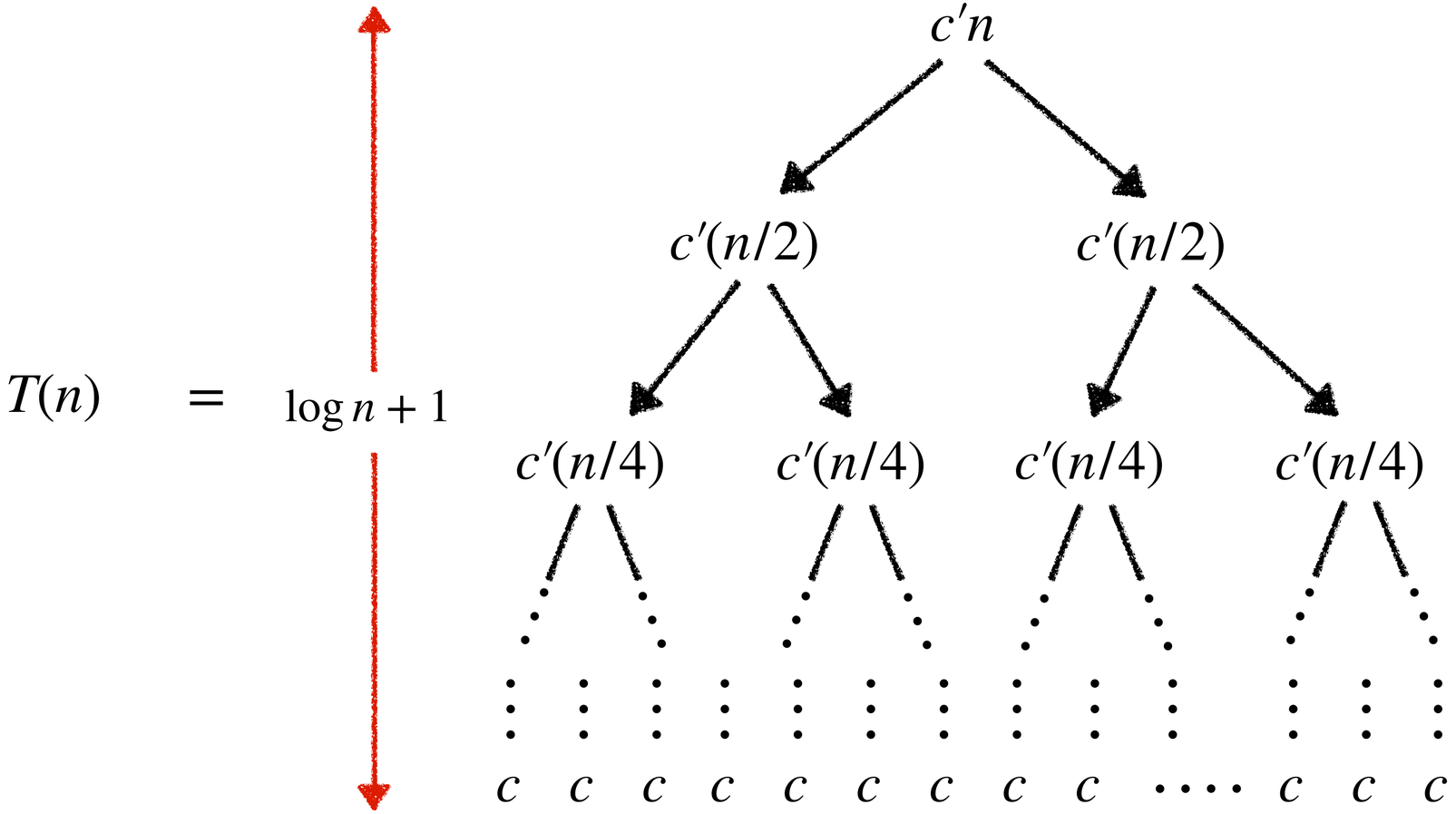

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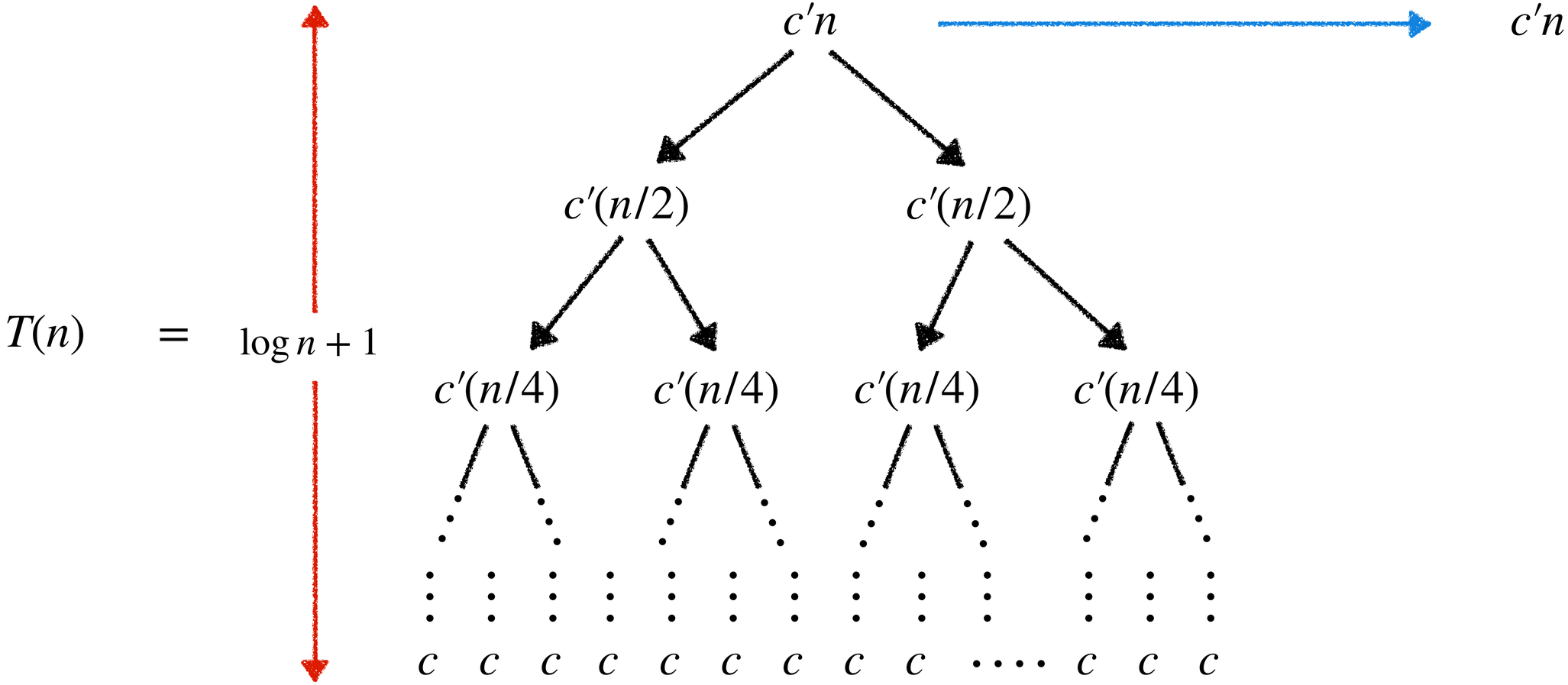

