Lecture 14

Introduction to Graphs

Source: Introduction to Algorithms, CLRS

What’s a Graph?

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It’s just dots and connections.

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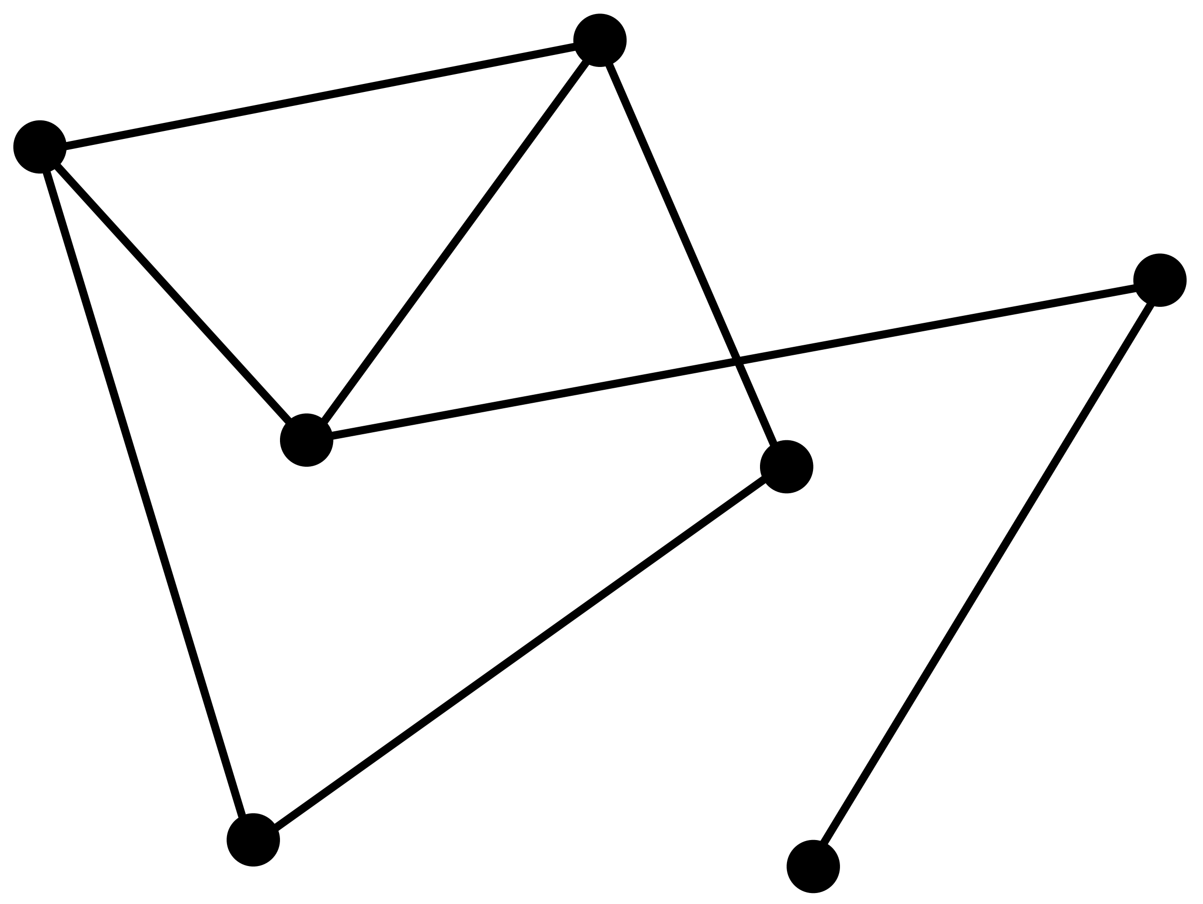

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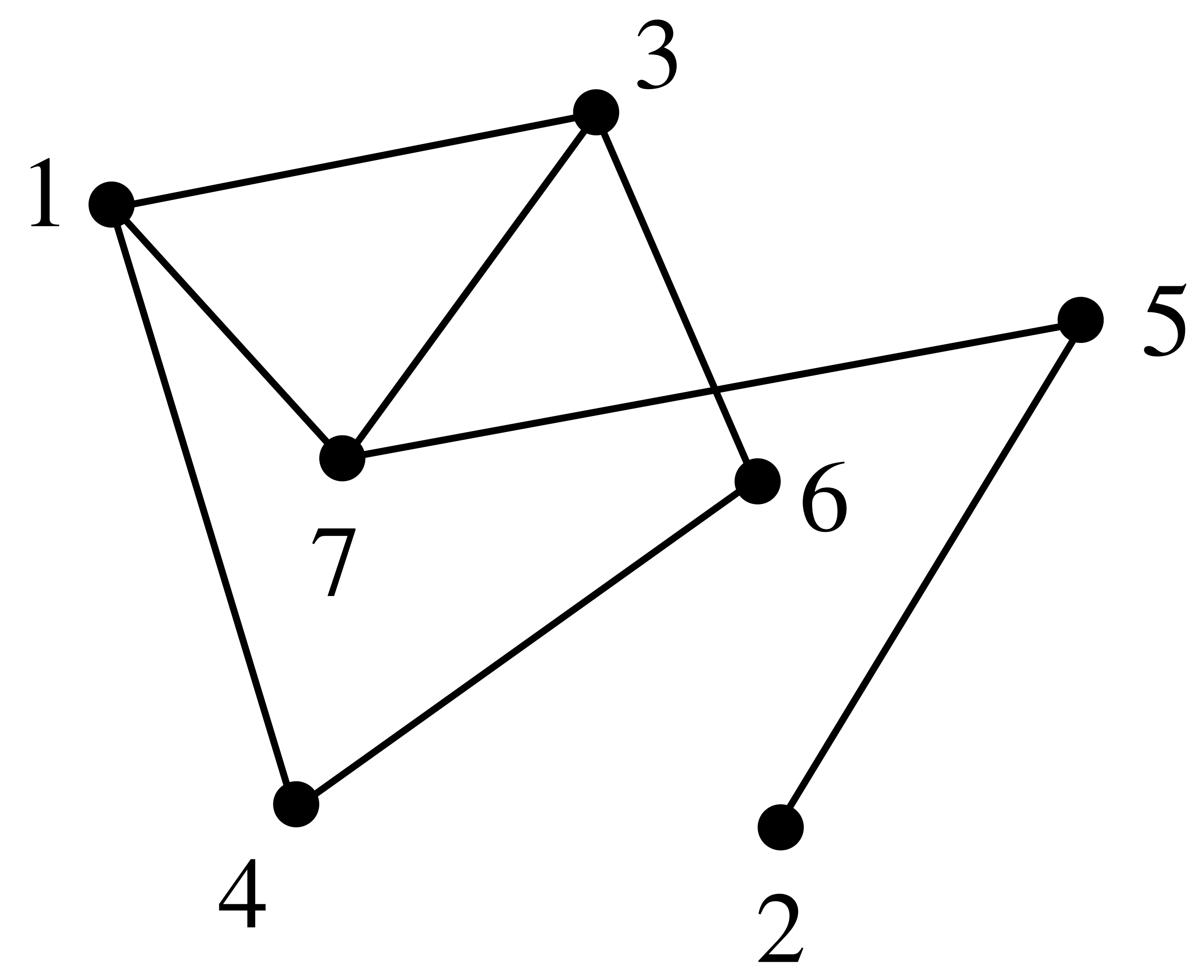

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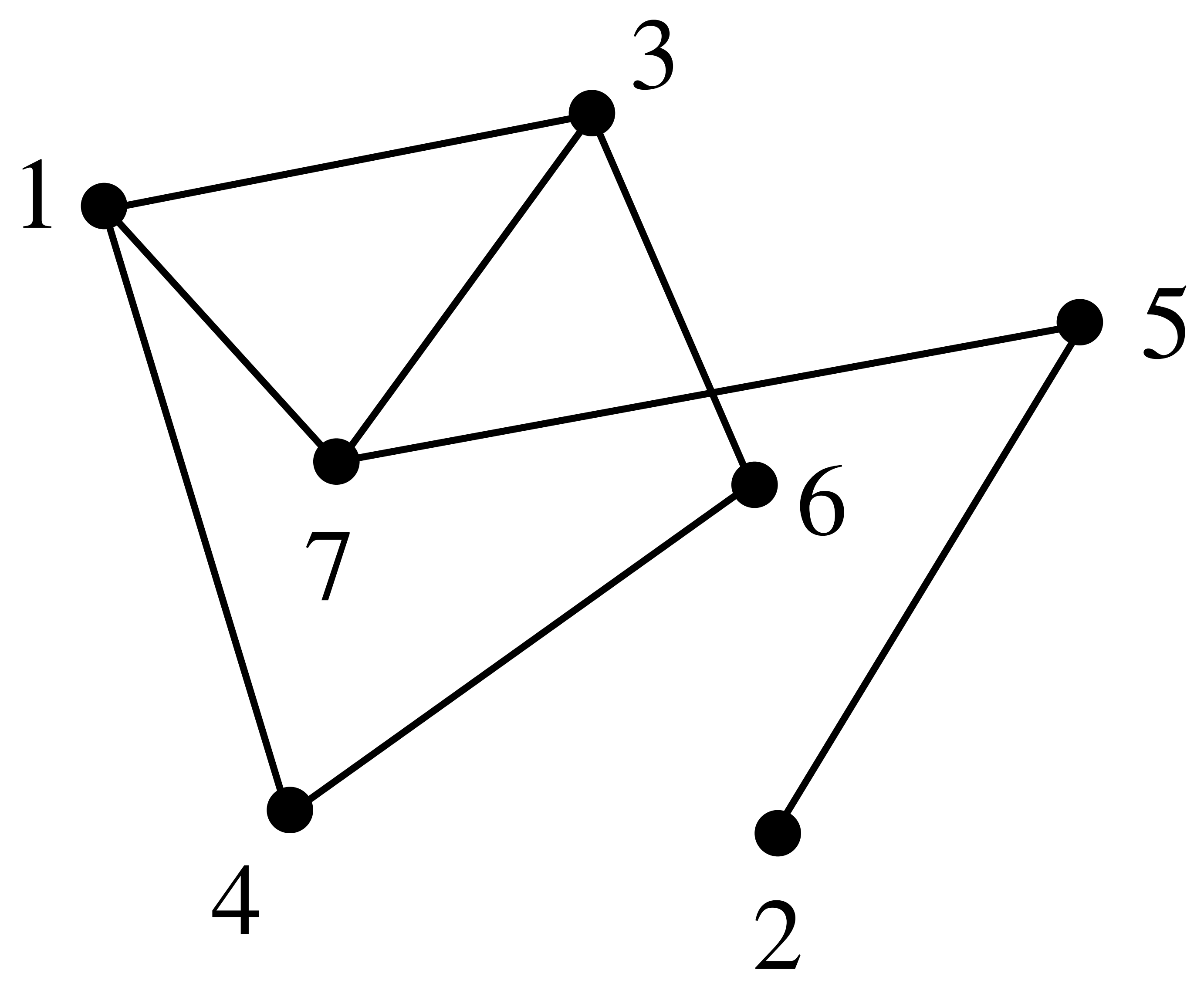

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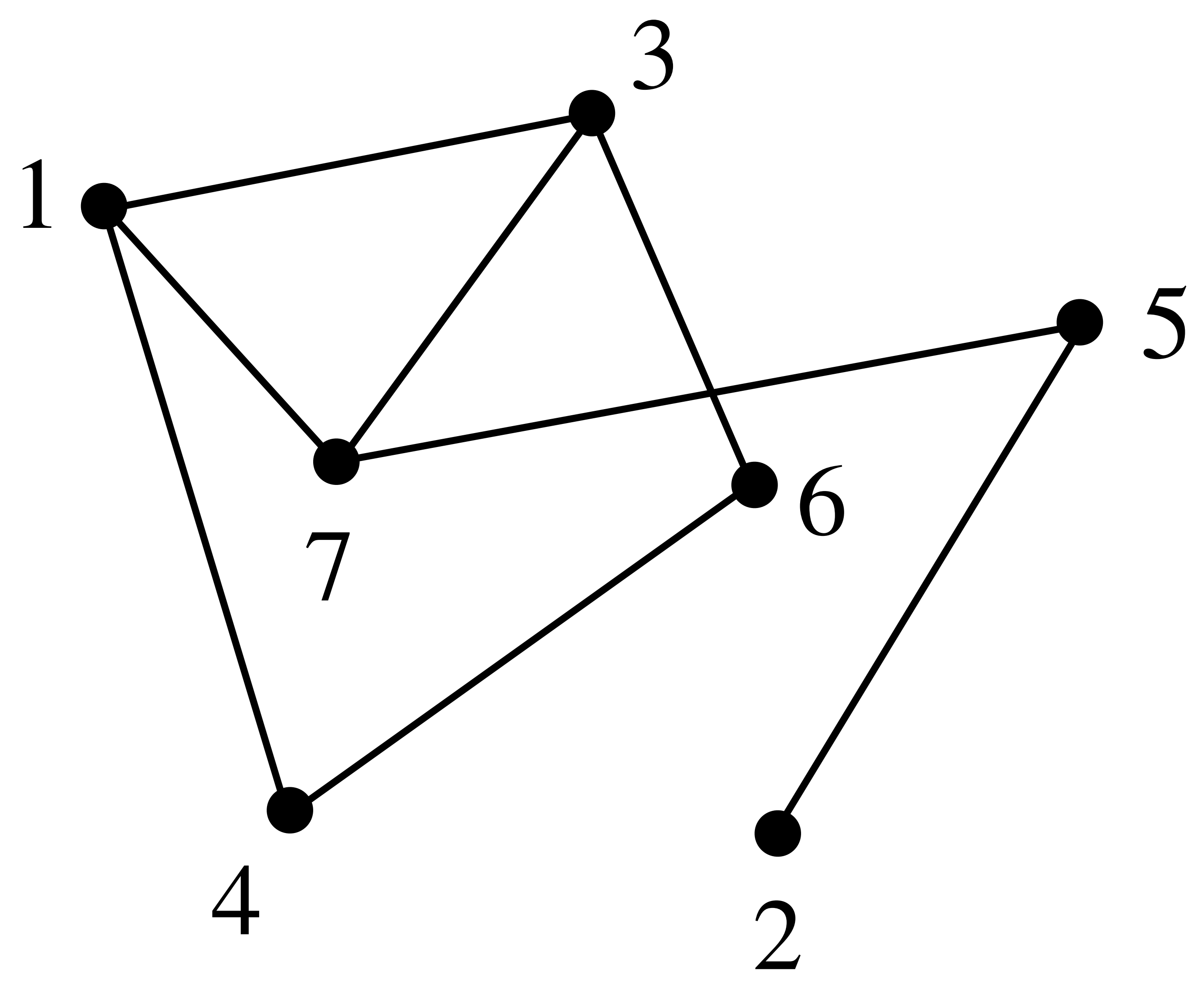

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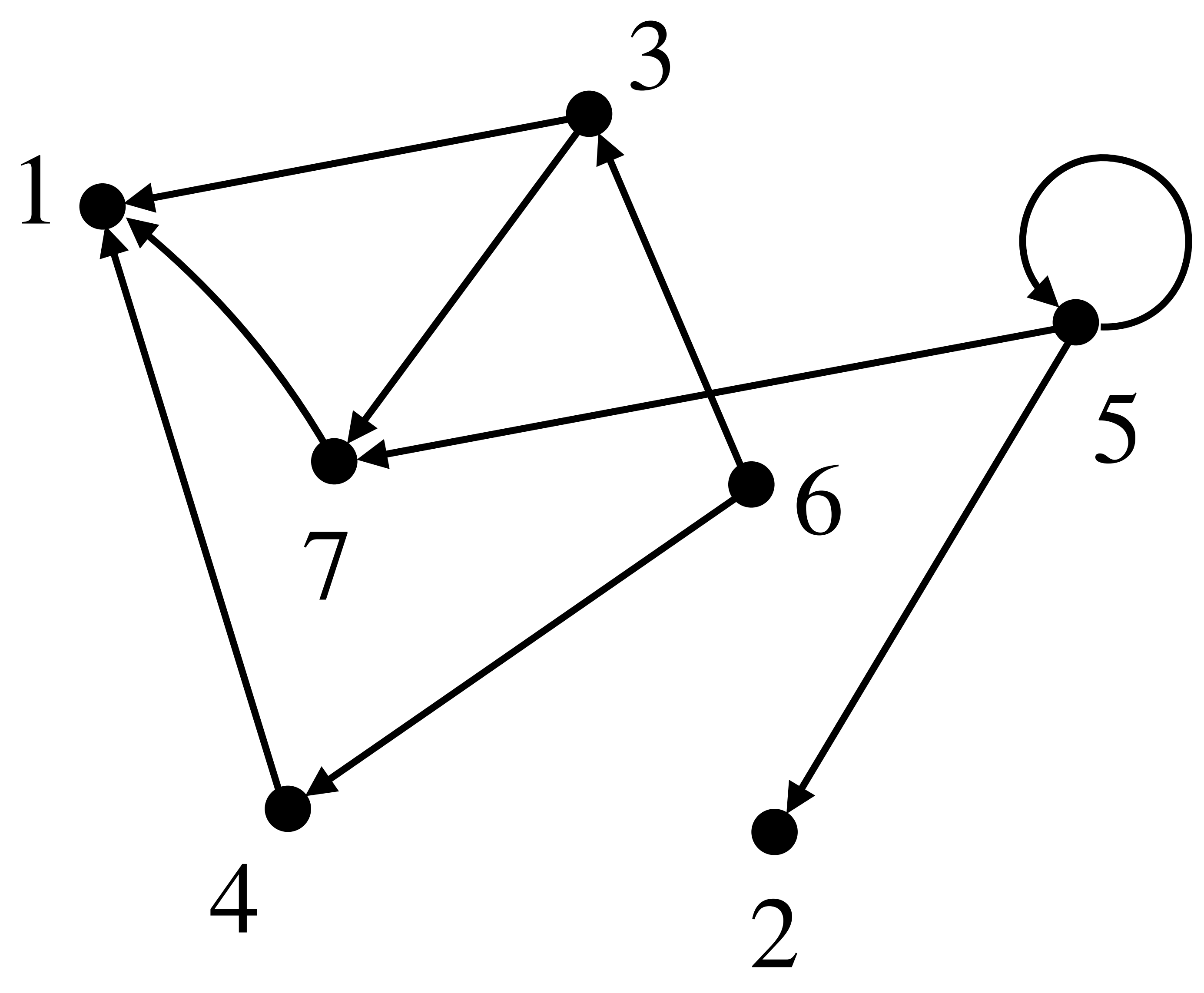

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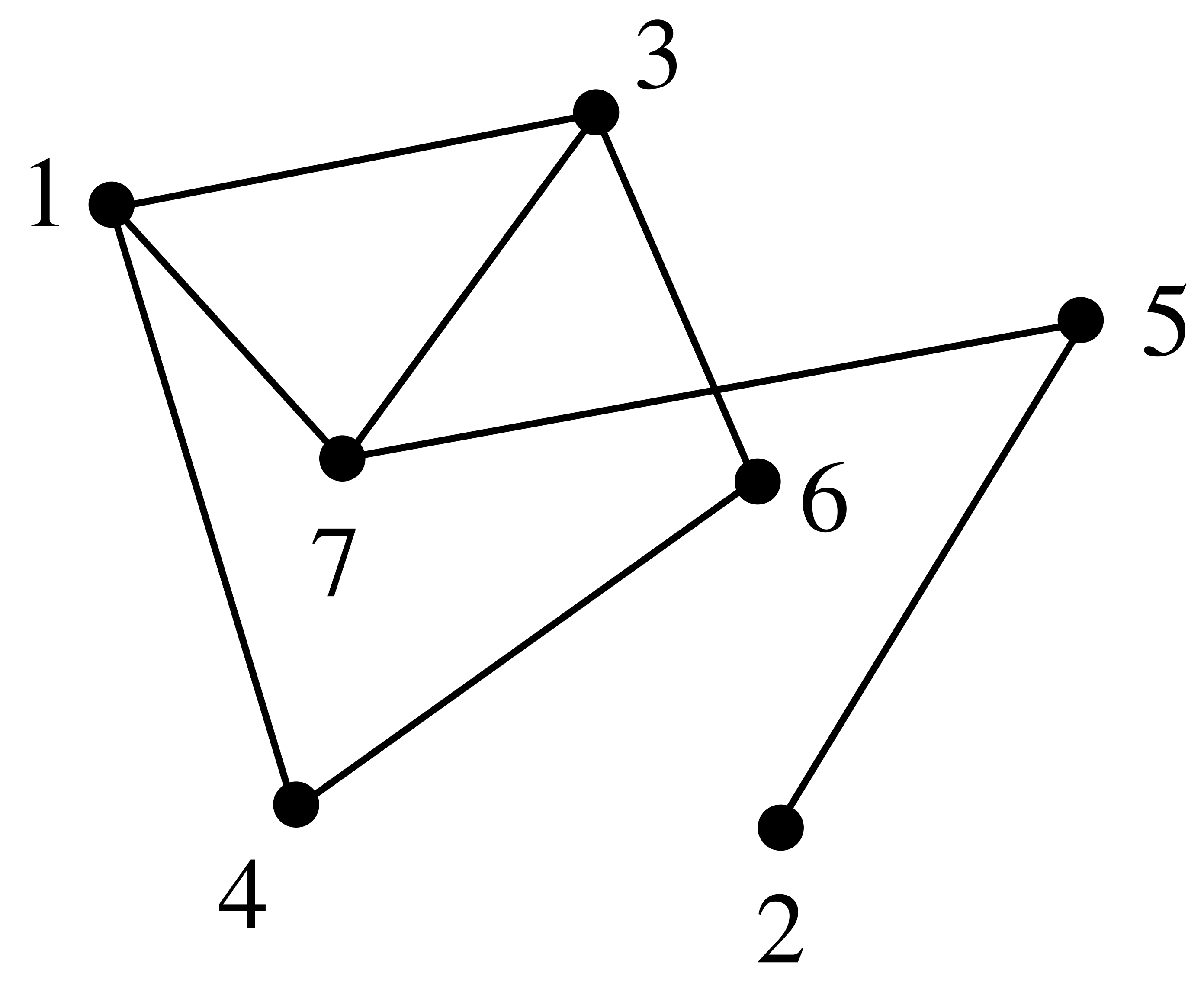

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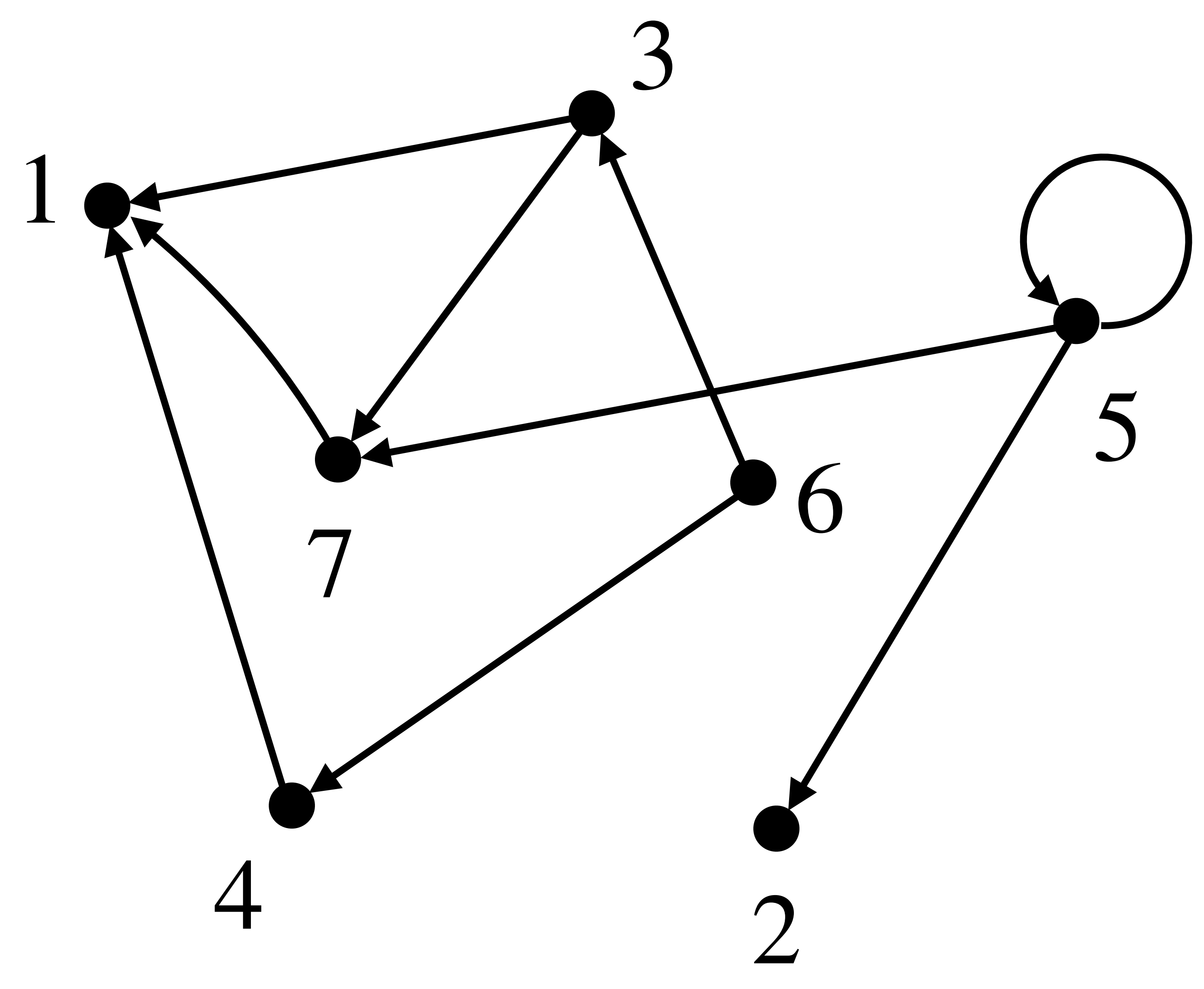
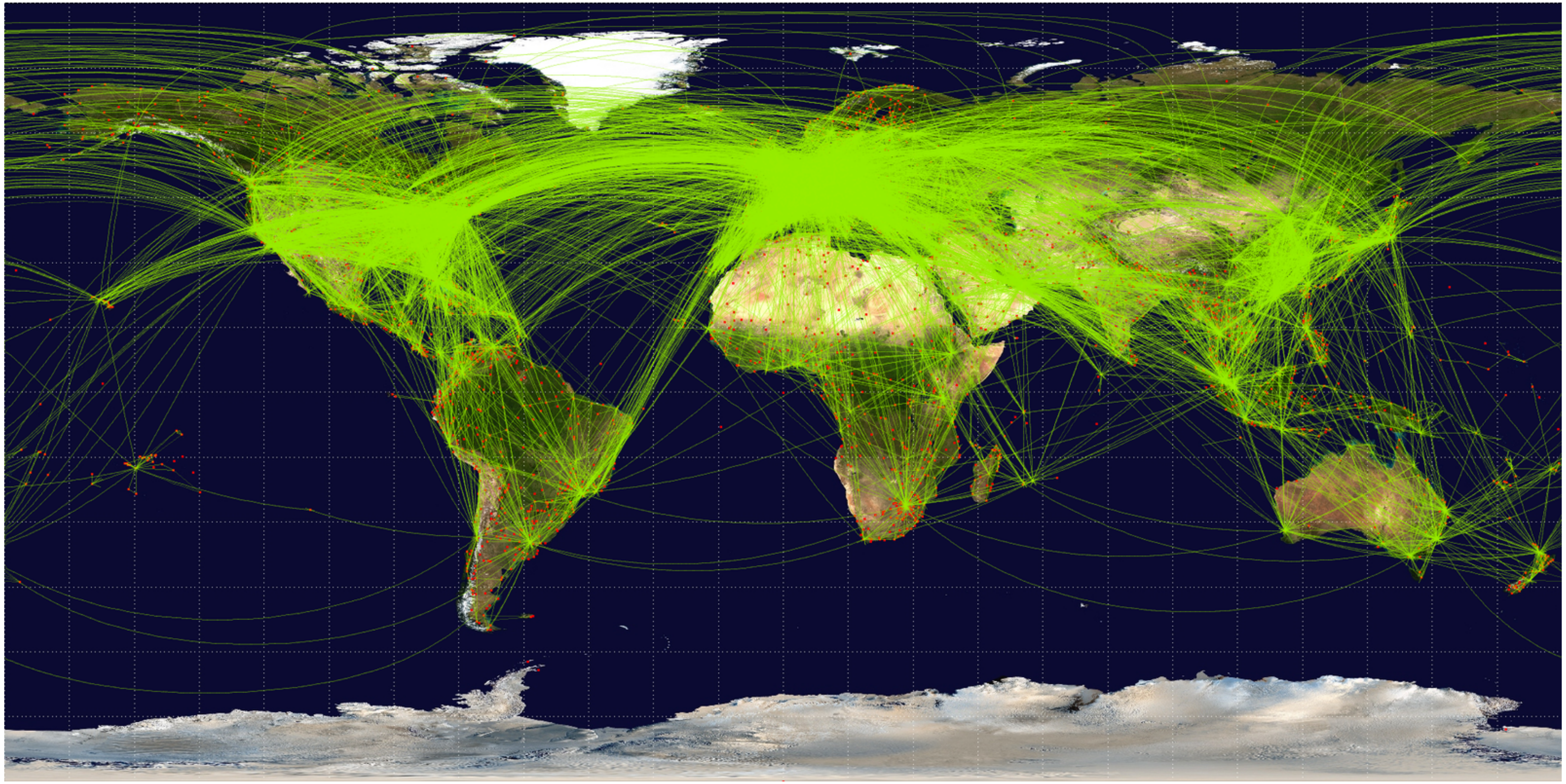


A directed graph

Graphs in Real Life



World airline map

Source: Wiki

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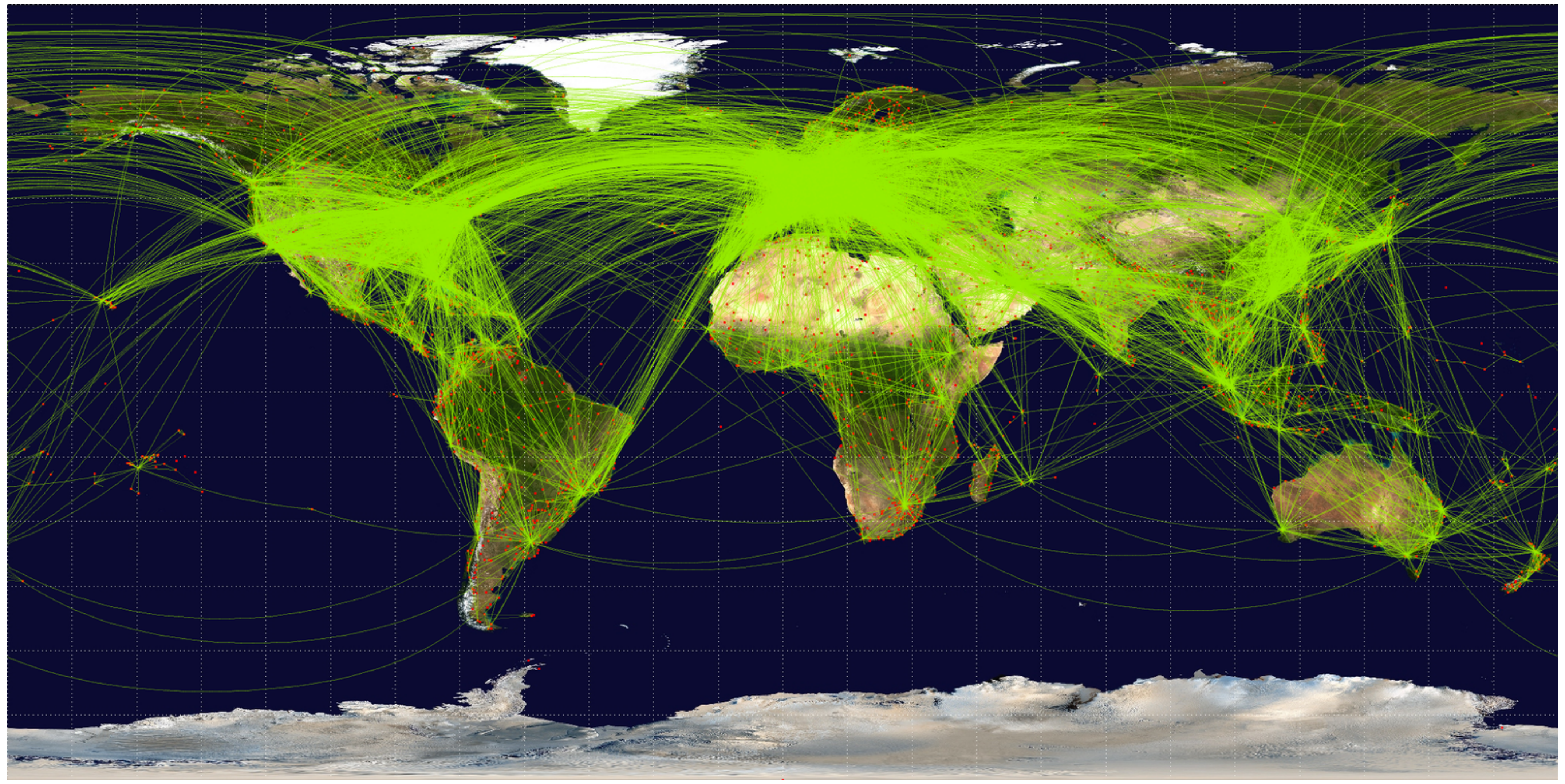

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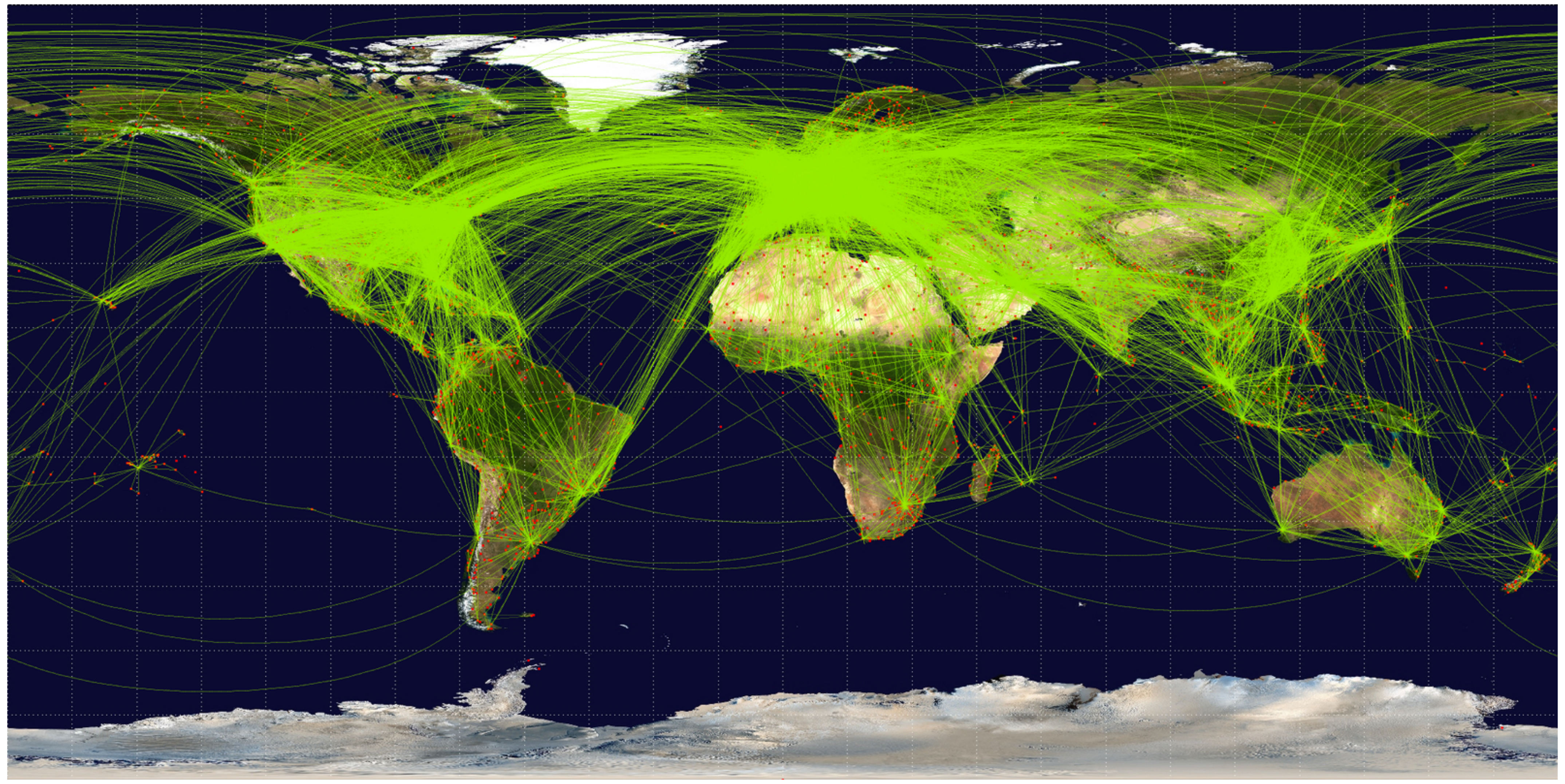

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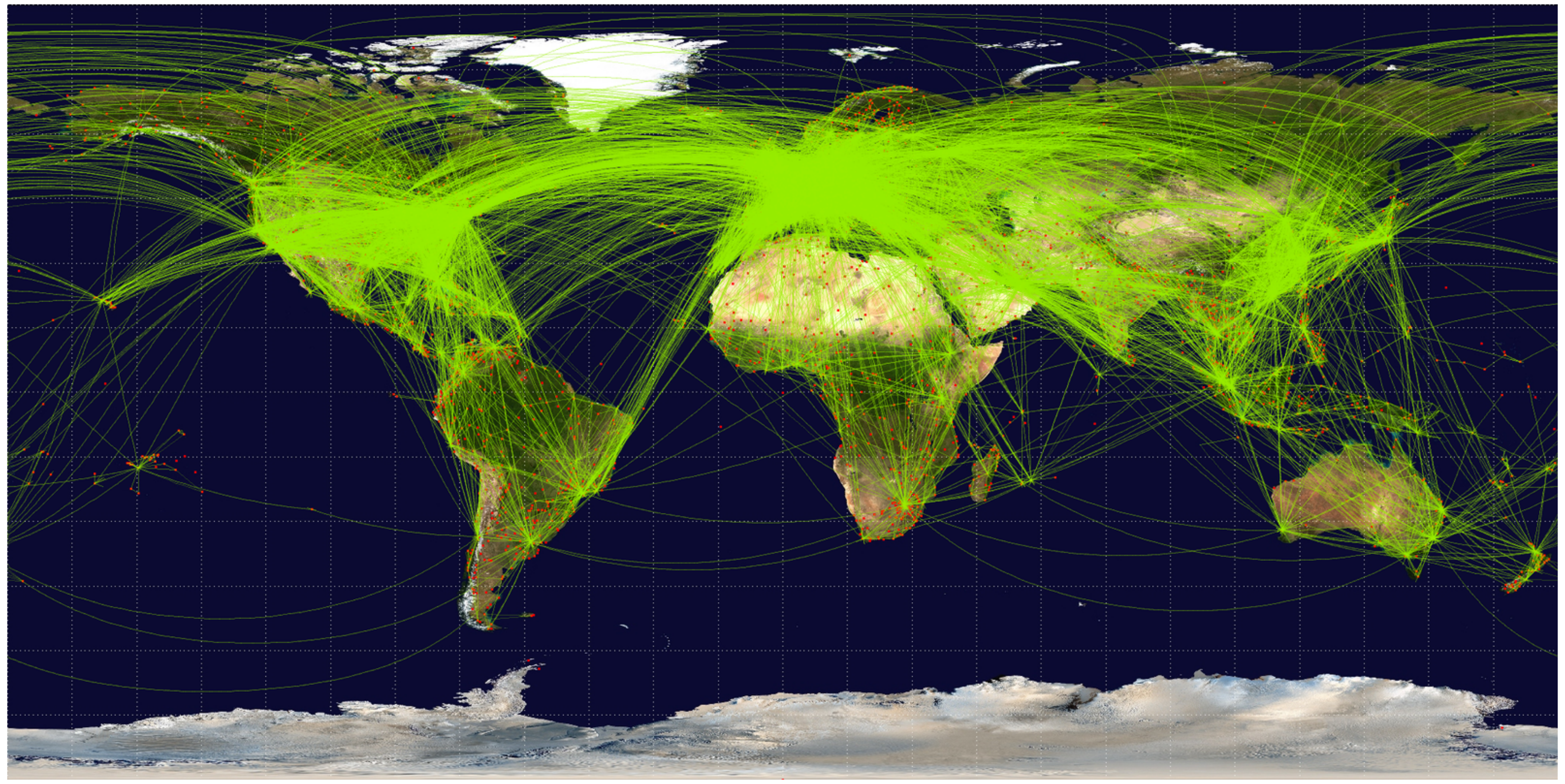
Possible queries:

- What's the shortest route from **Delhi** to **Berlin**?



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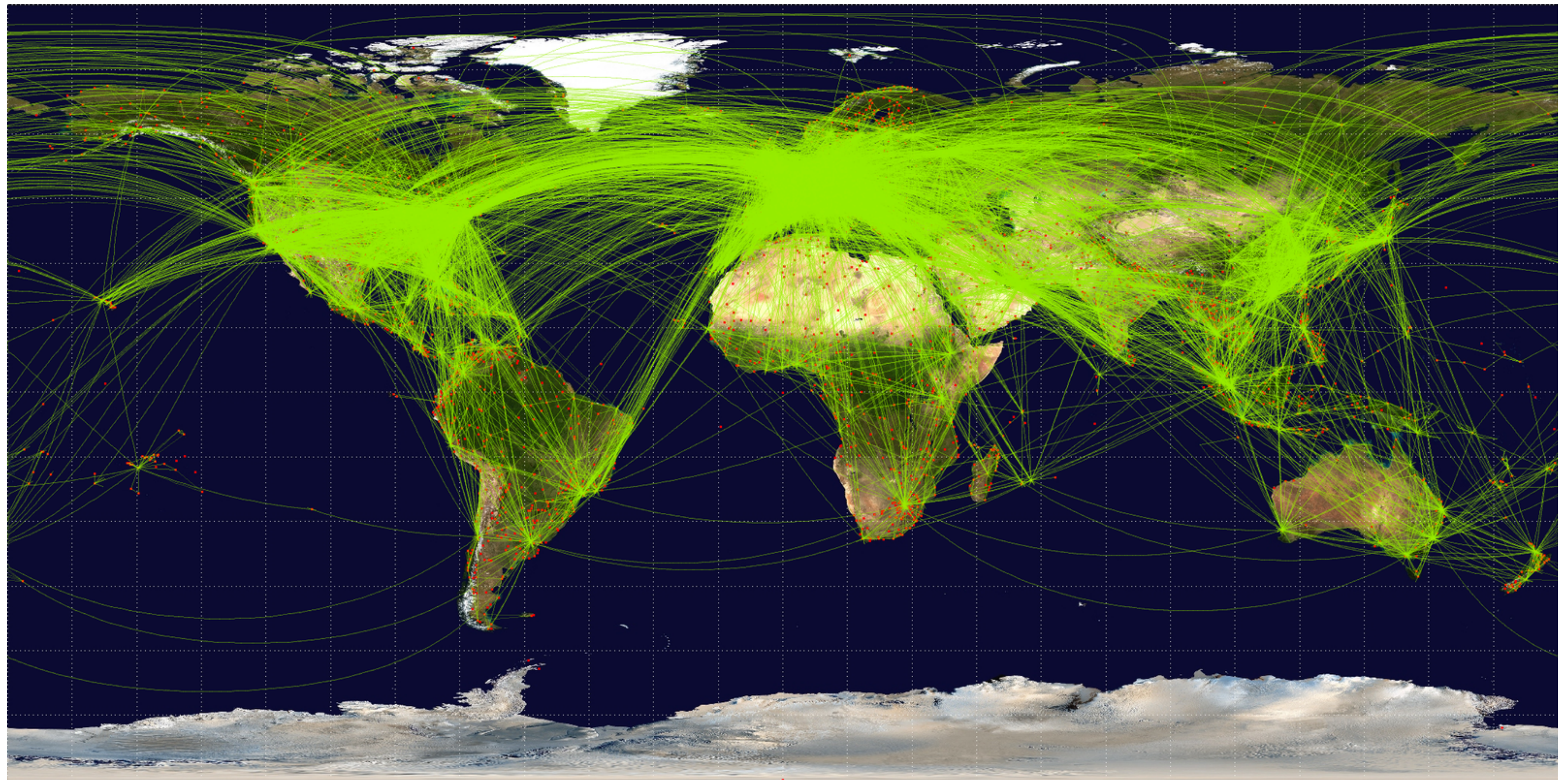
Possible queries:

- What's the shortest route from **Delhi** to **Berlin**?
- Is there a route from **Lucknow** to **Helsinki**?