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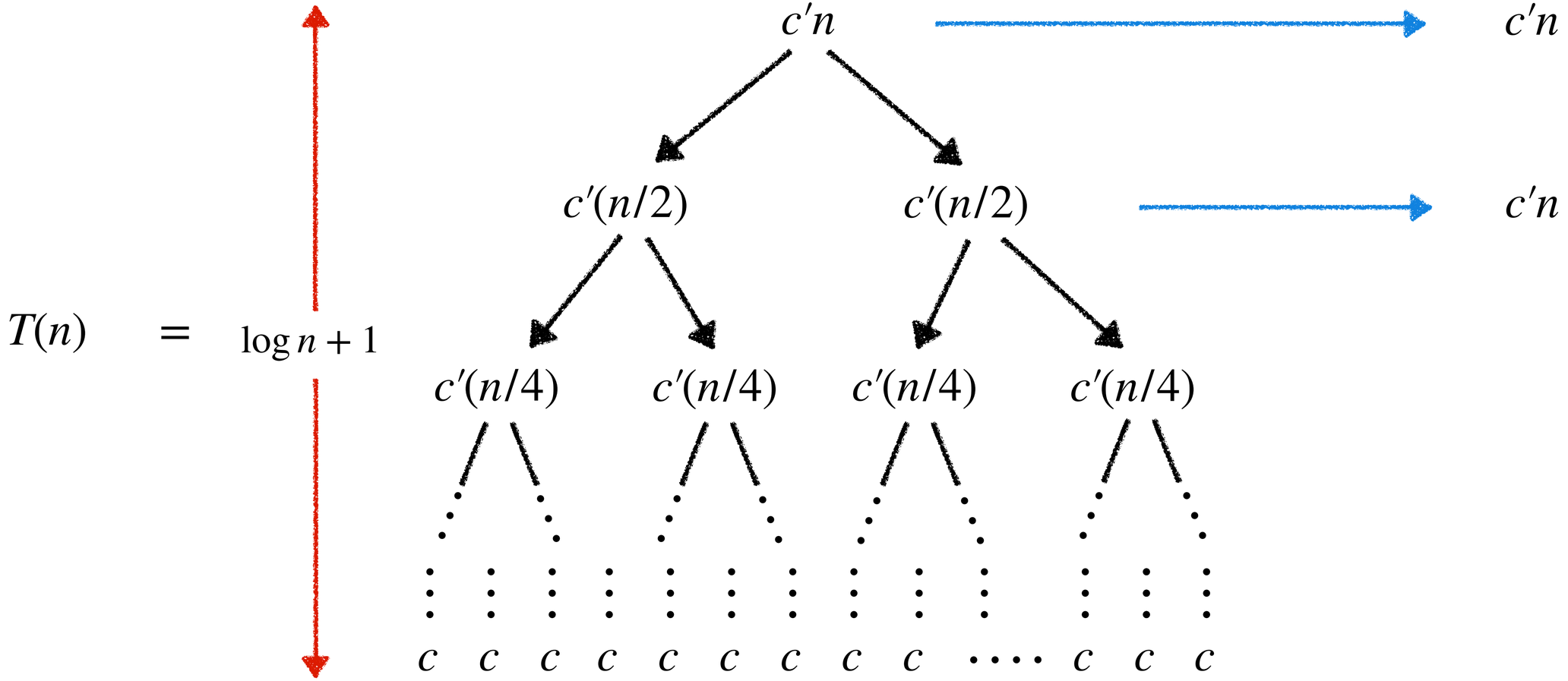

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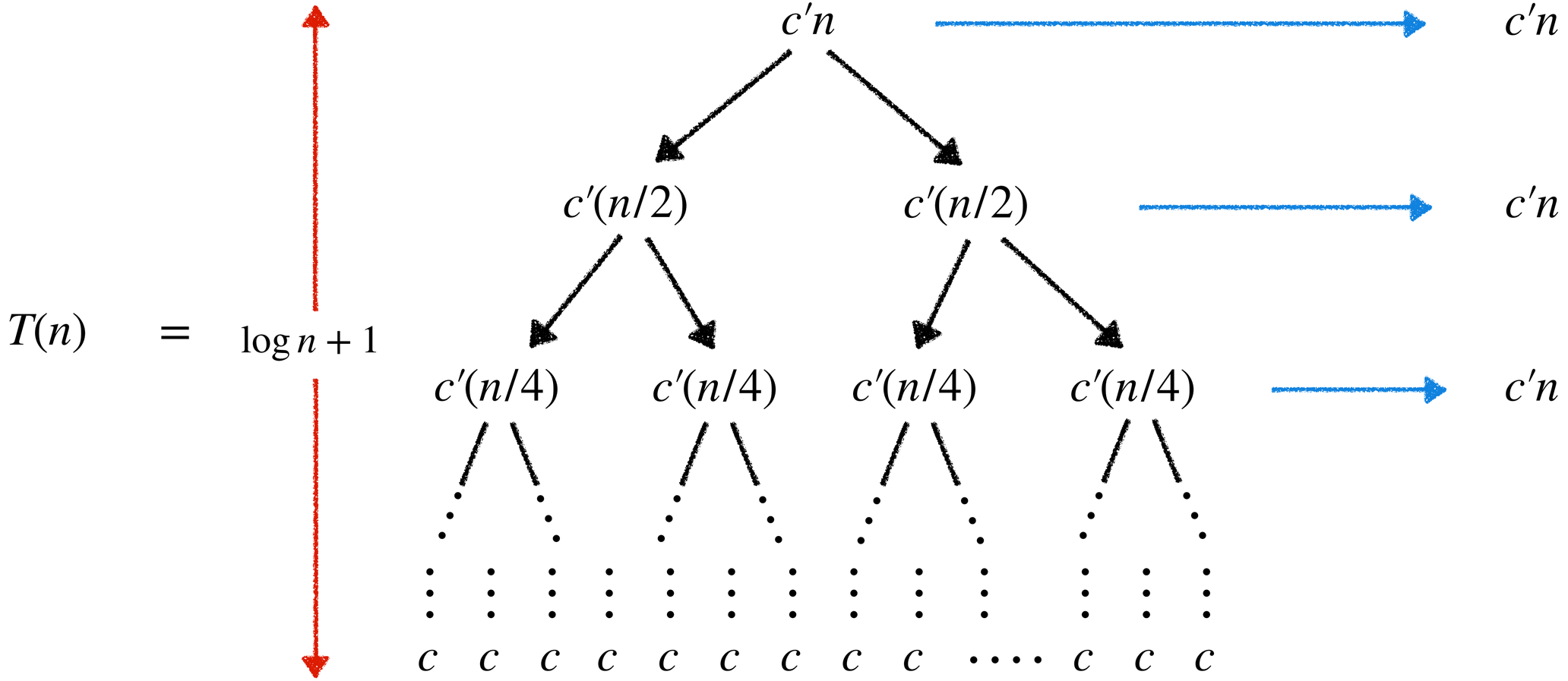