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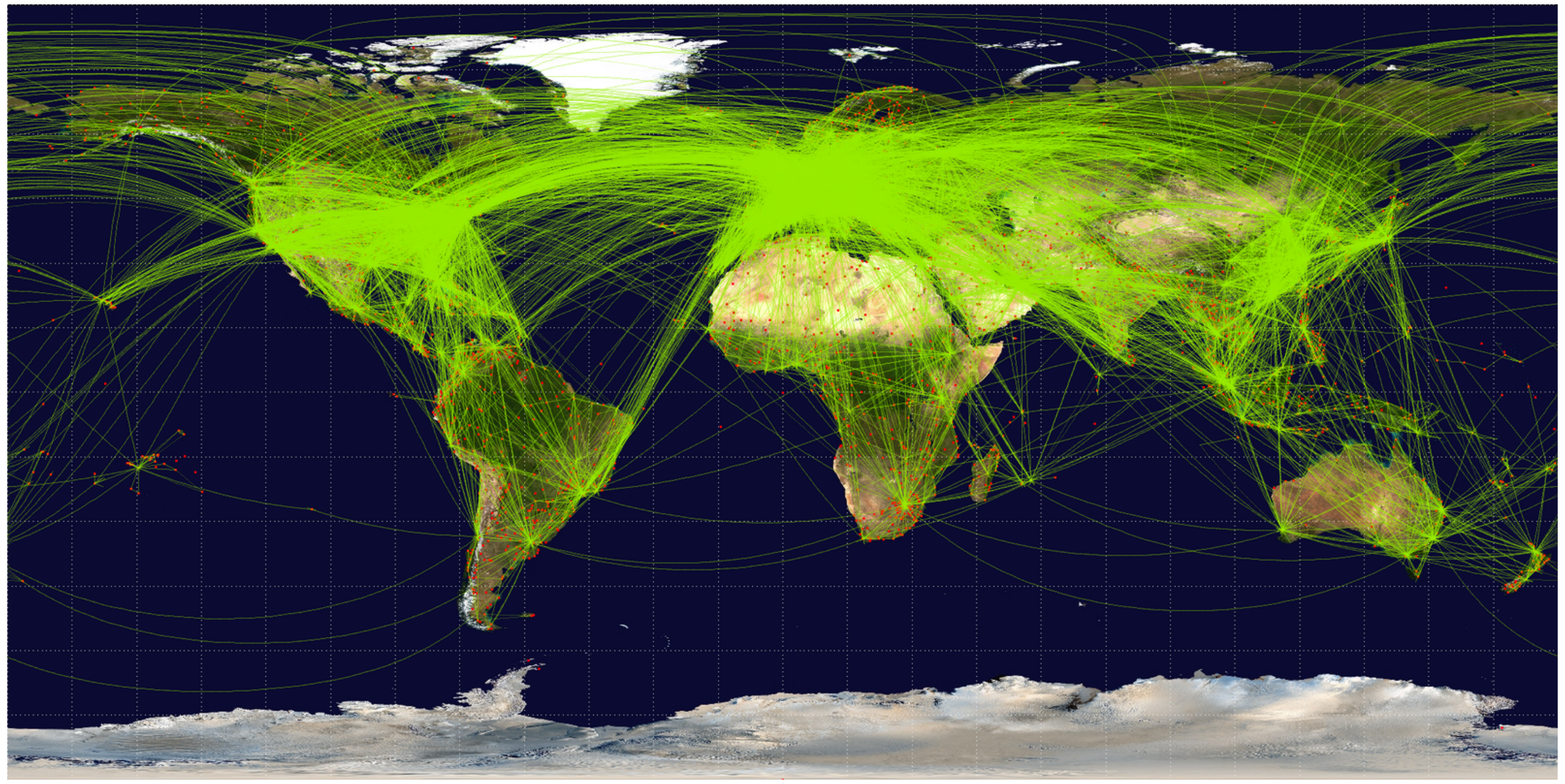
Possible queries:

- What's the shortest route from Delhi to Berlin?
- Is there a route from Lucknow to Helsinki?
- How many cities have a direct flight from Mumbai?



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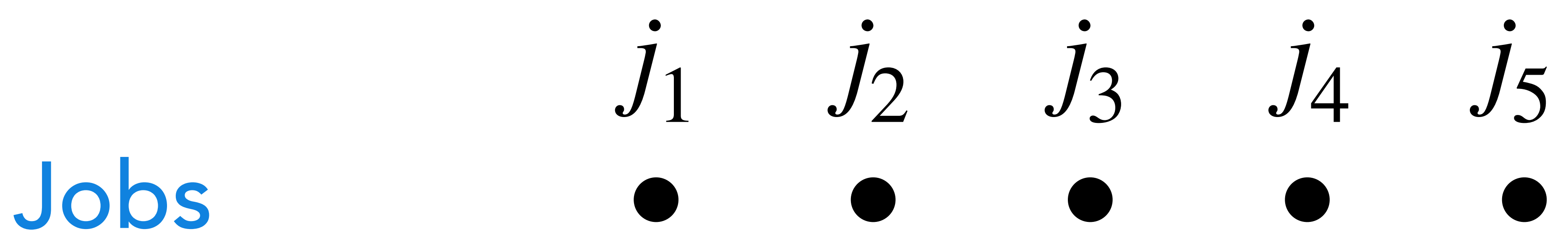

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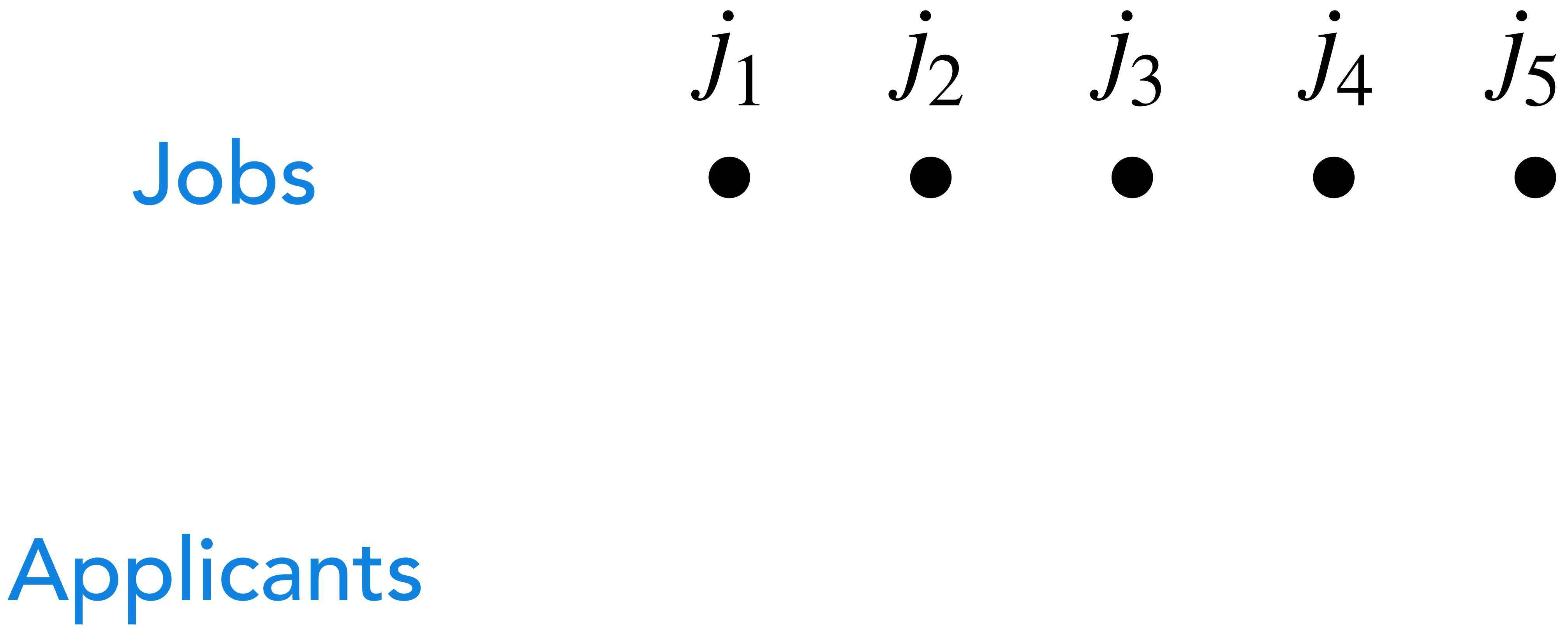

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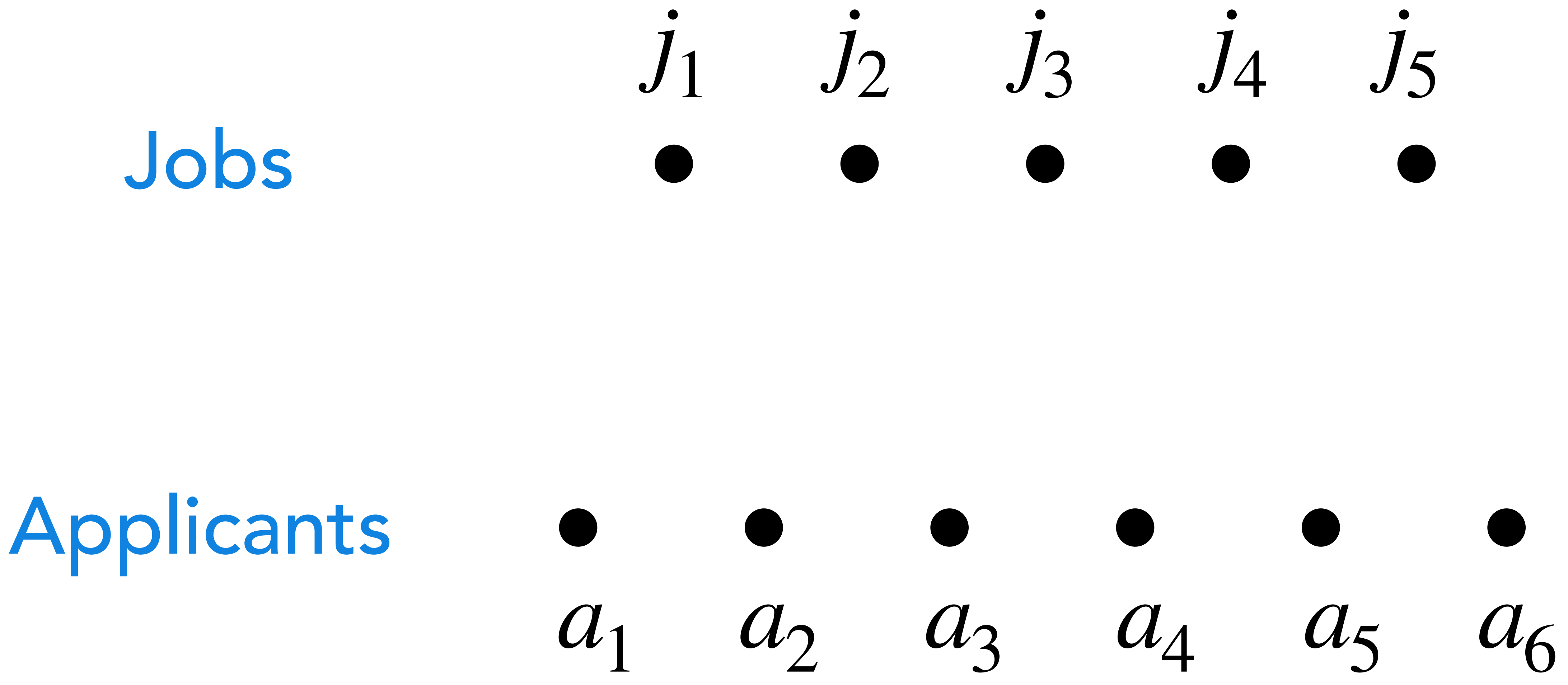

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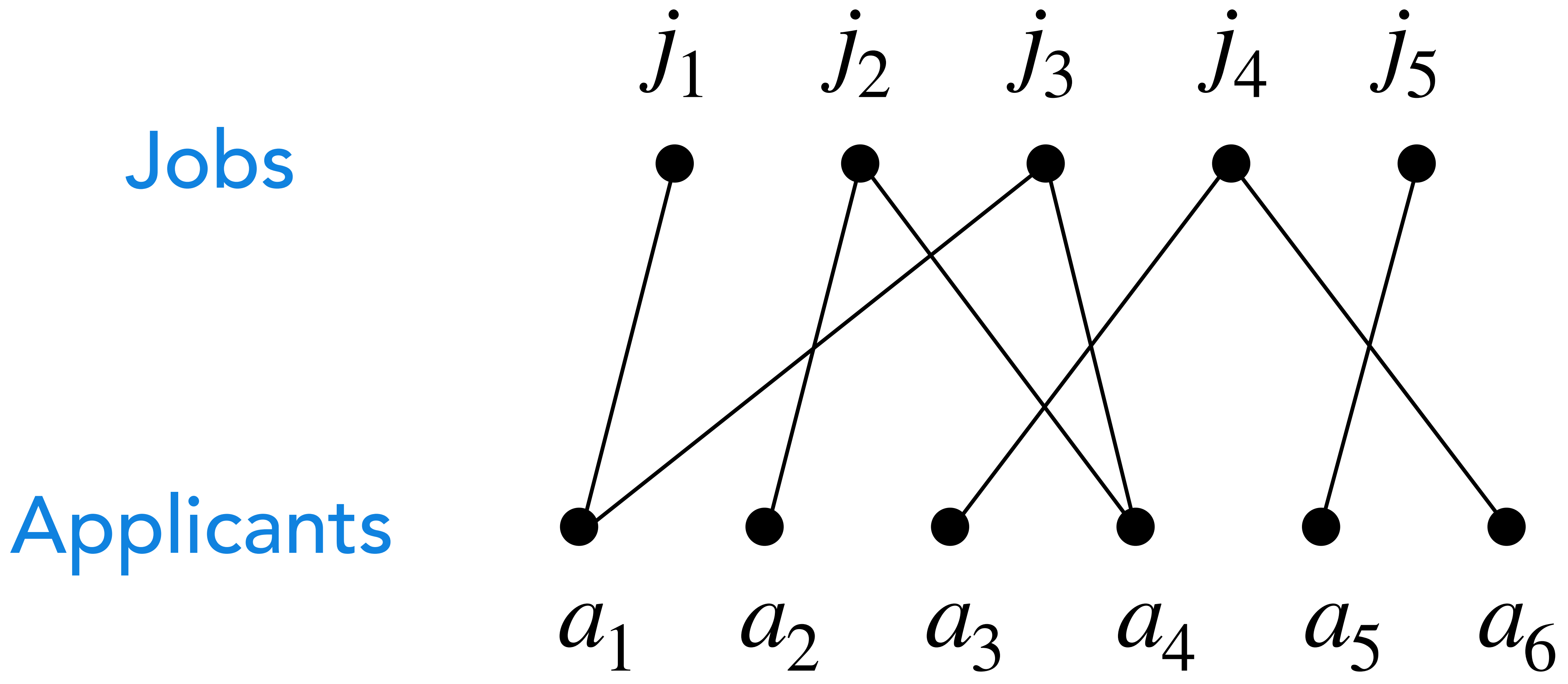

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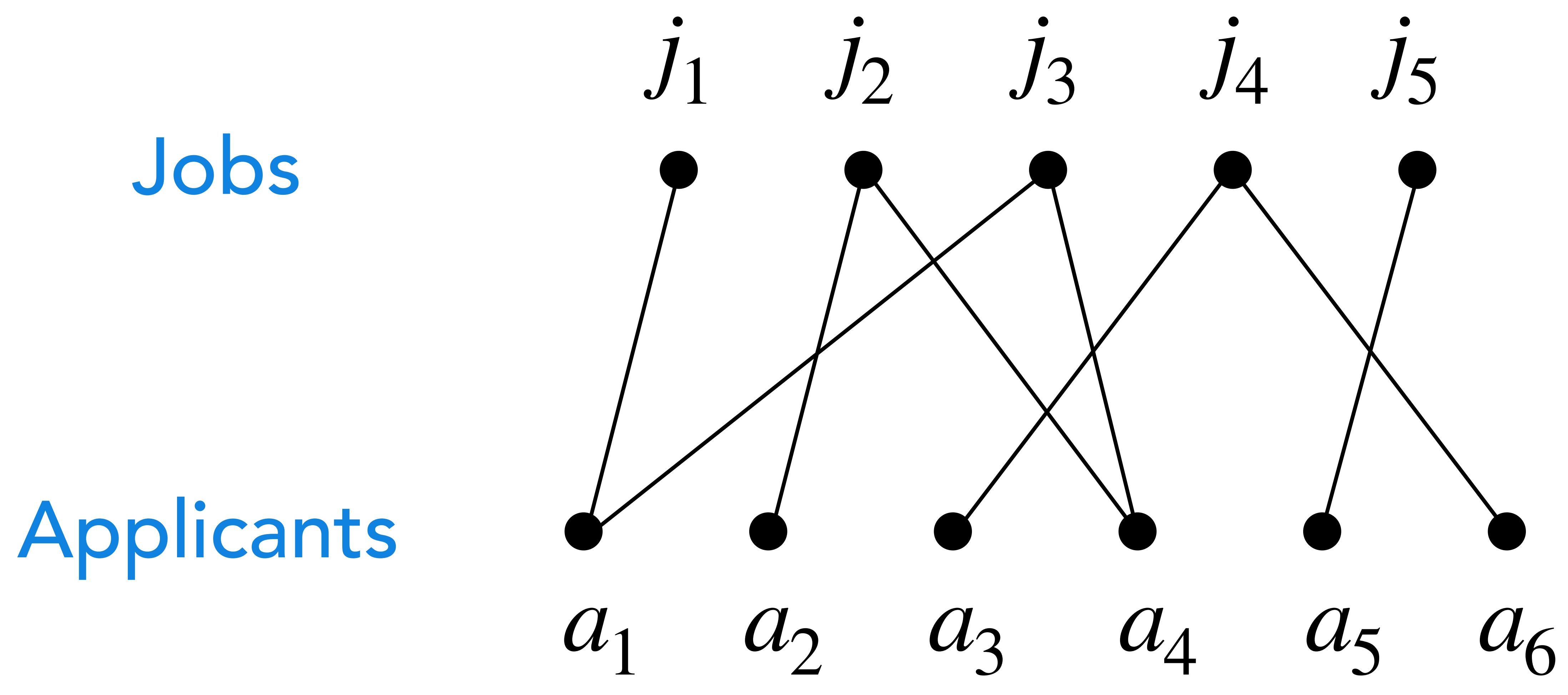

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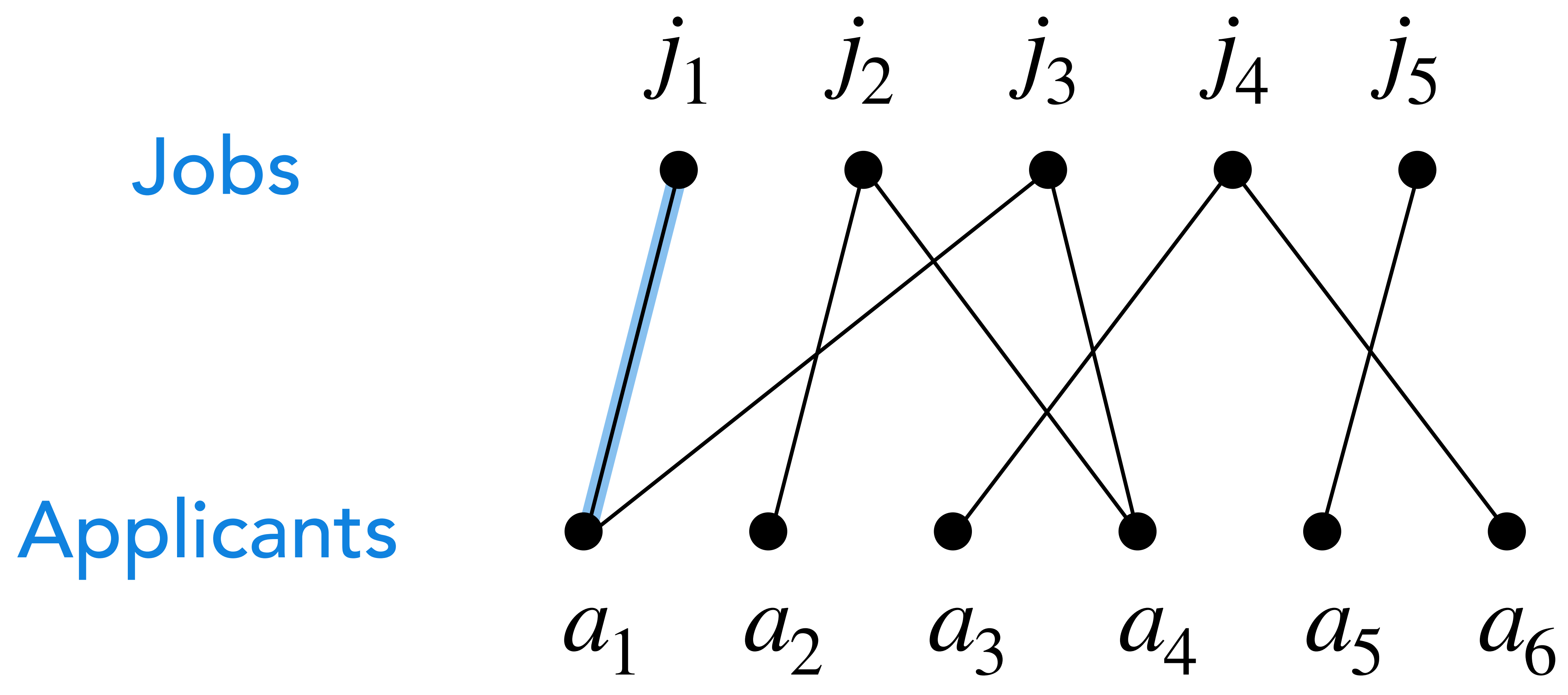

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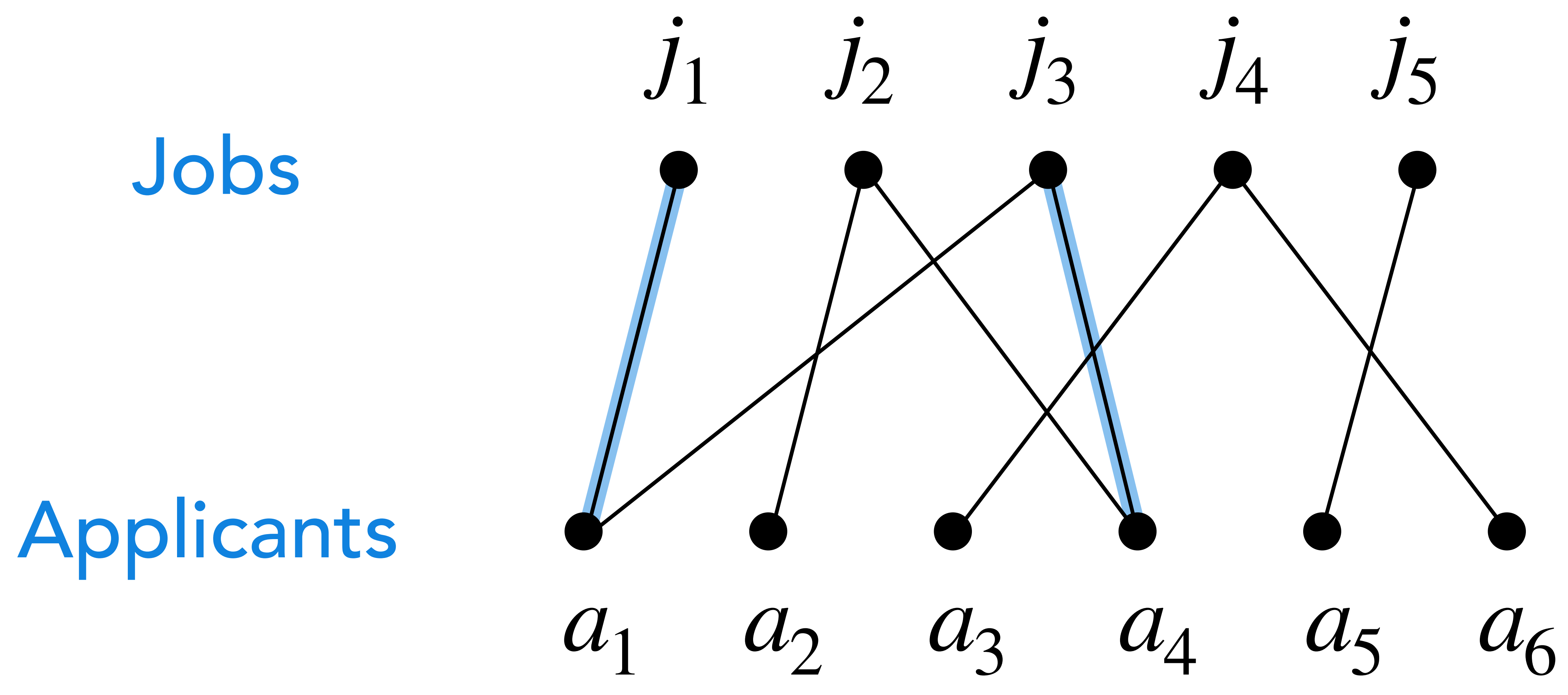

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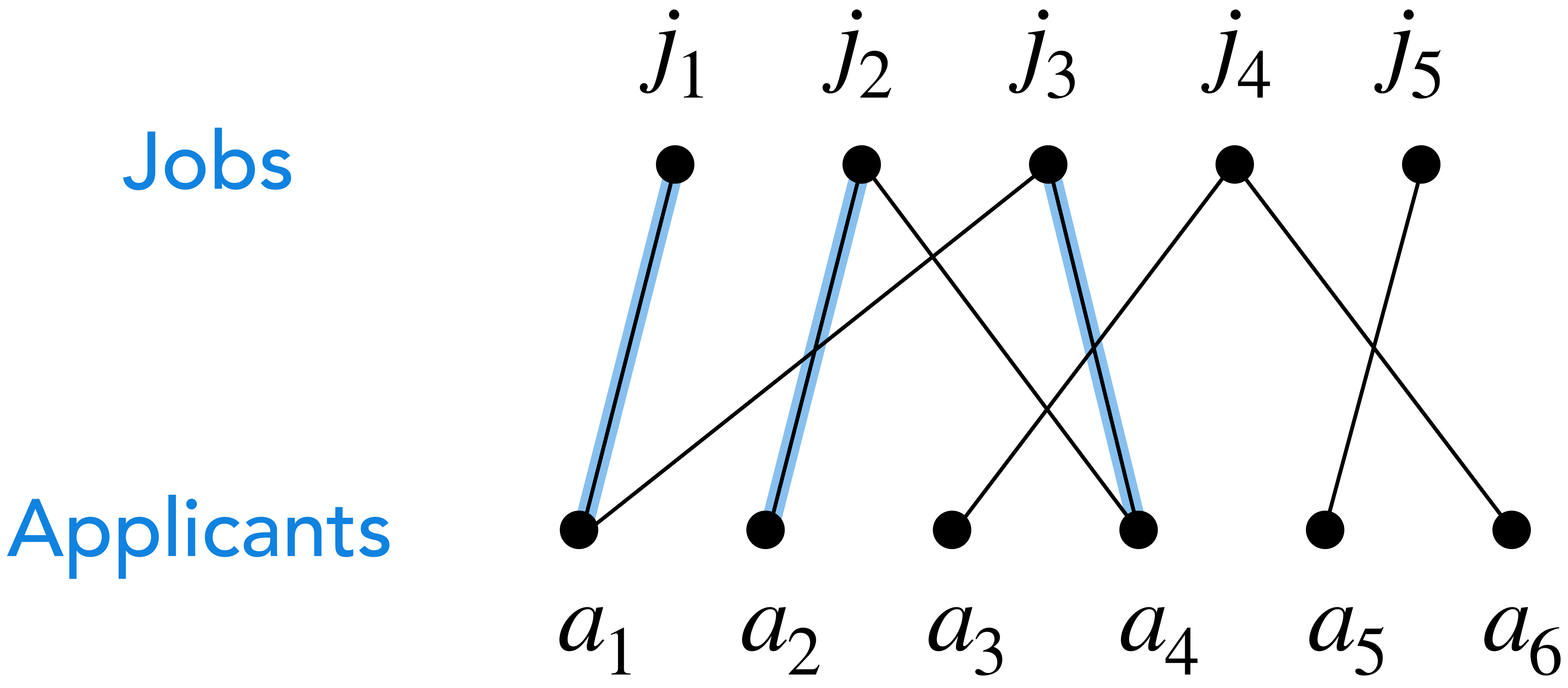

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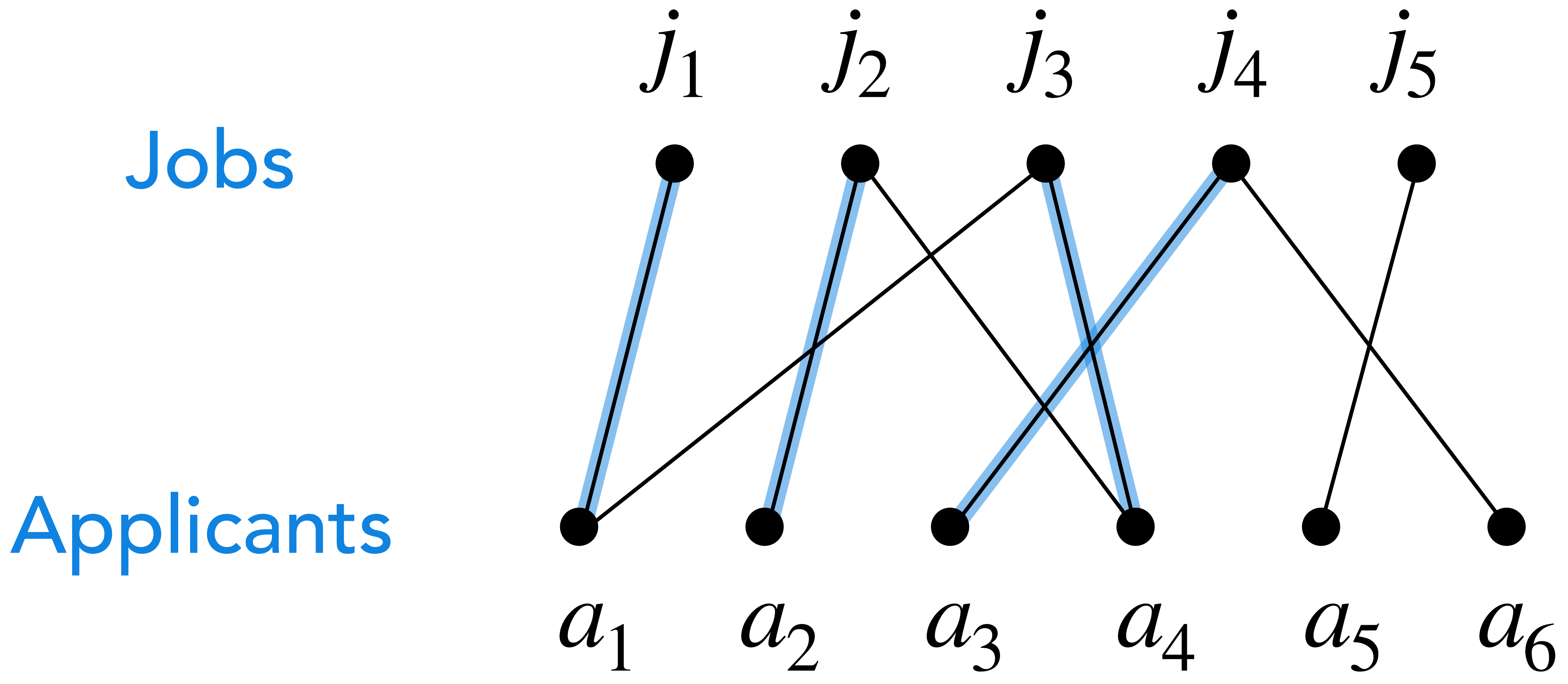

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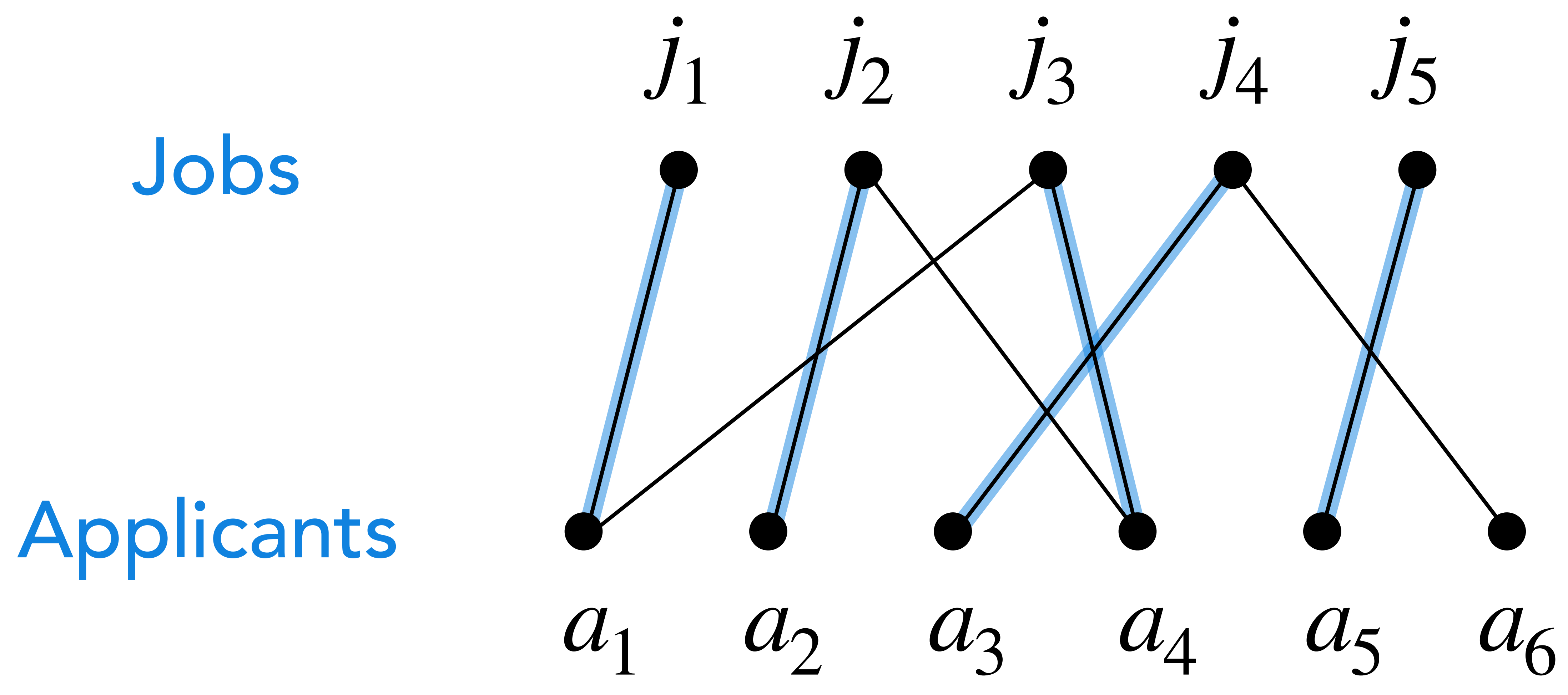

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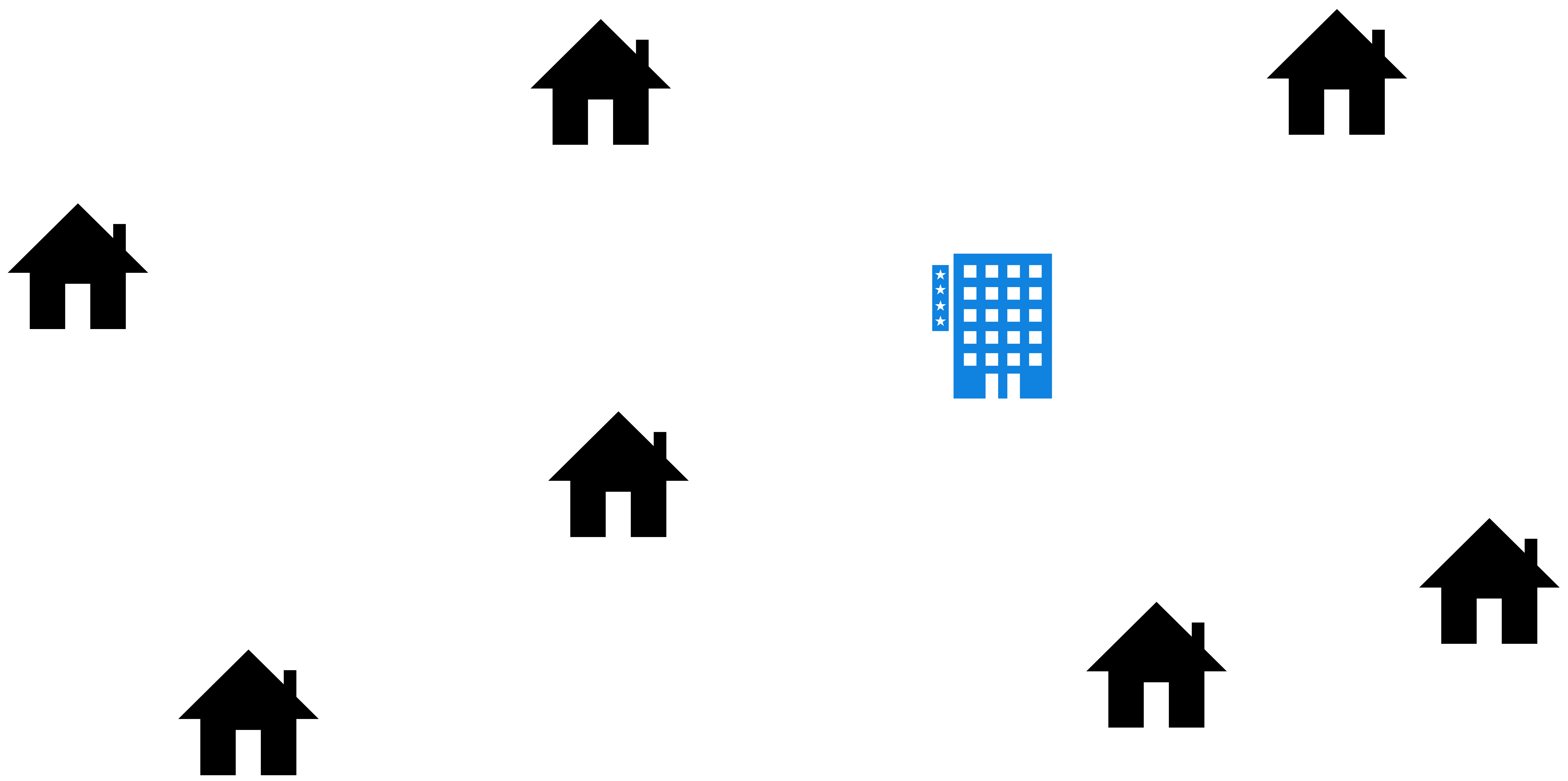

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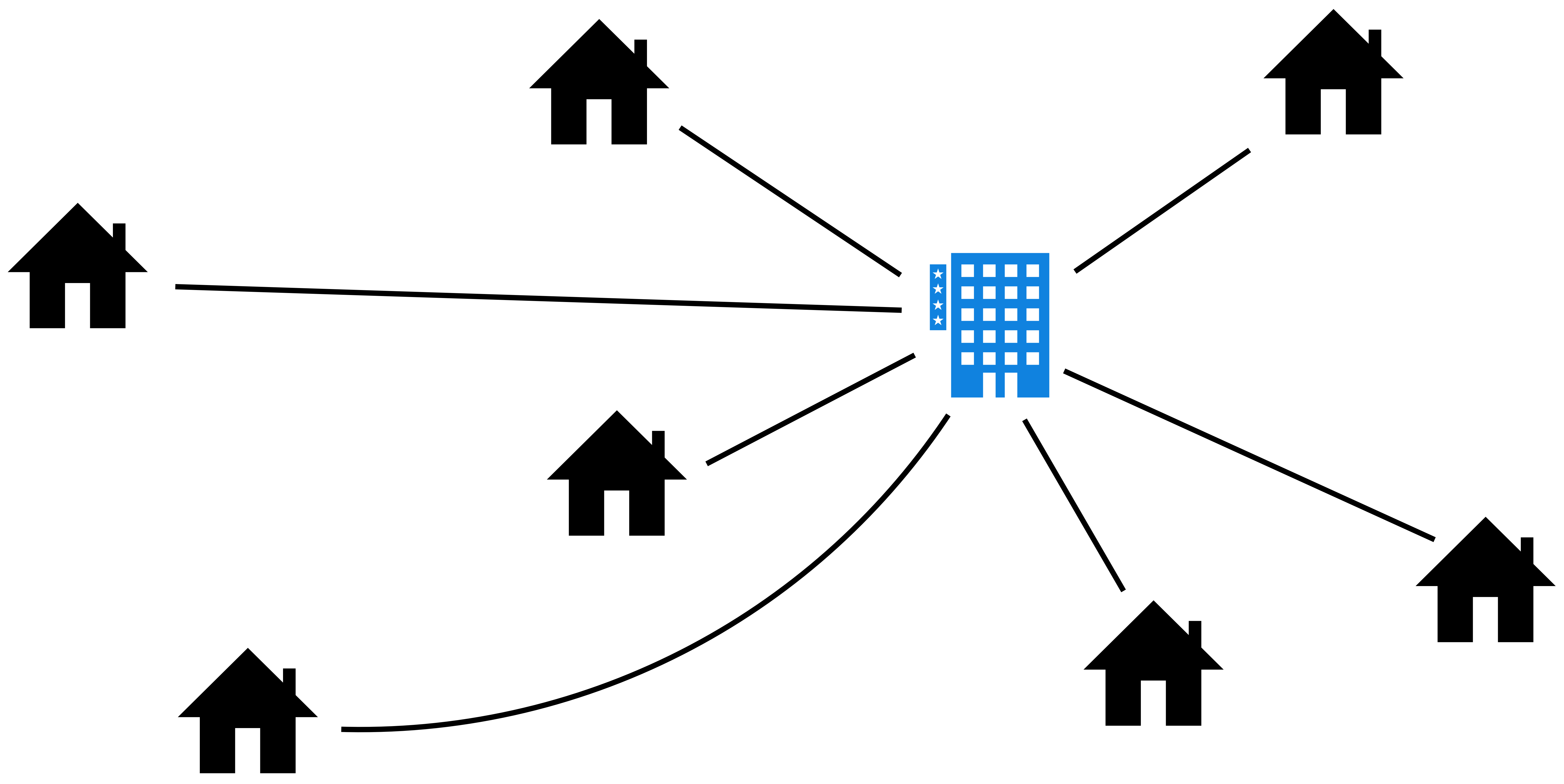

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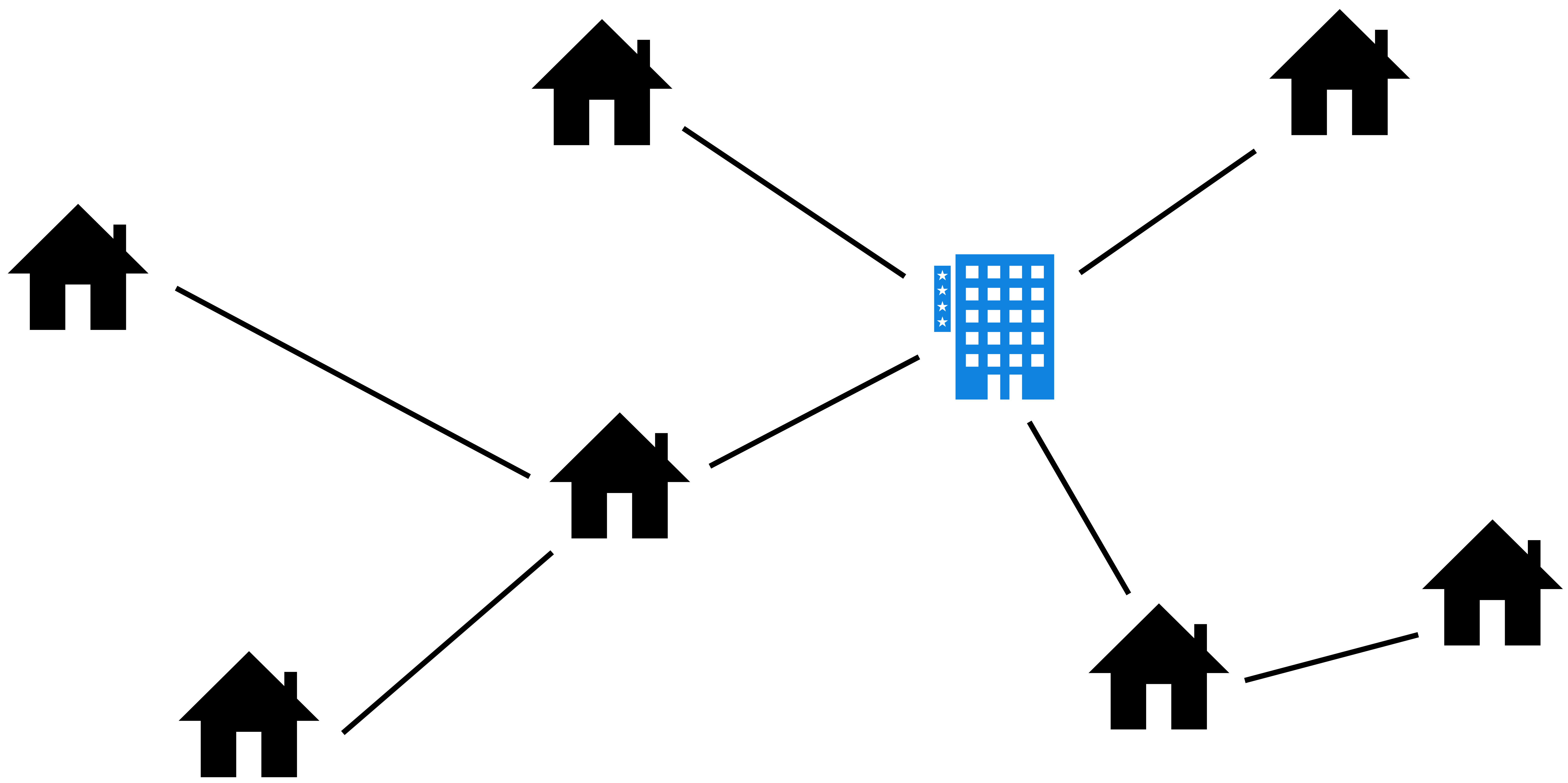