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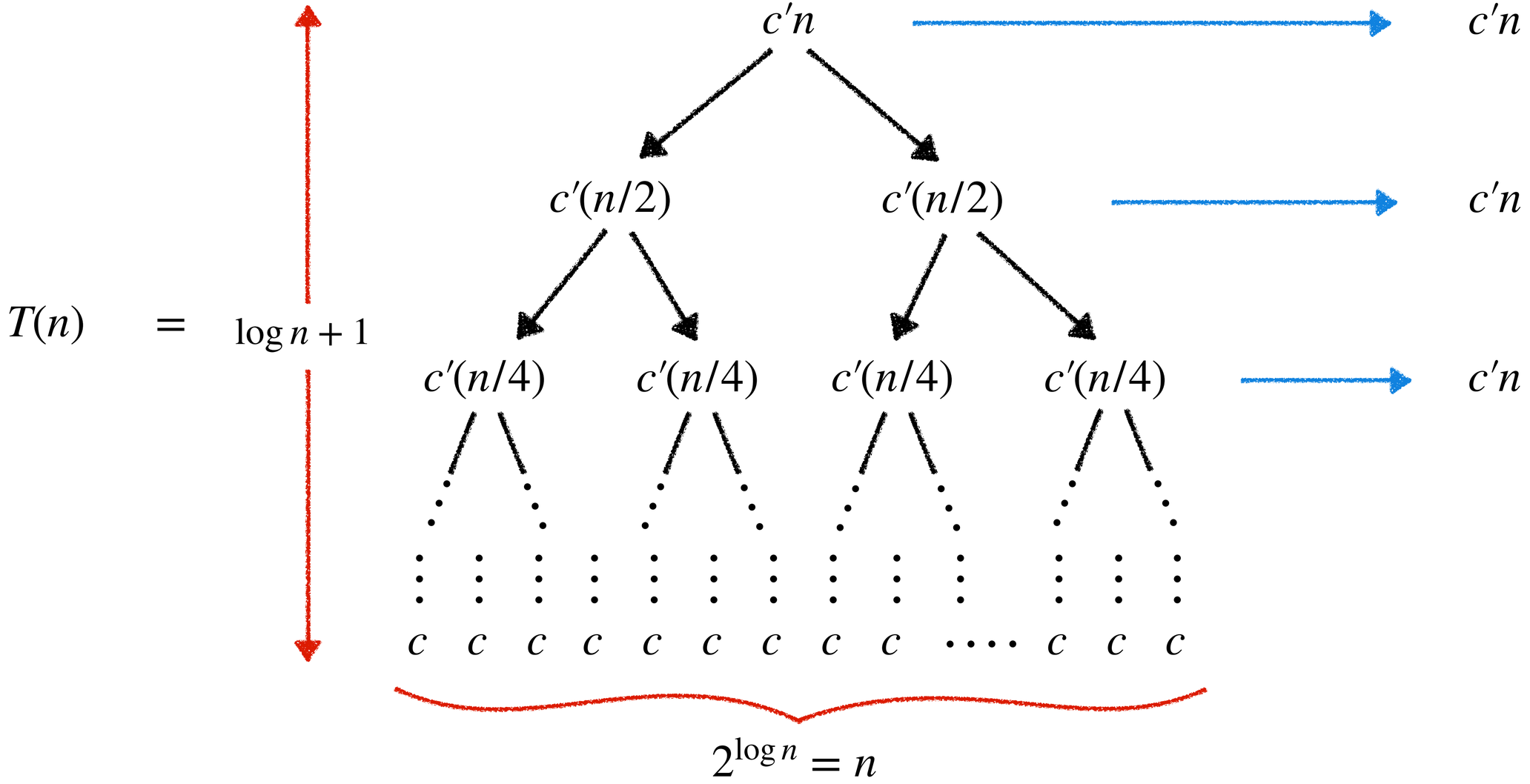

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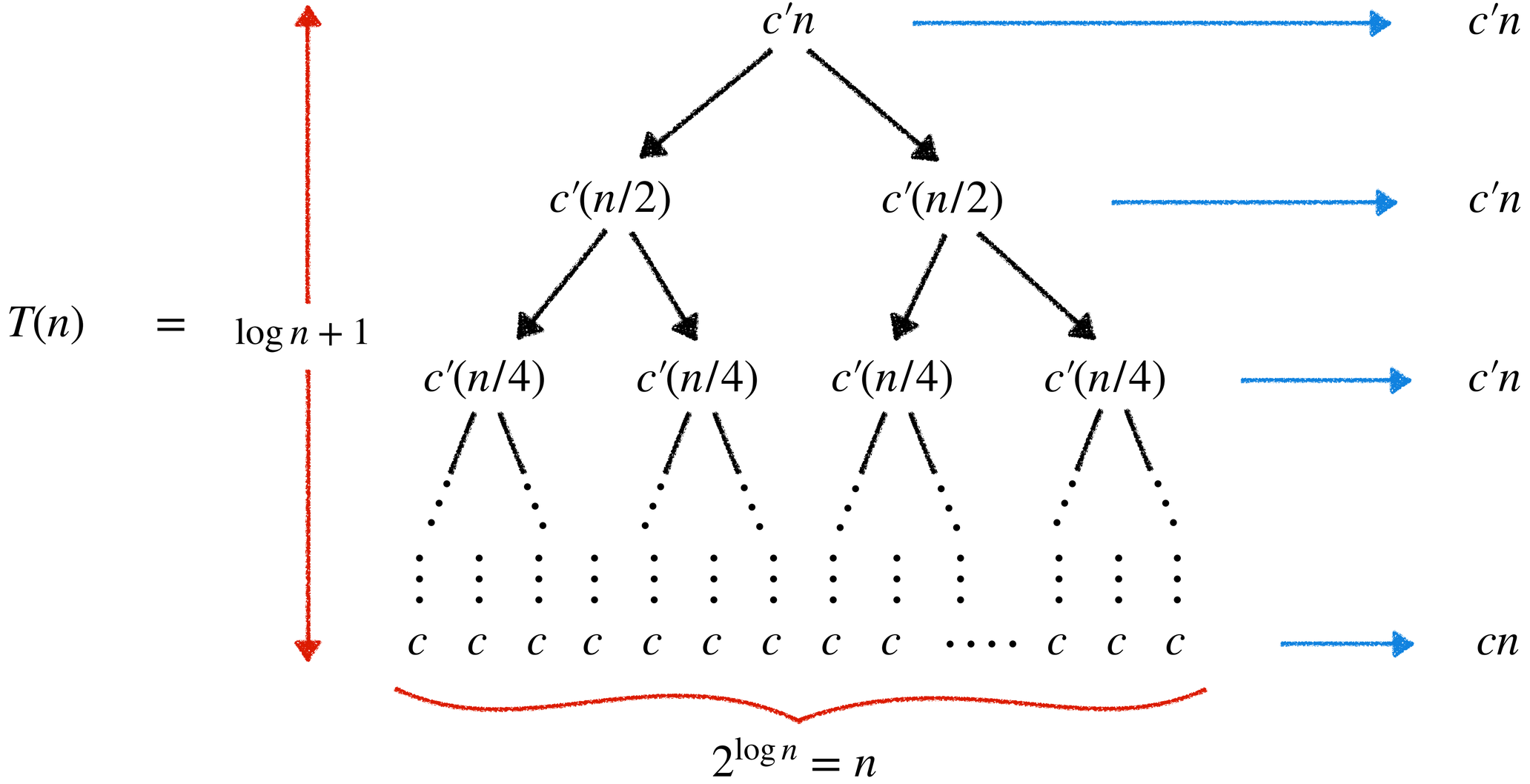

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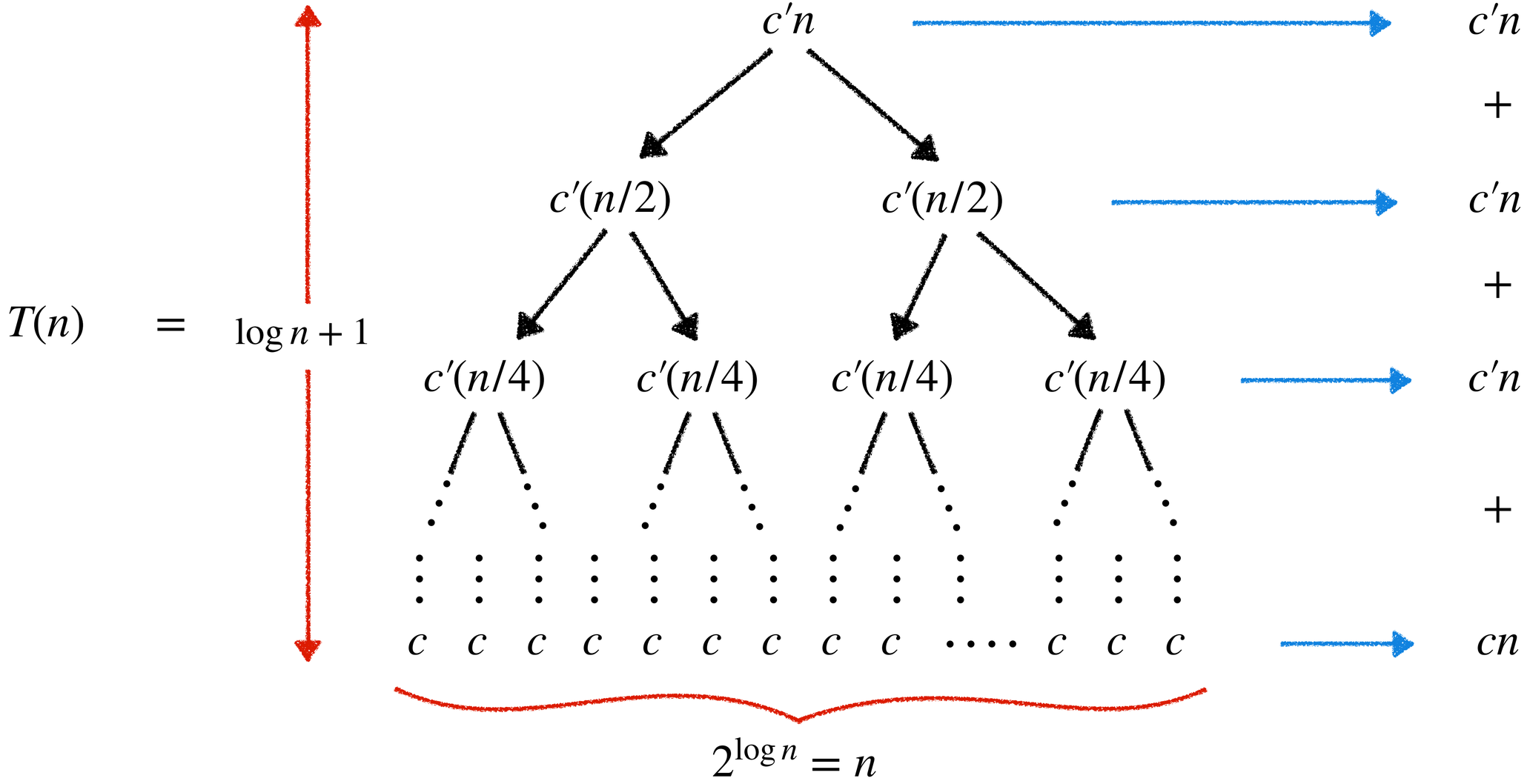