Graphs: Basics

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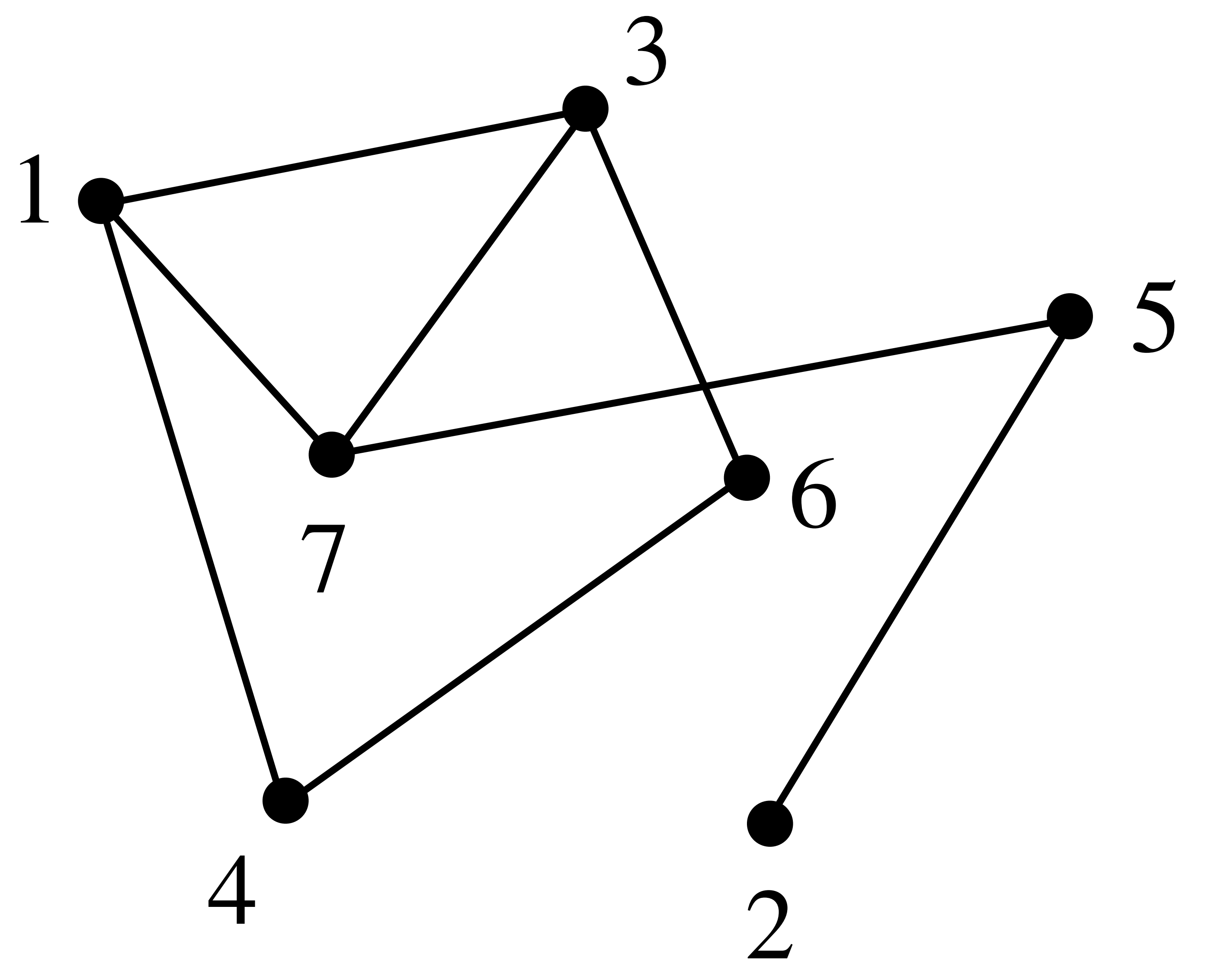

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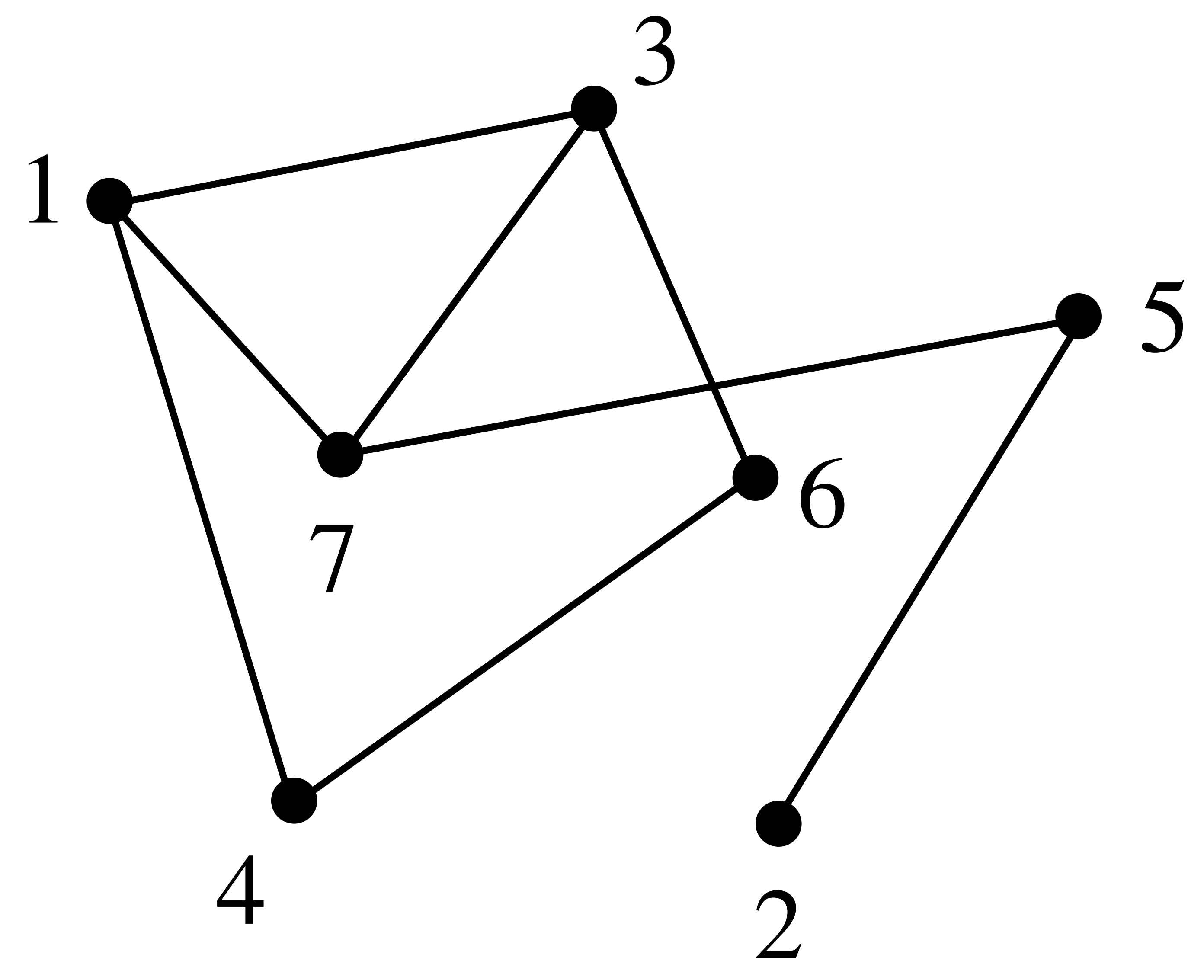

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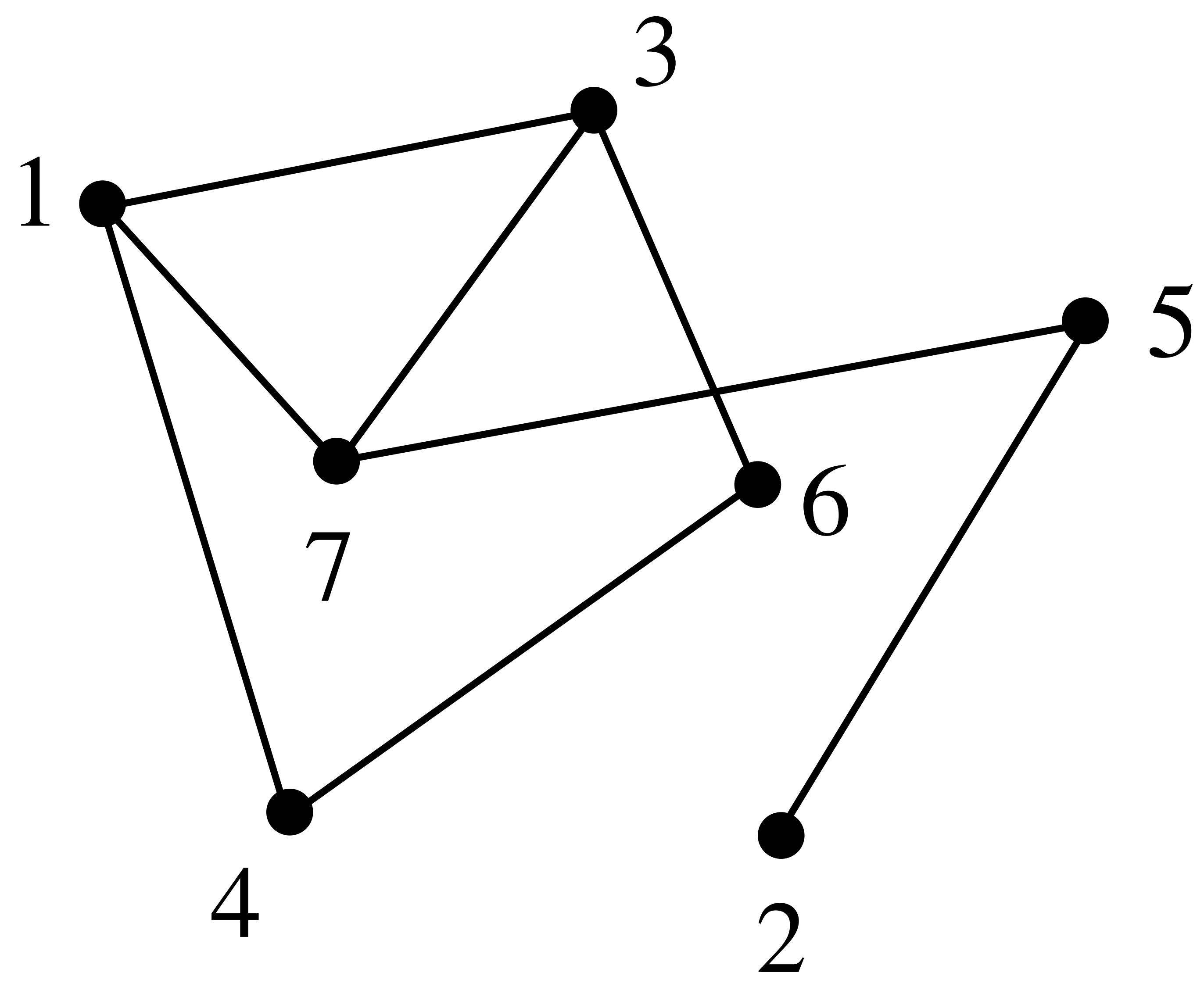

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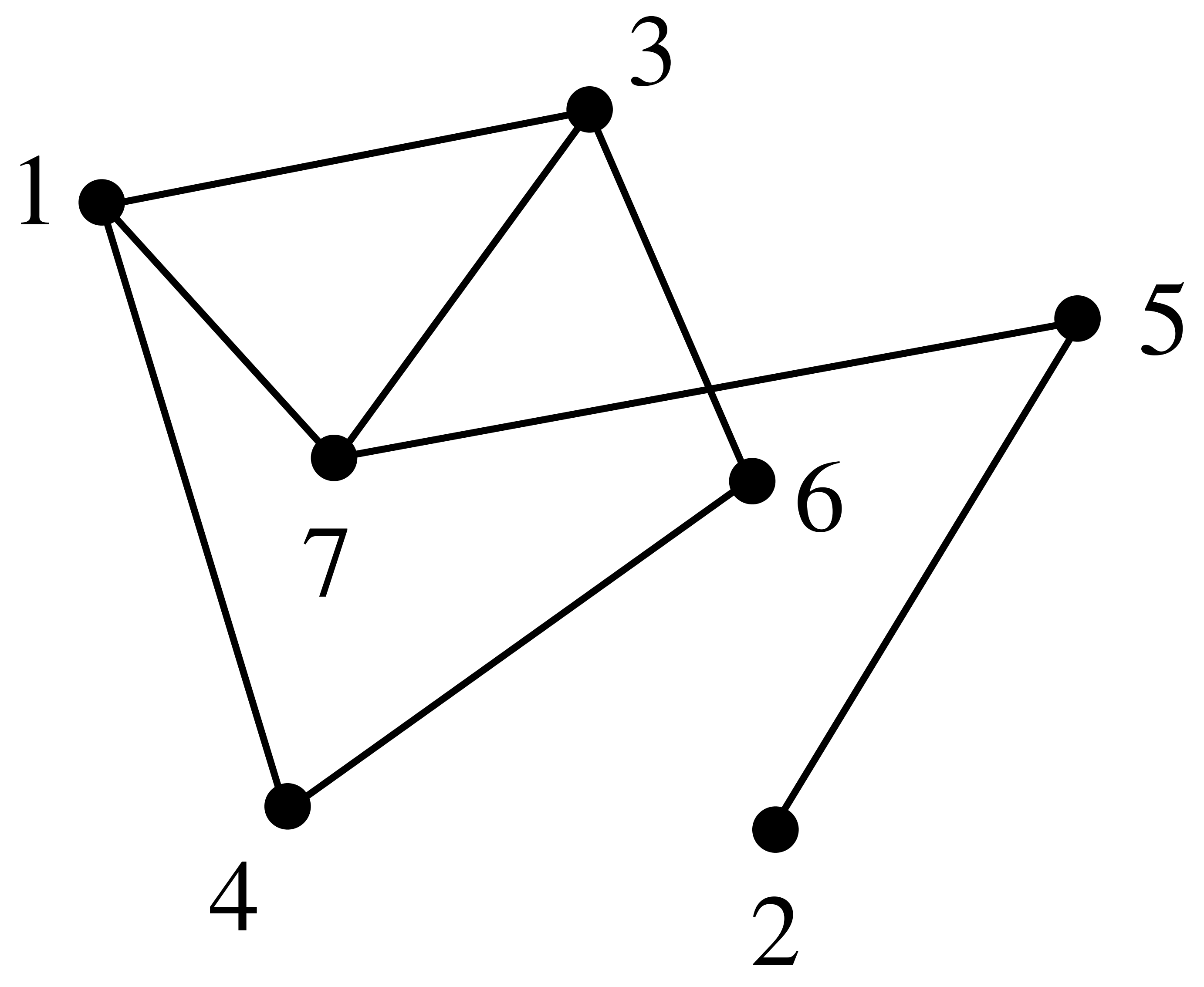

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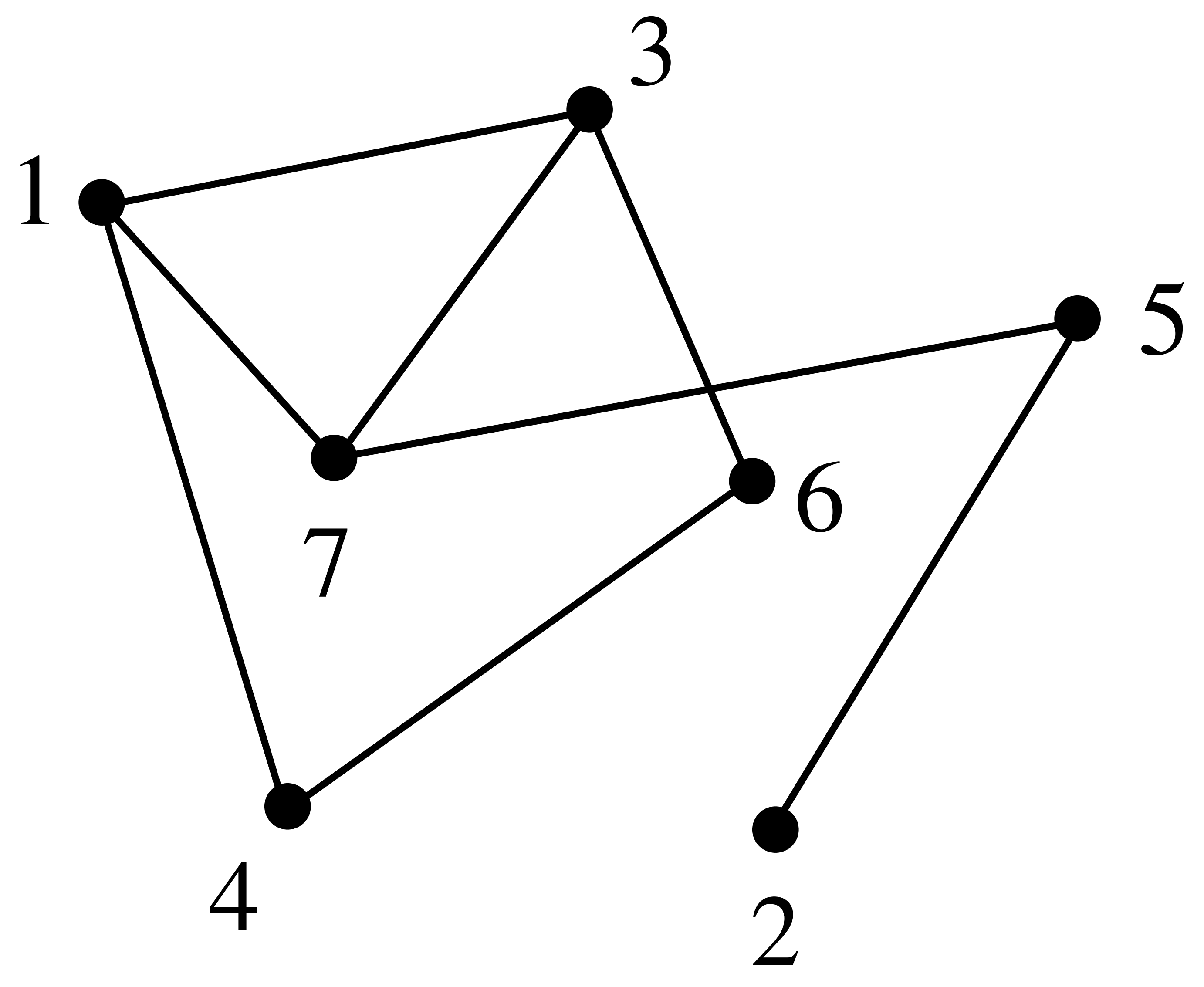

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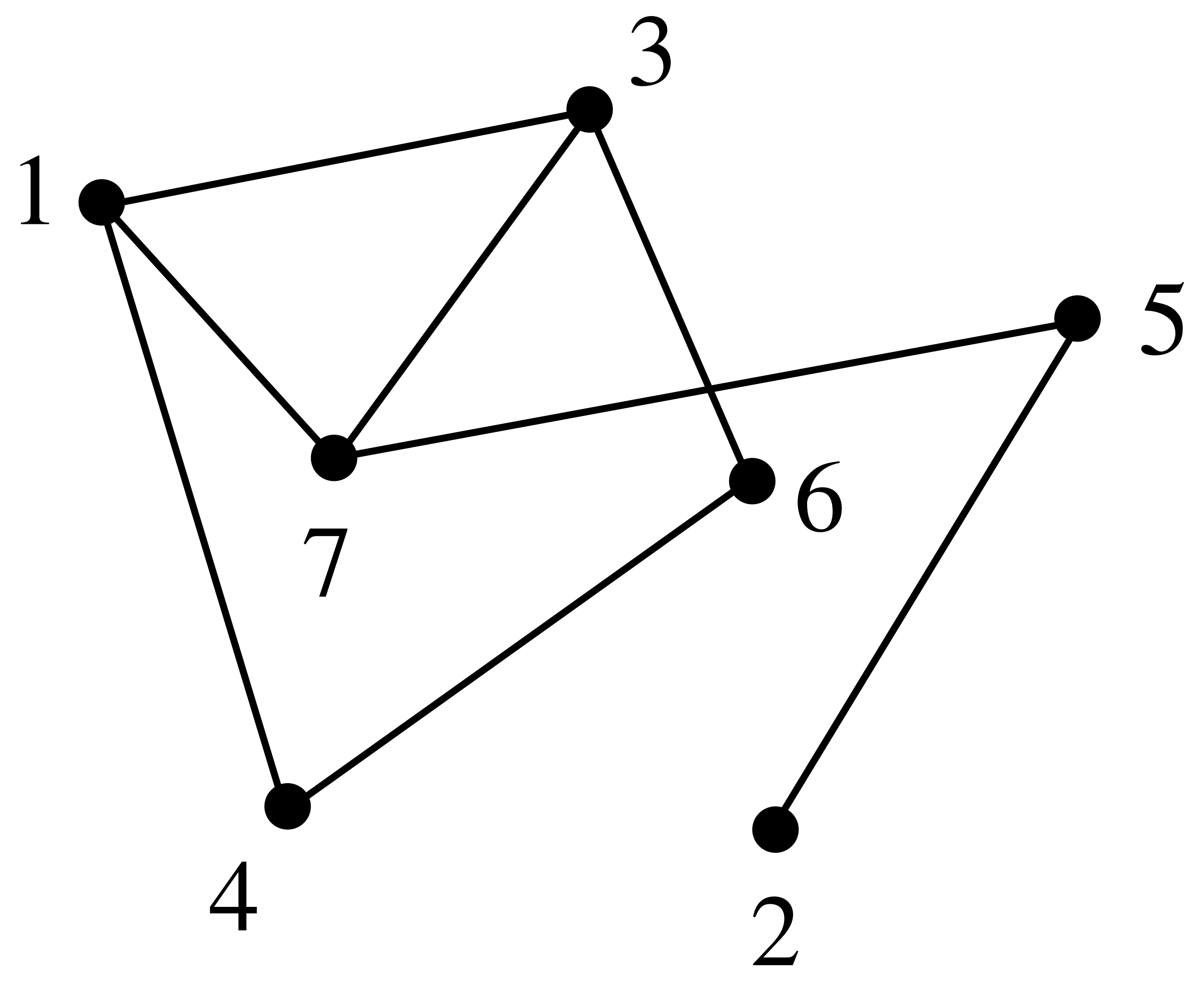

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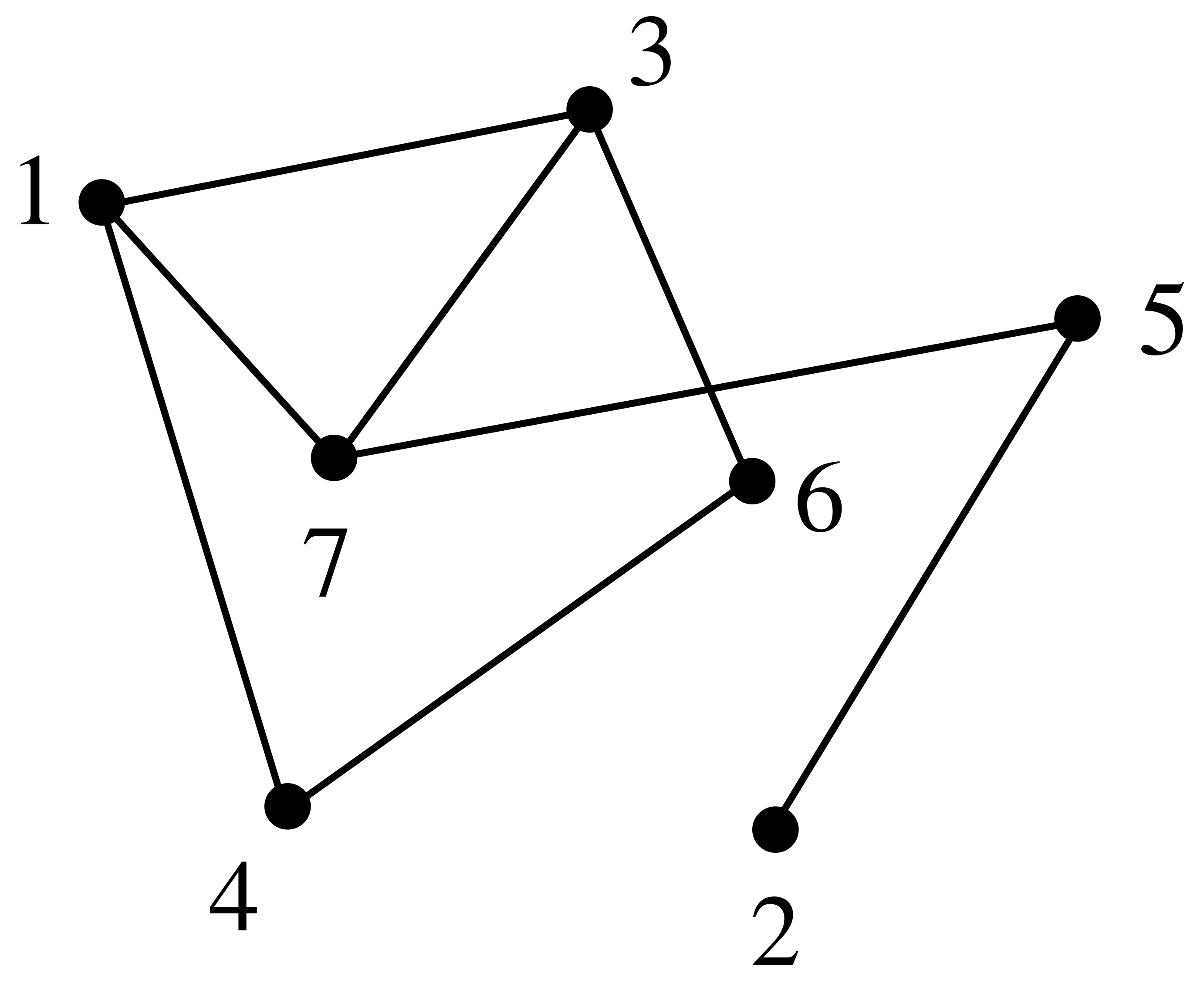

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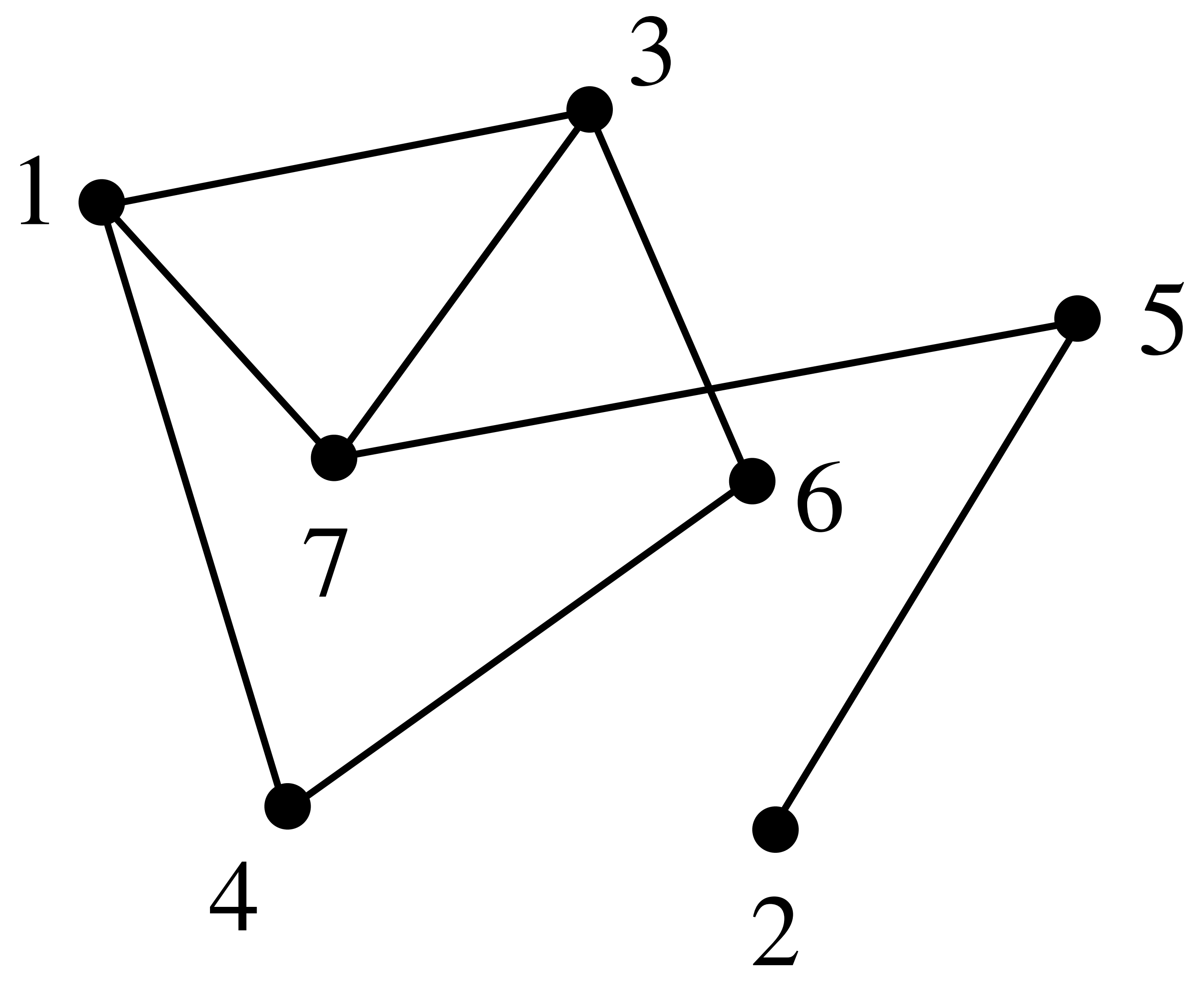

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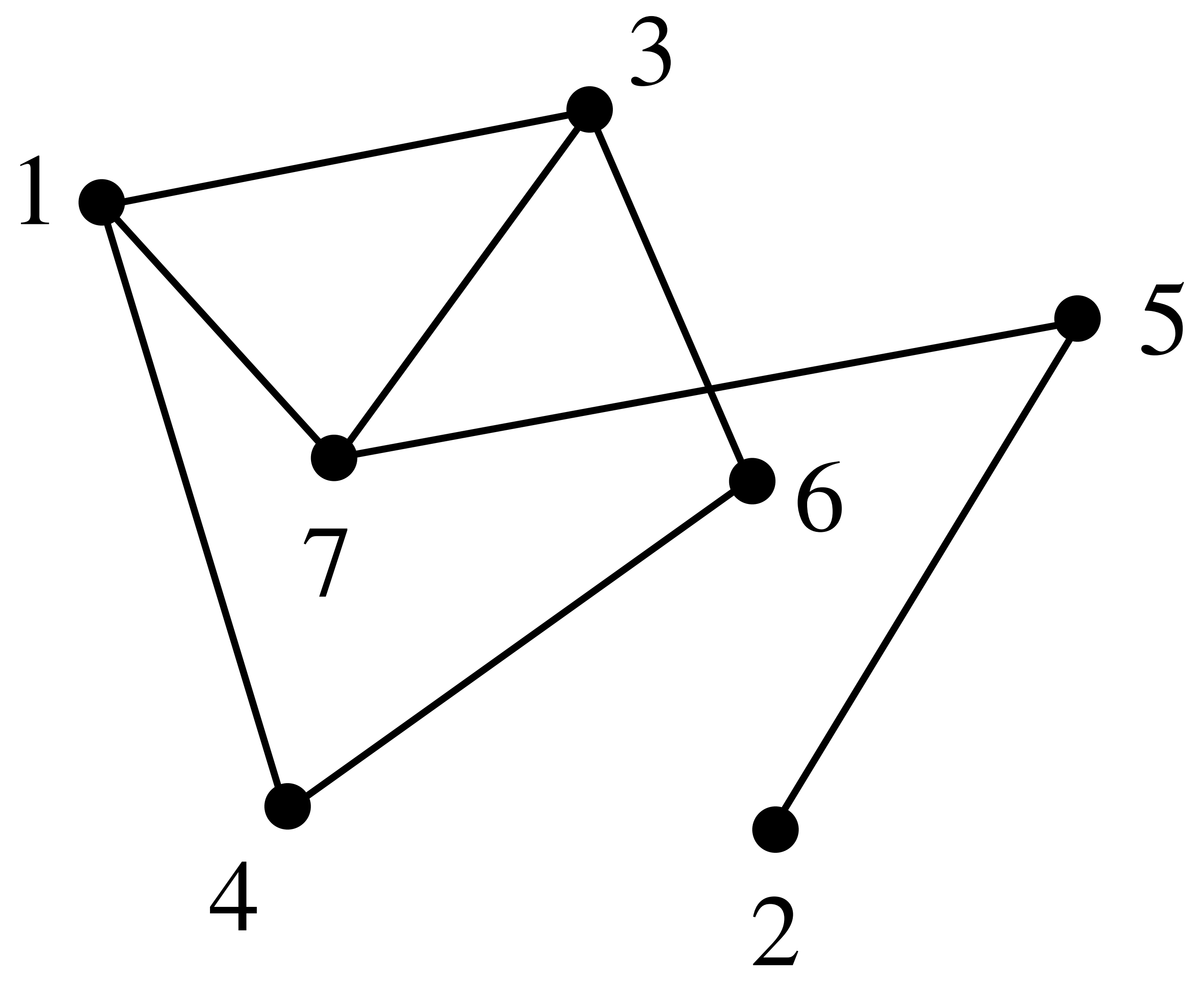

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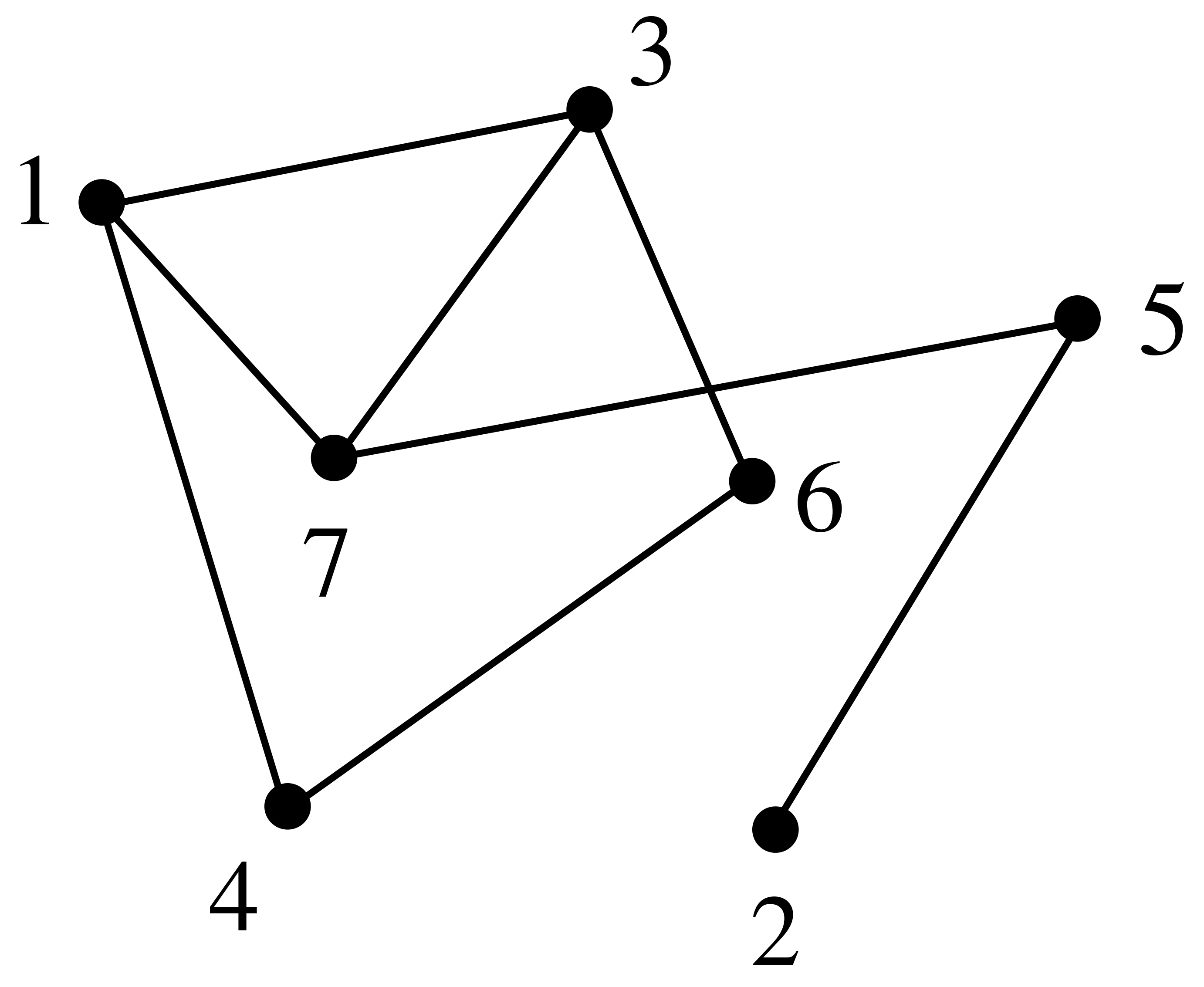

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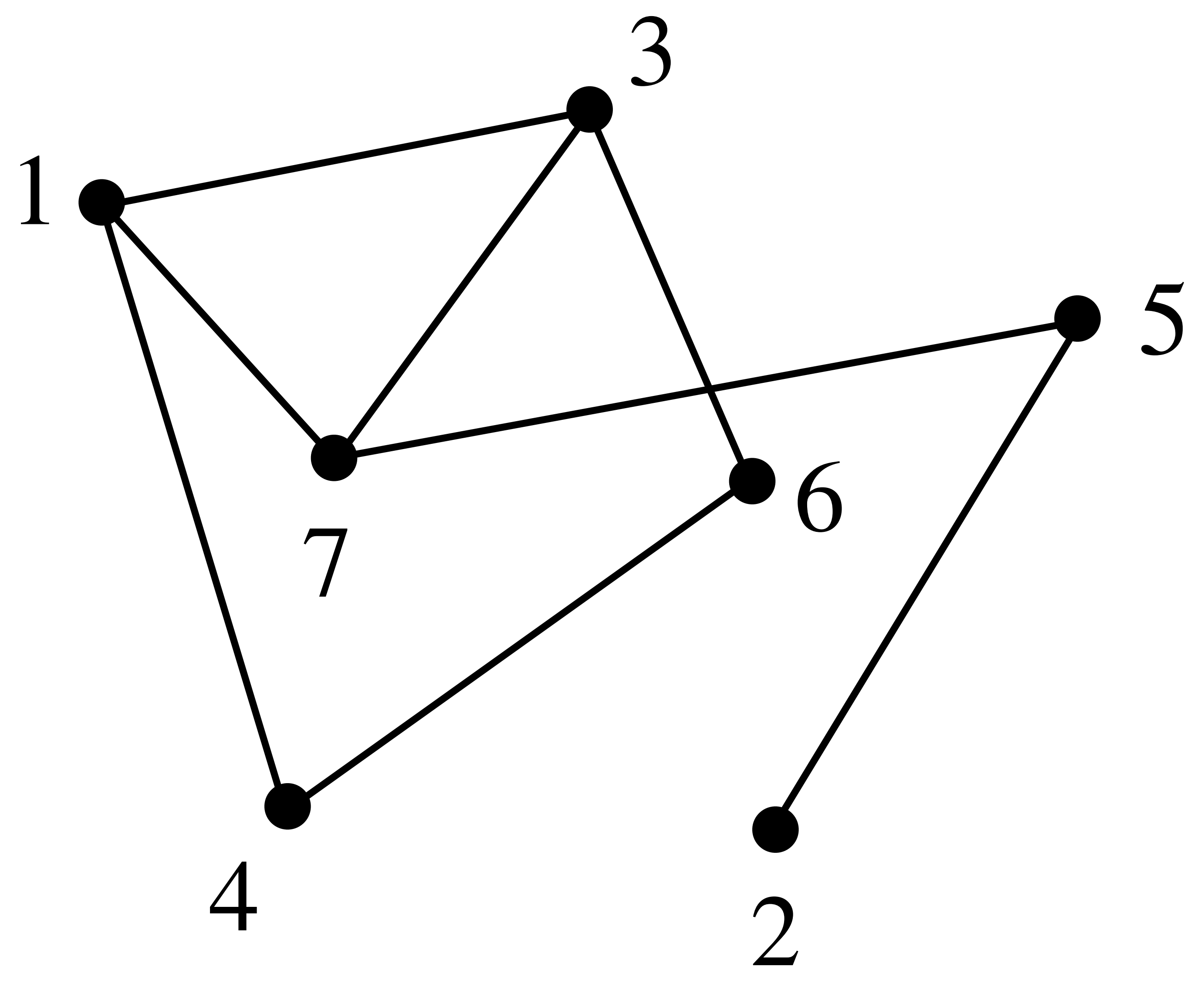

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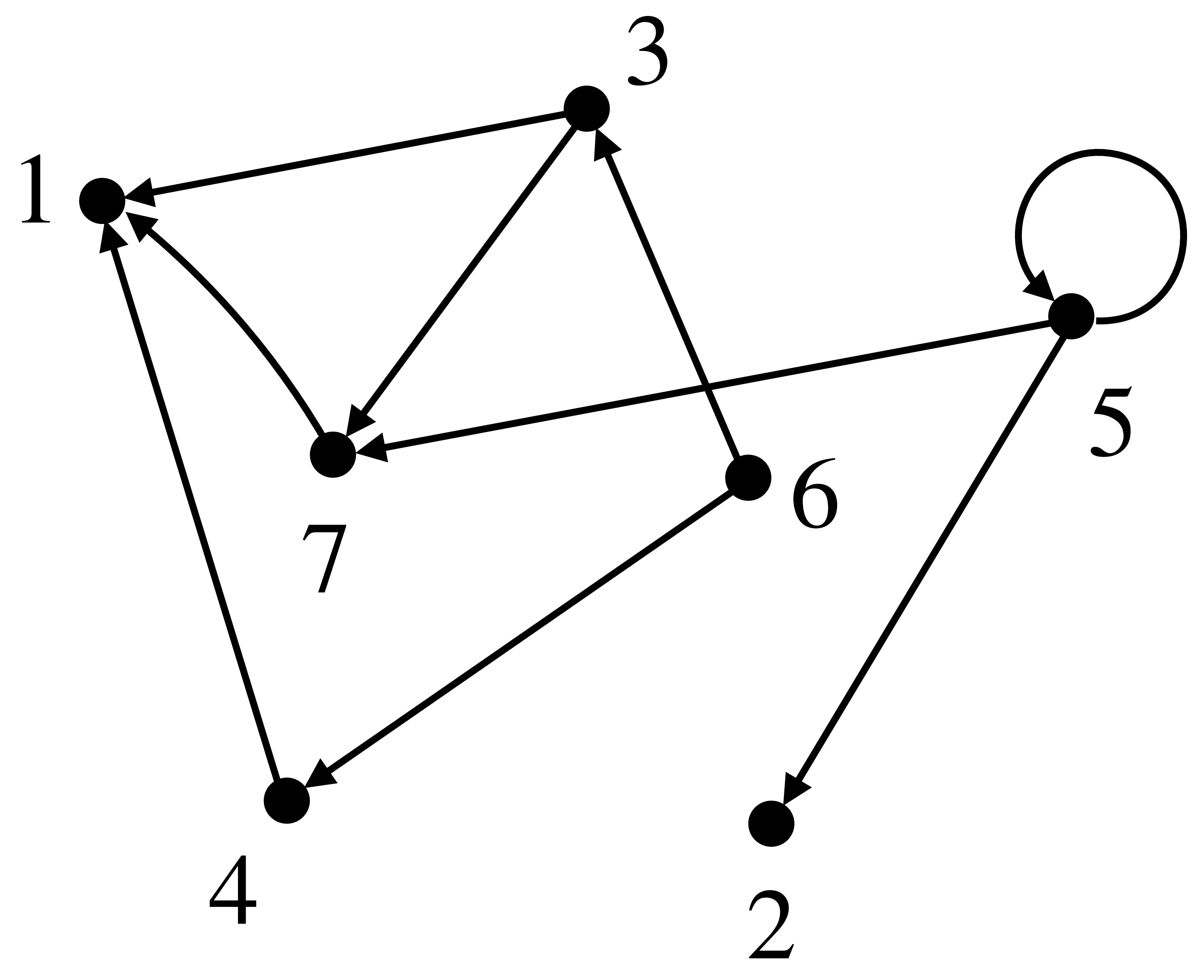