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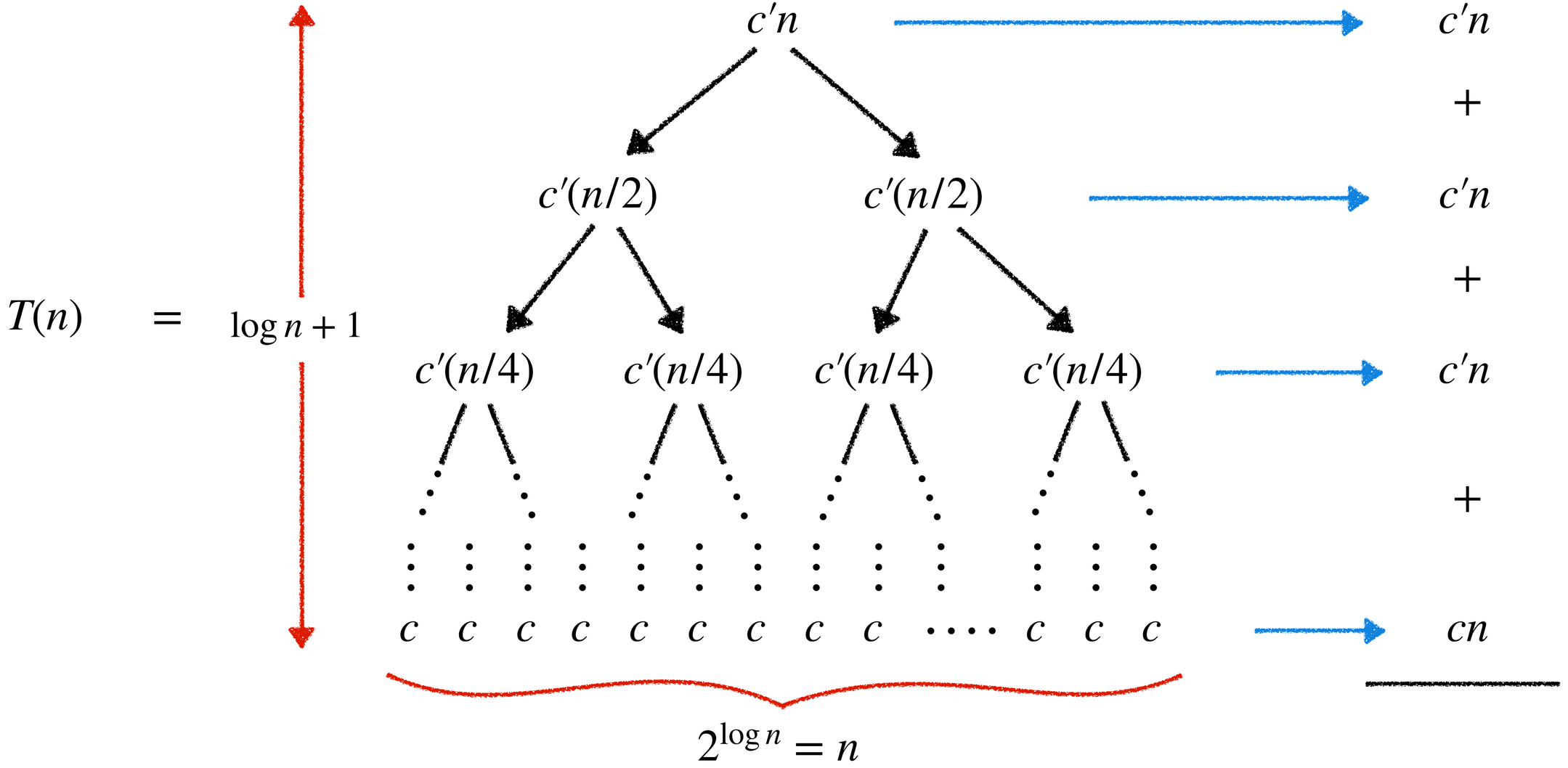

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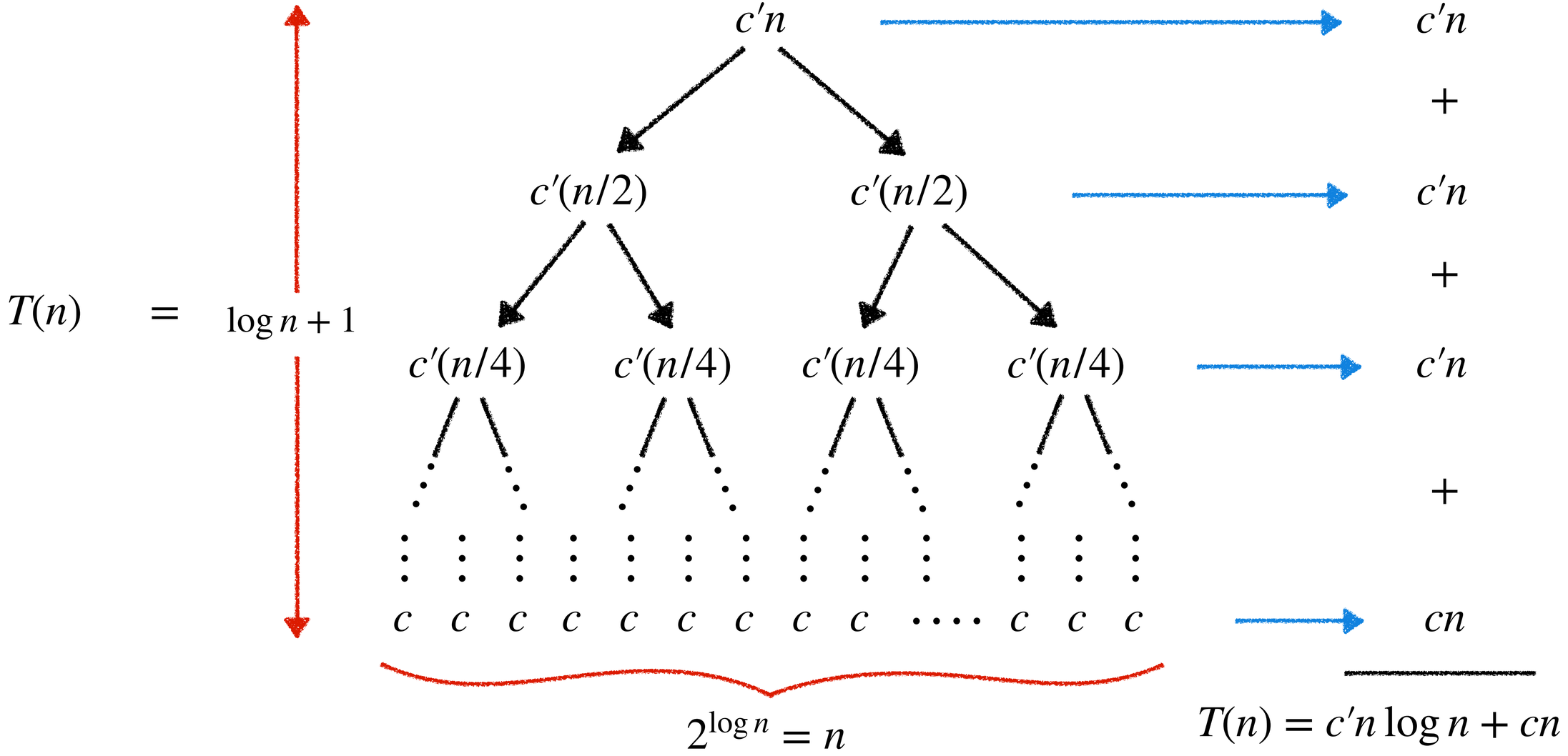