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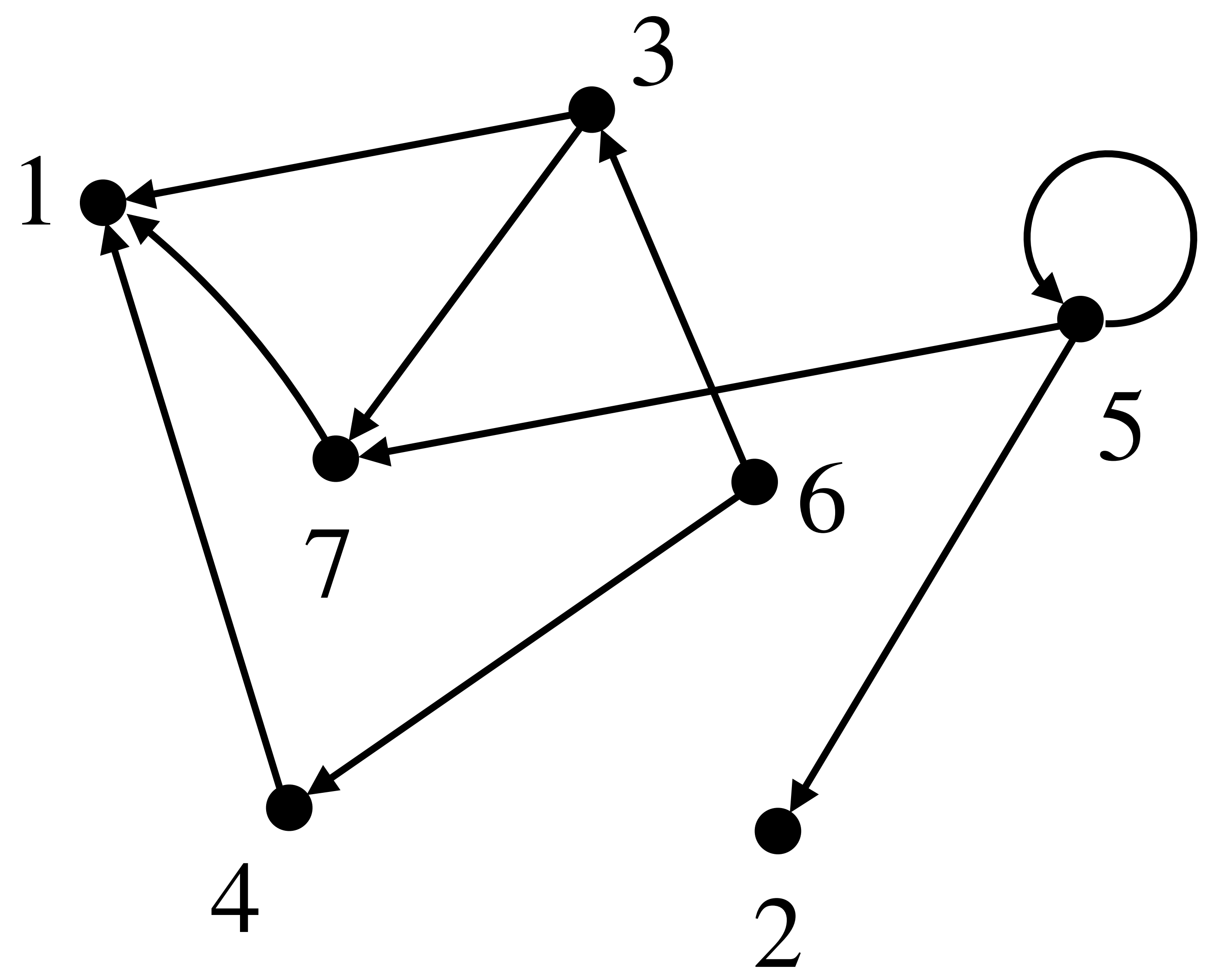

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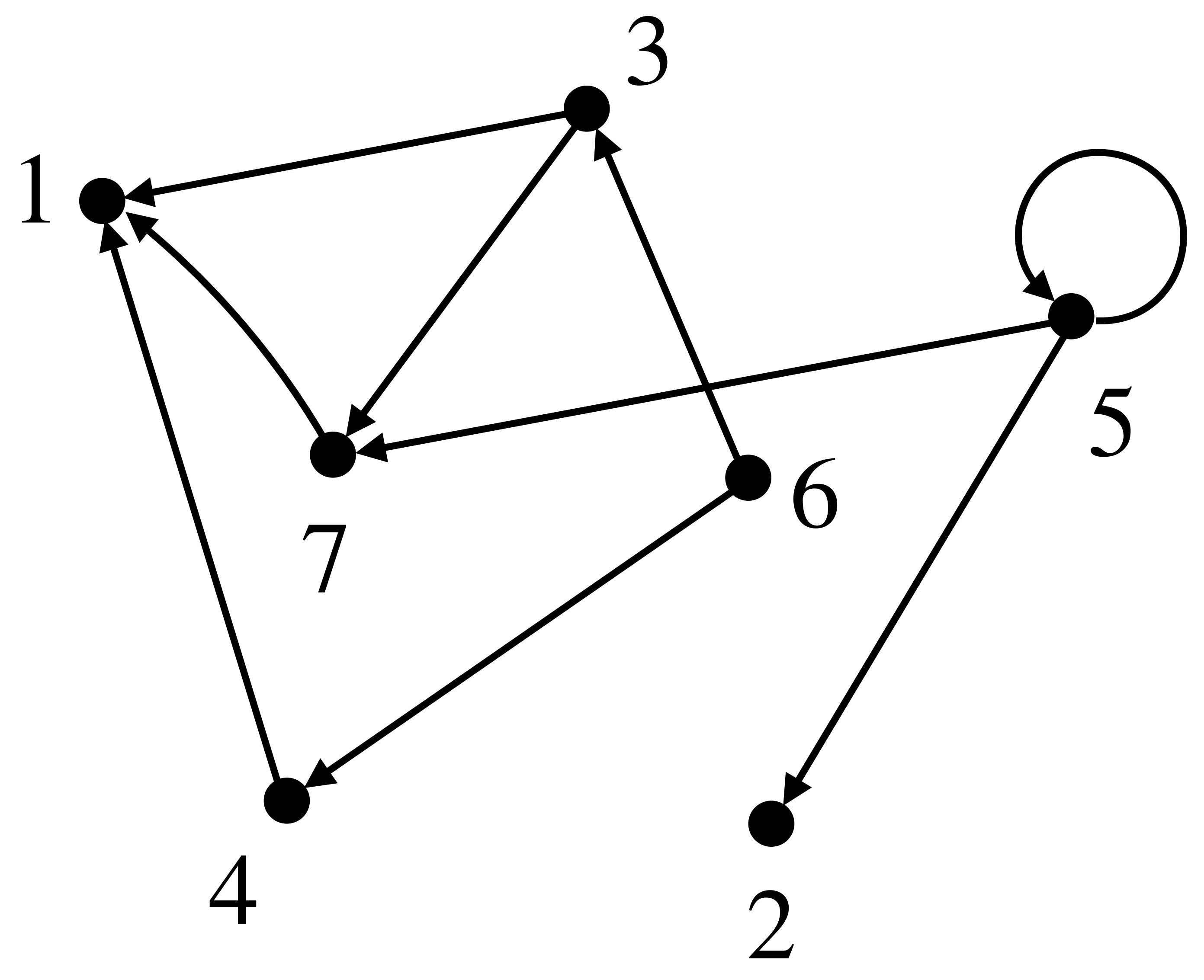

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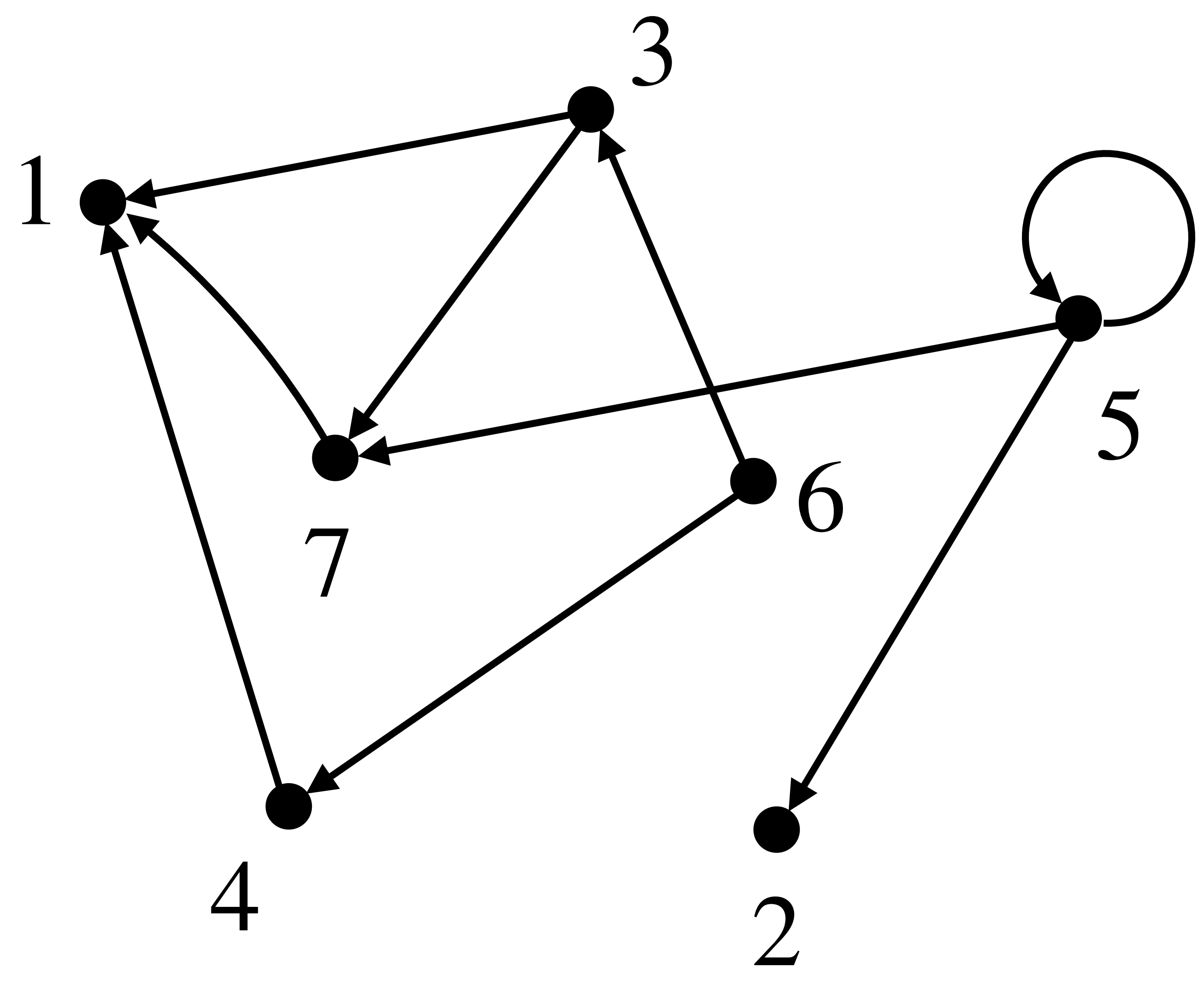

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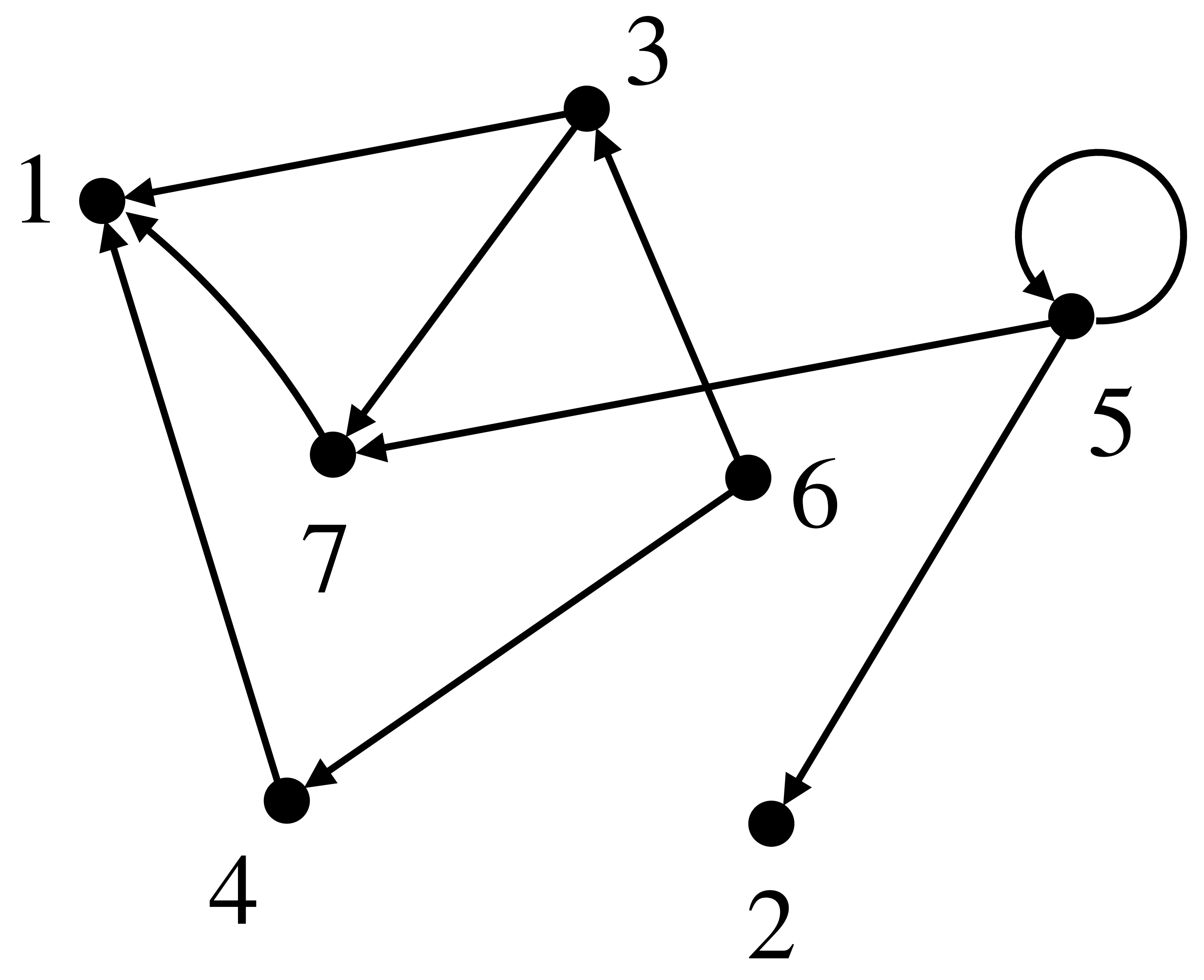

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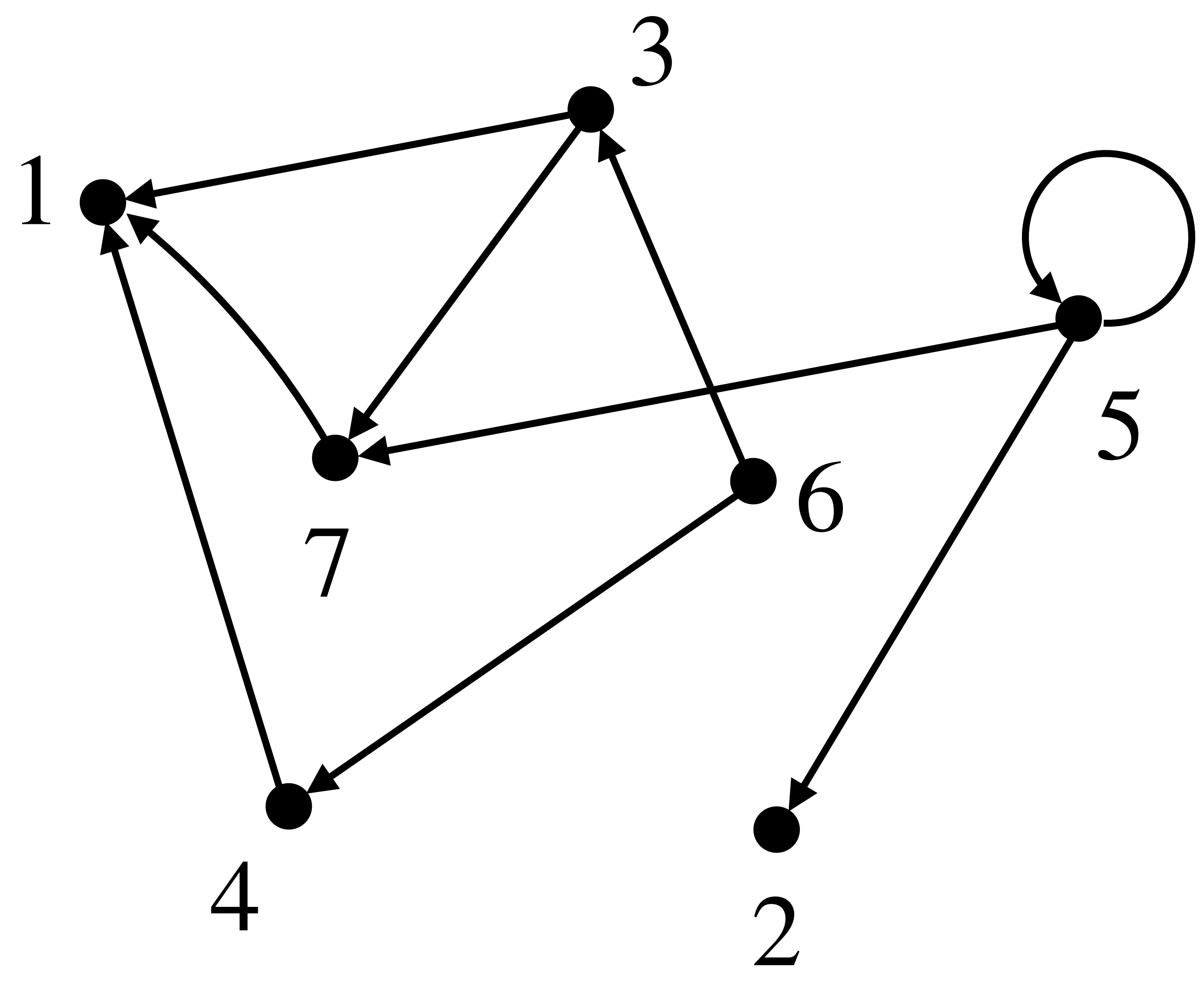

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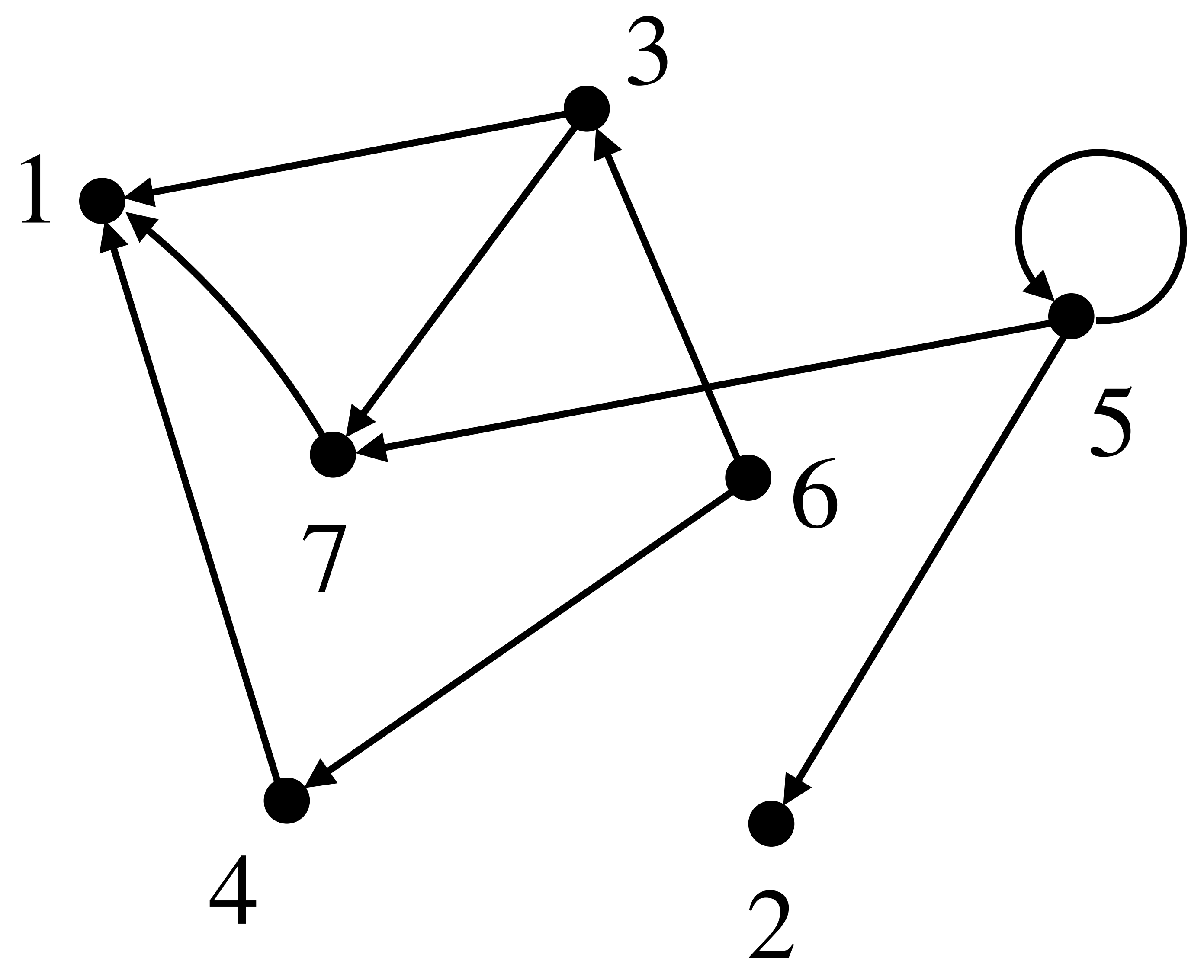

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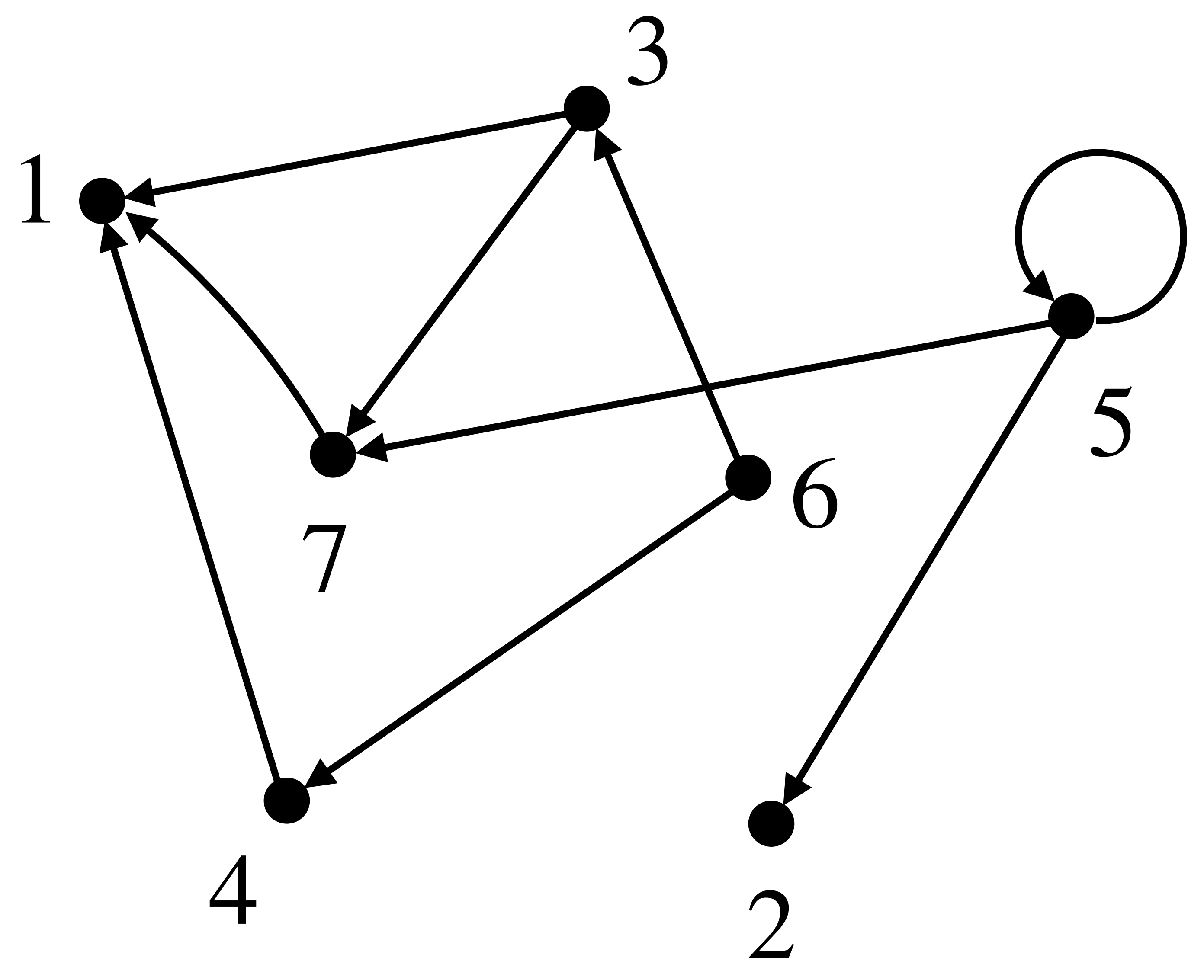

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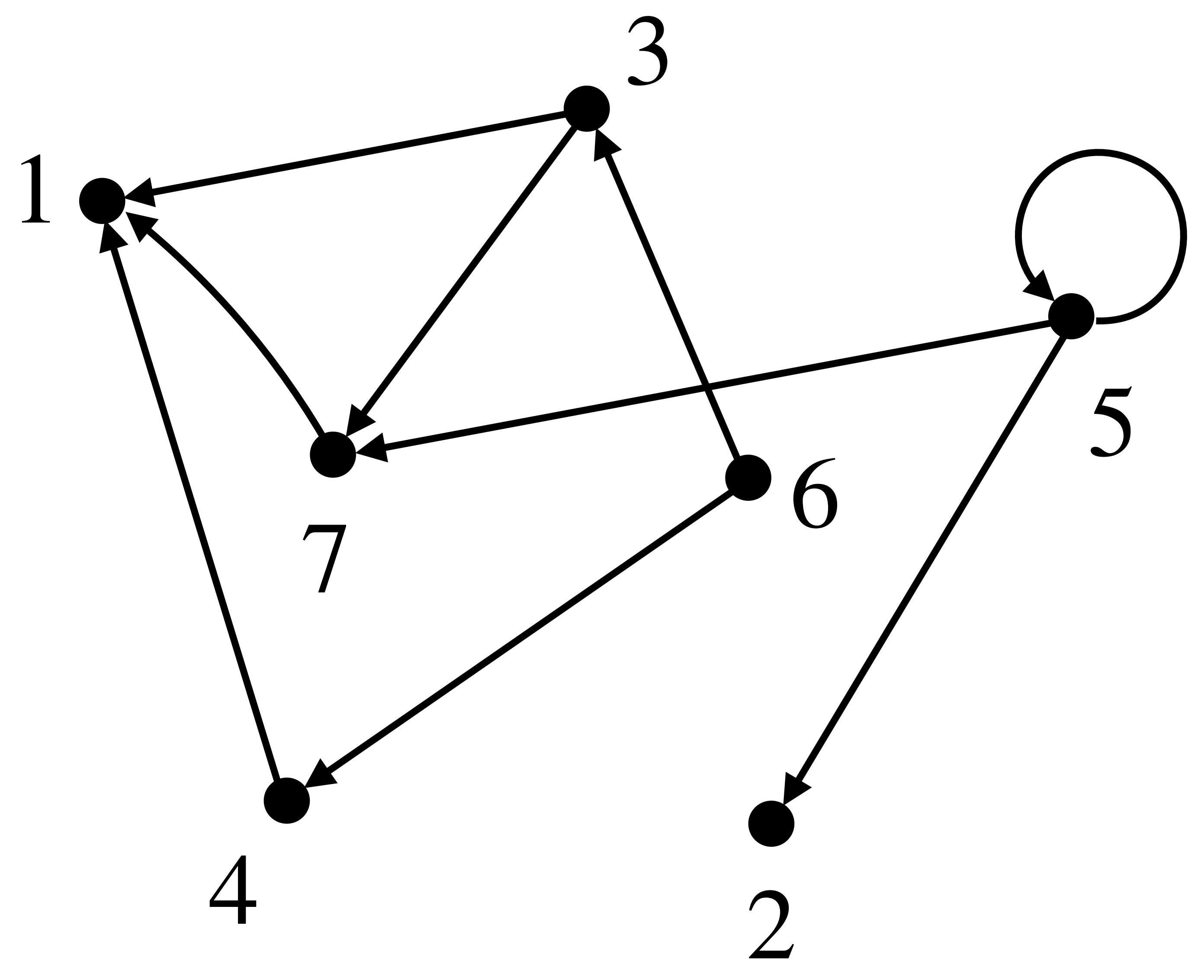

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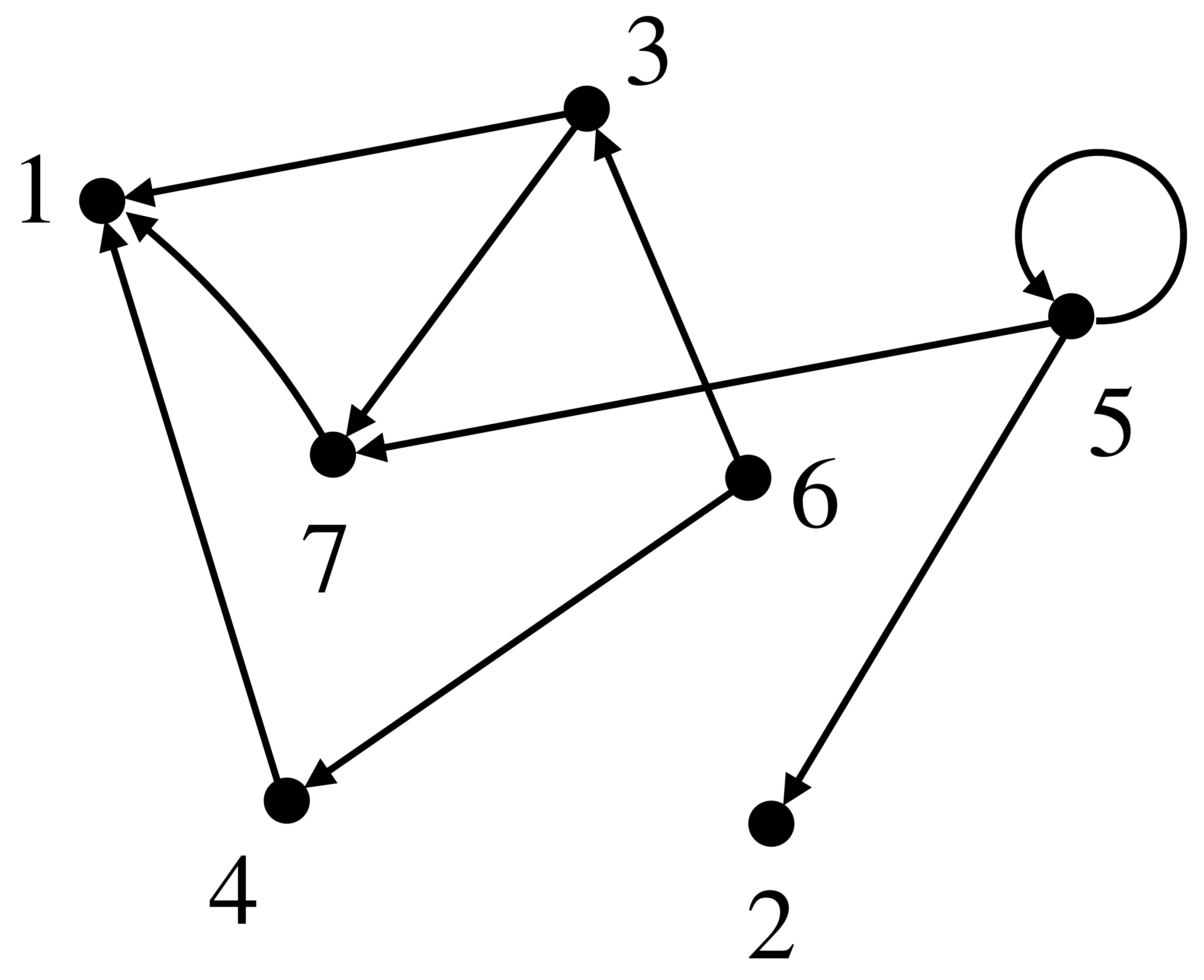

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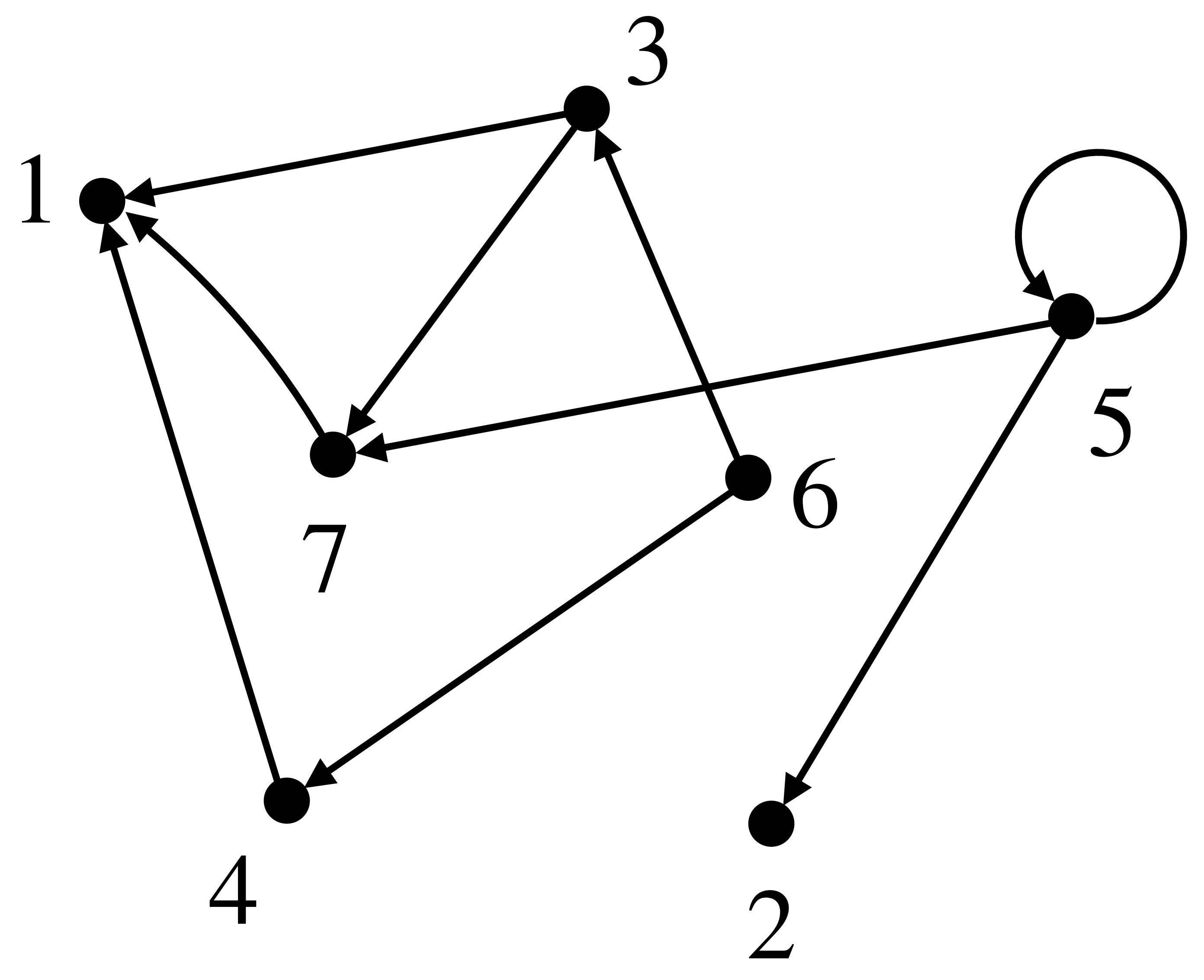

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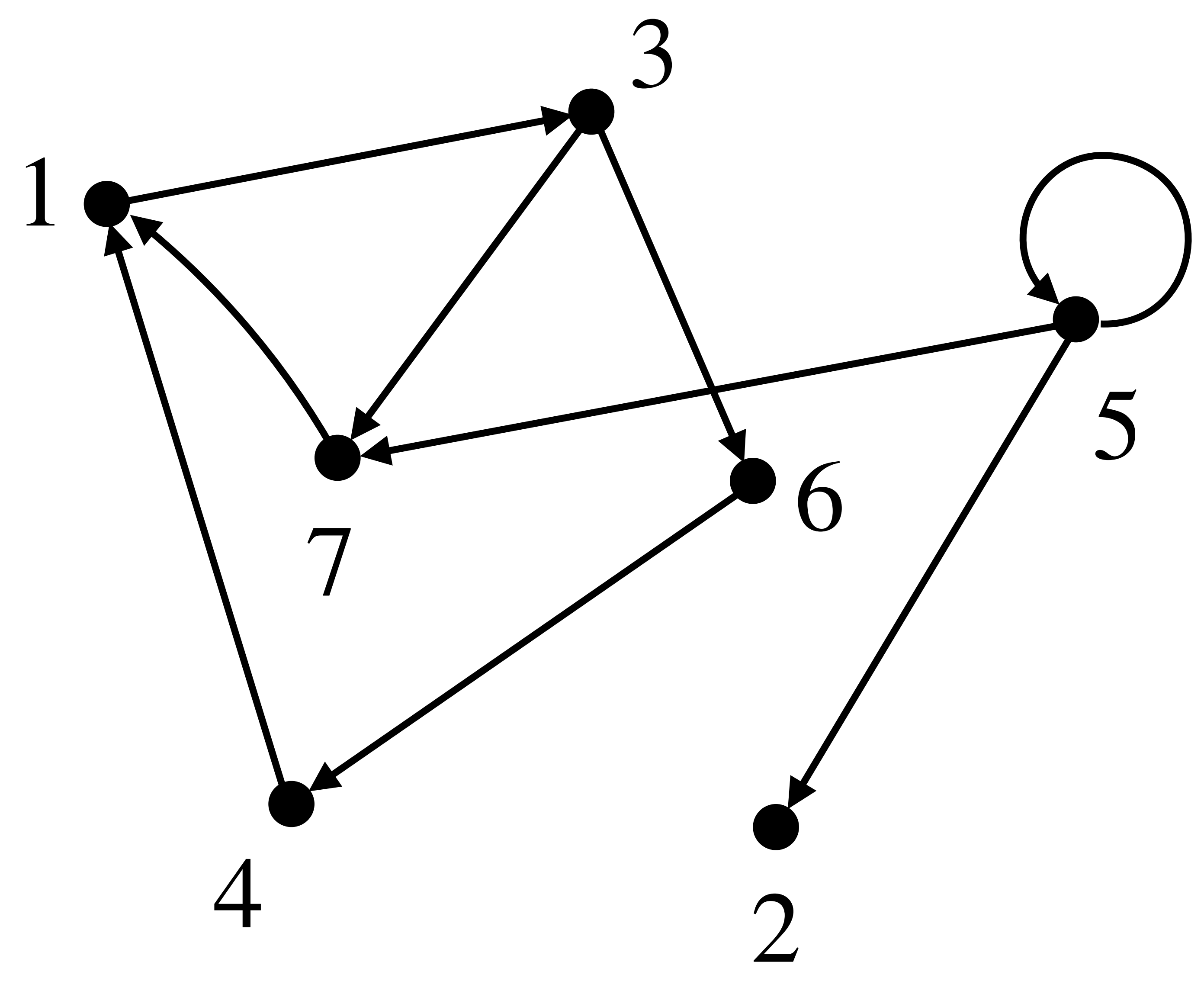

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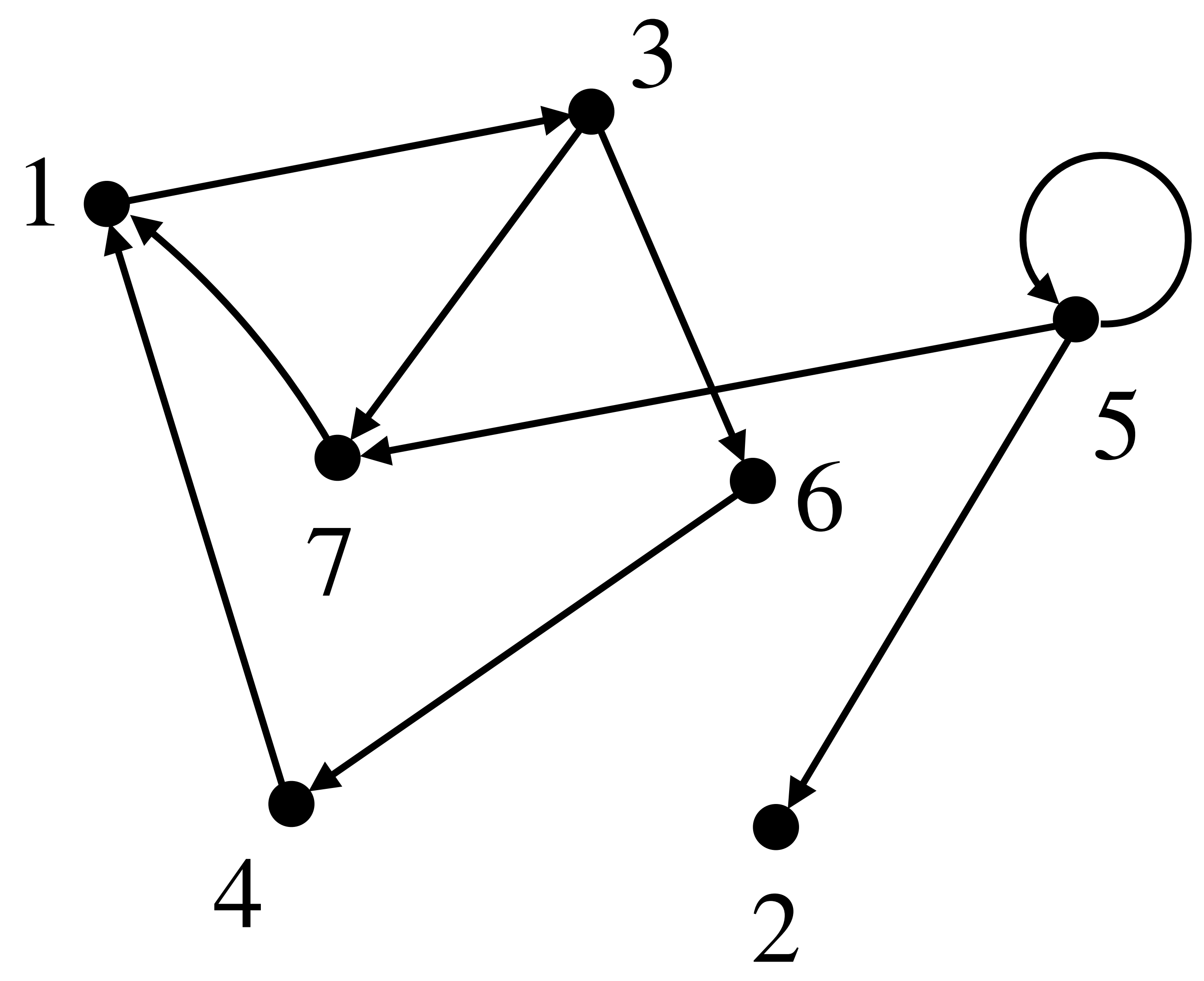

1 and 3 are adjacent

Graphs: Basic Terminology

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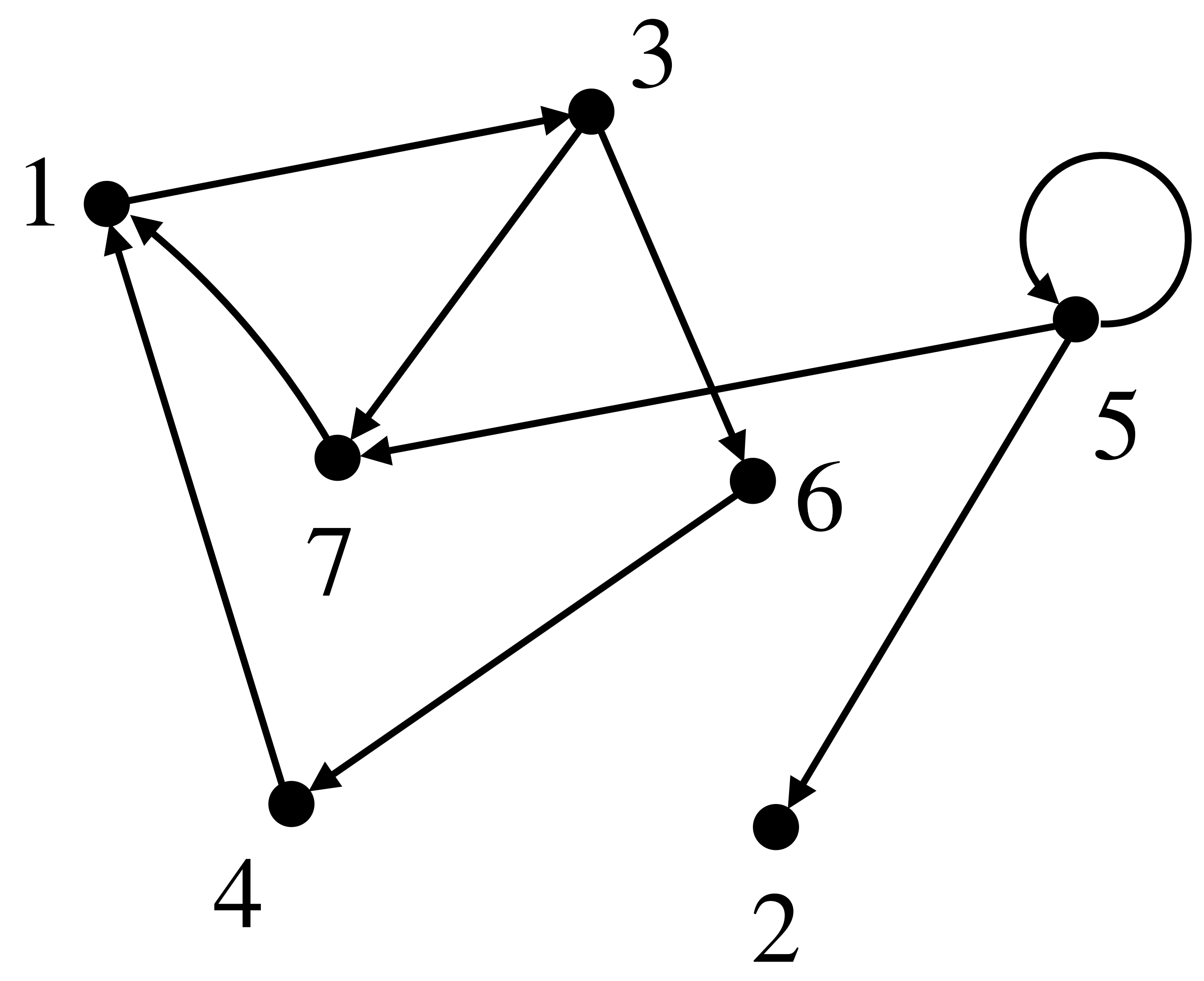

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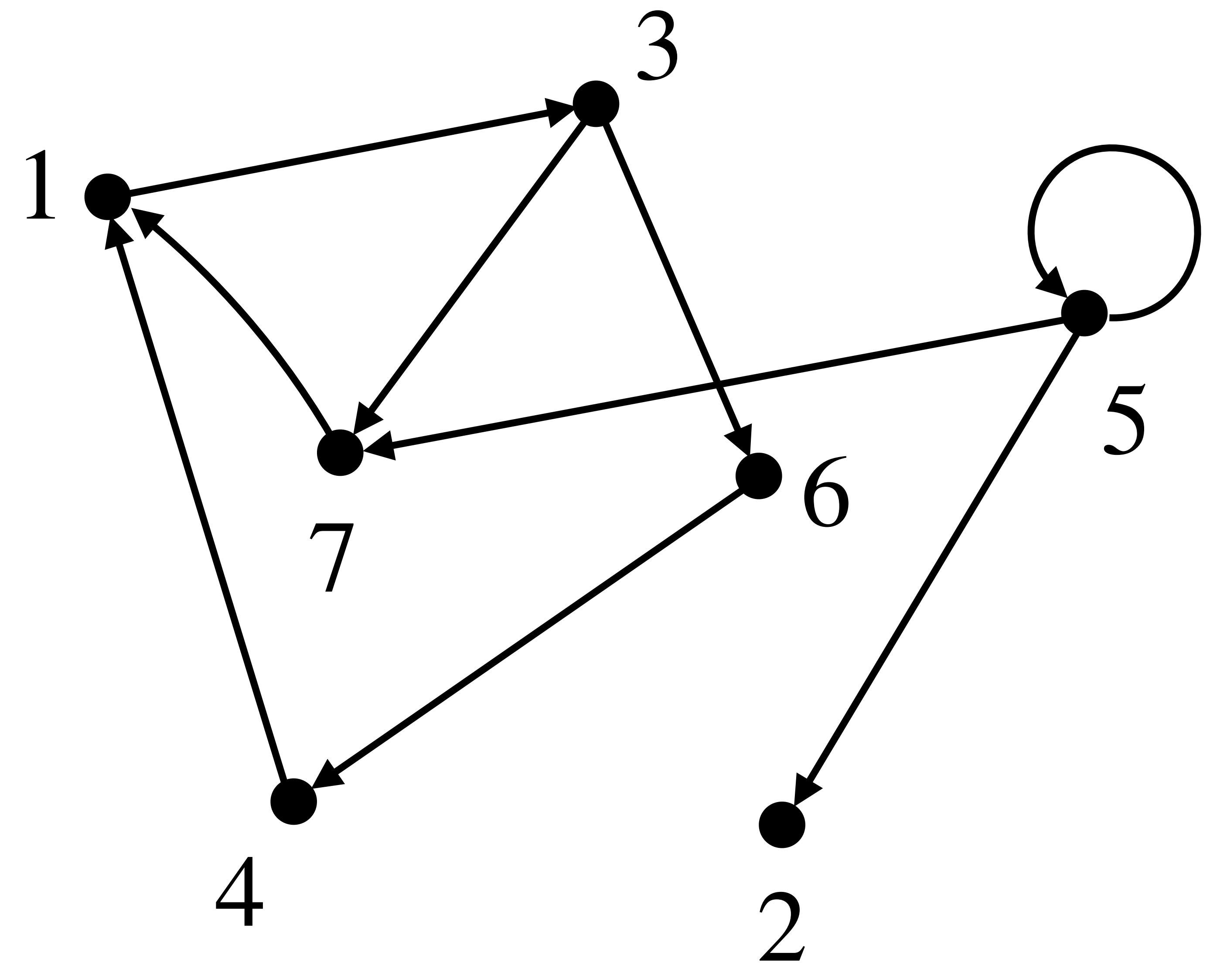

4 is incident on
(4,1), (6,4) edges

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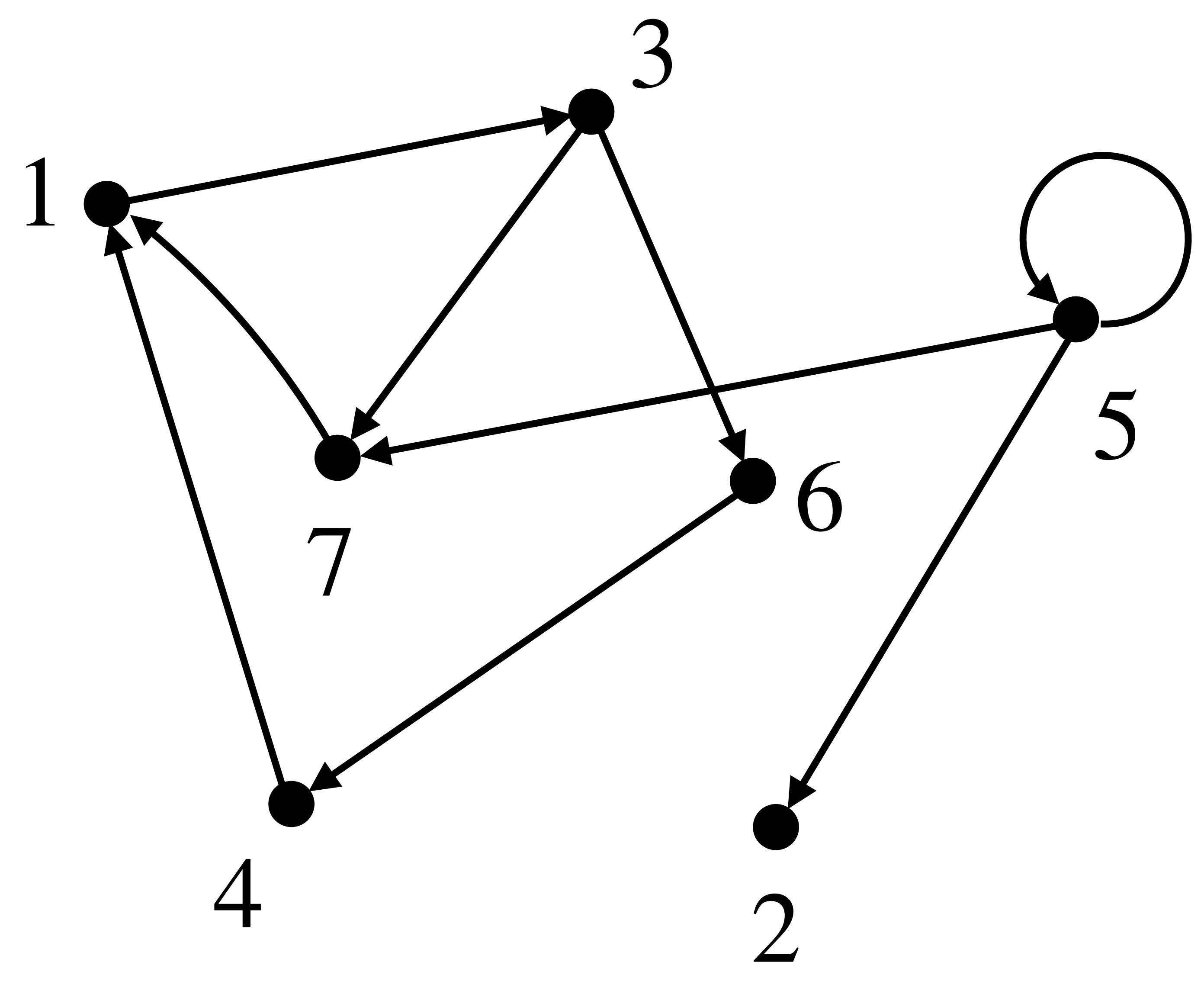

- The degree of a vertex is the number of edges it is incident with.

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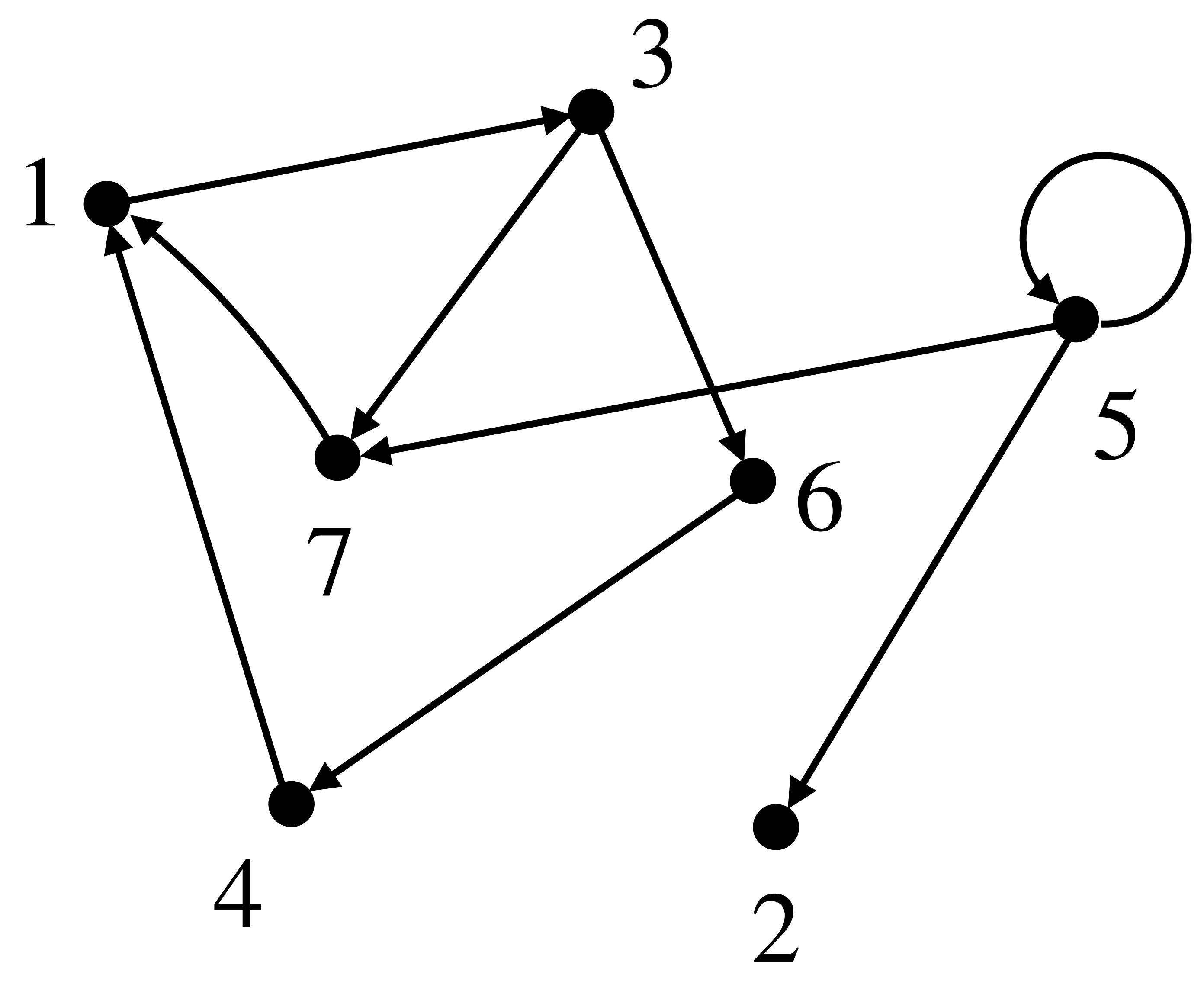

$degree(1) = 3$

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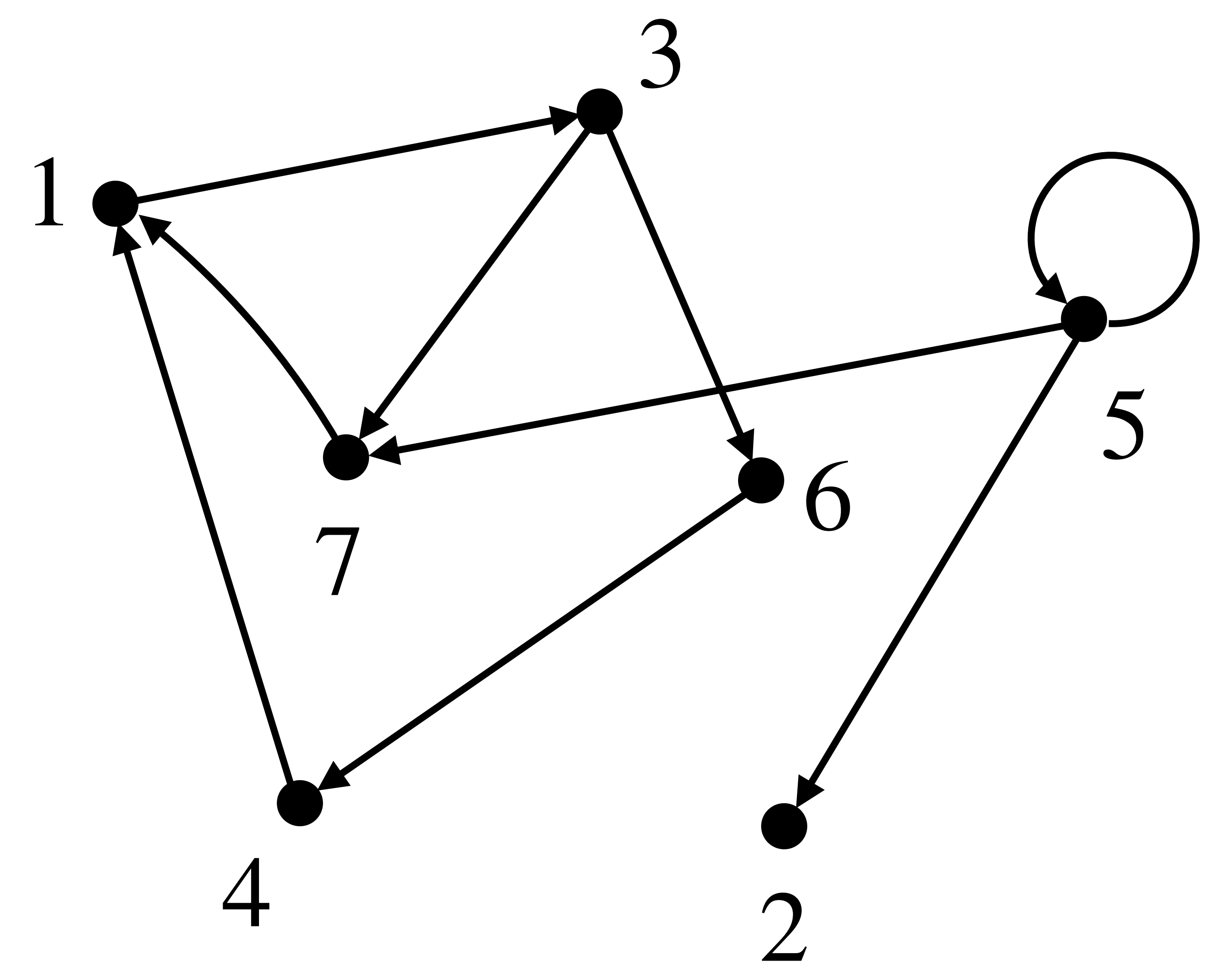

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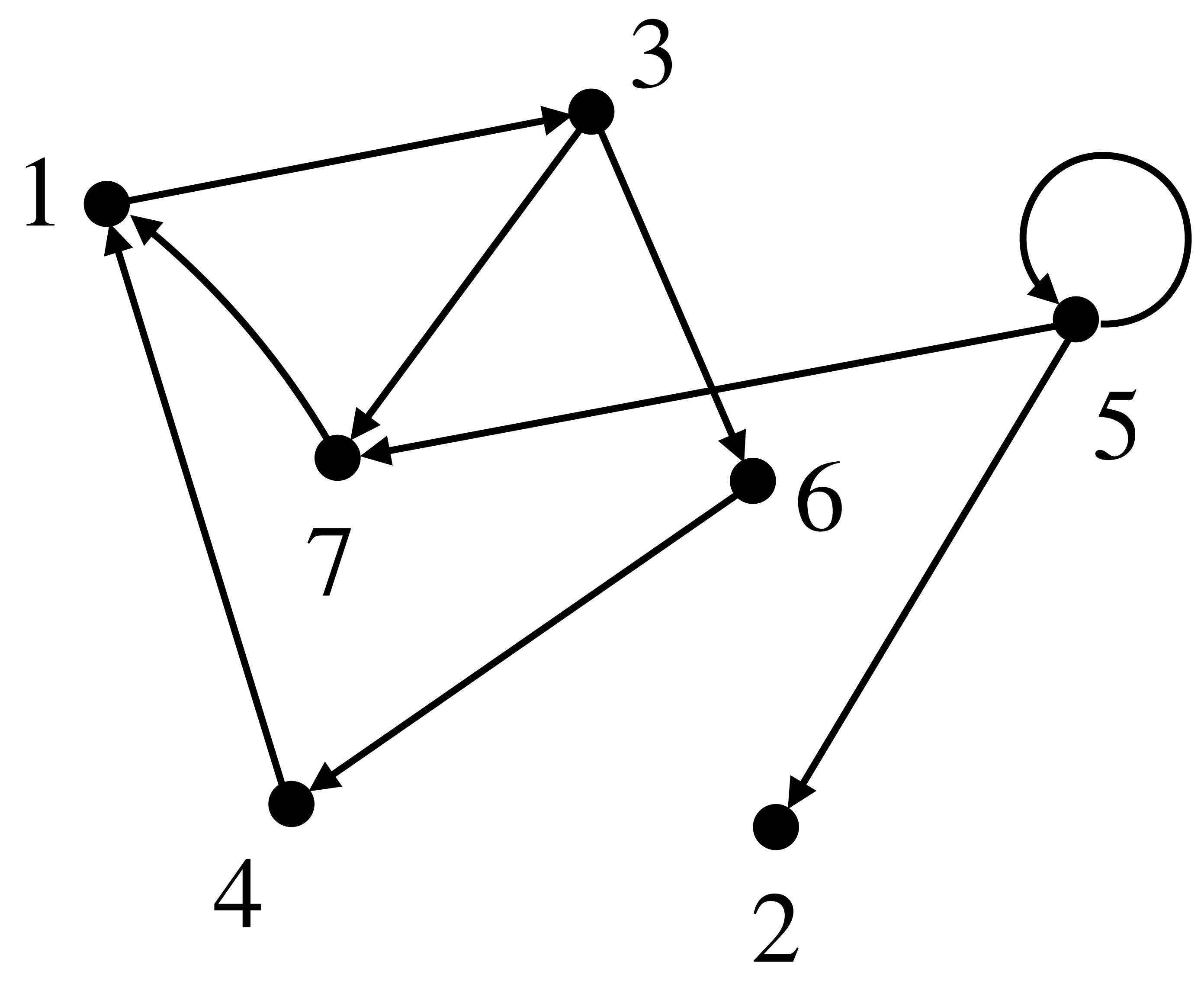

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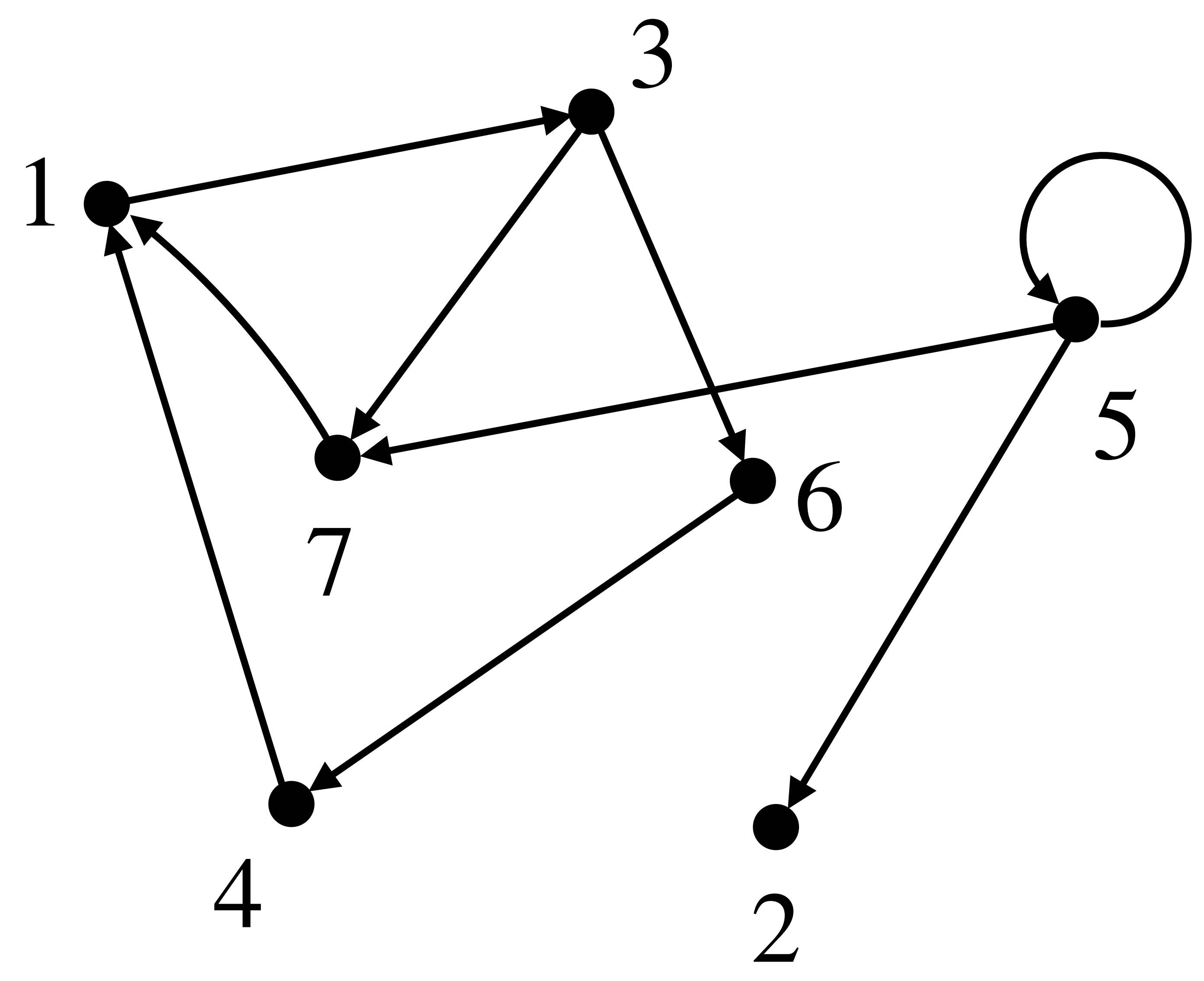

$\langle 5,7,1,3,6 \rangle$ is a path

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Graphs: Basic Terminology

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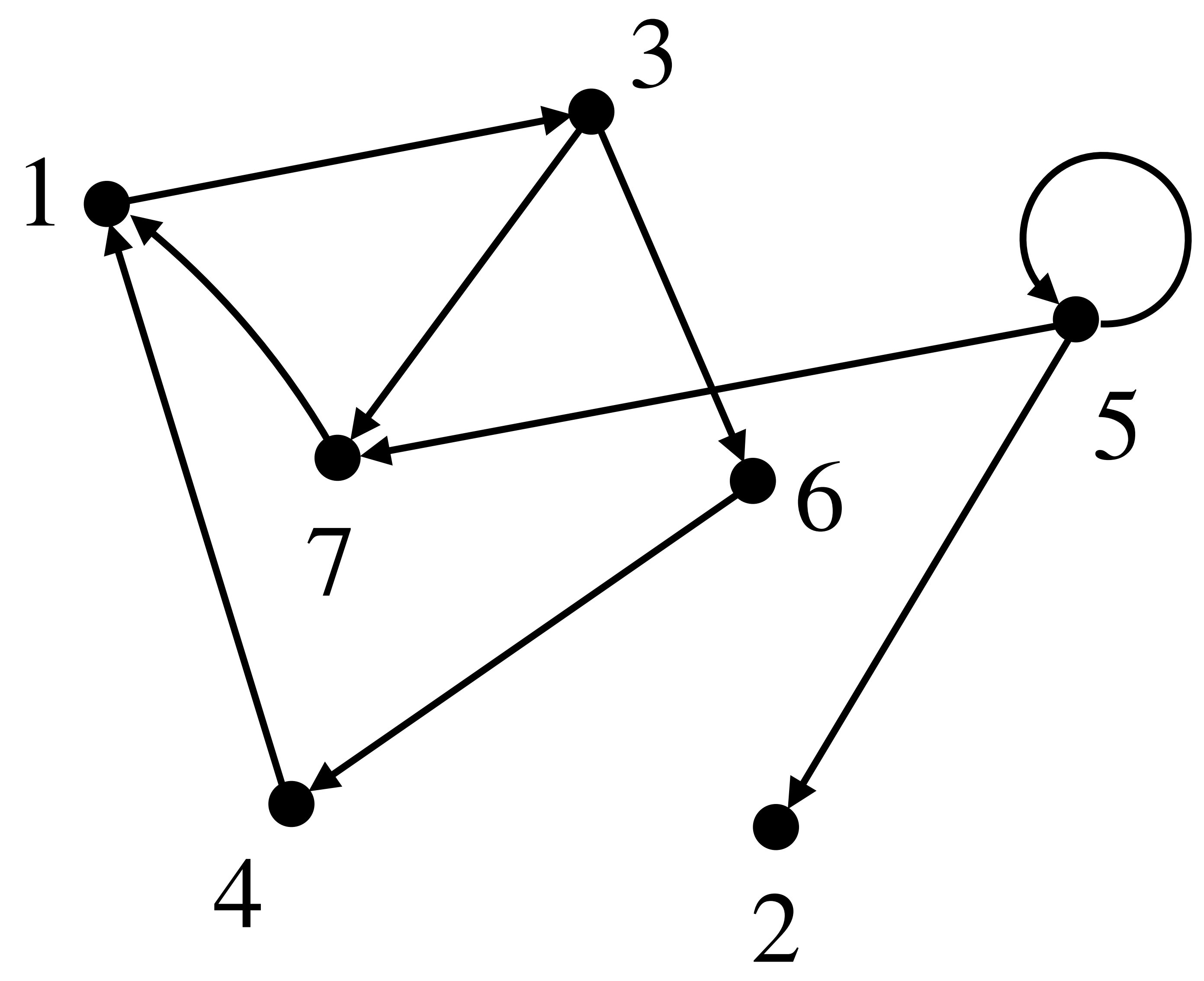

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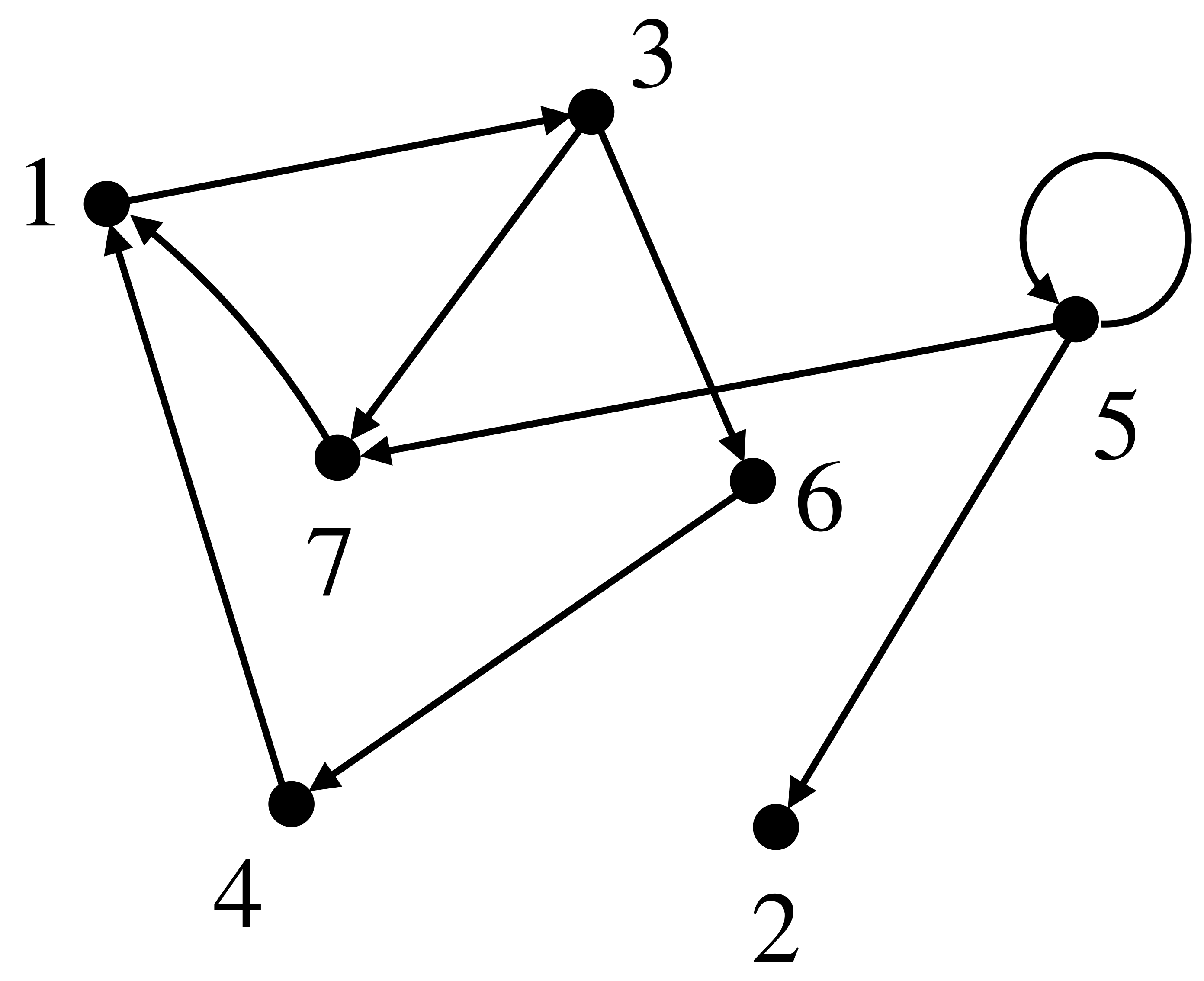
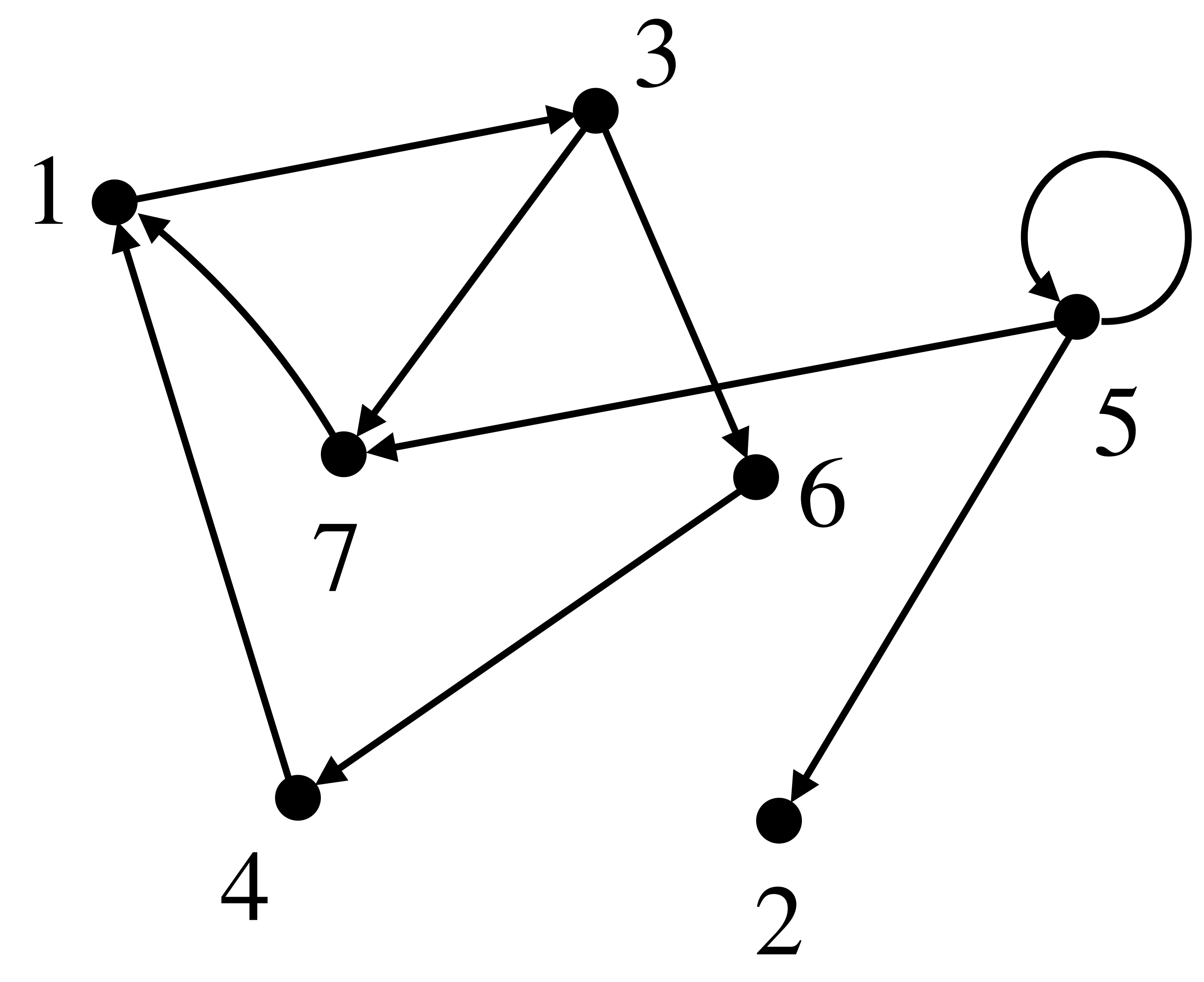


$\langle 5,7,1,3,6 \rangle$ is a path of length 4

Graphs: Basic Terminology

- The length of a path is the number of edges in it. The distance from a vertex i to a vertex j is the length of the shortest path from i to j.



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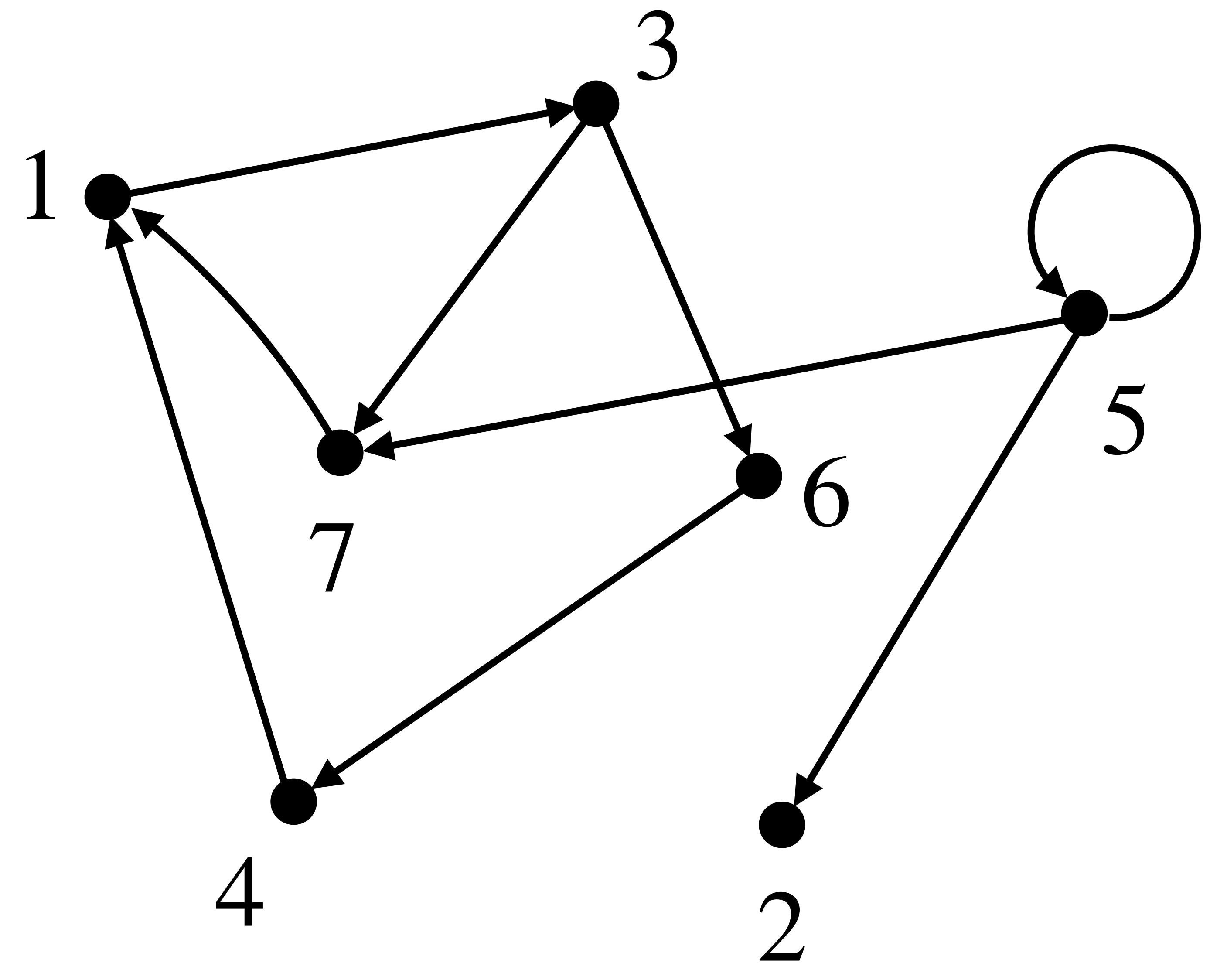

Distance from 6 to 1 is 2

Graphs: Basic Terminology

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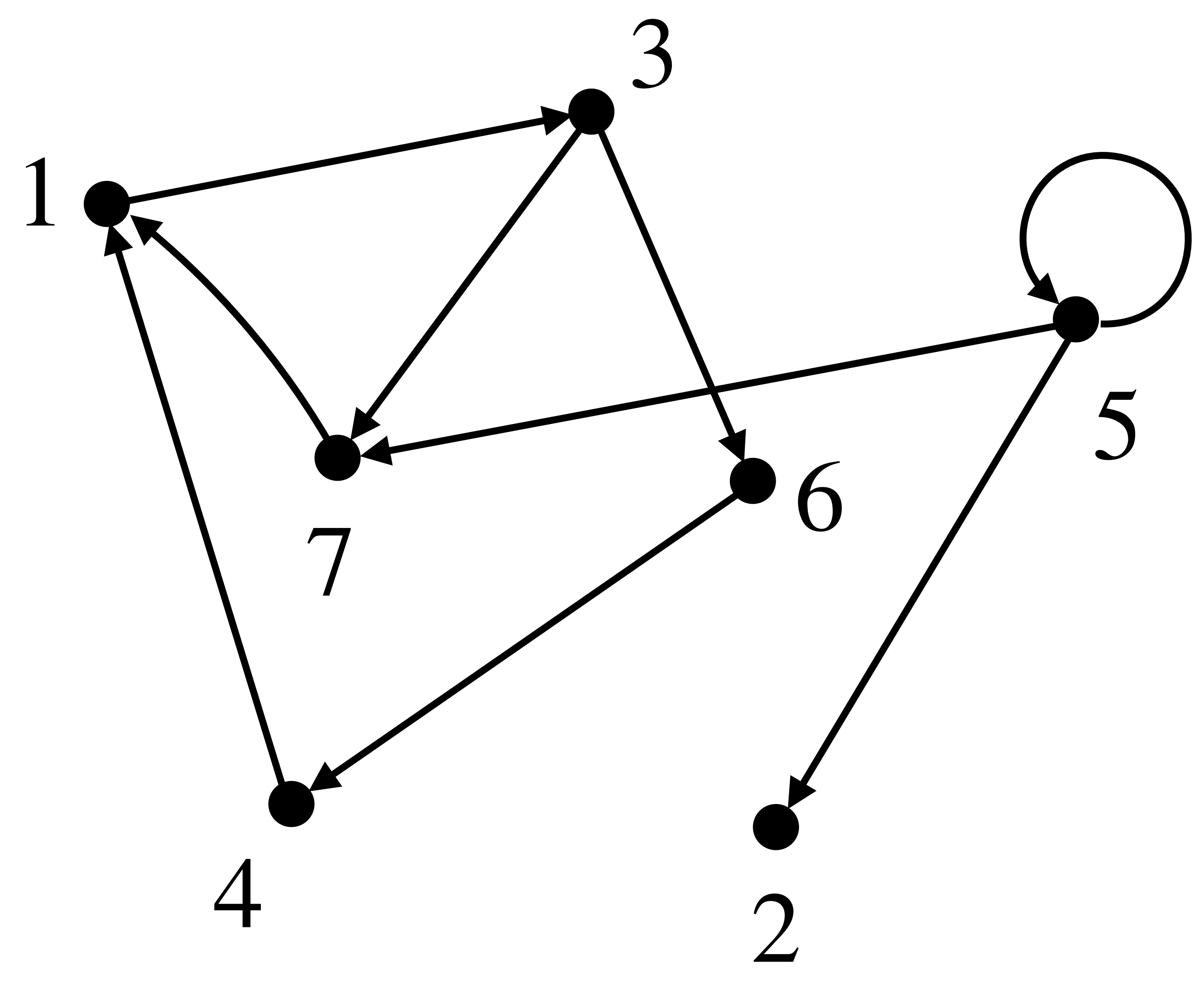

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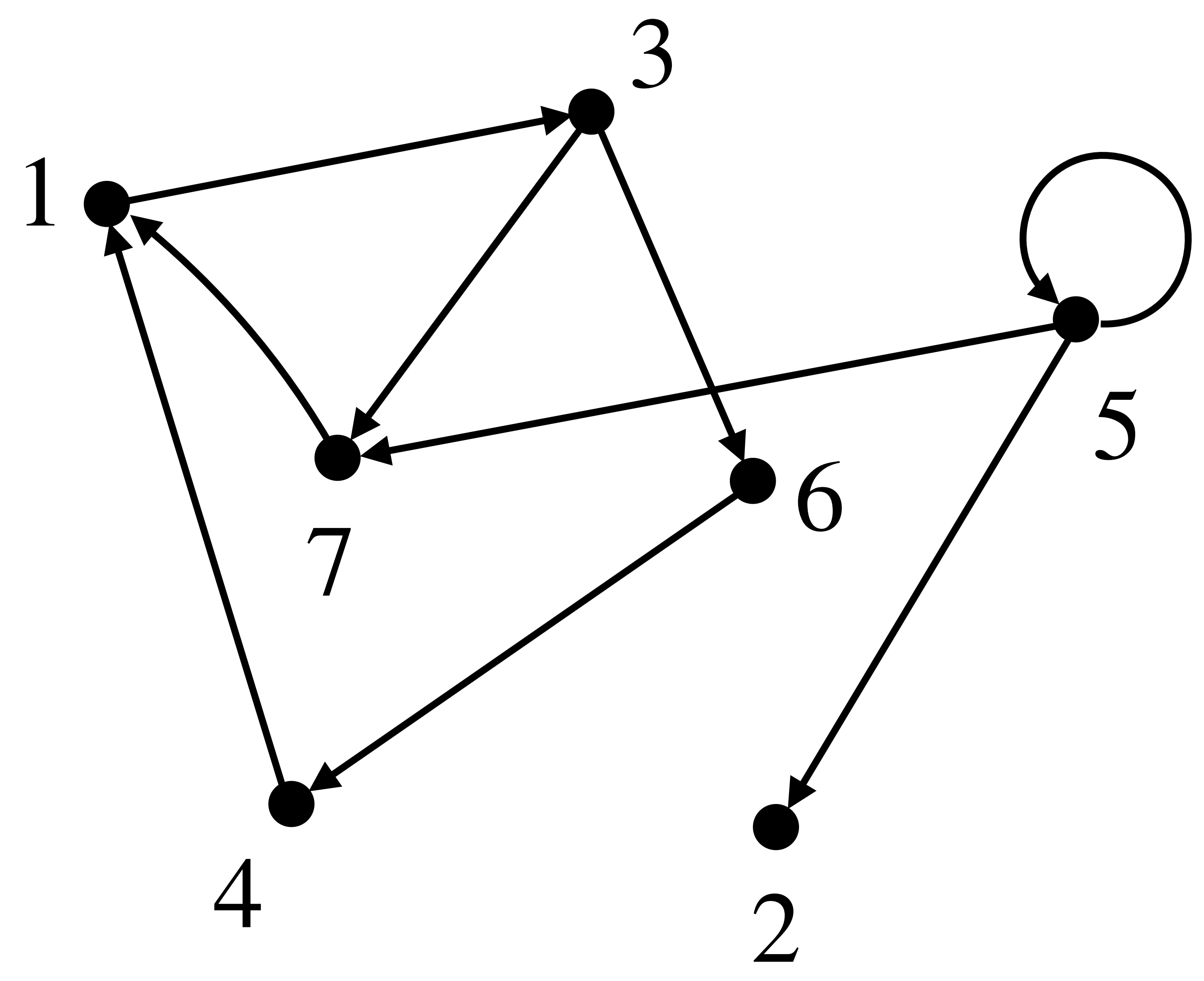

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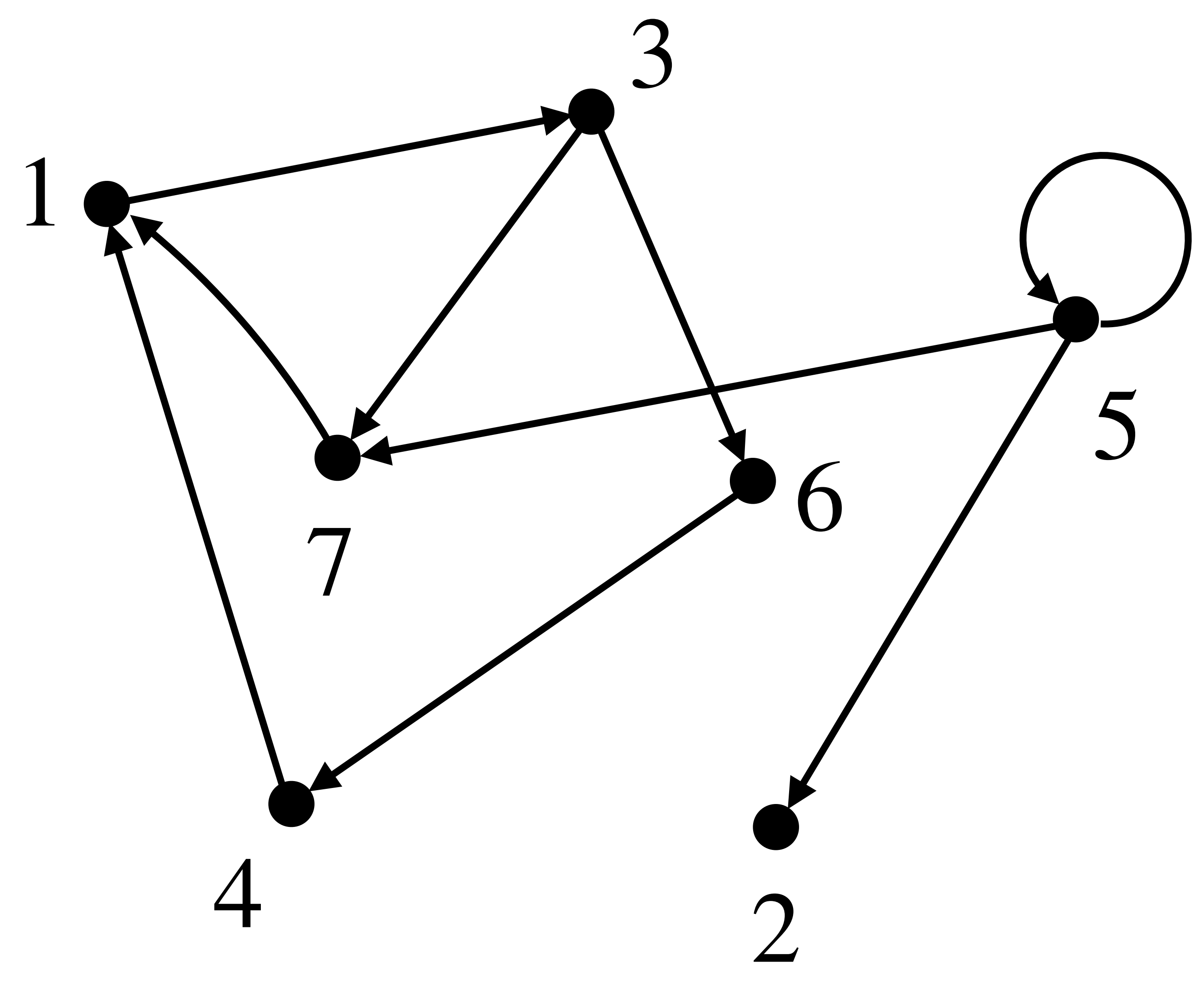

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Graphs: Representation

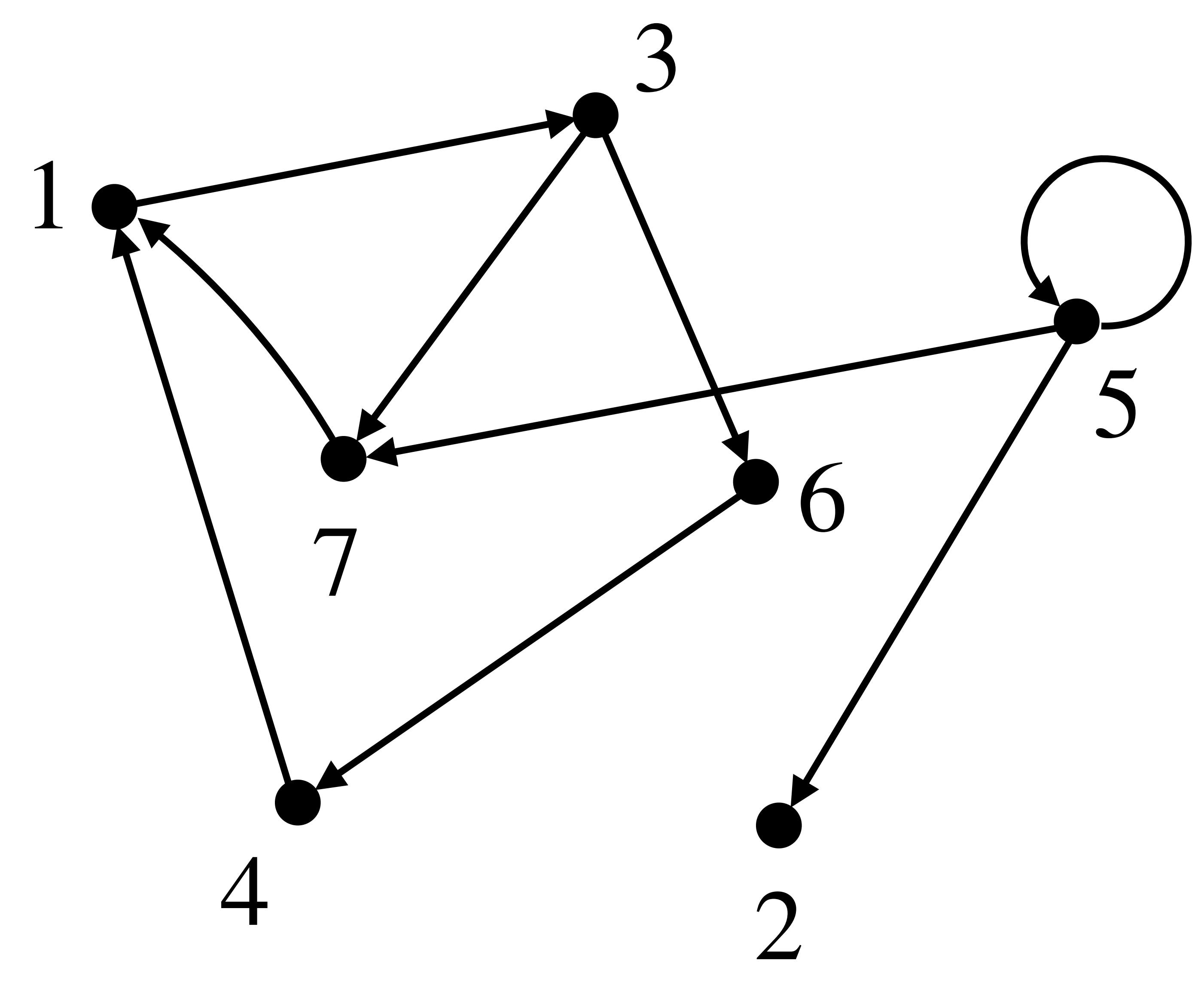
Graphs: Representation

Adjacency-list representation:

Graphs: Representation

Adjacency-list representation:

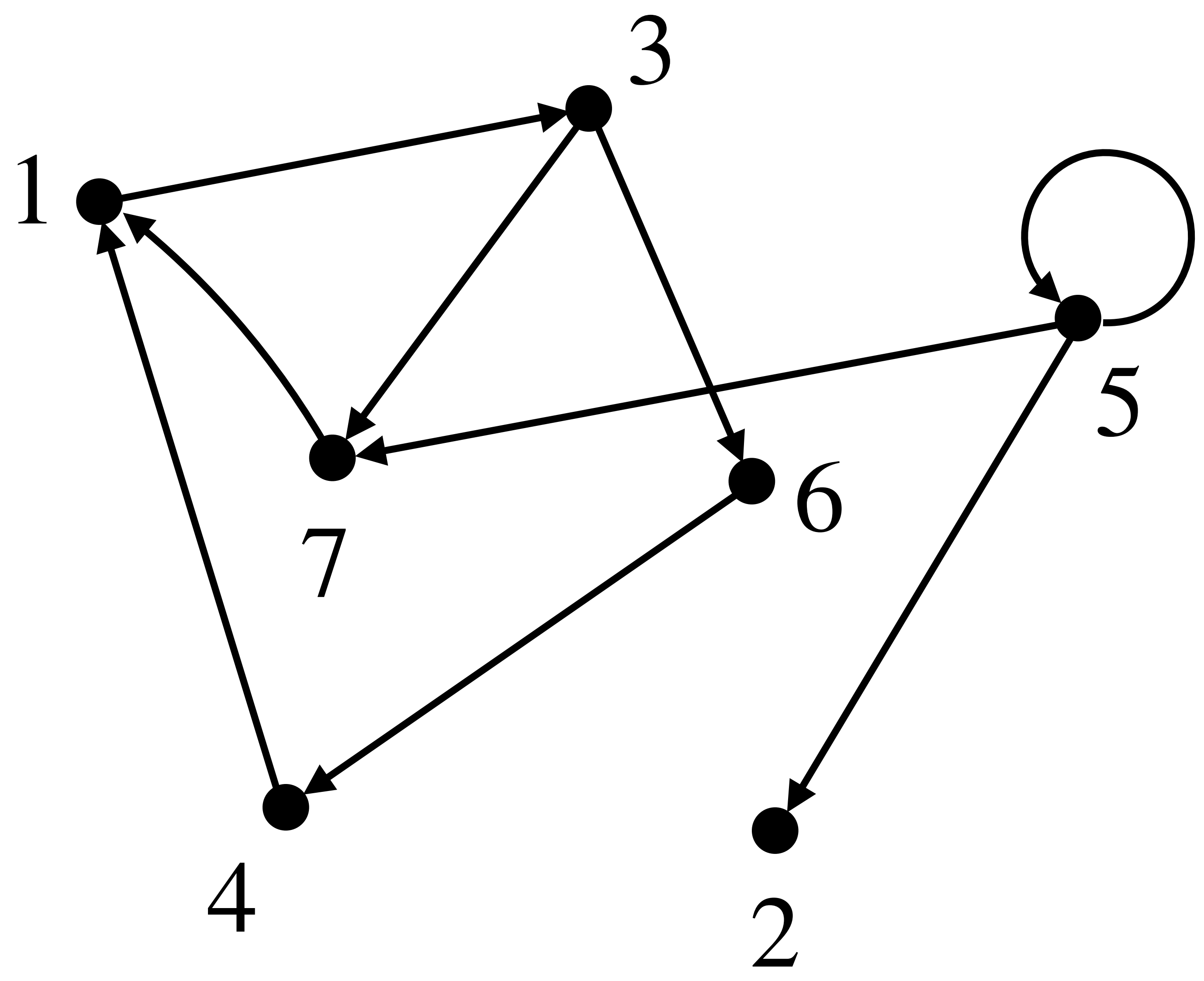
Graph G



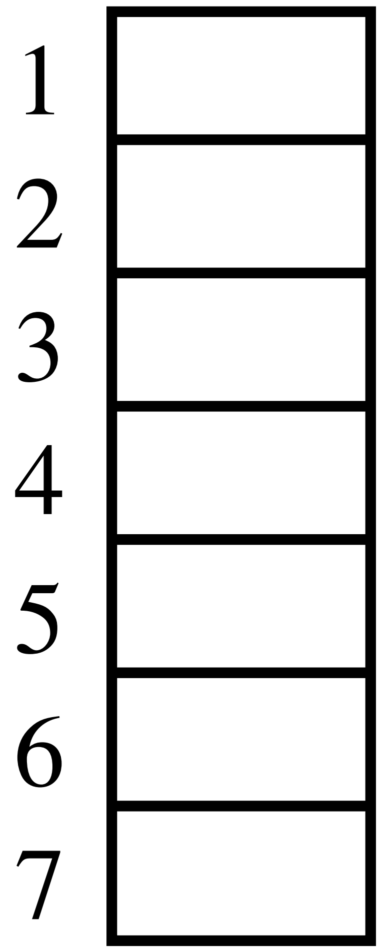
Graphs: Representation

Adjacency-list representation:

Graph G



G's adjacency list



Graphs: Representation

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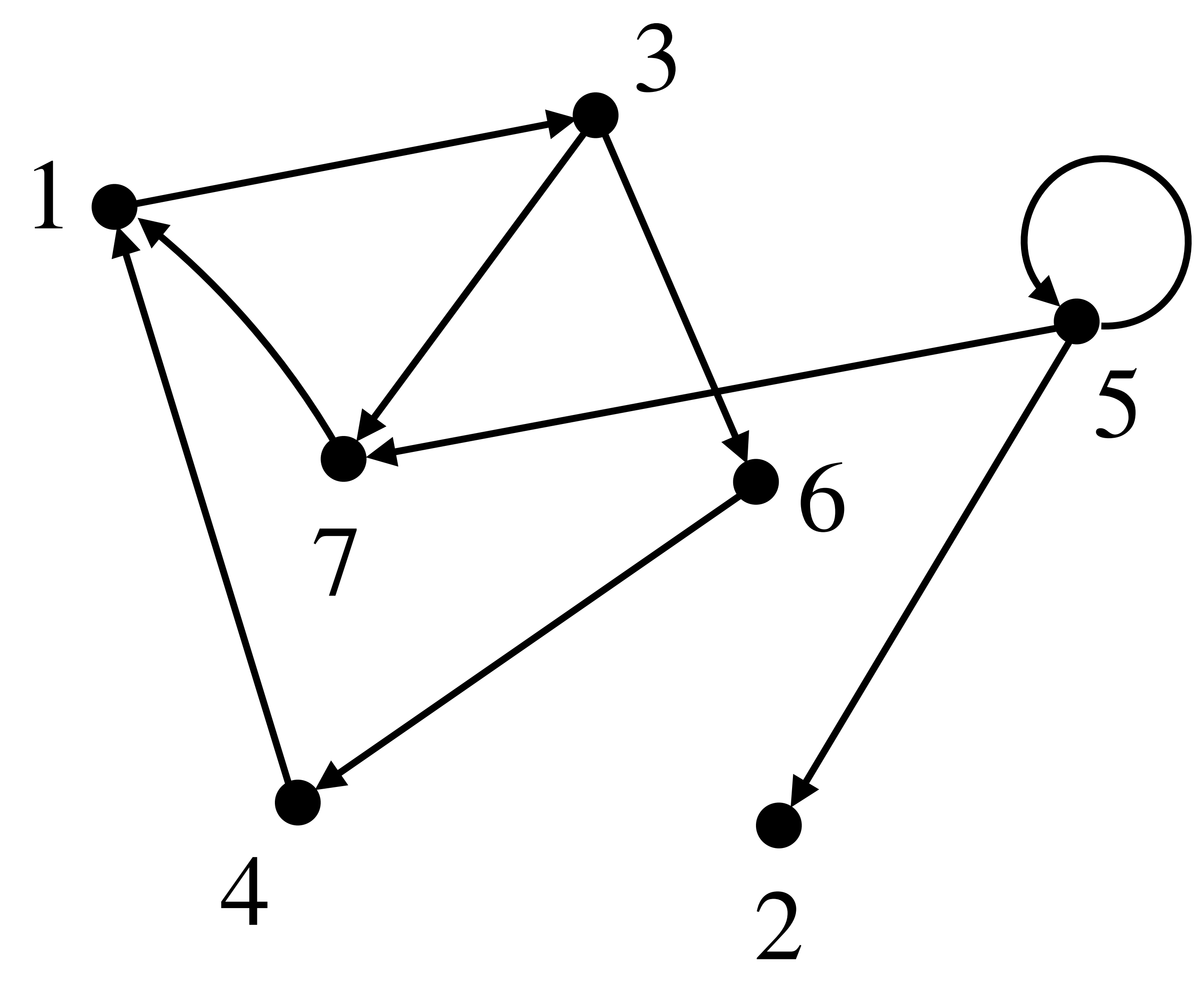

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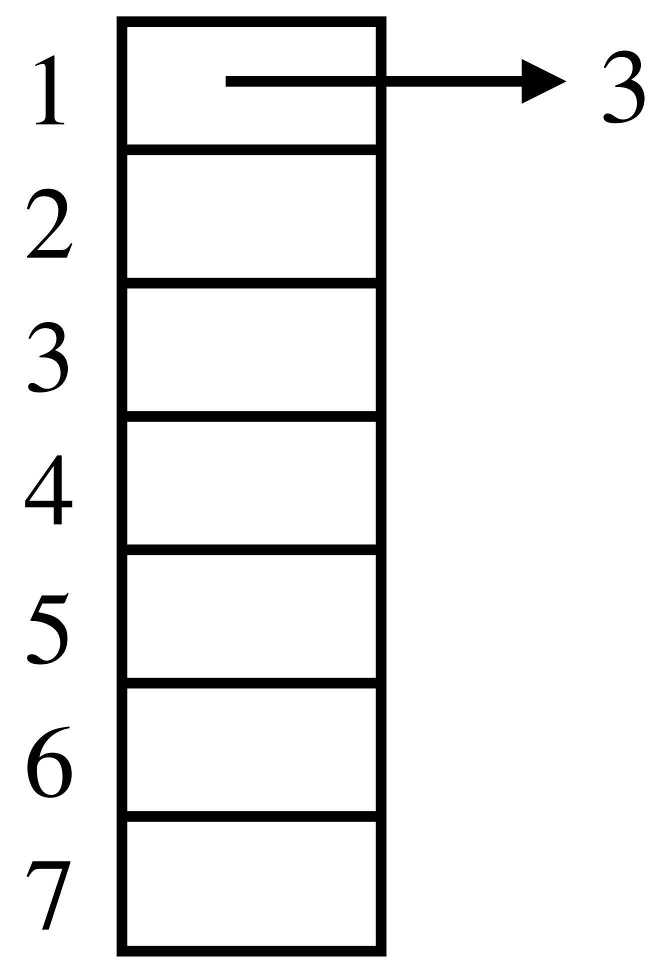

Graphs: Representation

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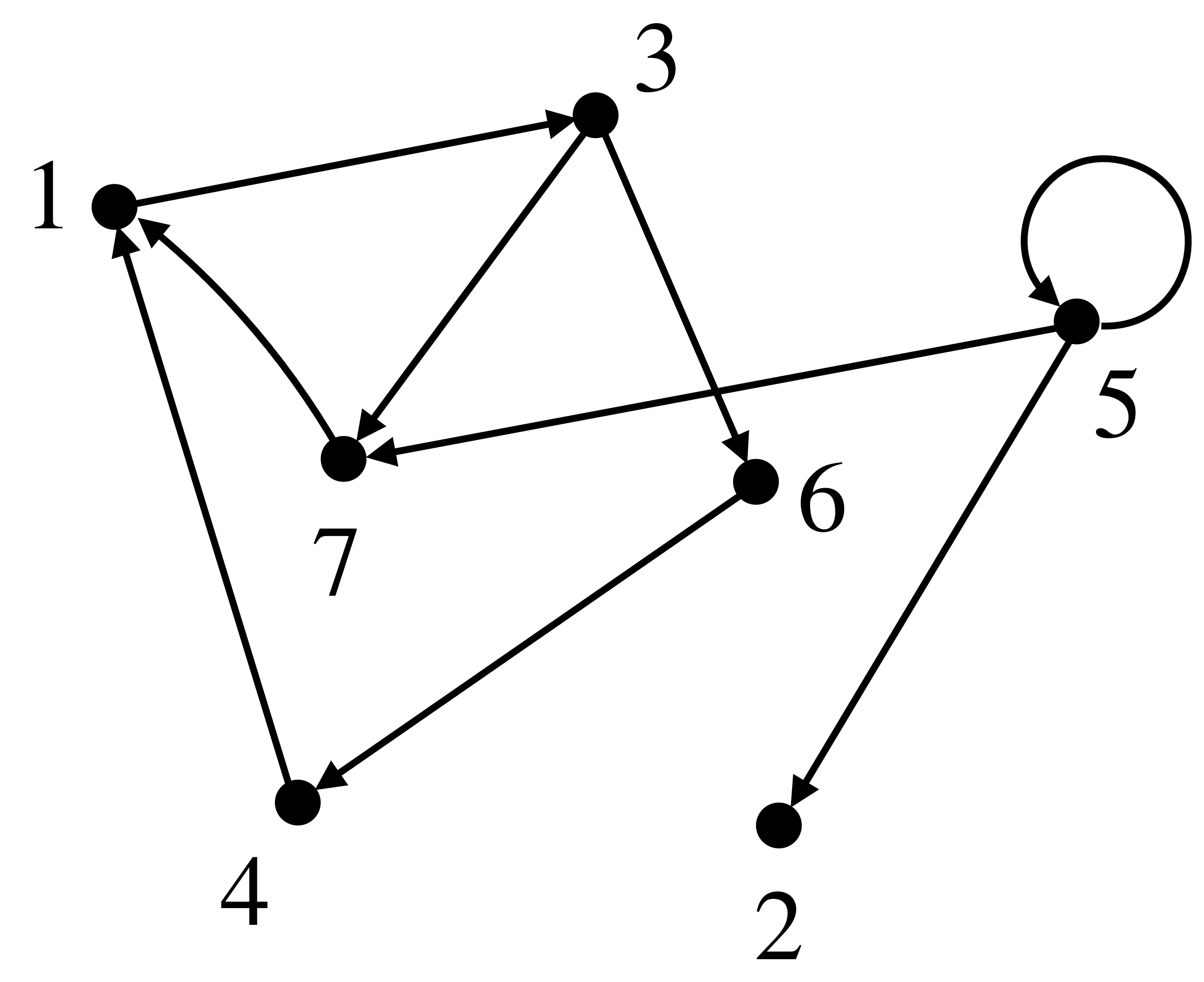

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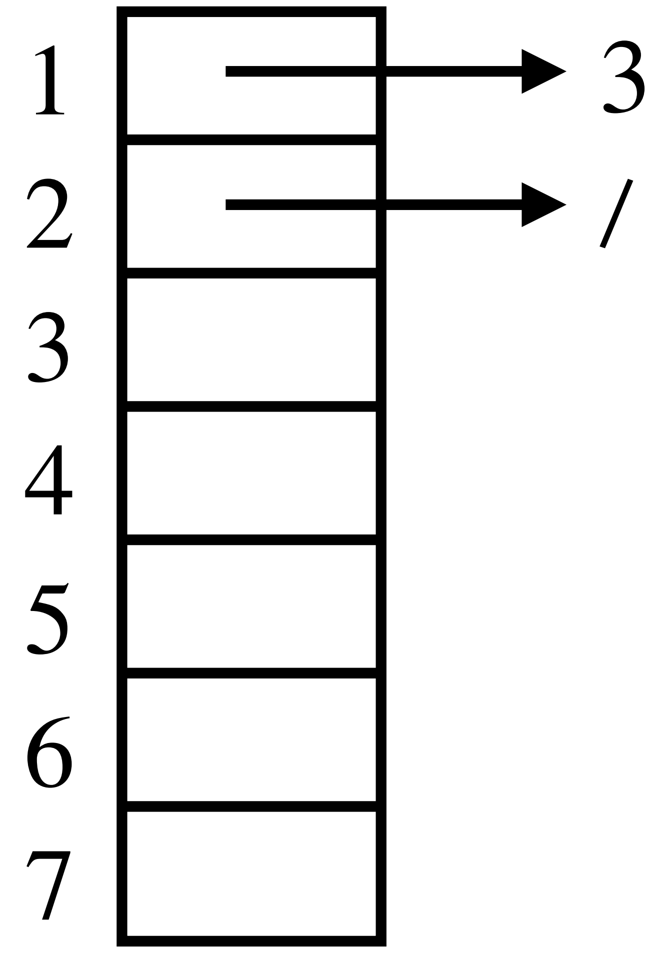

Graphs: Representation

Adjacency-list representation:

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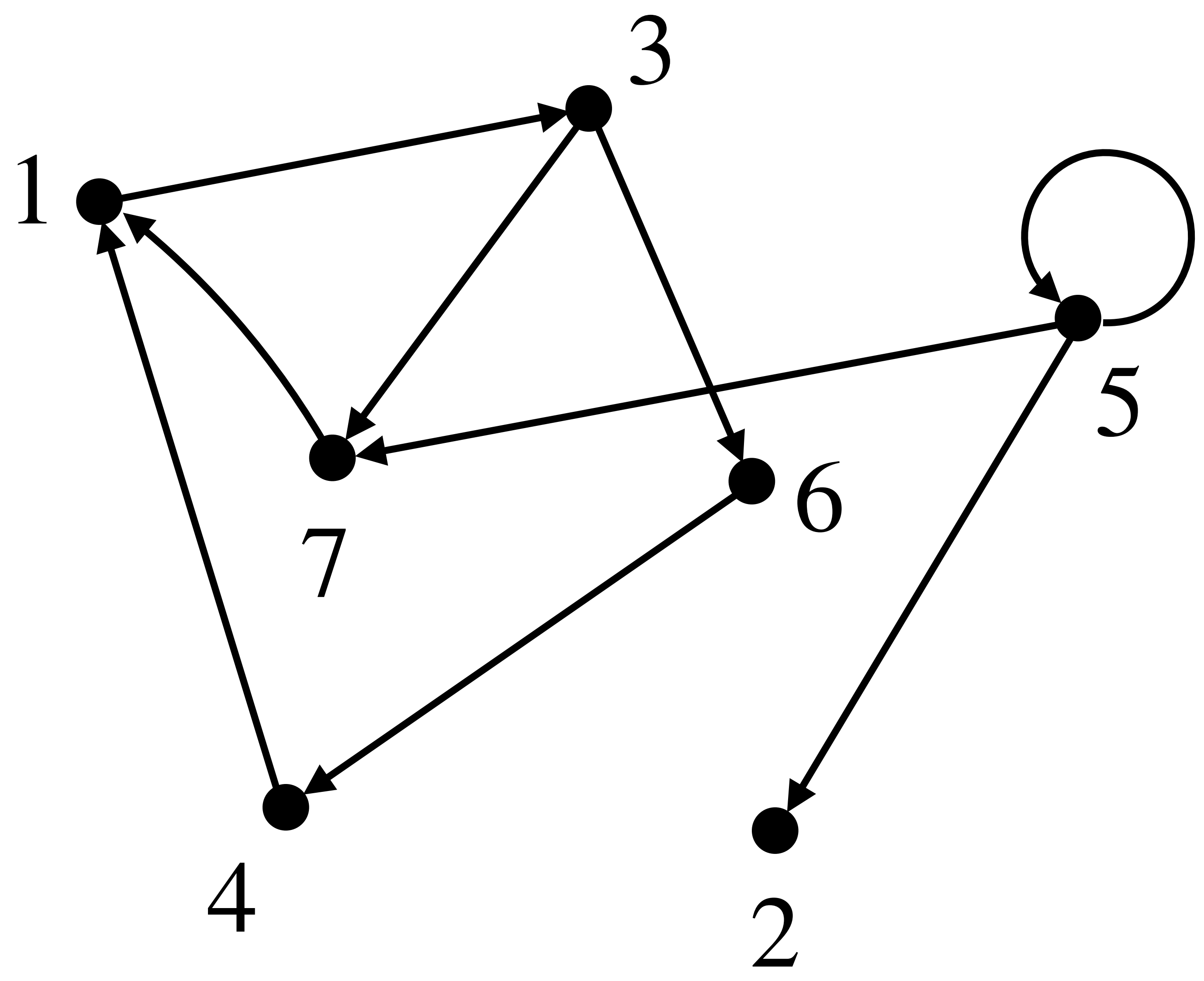

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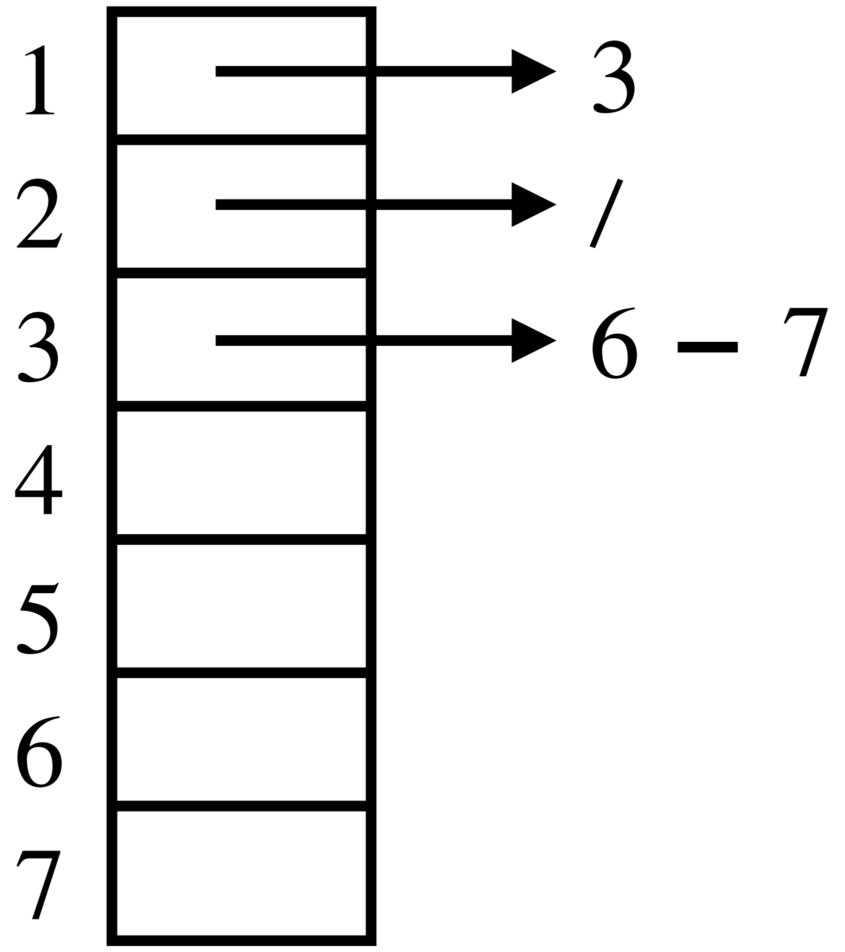

Graphs: Representation

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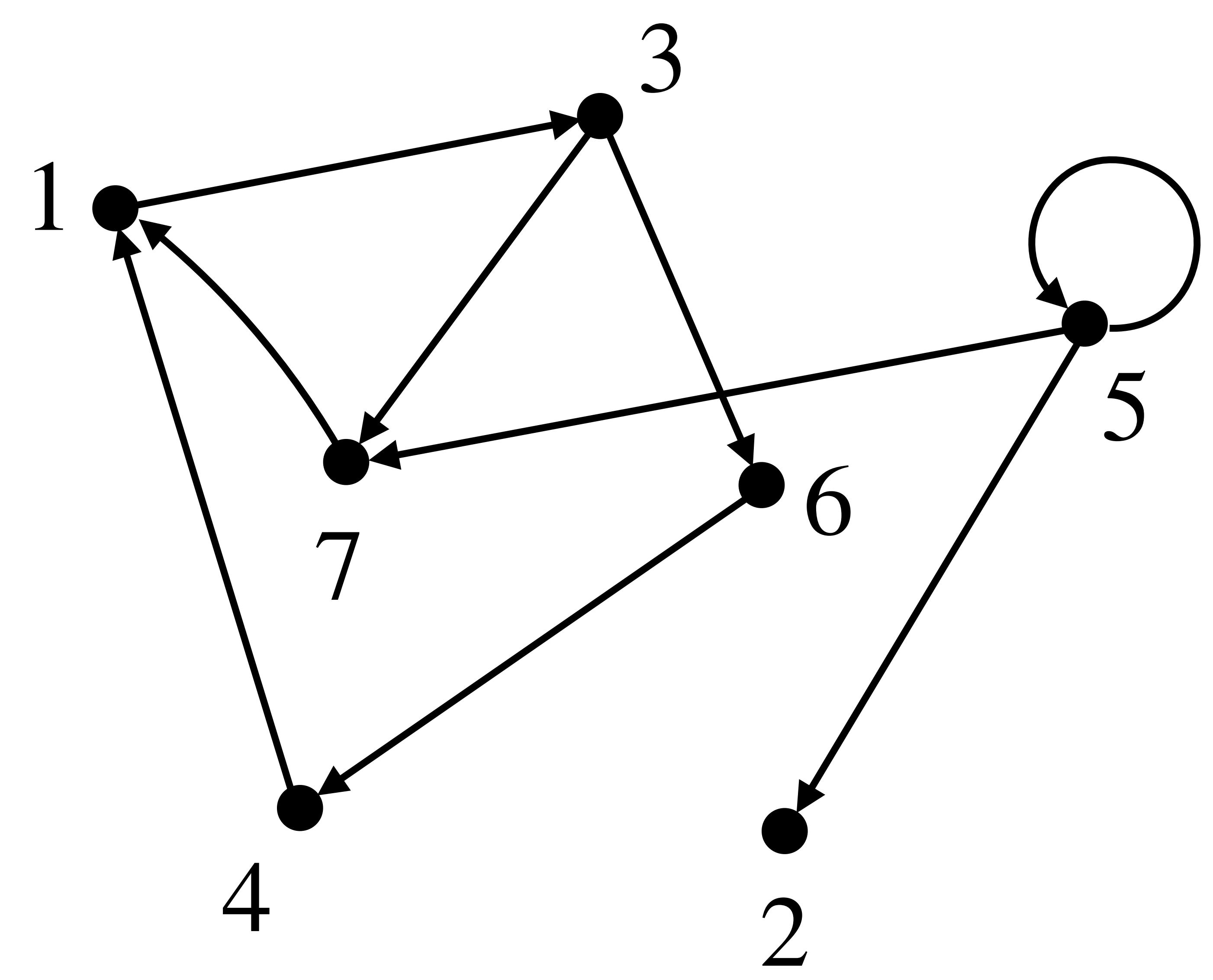

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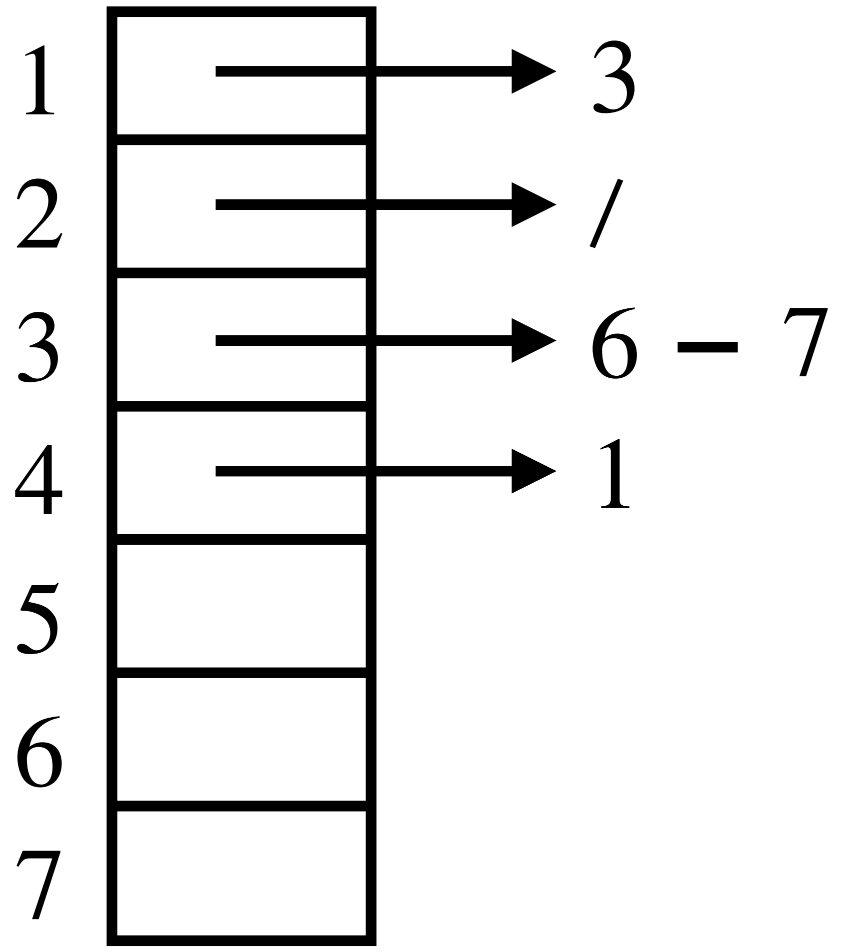

Graphs: Representation

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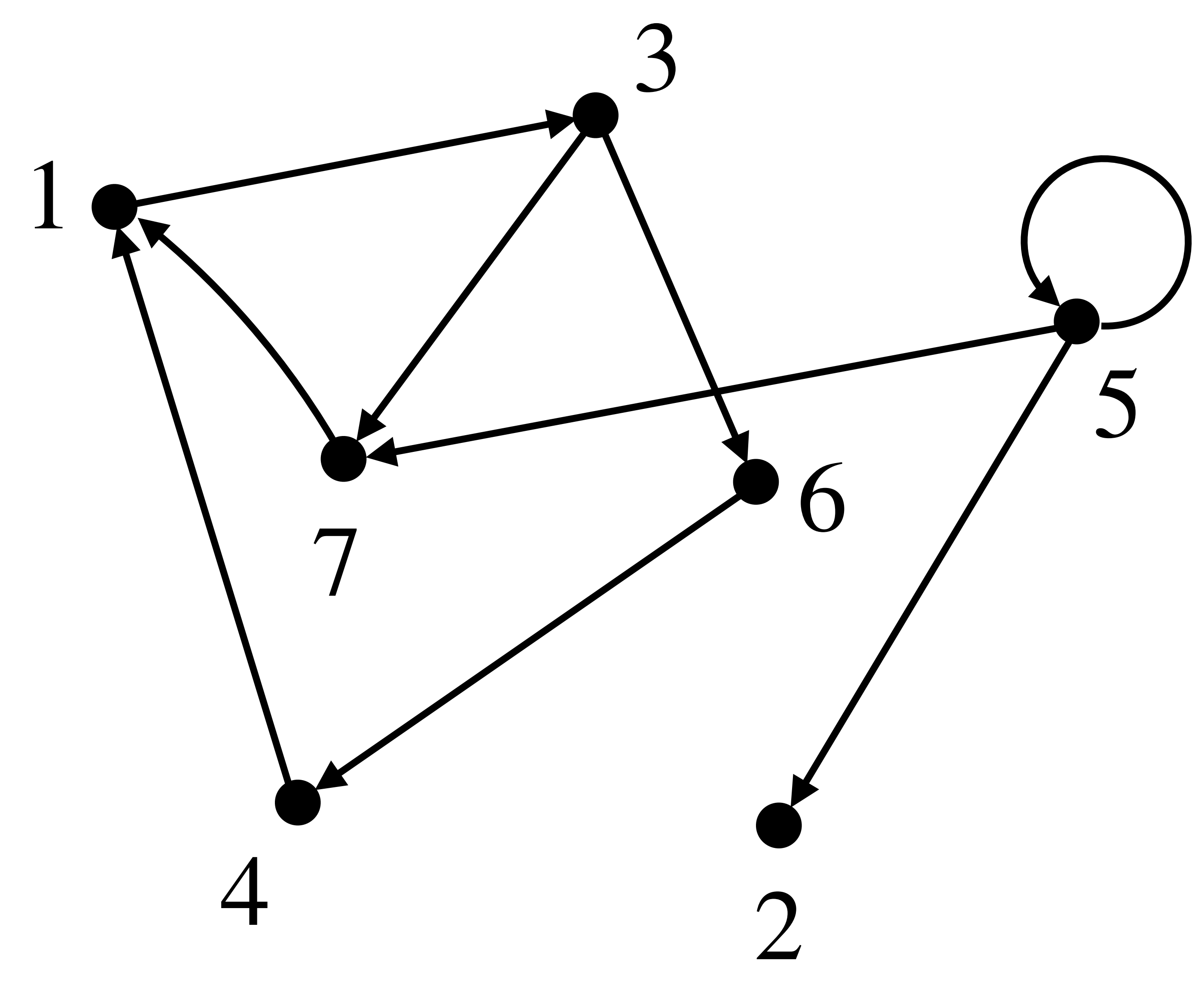

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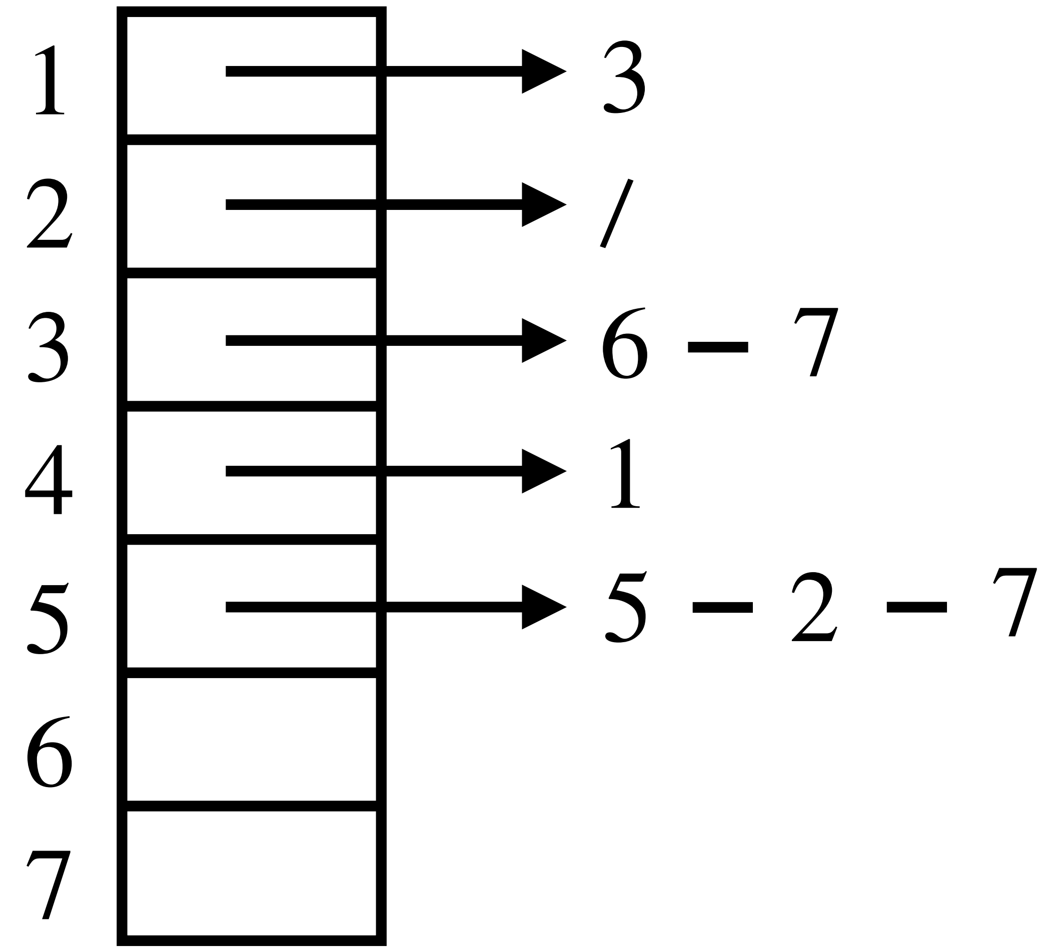

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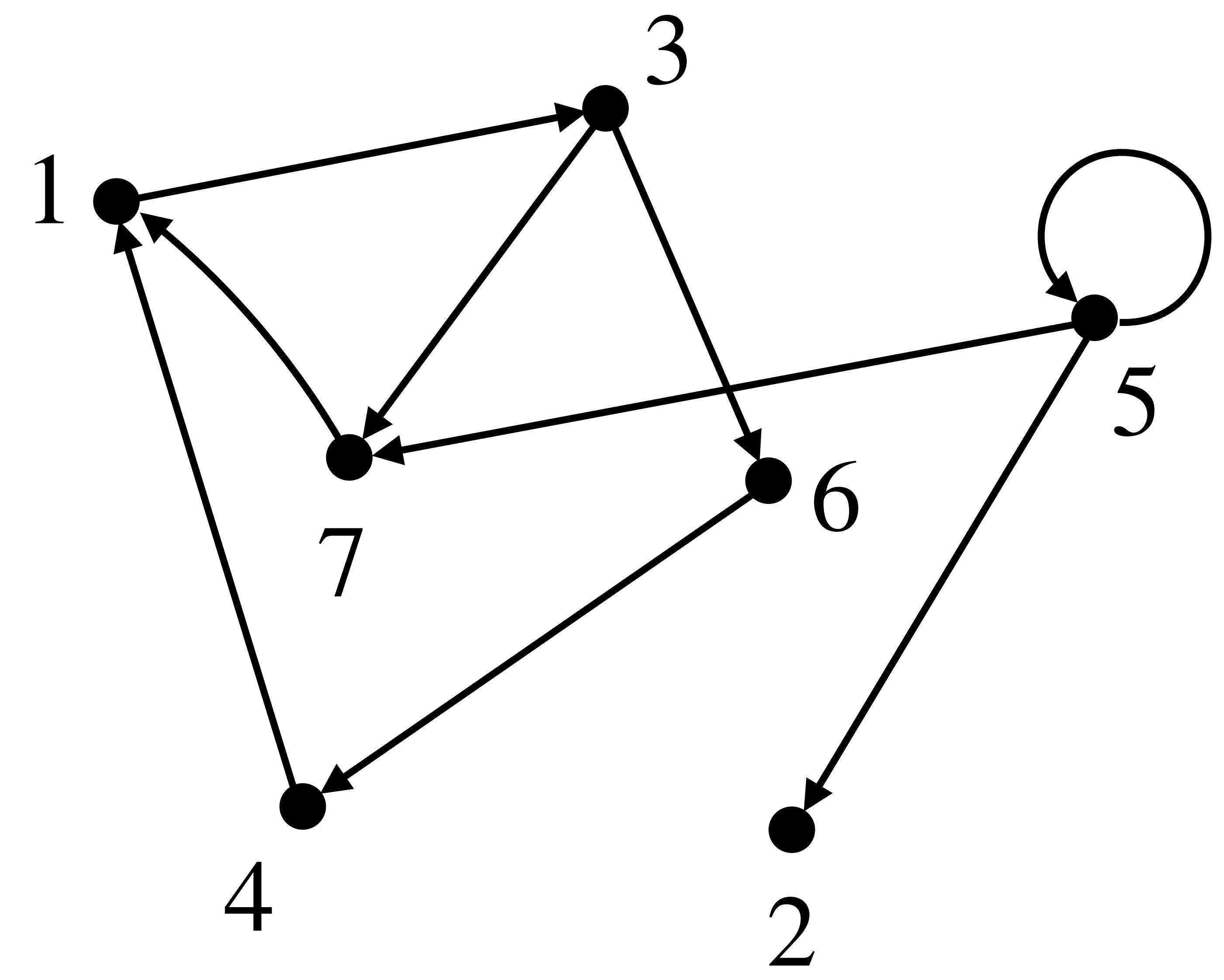

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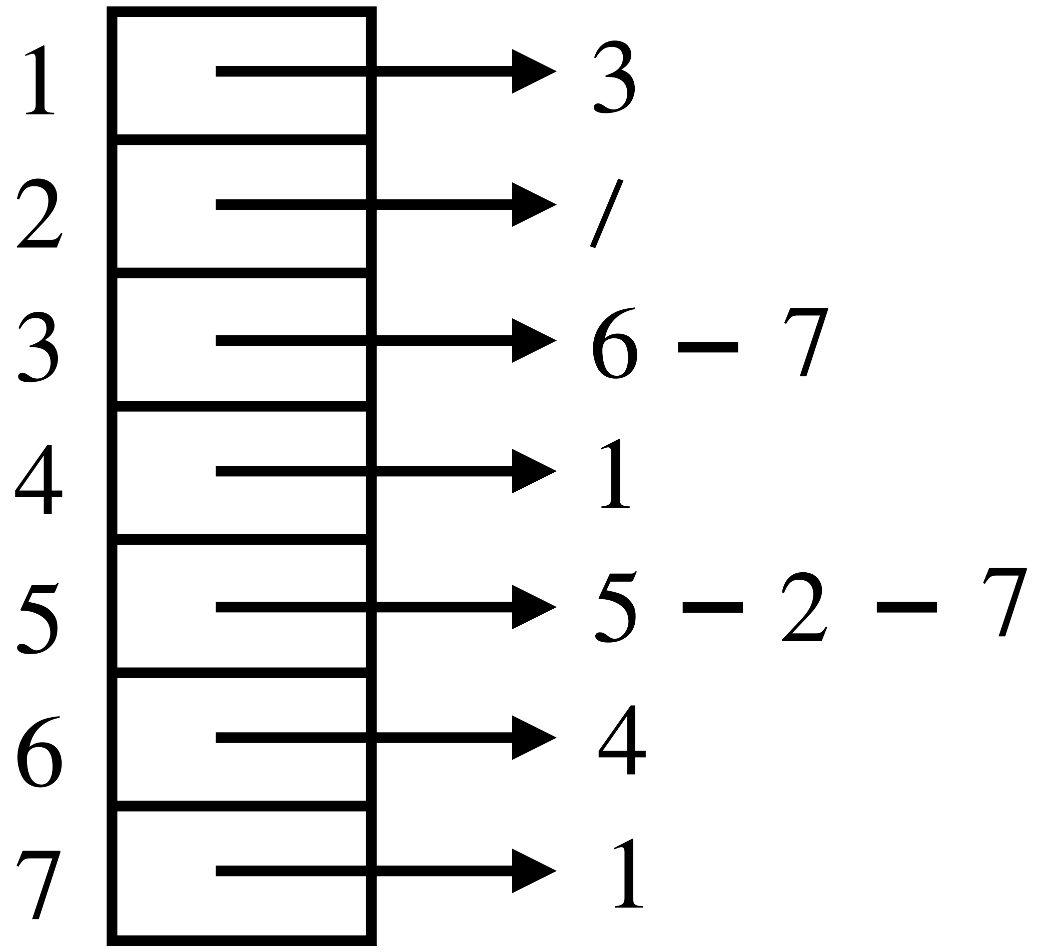

Graphs: Representation

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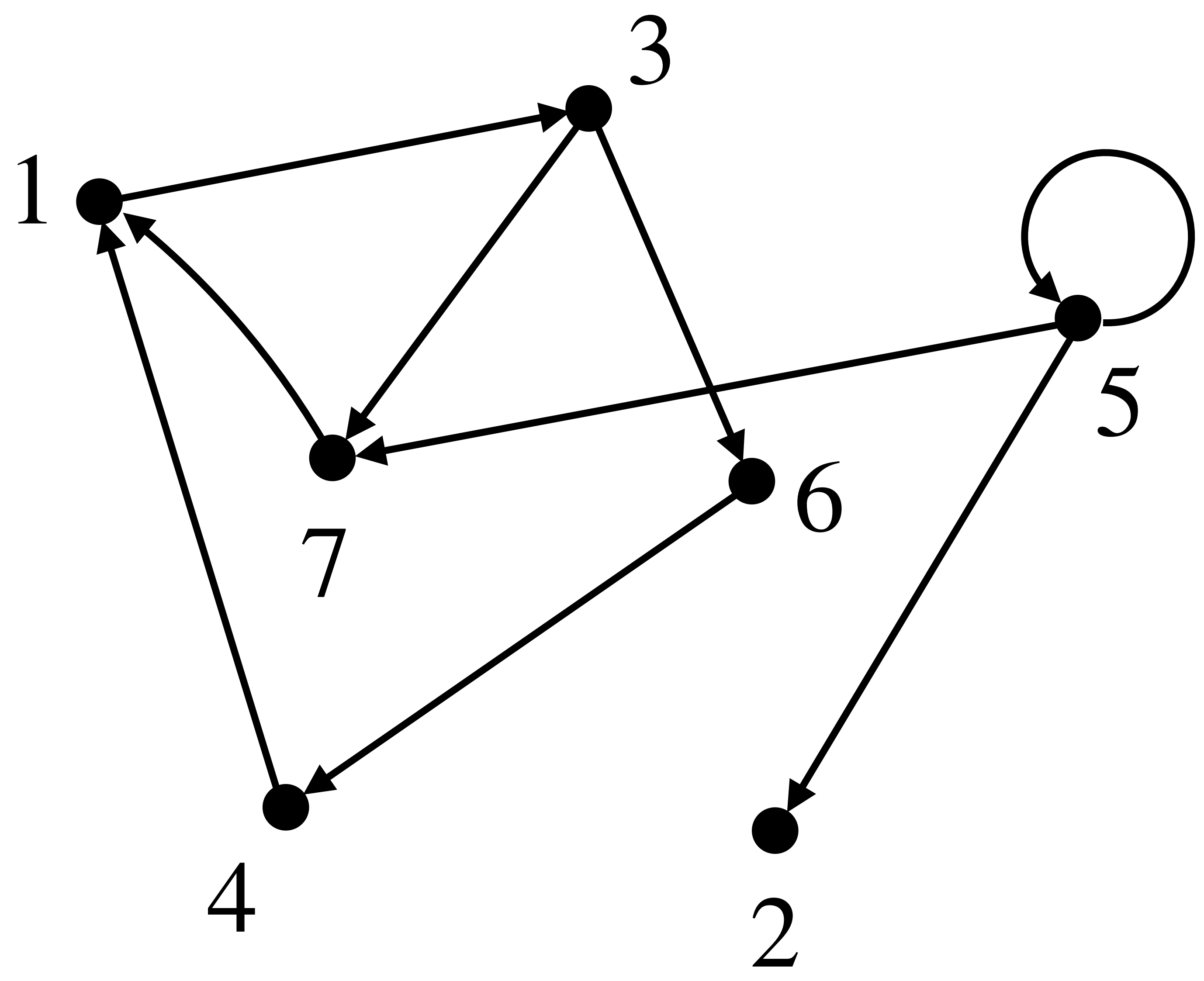


G's adjacency list

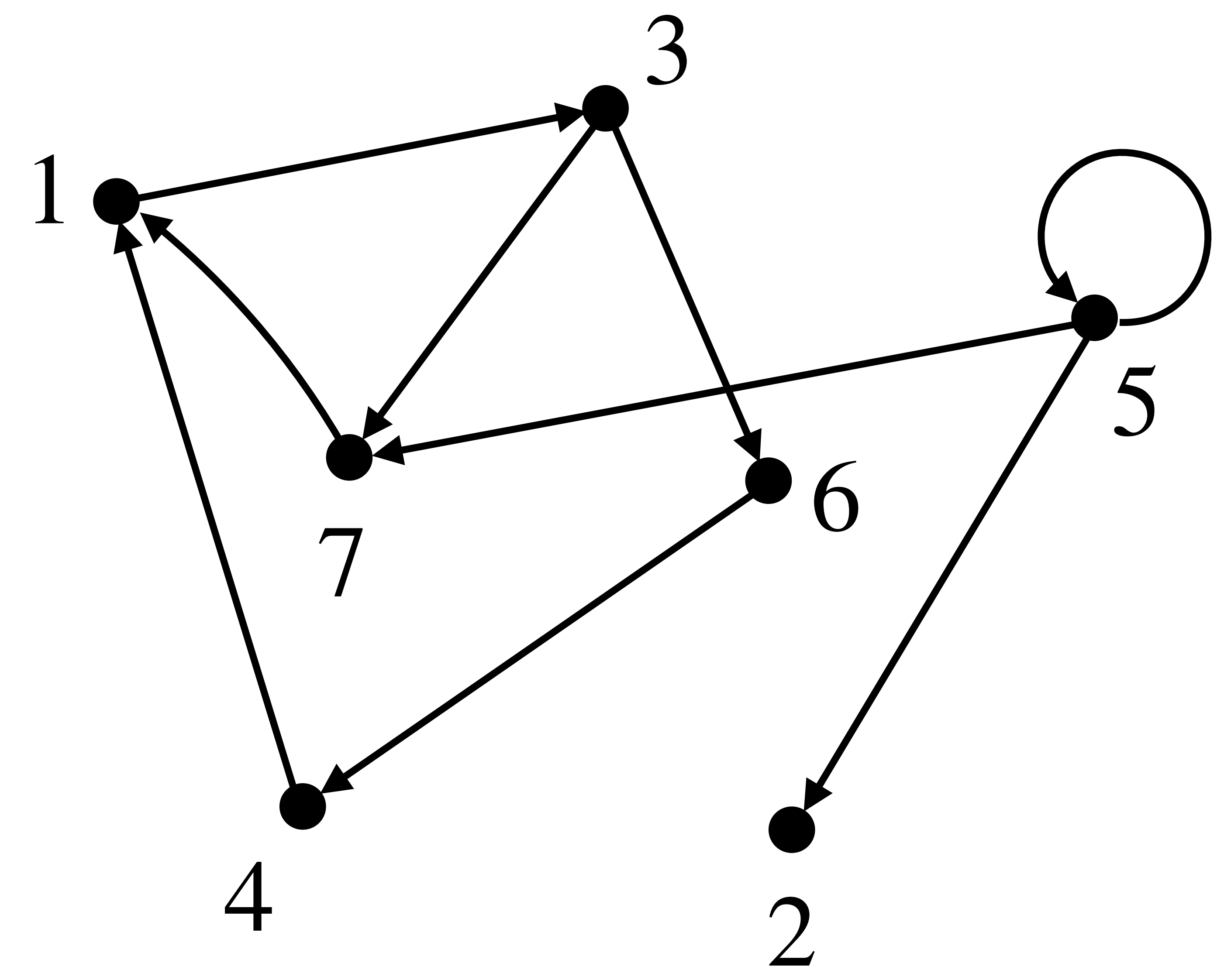
1	→ 3
2	→ /
3	→ 6 − 7
4	→ 1
5	→ 5 − 2 − 7
6	→ 4
7	→ 1

The adjacency-list representation of a graph $G = (V, E)$ consists of an array Adj of $|V|$ lists,

Graphs: Representation

Adjacency-list representation:

Graph G



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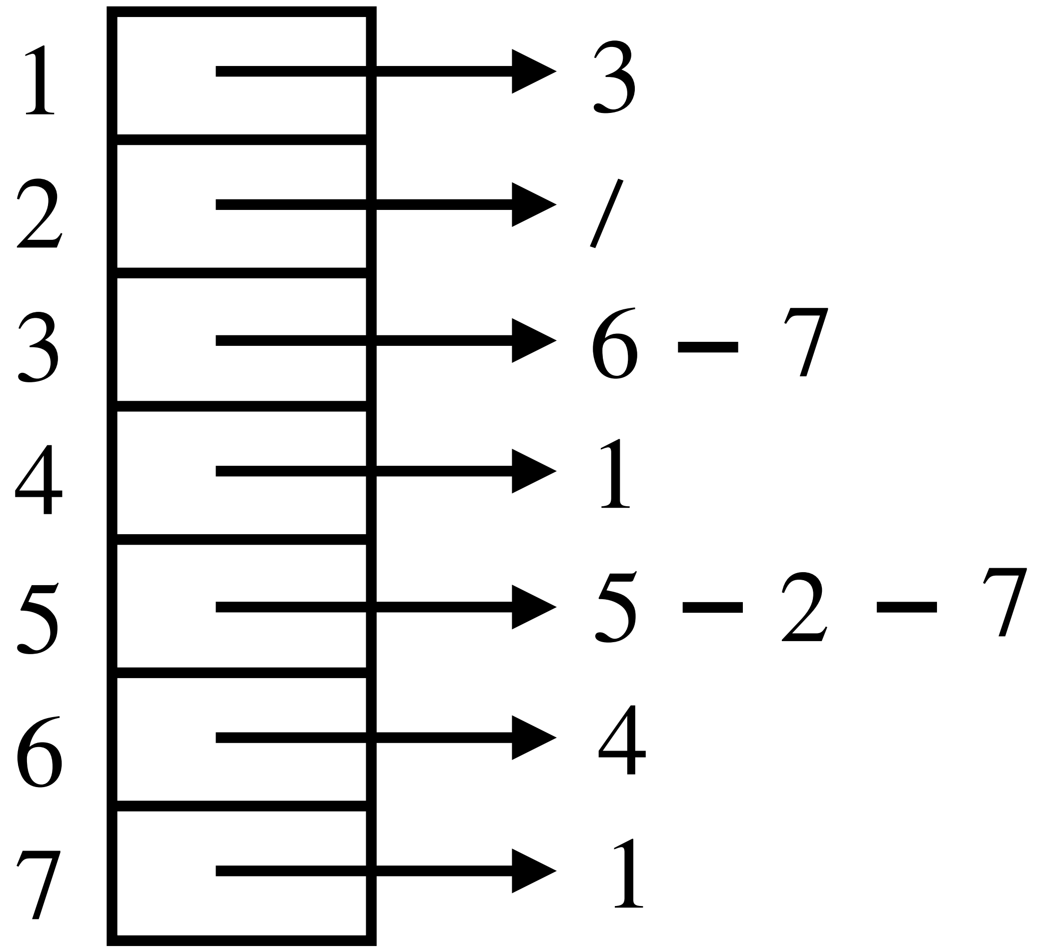

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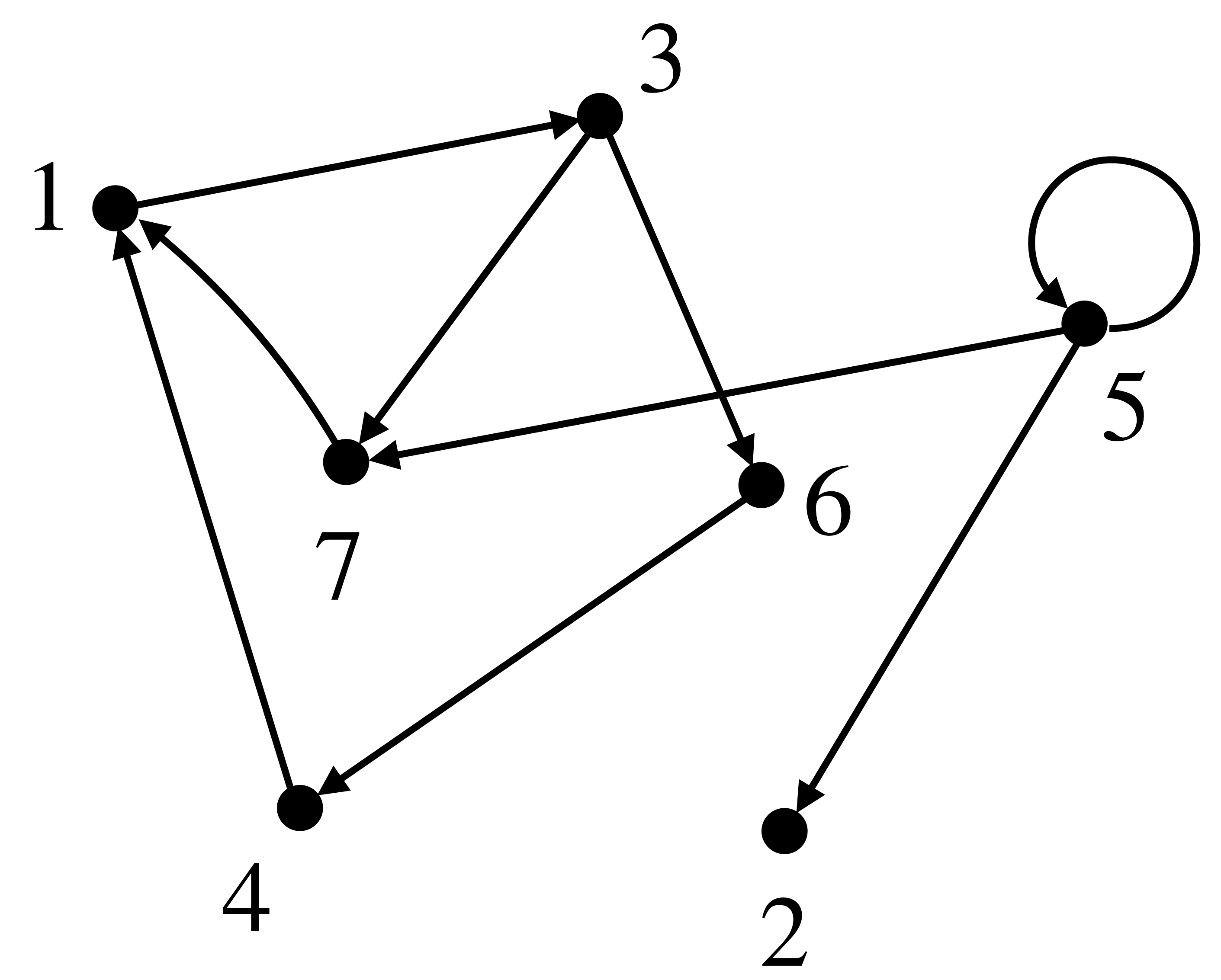
Graphs: Representation

Adjacency-list representation:

Graph G



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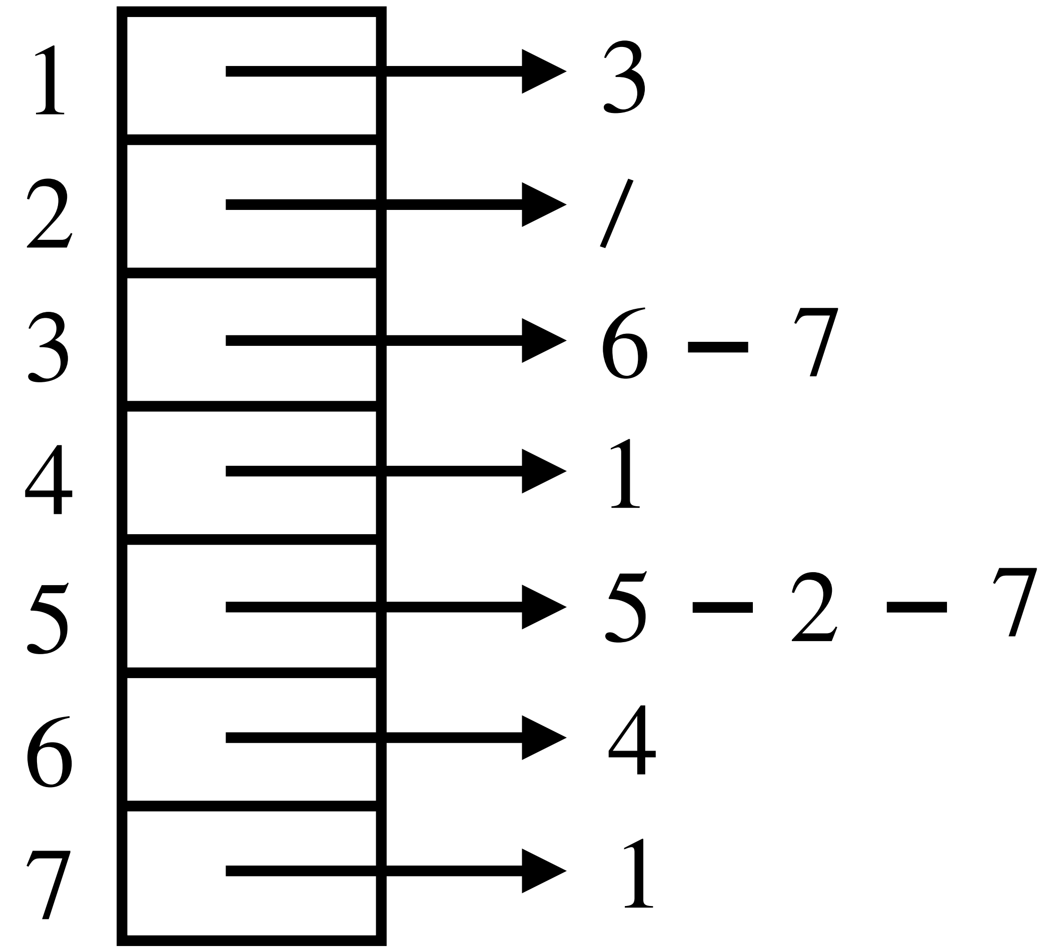

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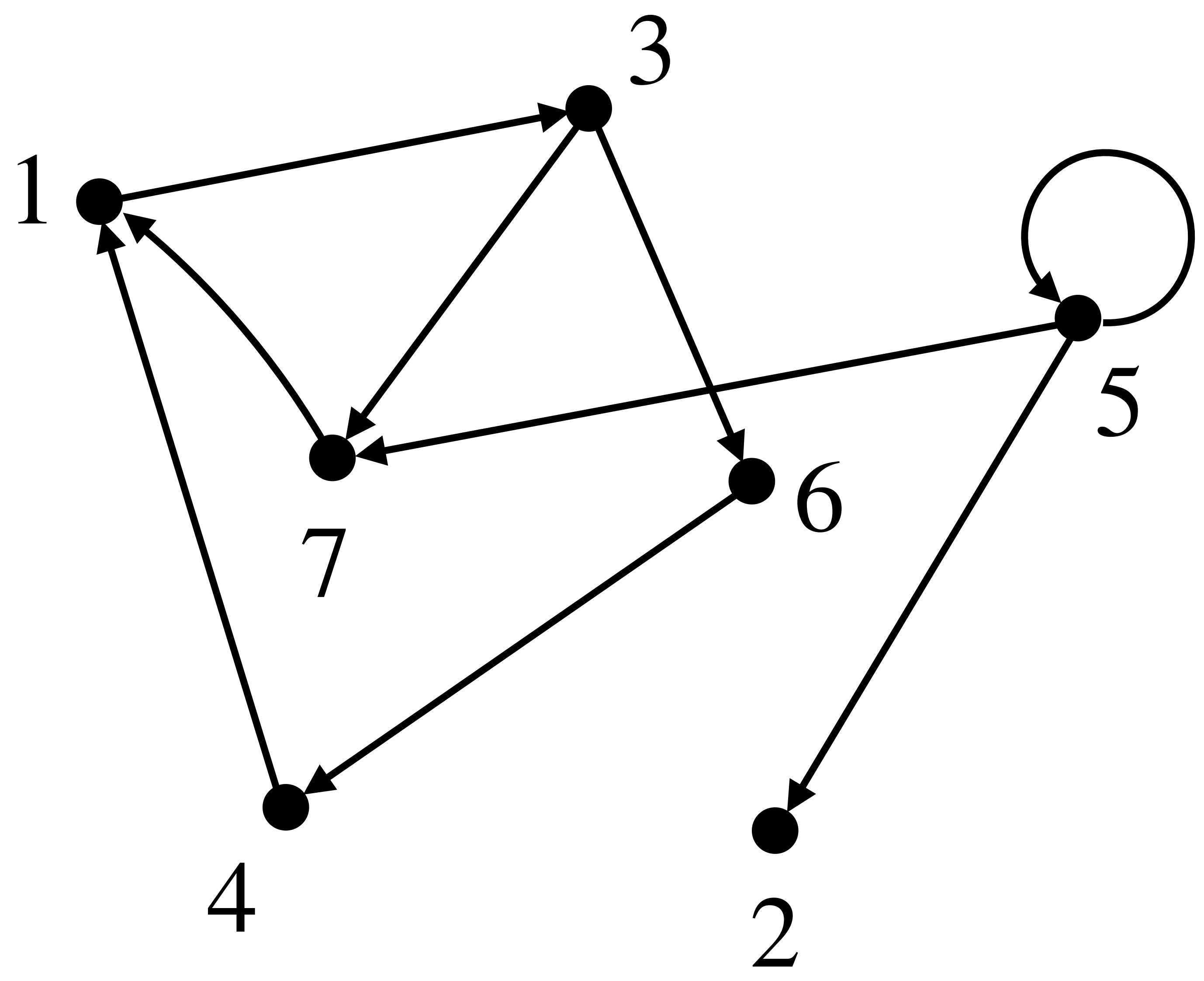
Graphs: Representation

Adjacency-list representation:

Graph G



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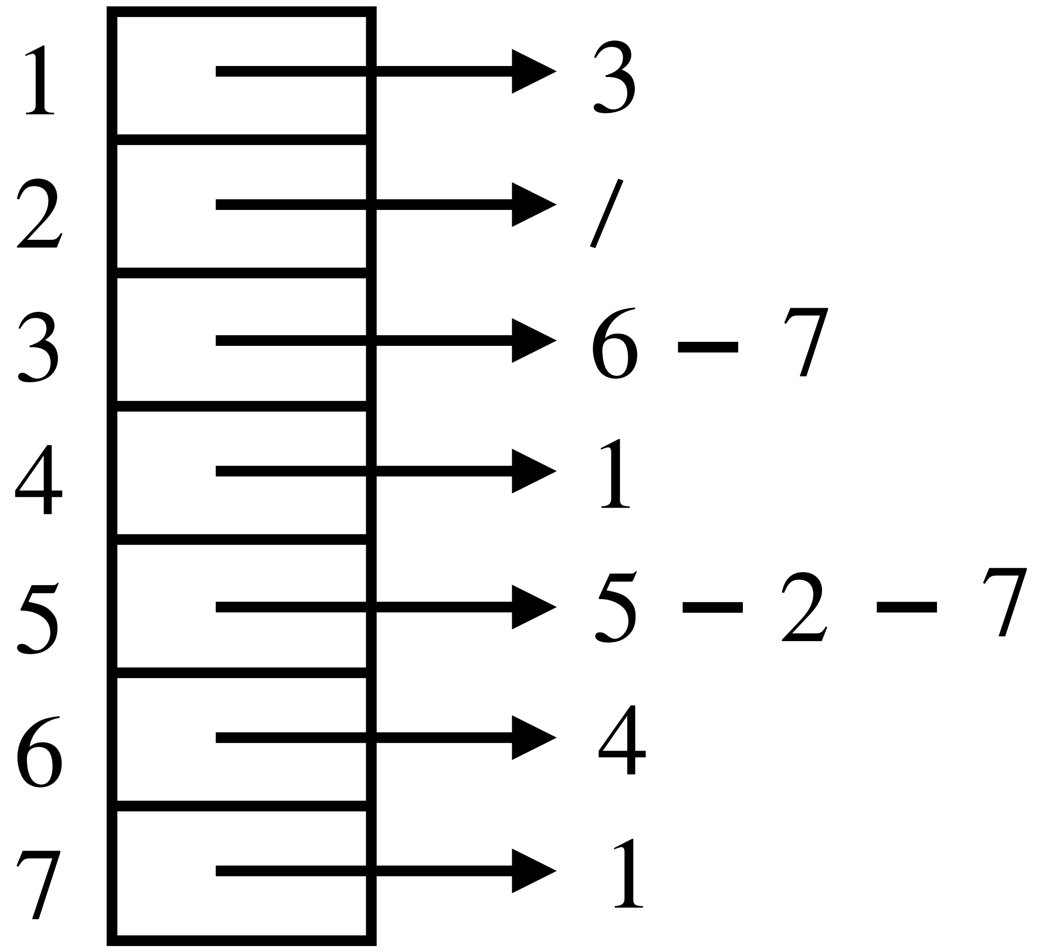

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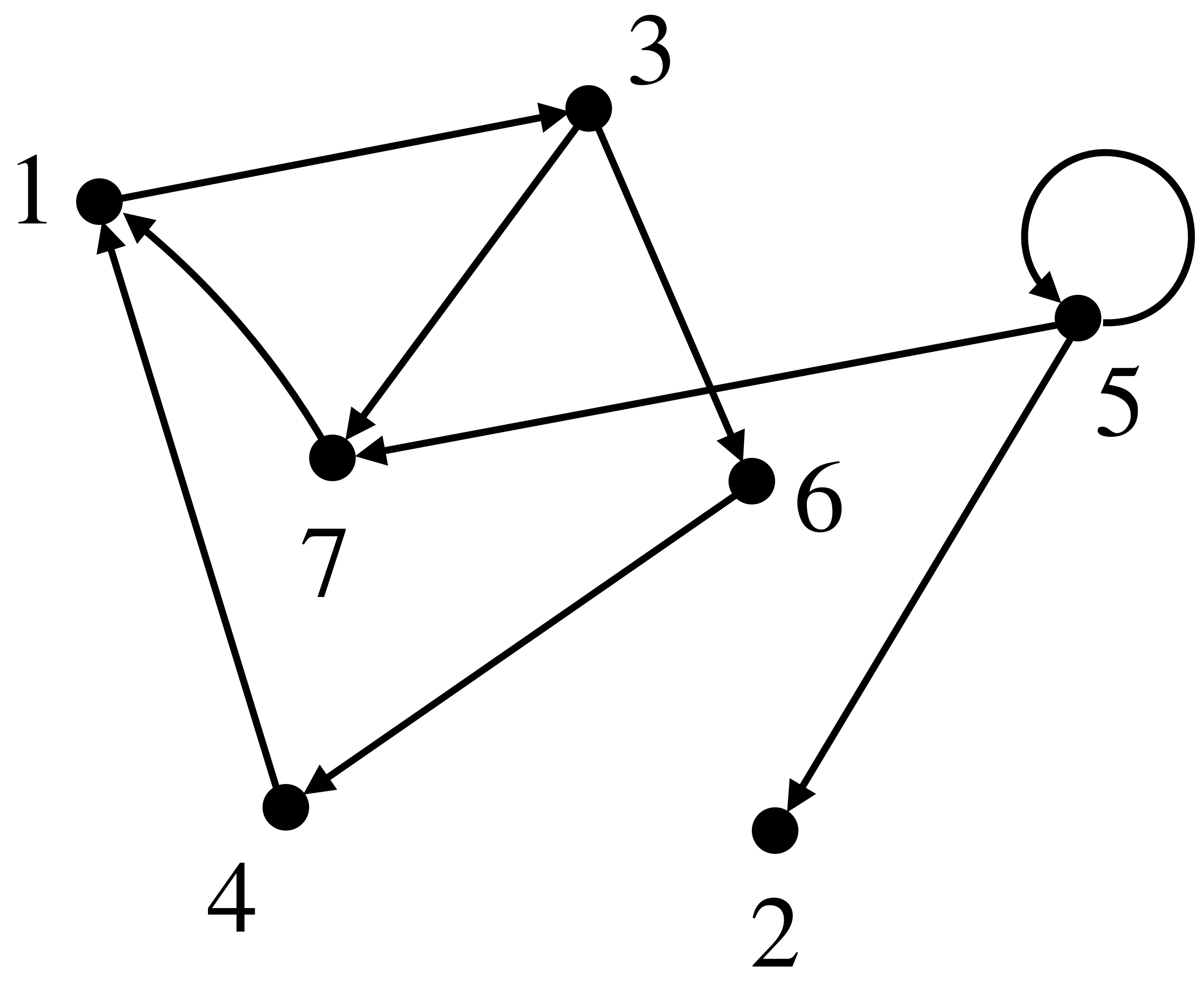
Graphs: Representation

Adjacency-list representation:

Graph G



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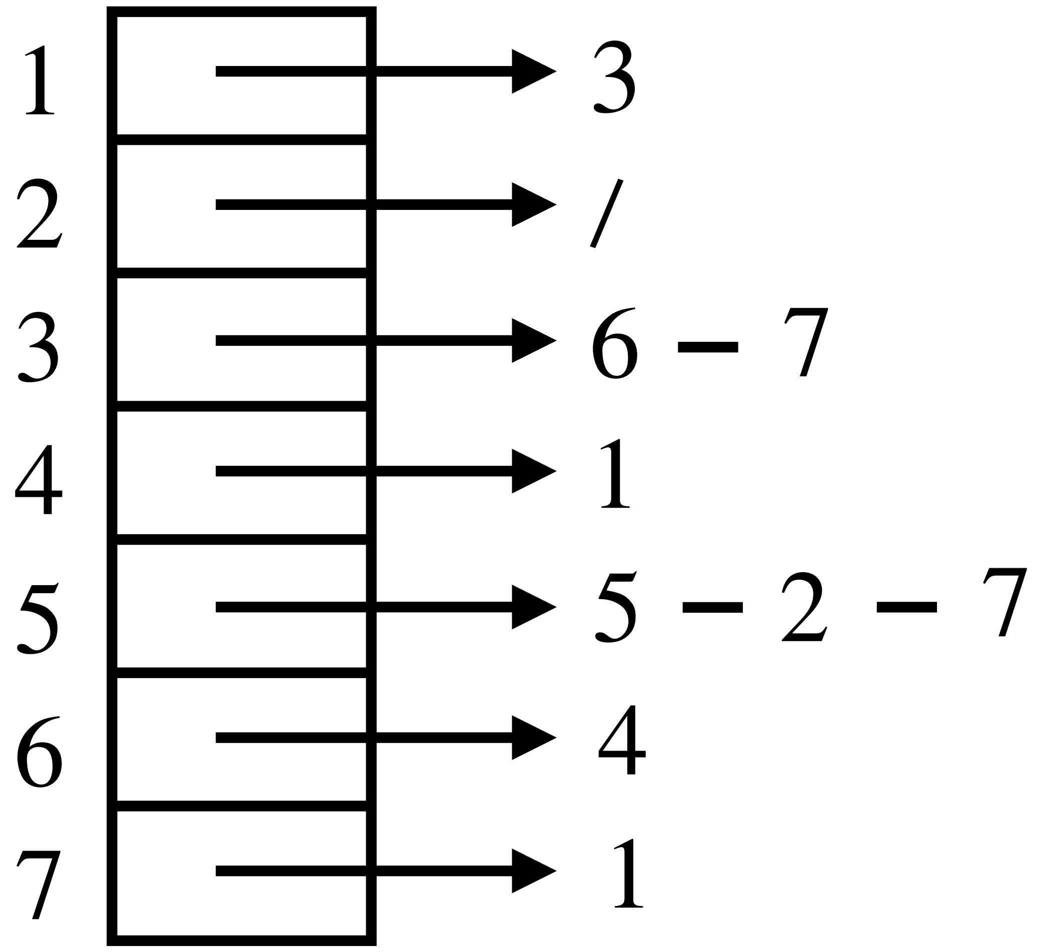

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Note: In case of undirected graphs, for an edge $\{i, j\}$, $i \in Adj[j]$ and $j \in Adj[i]$.

Graphs: Representation

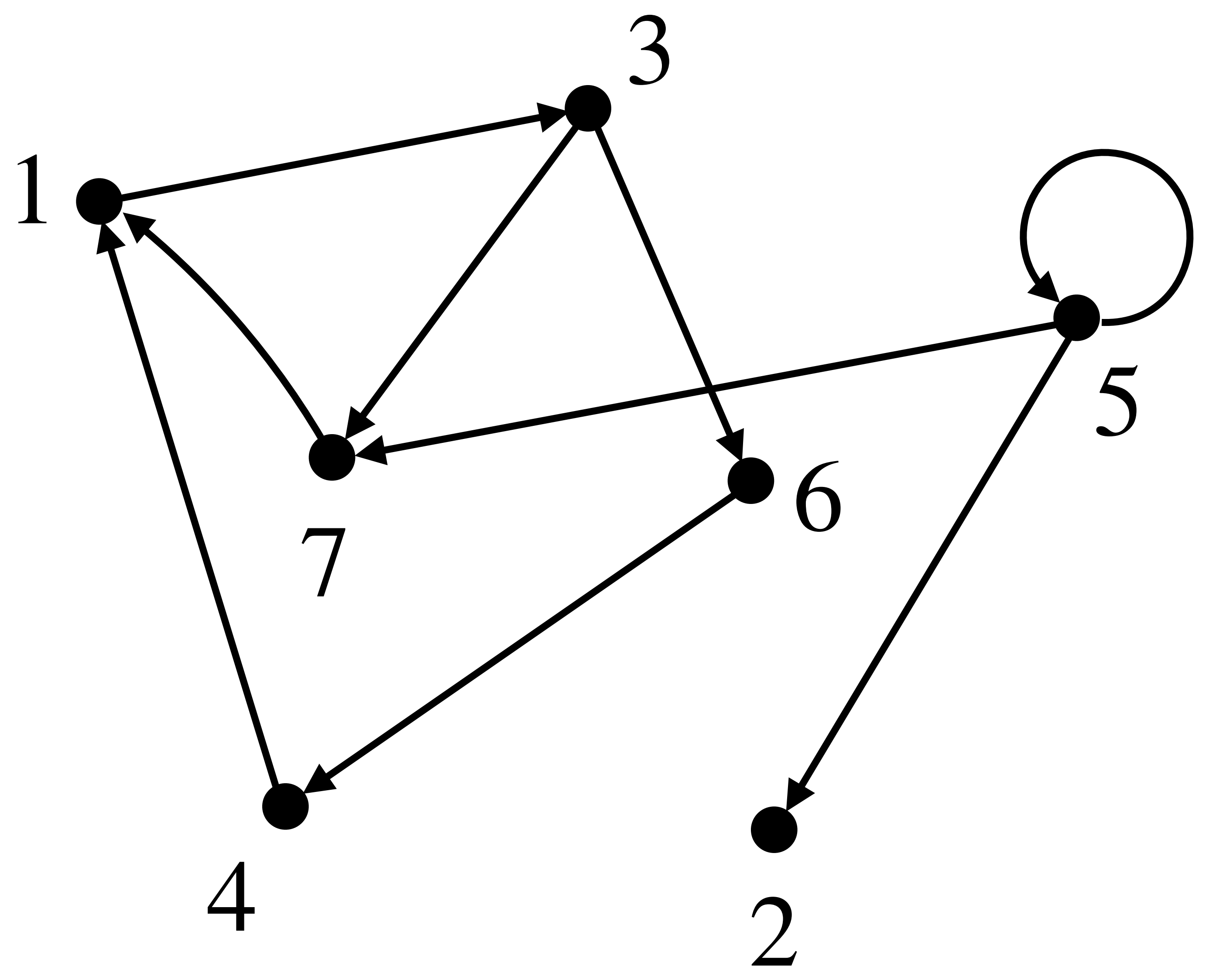
Graphs: Representation

Adjacency-matrix representation:

Graphs: Representation

Adjacency-matrix representation:

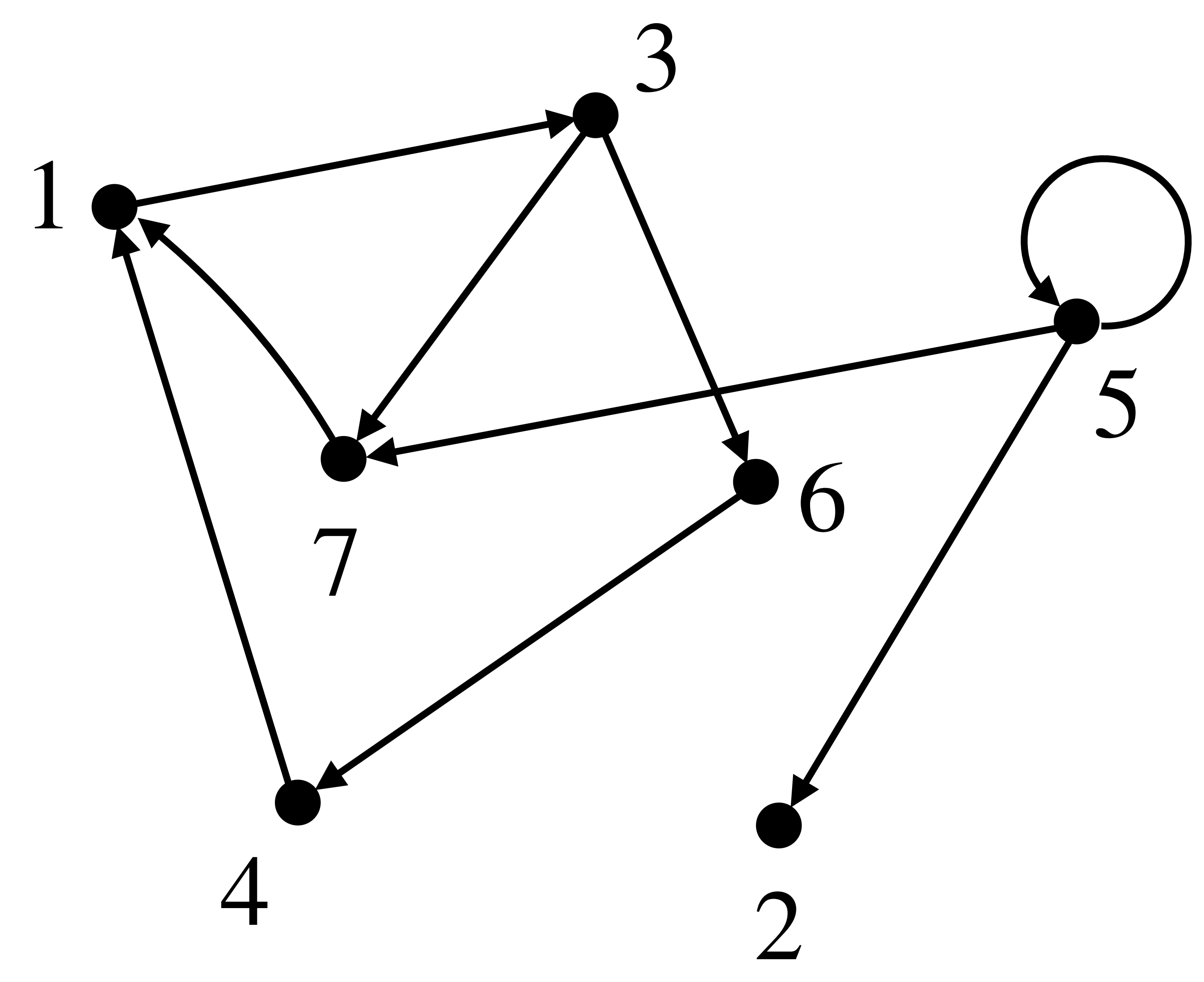
Graph G



Graphs: Representation

Adjacency-matrix representation:

Graph G



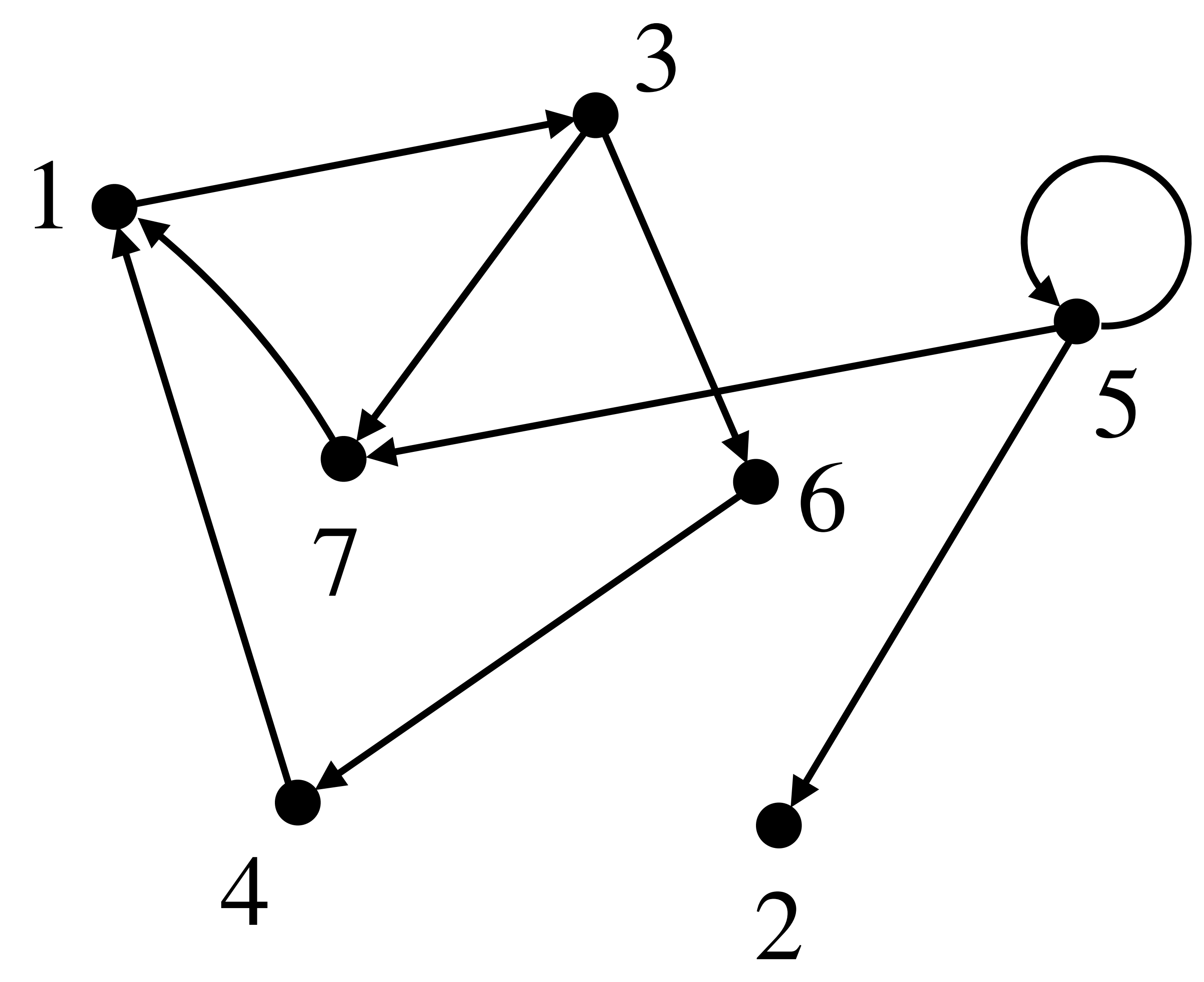
G's adjacency matrix

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

Graphs: Representation

Adjacency-matrix representation:

Graph G



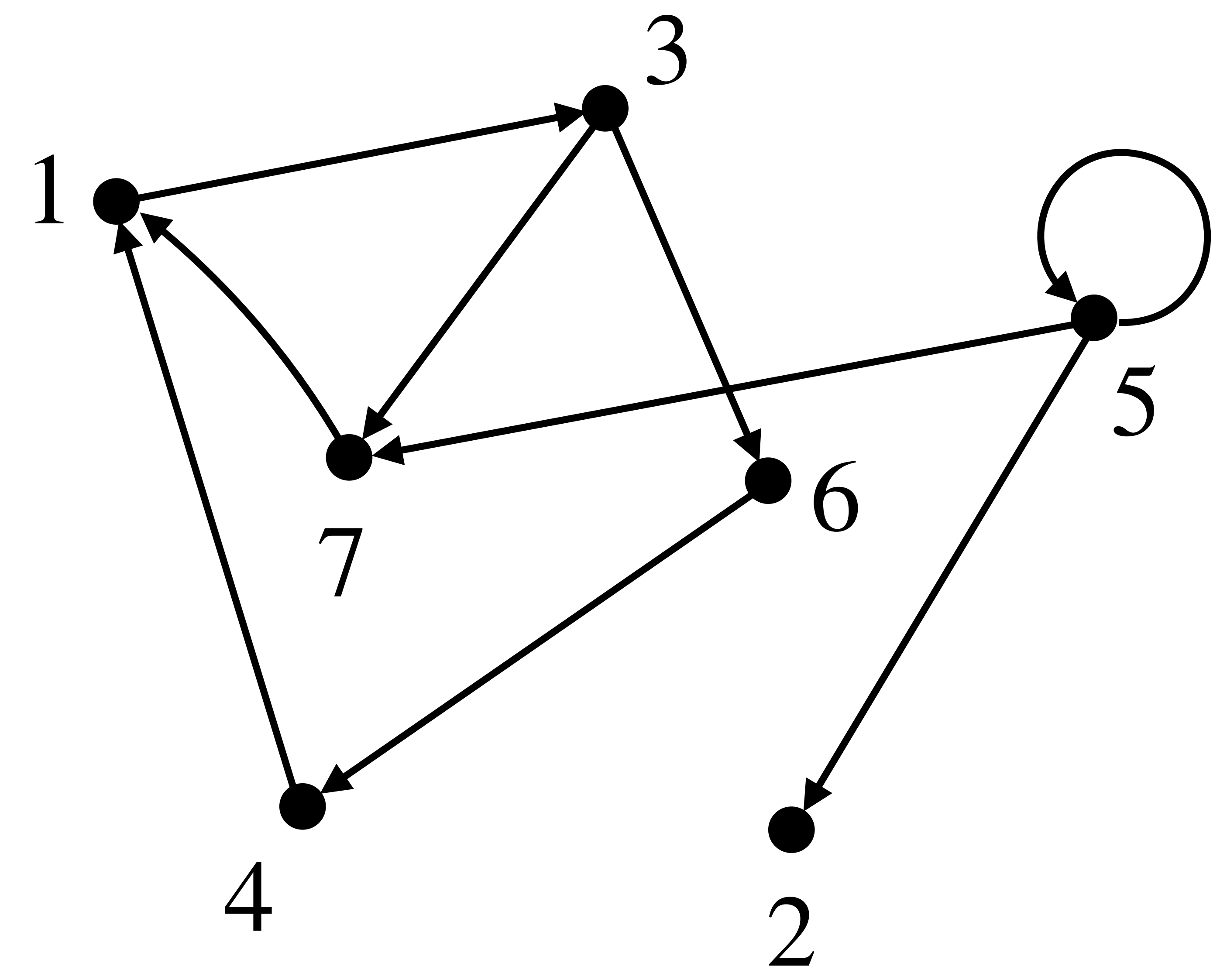
G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2							
3							
4							
5							
6							
7							

Graphs: Representation

Adjacency-matrix representation:

Graph G



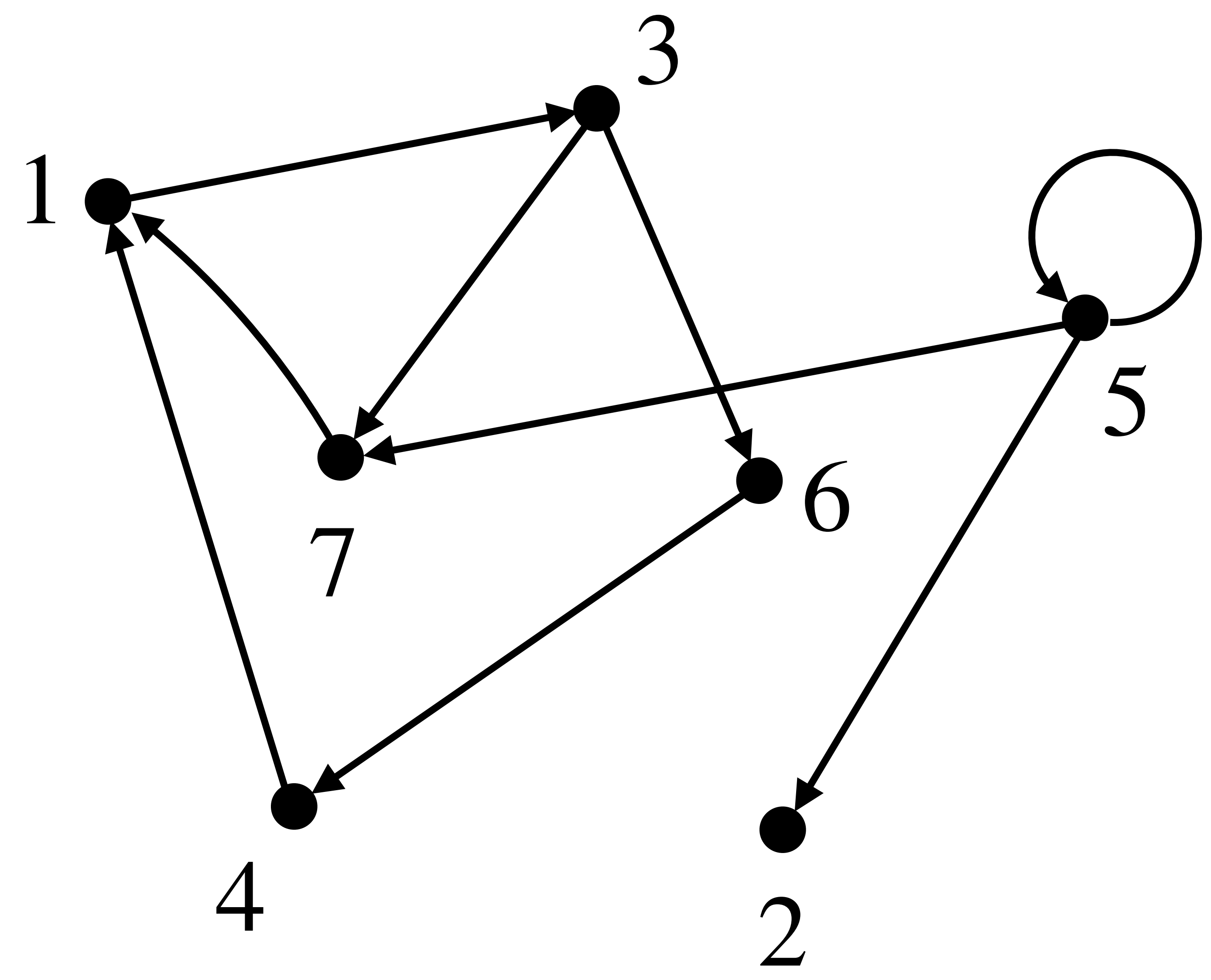
G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3							
4							
5							
6							
7							

Graphs: Representation

Adjacency-matrix representation:

Graph G



G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4							
5							
6							
7							

Graphs: Representation

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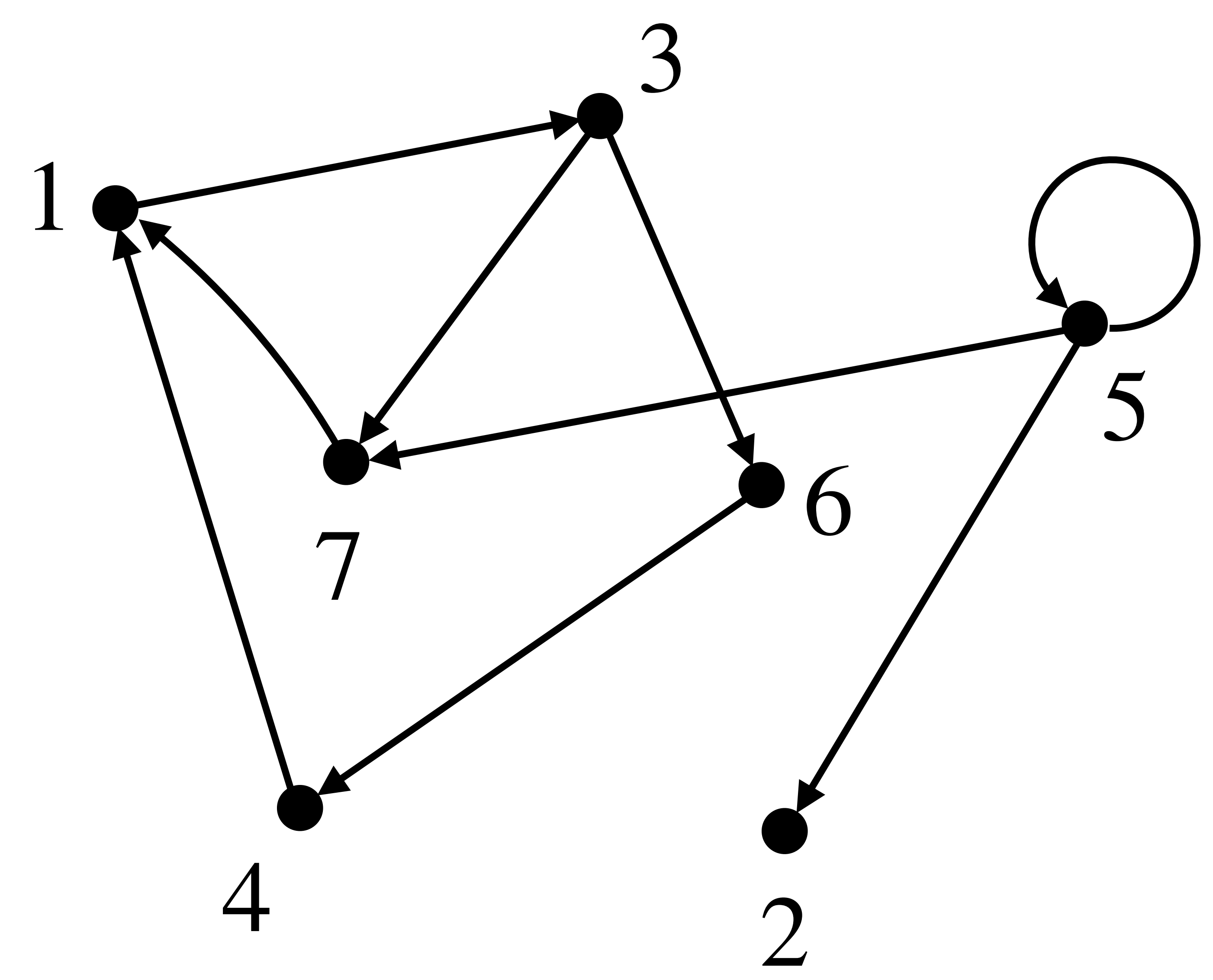
Graph G



G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4	1	0	0	0	0	0	0
5							
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Graphs: Representation

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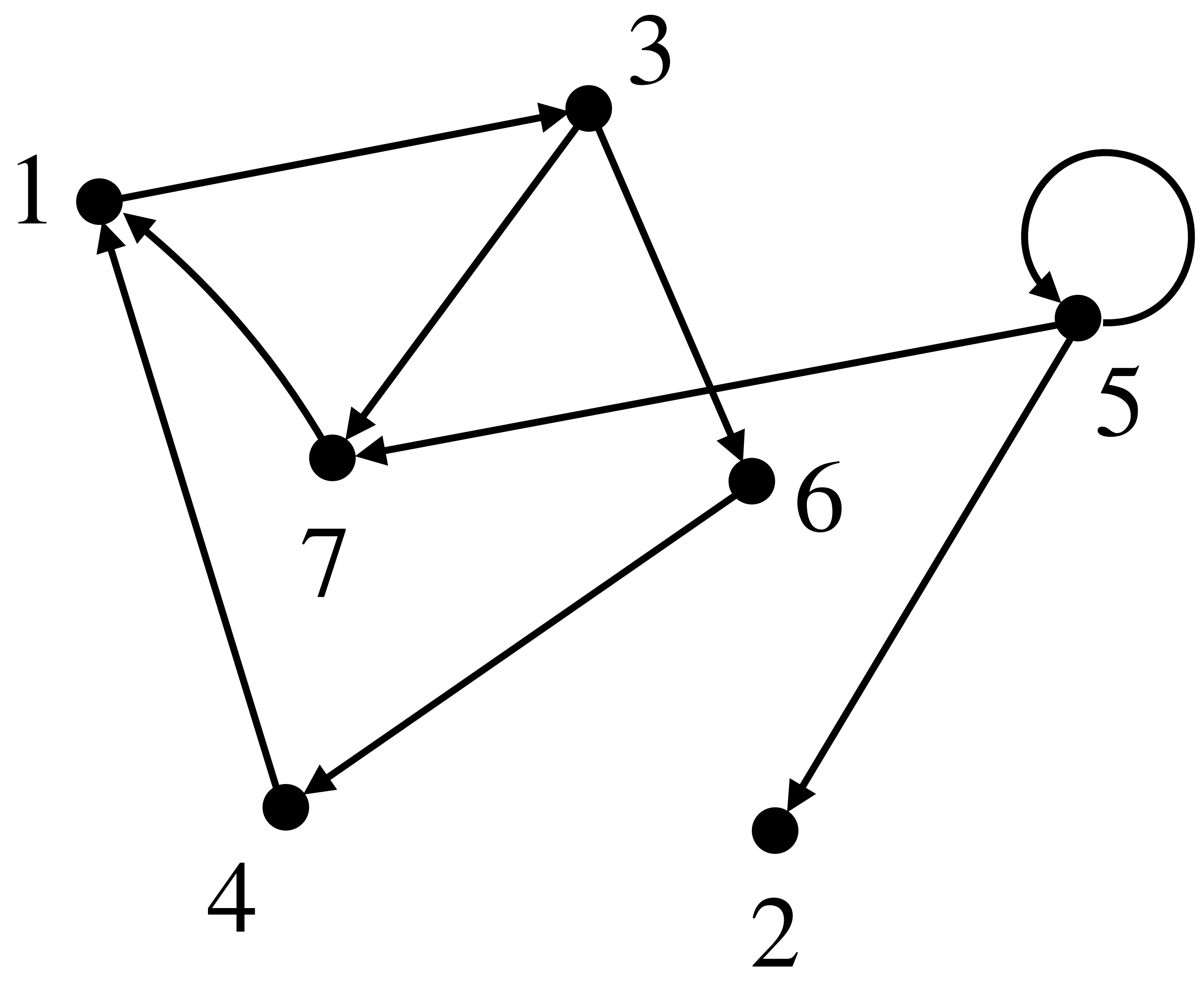
Graph G



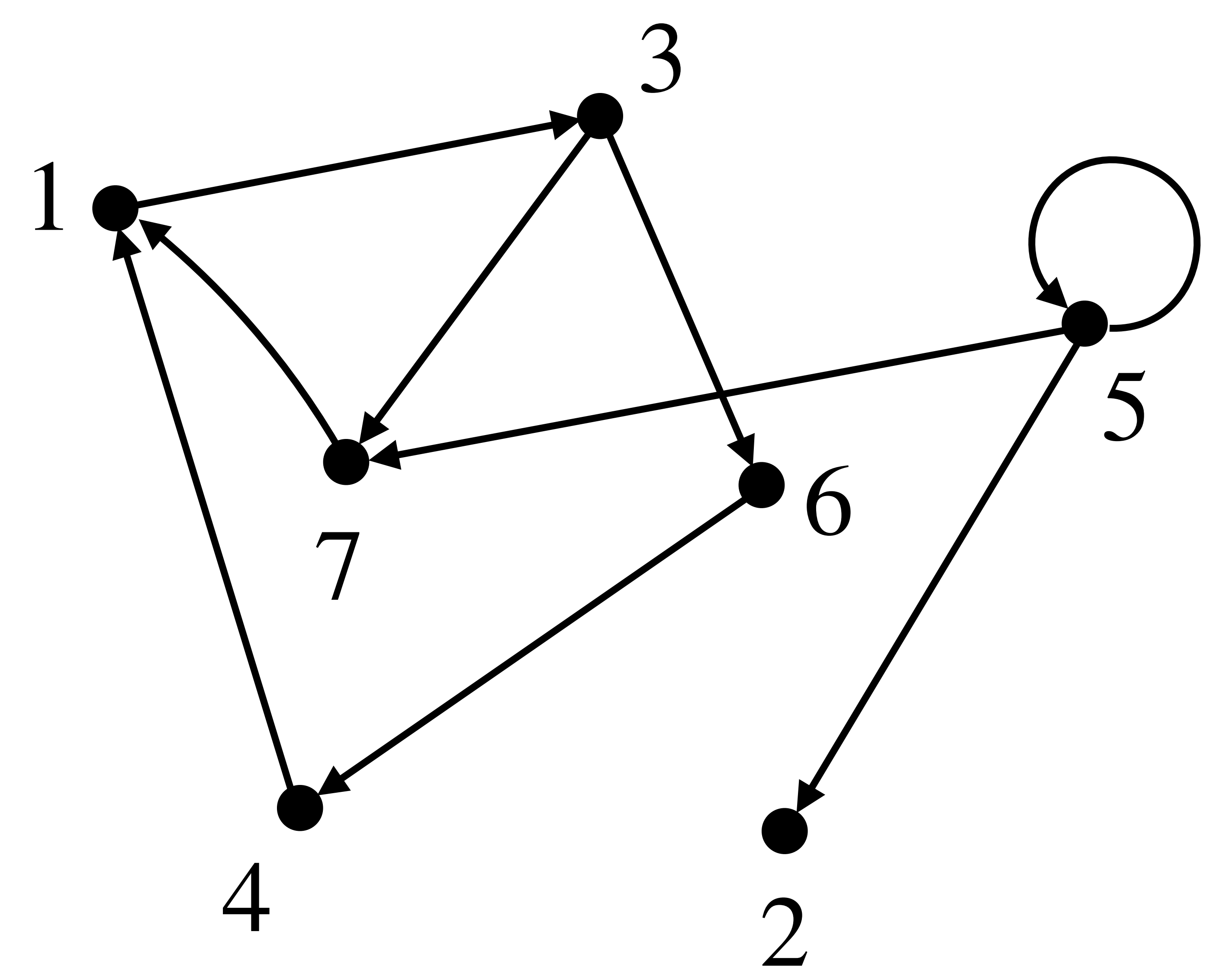
G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4	1	0	0	0	0	0	0
5	0	1	0	0	1	0	1
6							
7							

Graphs: Representation

Adjacency-matrix representation:

Graph G



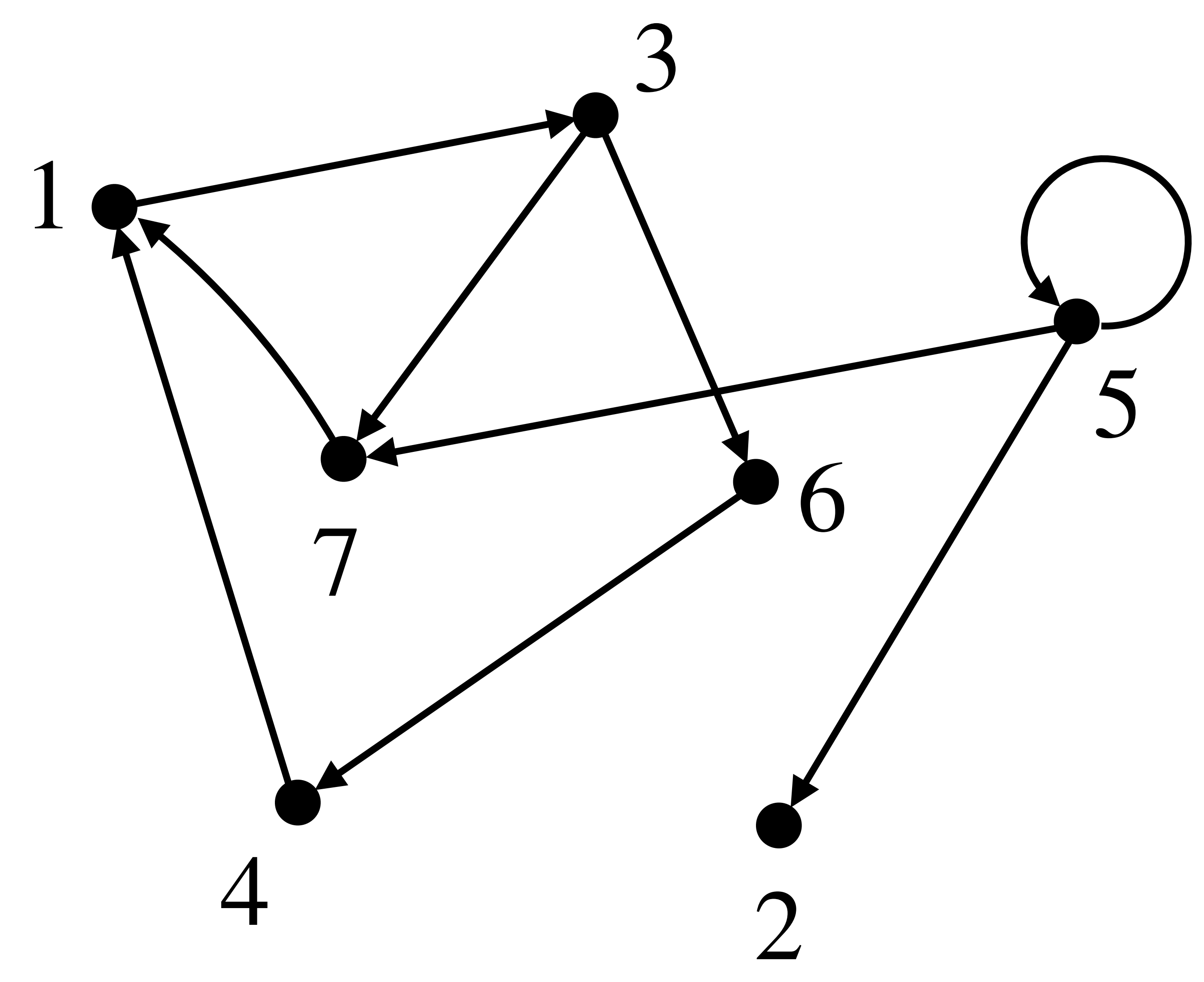
G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4	1	0	0	0	0	0	0
5	0	1	0	0	1	0	1
6	0	0	0	1	0	0	0
7							

Graphs: Representation

Adjacency-matrix representation:

Graph G



G's adjacency matrix

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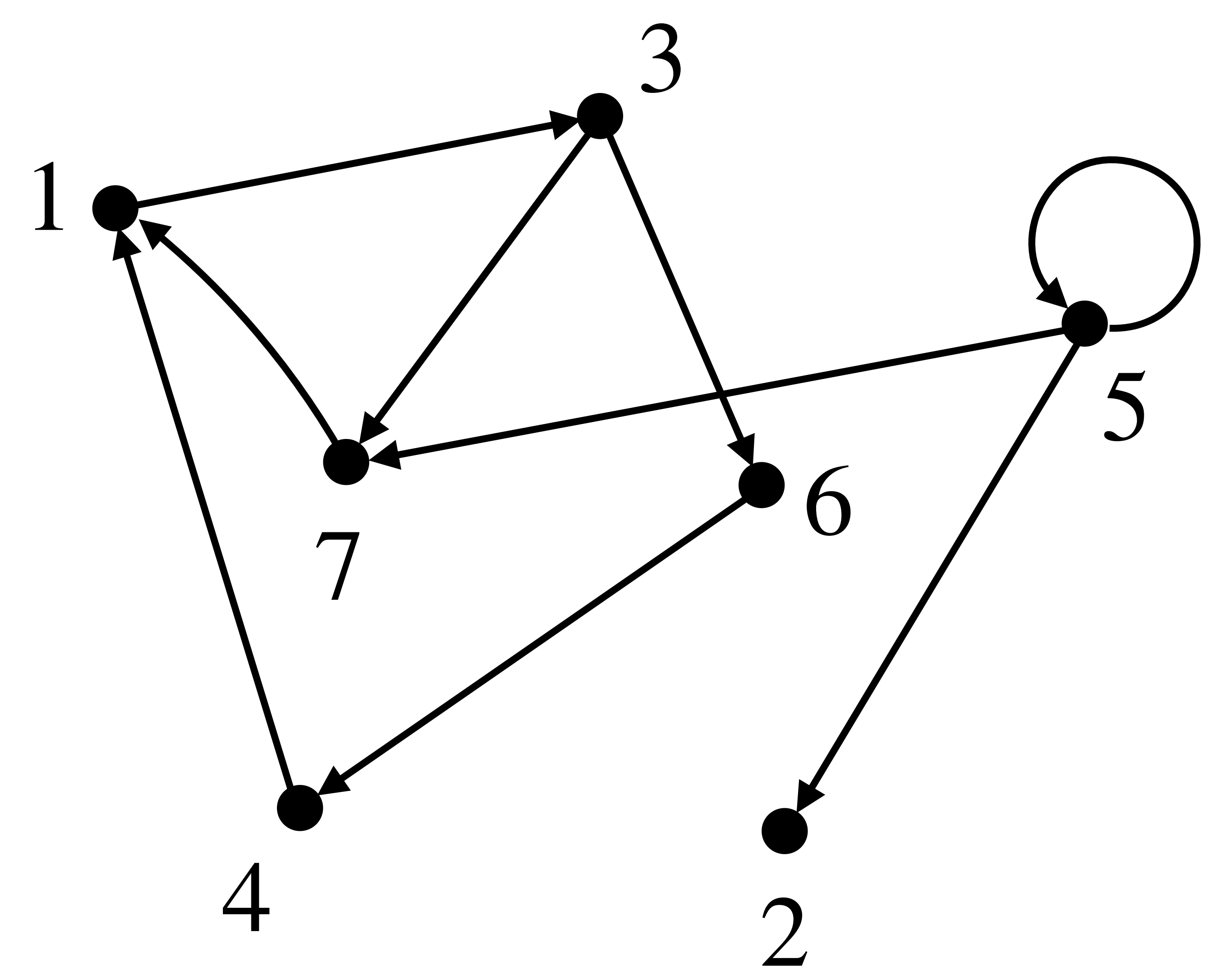
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4	1	0	0	0	0	0	0
5	0	1	0	0	1	0	1
6	0	0	0	1	0	0	0
7	1	0	0	0	0	0	0

Graphs: Representation

Adjacency-matrix representation:

Graph G



G's adjacency matrix

	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1
4	1	0	0	0	0	0	0
5	0	1	0	0	1	0	1
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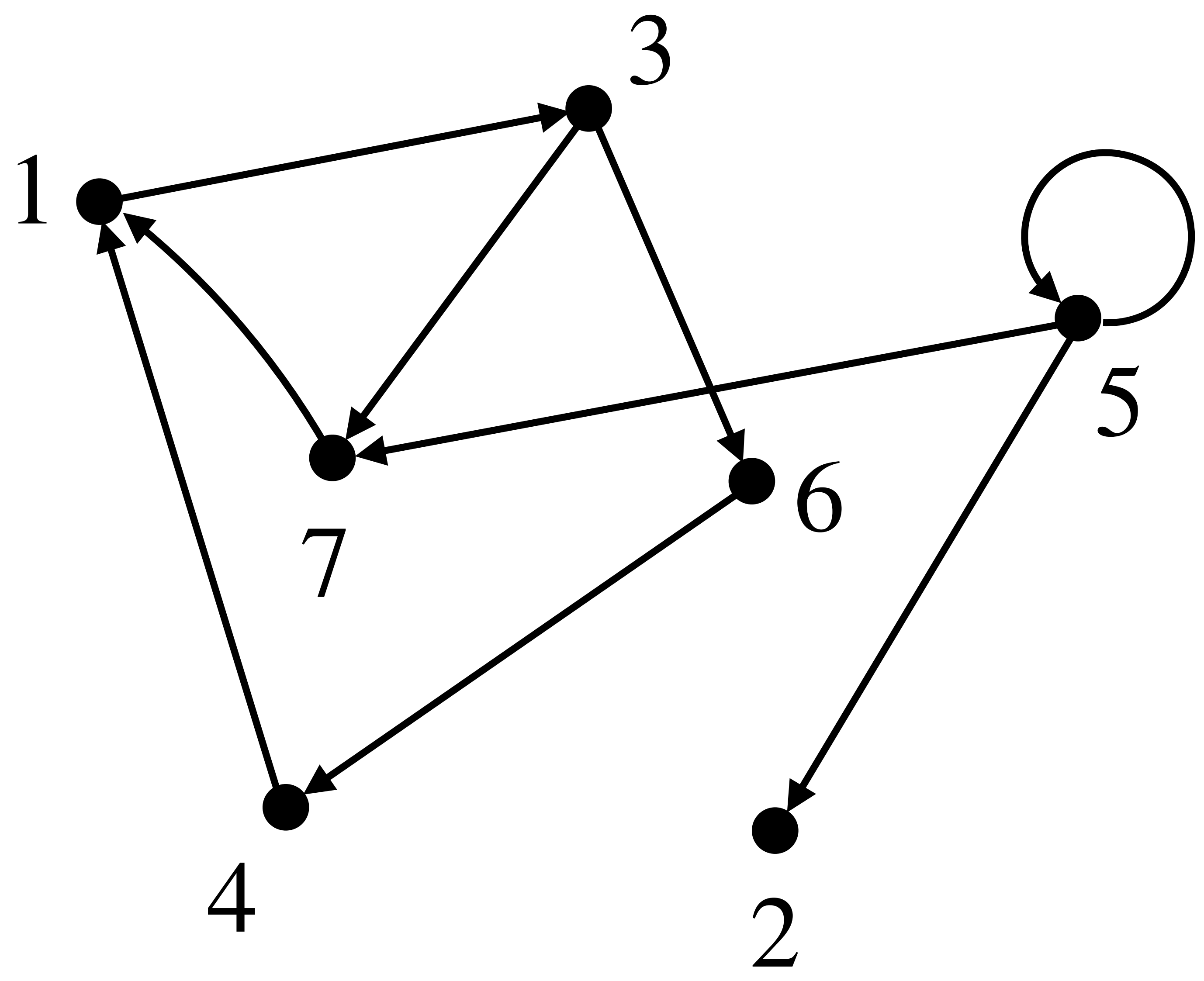

G's adjacency matrix

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1	0	0	1	0	0	0	0
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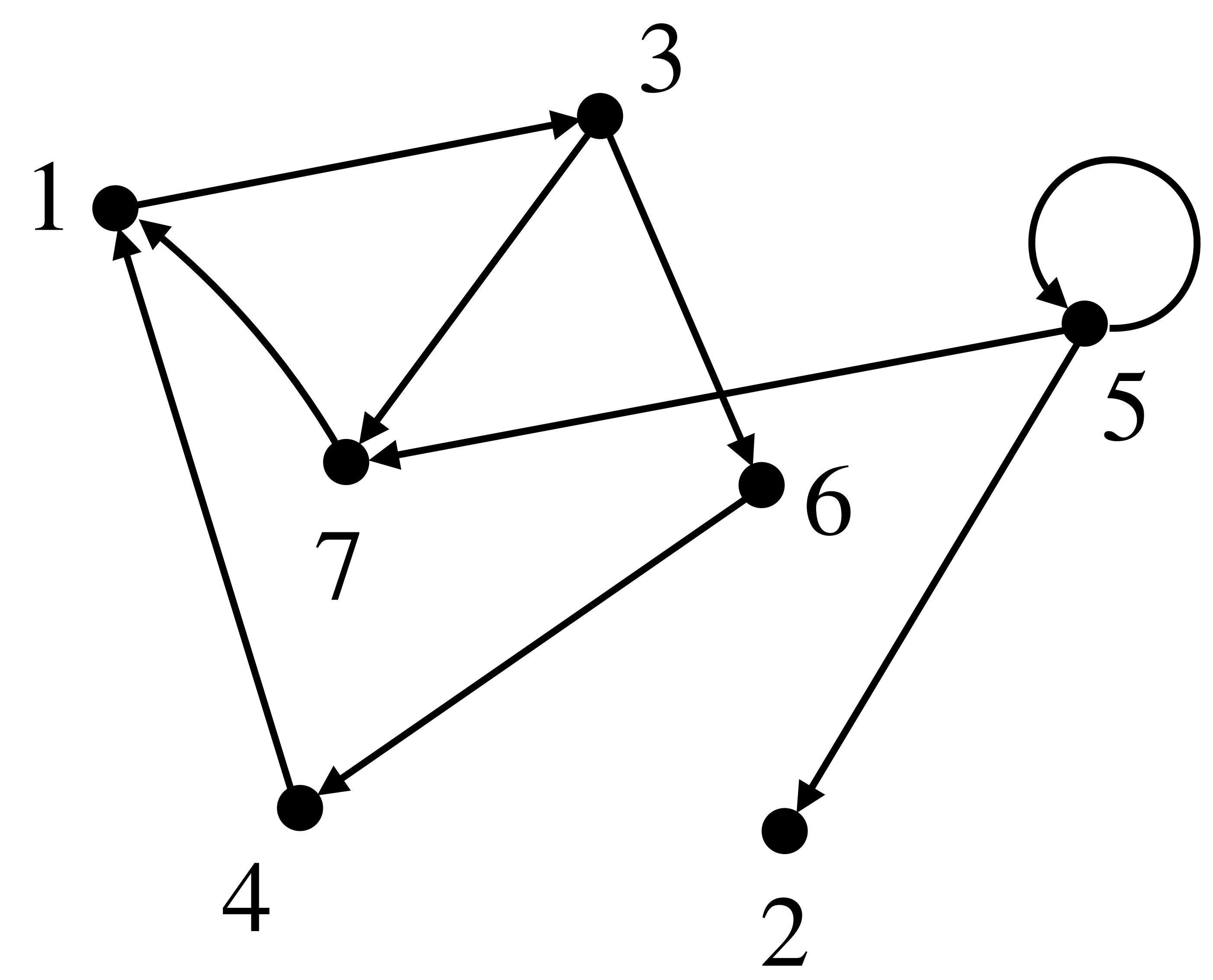
4	1	0	0	0	0	0	0
5	0	1	0	0	1	0	1
6	0	0	0	1	0	0	0
7	1	0	0	0	0	0	0

The adjacency-matrix representation of a graph $G = (V, E)$ assumes that the vertices are numbered $1,2,\ldots,|V|$ in an arbitrary manner.

Graphs: Representation

Adjacency-matrix representation:

Graph G



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3	0	0	0	0	0	1	1
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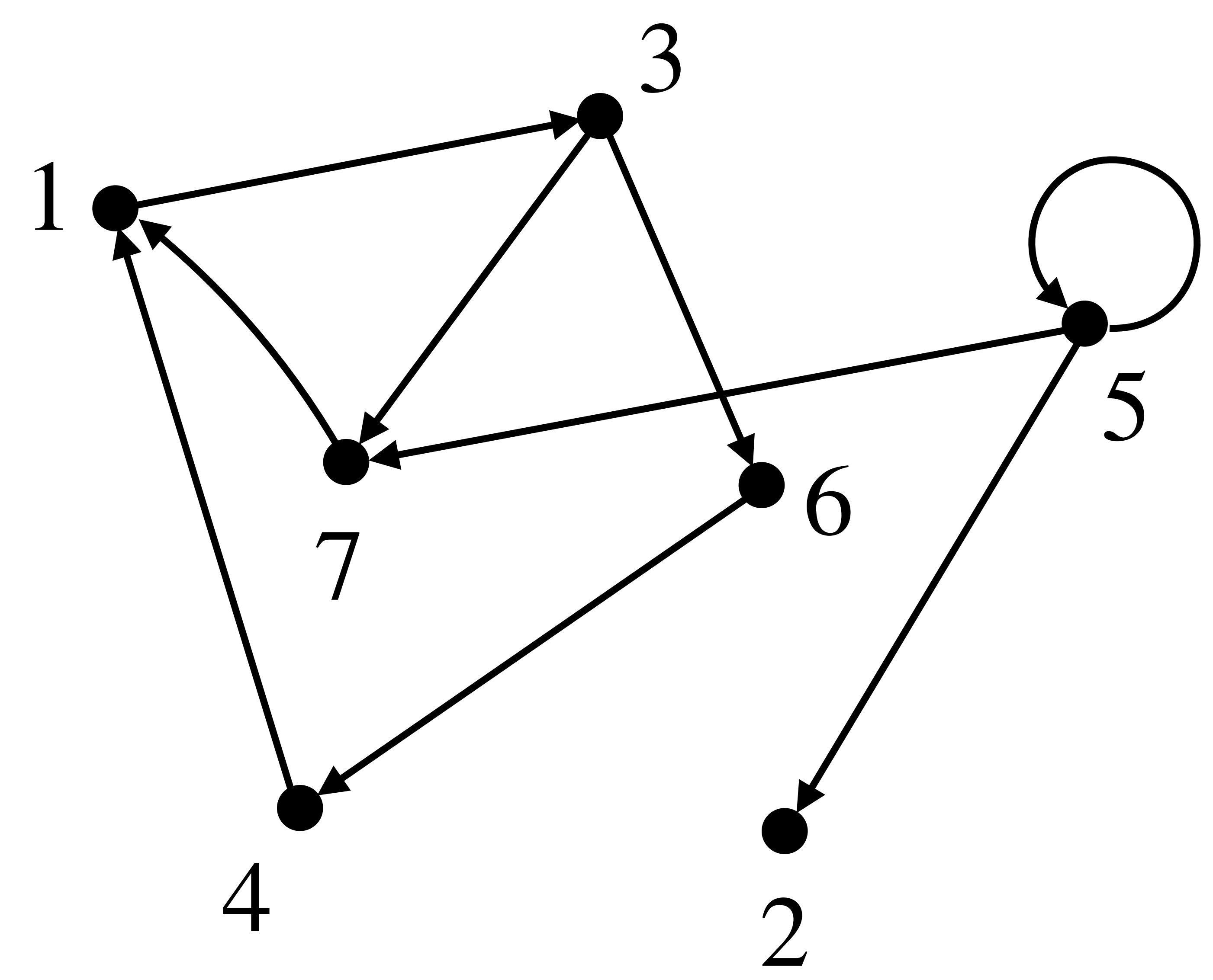

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The adjacency-matrix representation of a graph $G = (V, E)$ assumes that the vertices are numbered $1,2,\ldots,|V|$ in an arbitrary manner. Then, the adjacency matrix representation of a graph G consists of a $|V| \times |V|$ matrix A such that $A[i][j] = 1$, if $(i, j) \in E$, else $A[i][j] = 0$.

Graphs: Representation

Adjacency-matrix representation:

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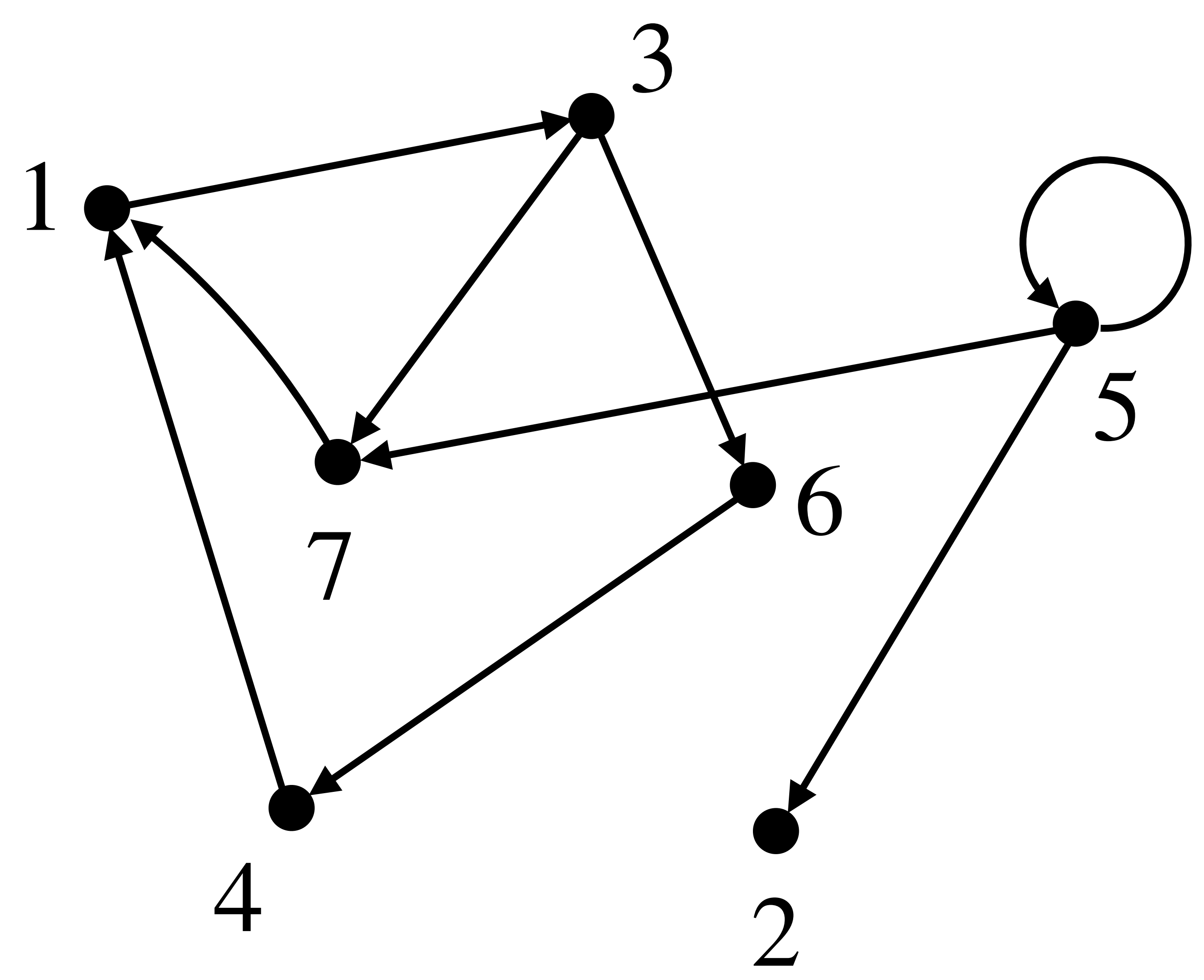

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Note: In case of undirected graphs, for an edge (i, j), $Adj[i][j] = Adj[j][i]$.

Graphs: Representation

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- In adjacency-list of a directed graph, sum of size of all the lists $= |E|$.
- If A is an adjacency-matrix of an undirected graph, then $A = A^T$.