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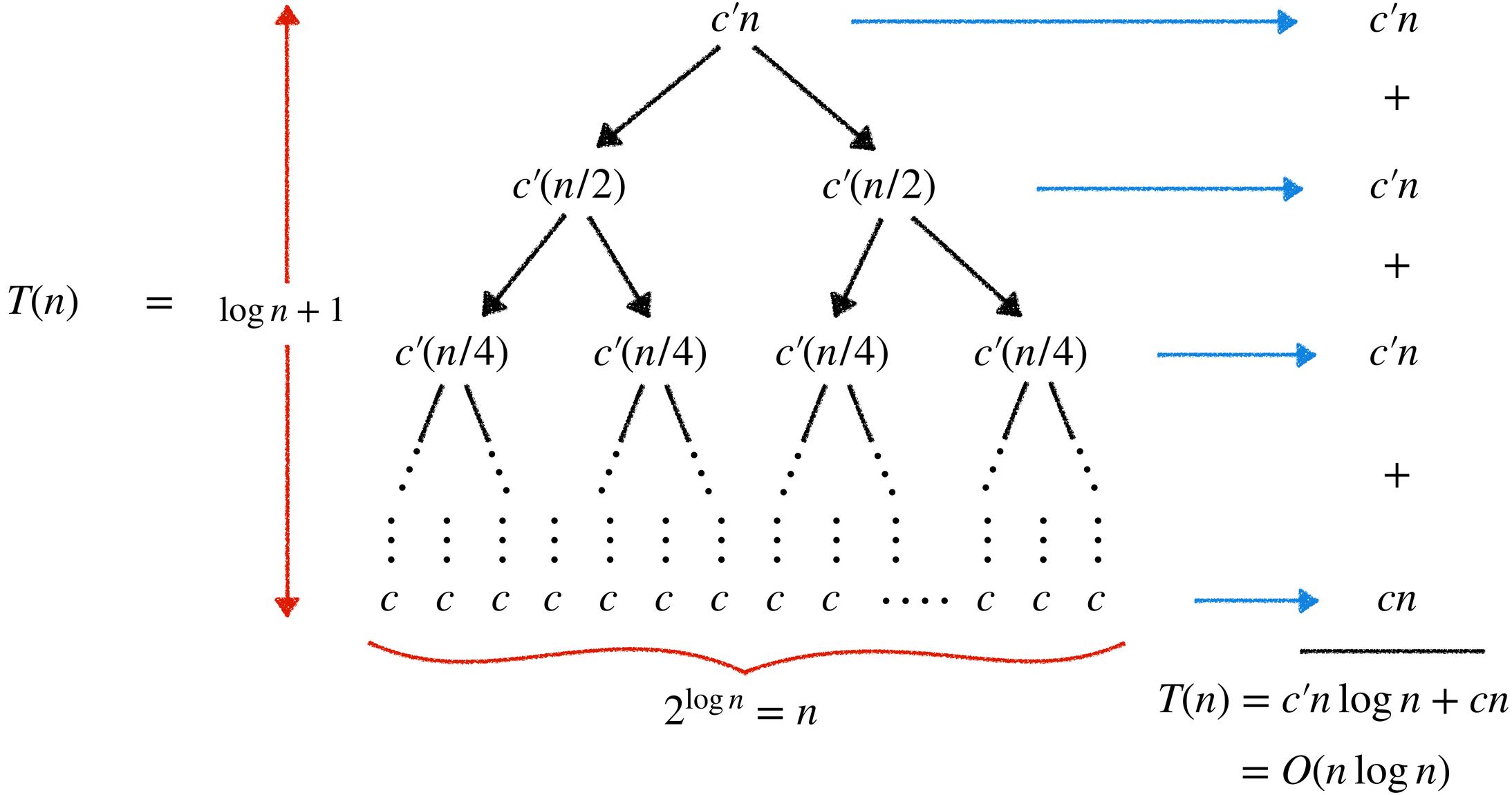

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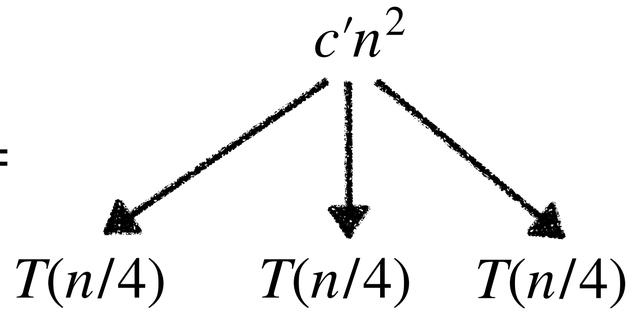
$T(n)$

Another Recurrence

$$T(n) = \begin{cases} c, & \text{if } n \leq 1 \\ 3.T(n/4) + c'n^2, & \text{otherwise} \end{cases}$$

Computing $T(n)$:

$T(n) =$



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Computing $T(n)$: