Lecture 12

Augmenting Data Structures, Disjoint-Set Data Structure

Source: Introduction to Algorithms, CLRS

Maintaining Subtree Sizes: Insertion

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Insertion phase:

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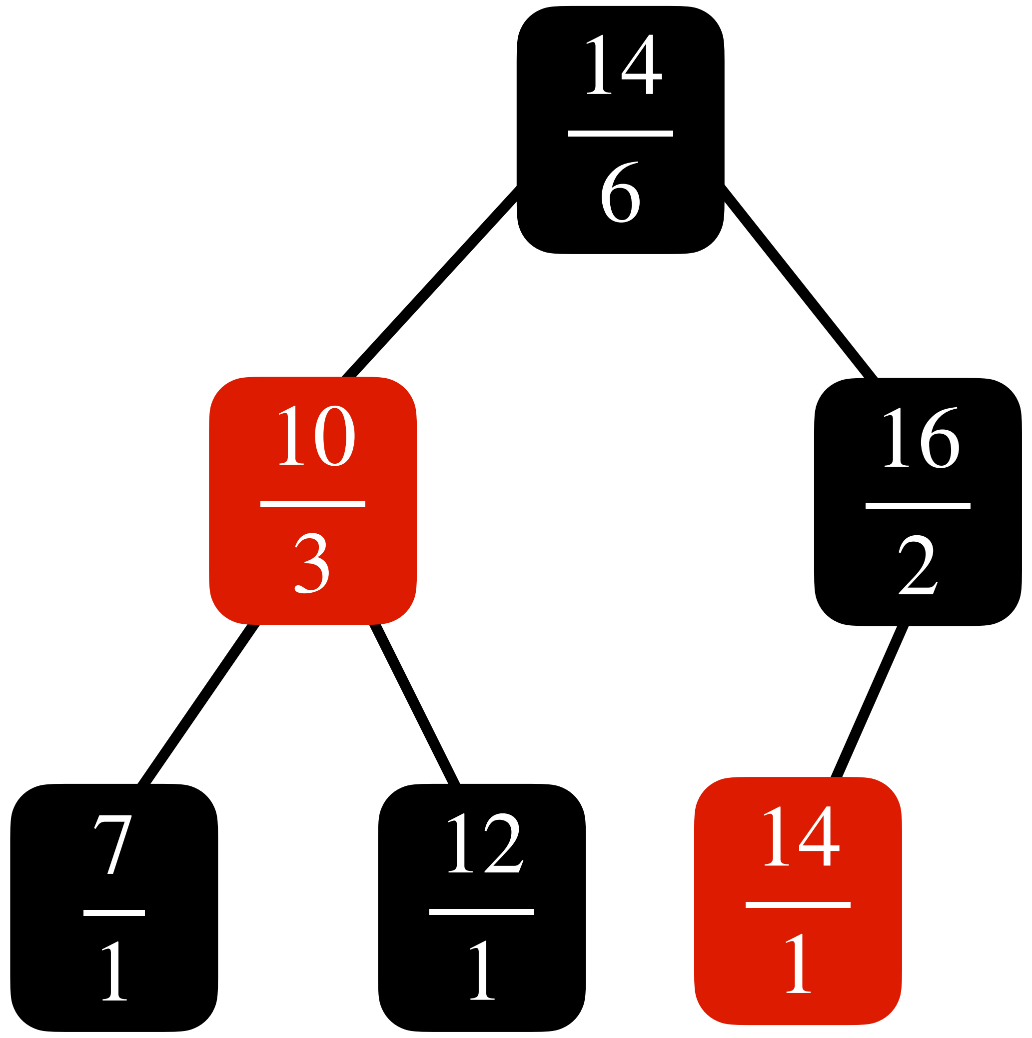

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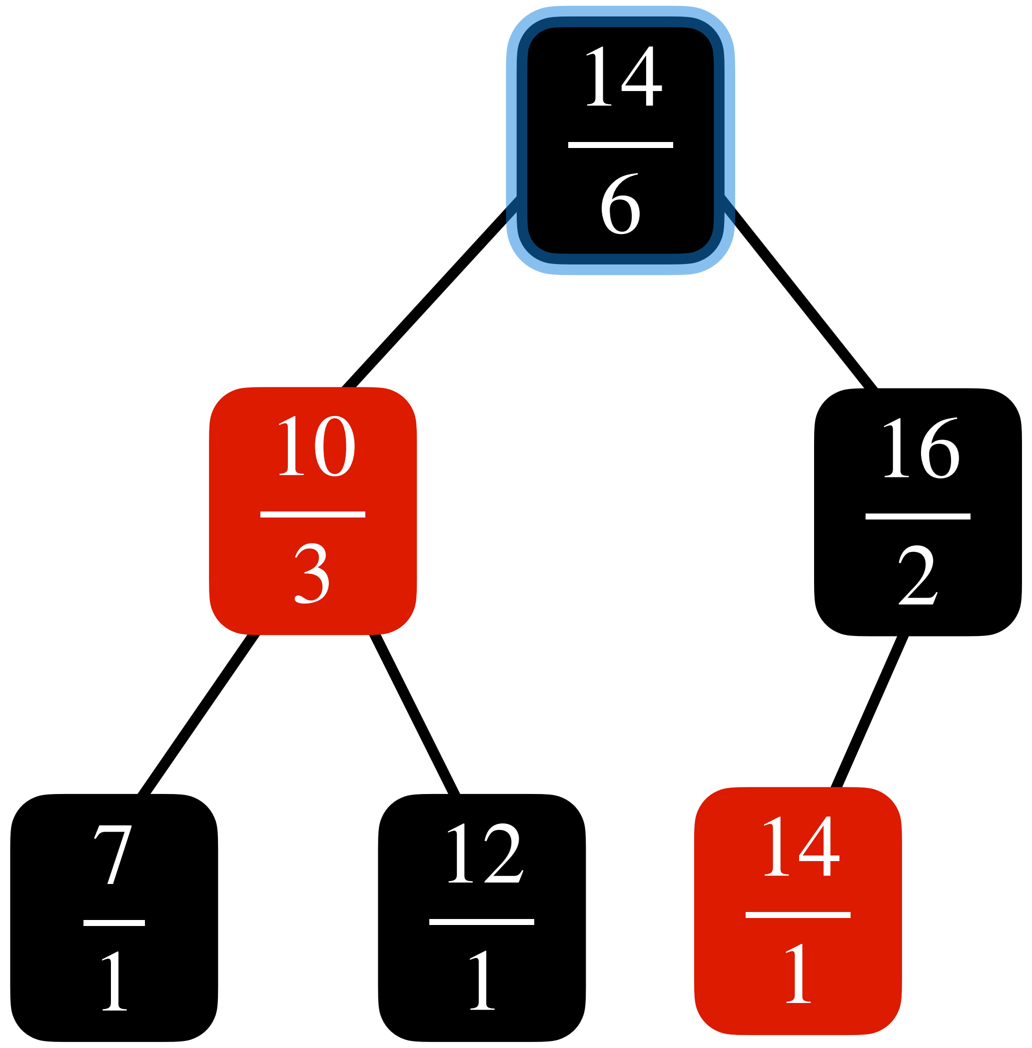

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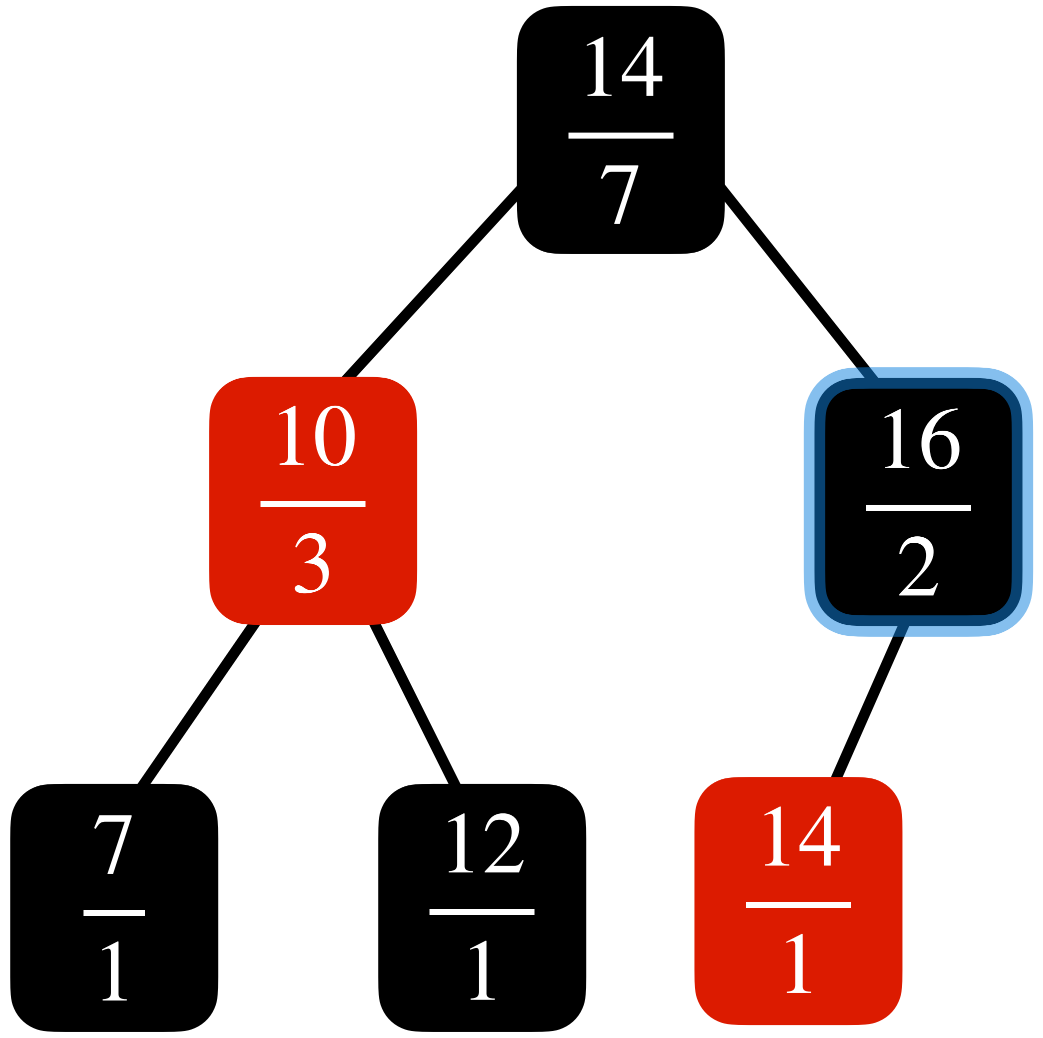

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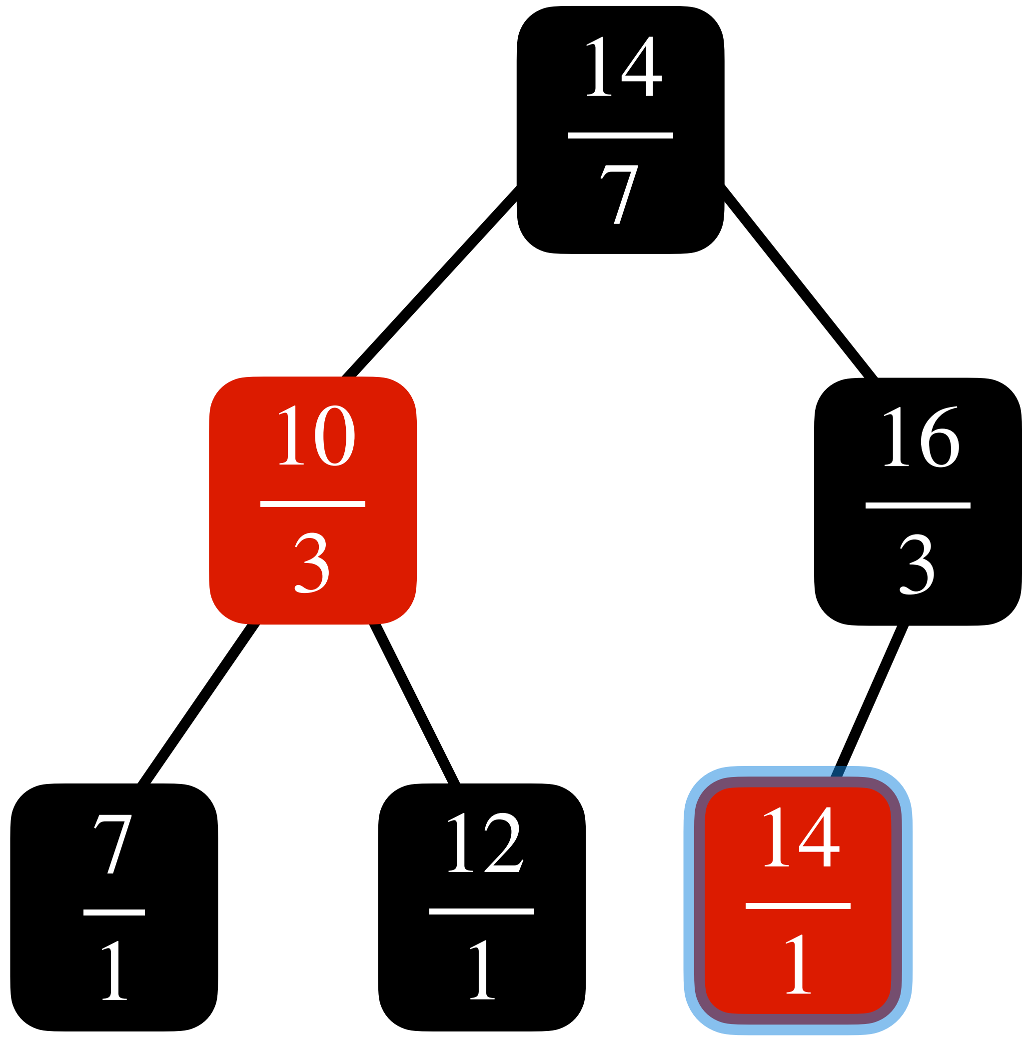

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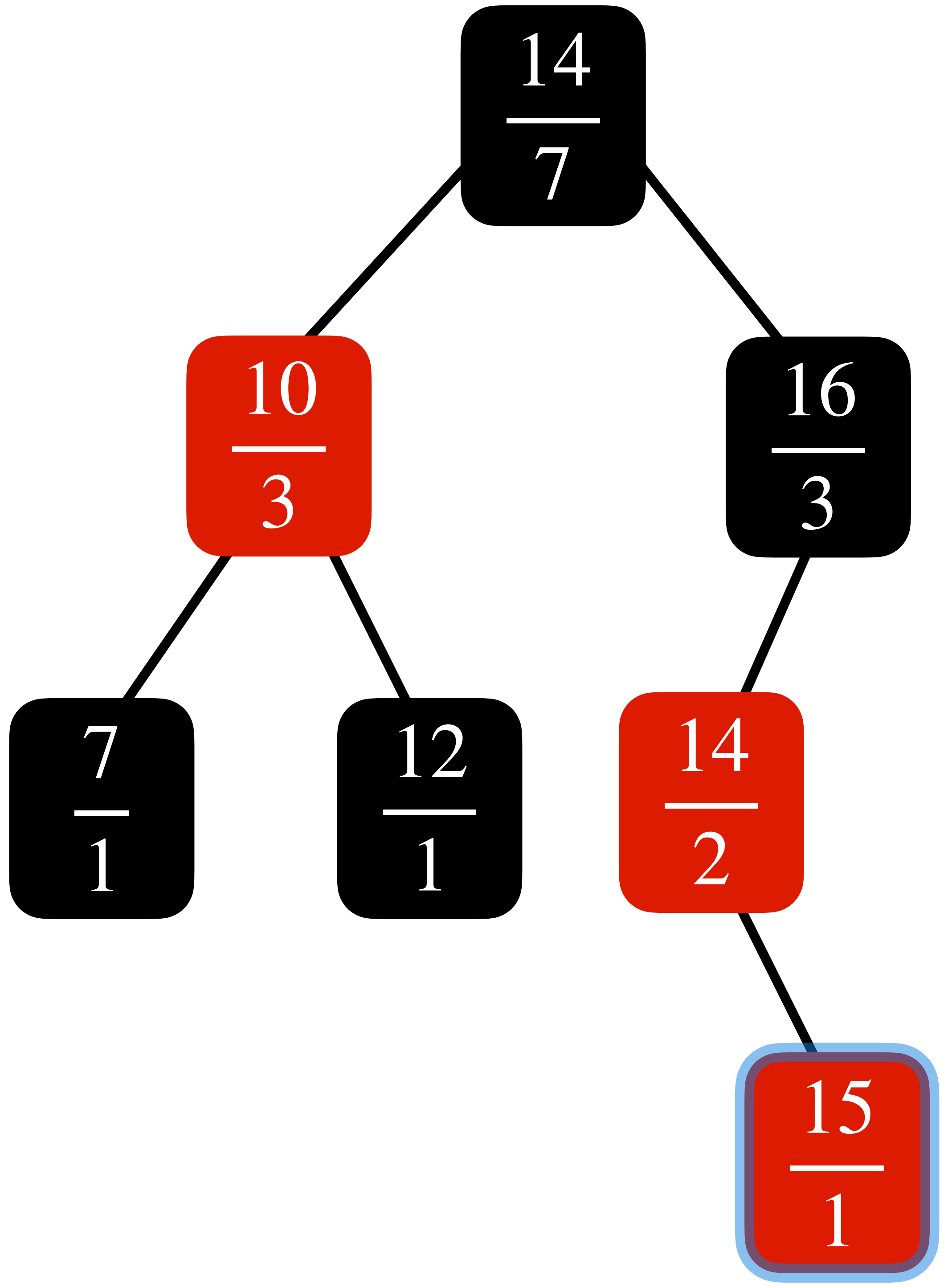

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Fix-up phase:

- Fix-ups involve only **rotations** and **recolouring**.
- **Recolouring** doesn’t require changing sizes.
- During **rotations** size changes are doable in constant time.

Maintaining Subtree Sizes: Deletion

DIY.

Finding Friend Groups on Facebook

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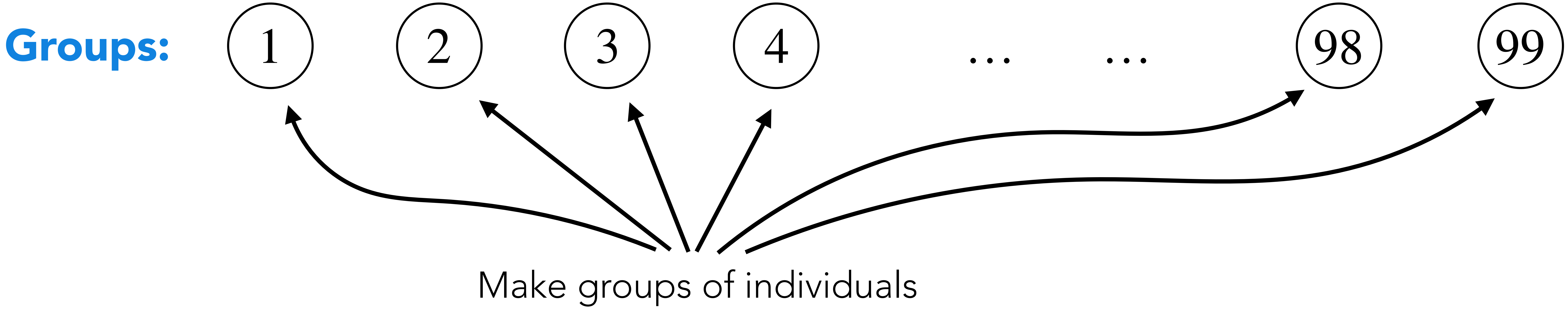
Users: 1 2 3 4 … … 98 99

Finding Friend Groups on Facebook



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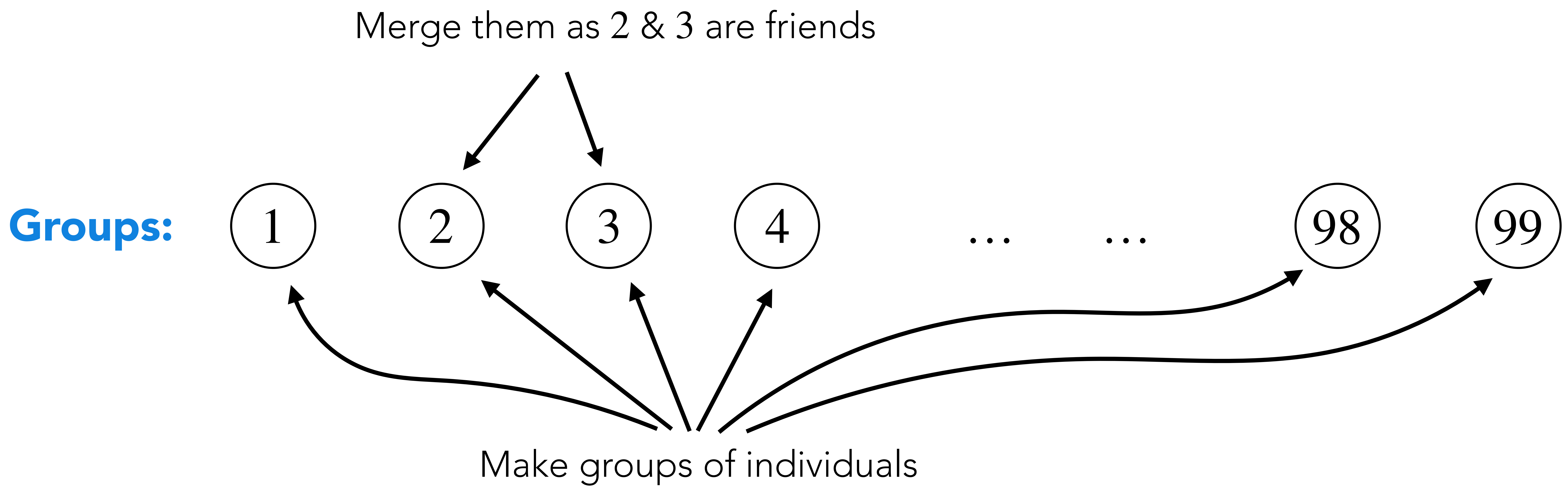

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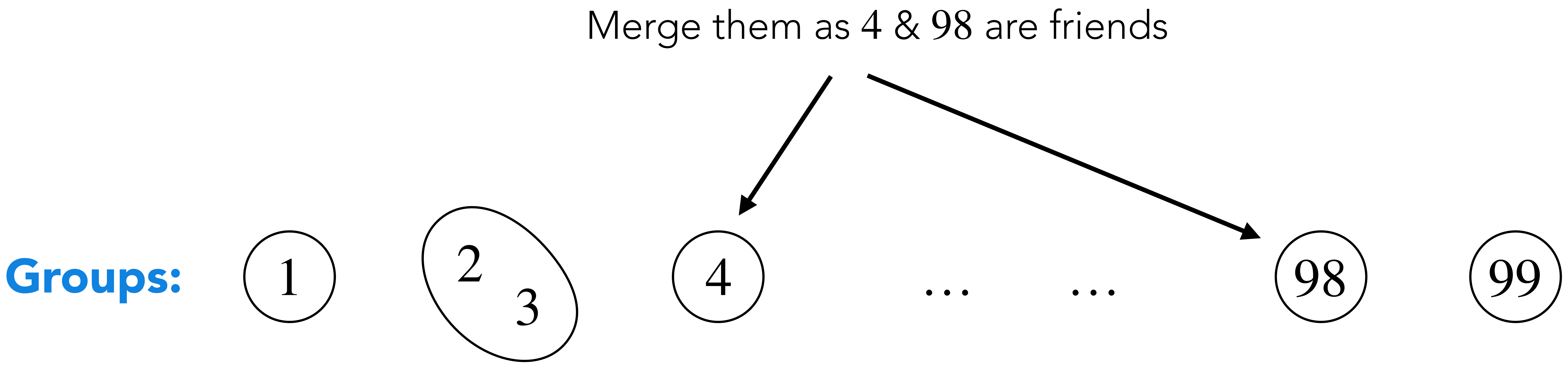

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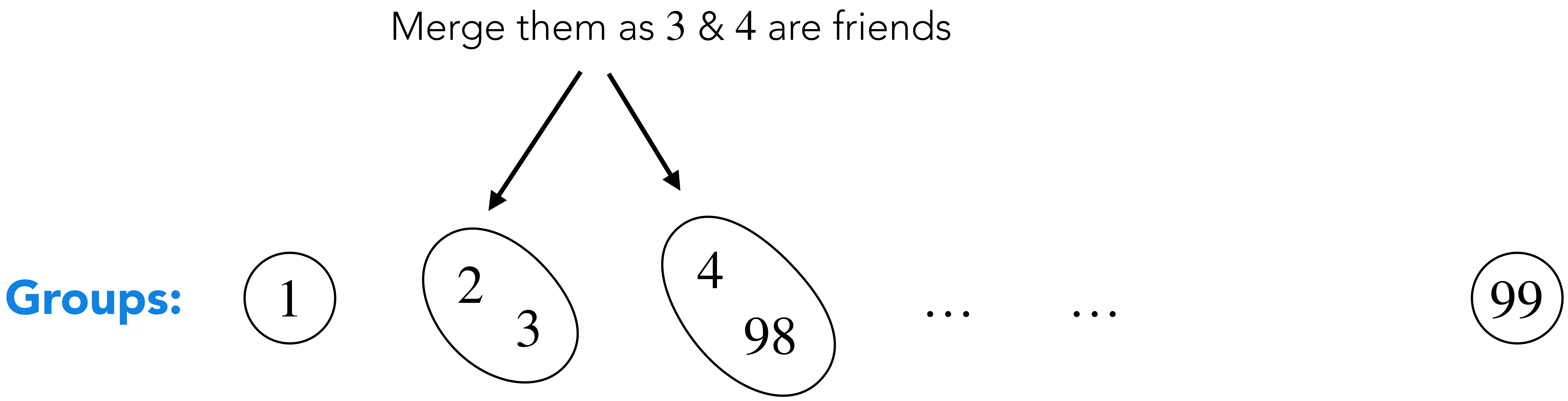

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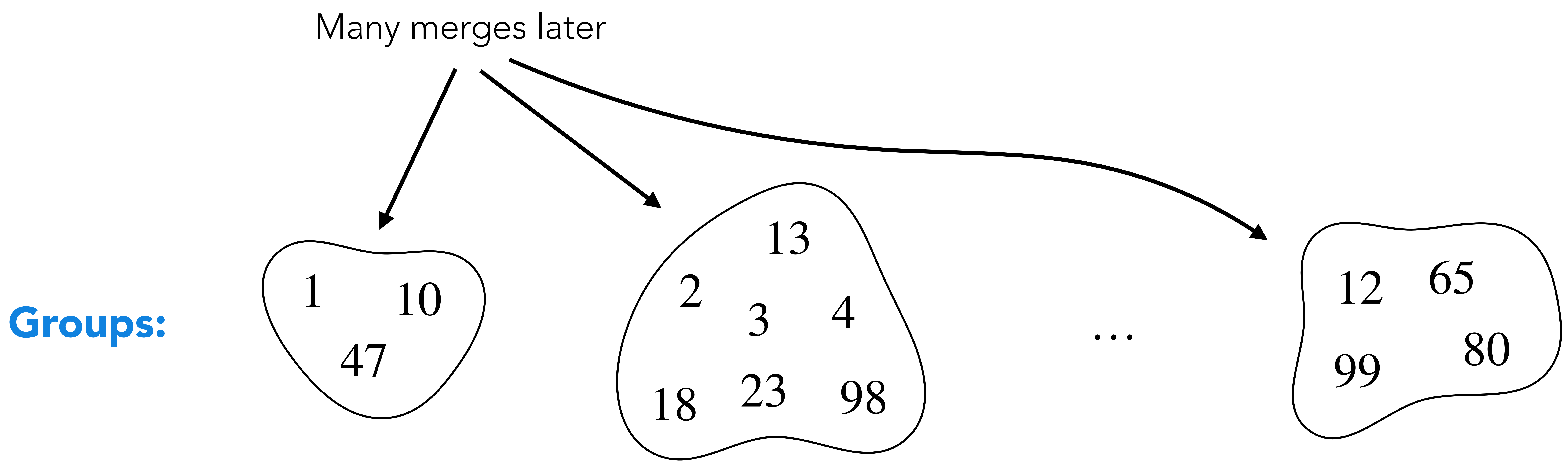

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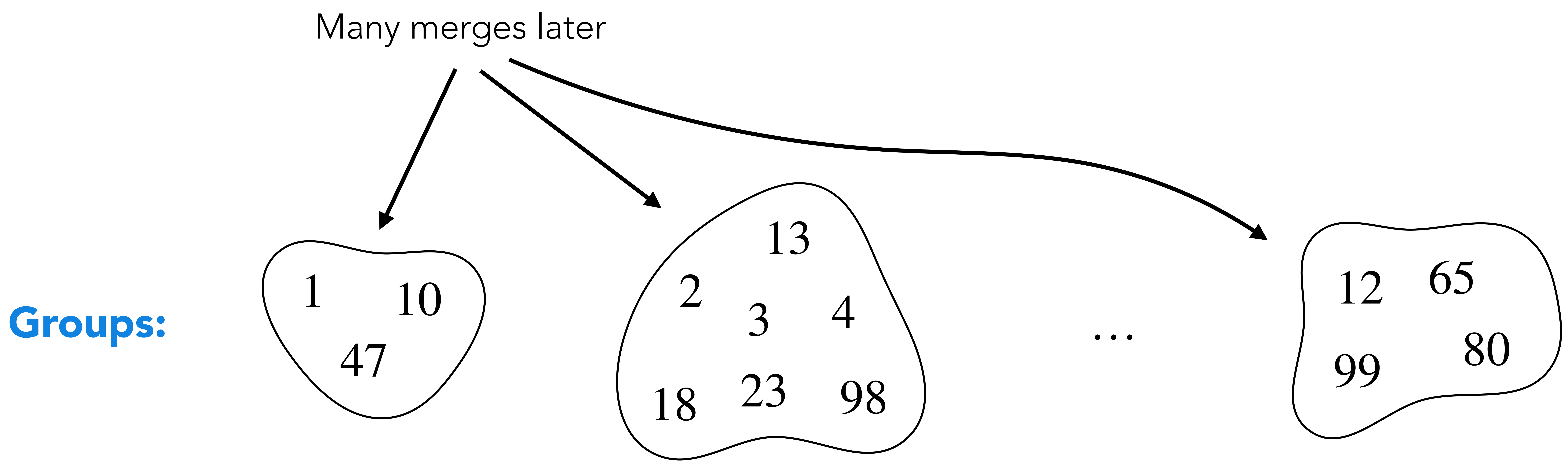

New Feature: To identify groups keep a representative for each group.

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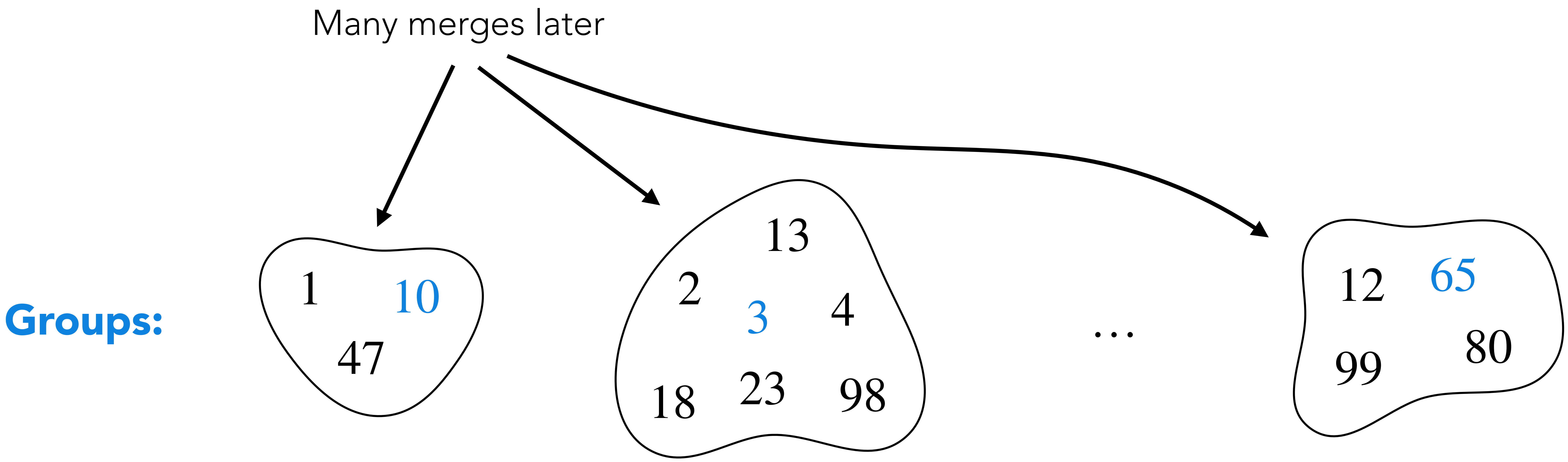

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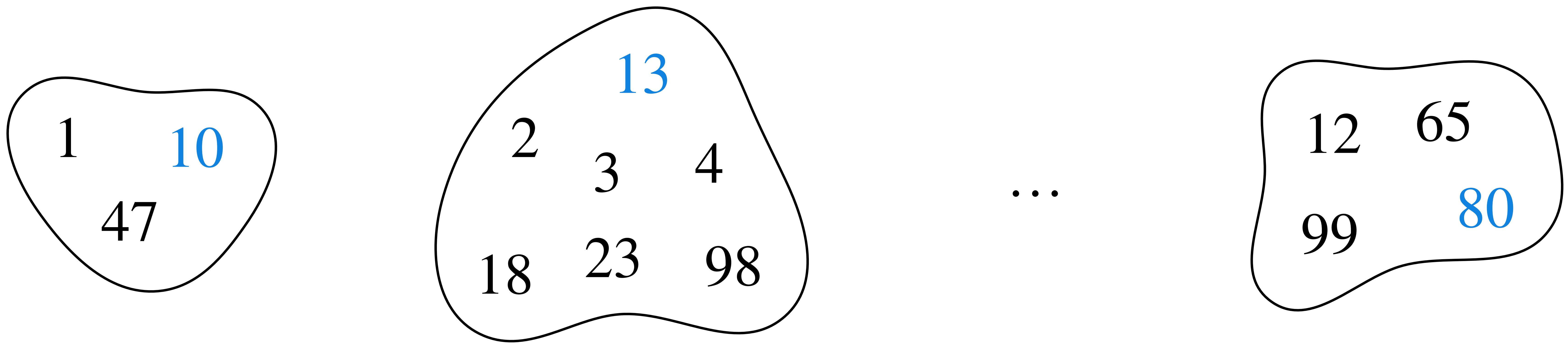

Goal: Design a data-structure so that:

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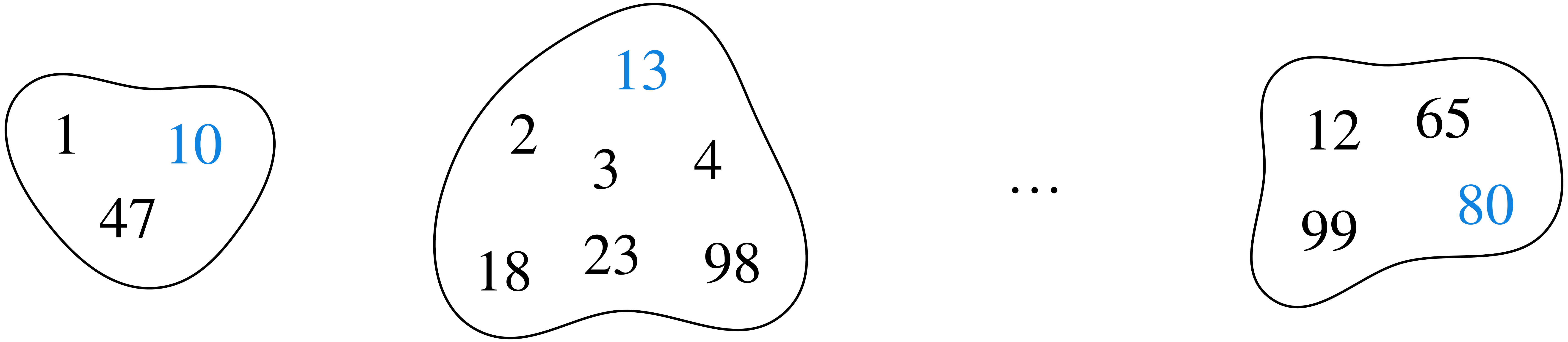

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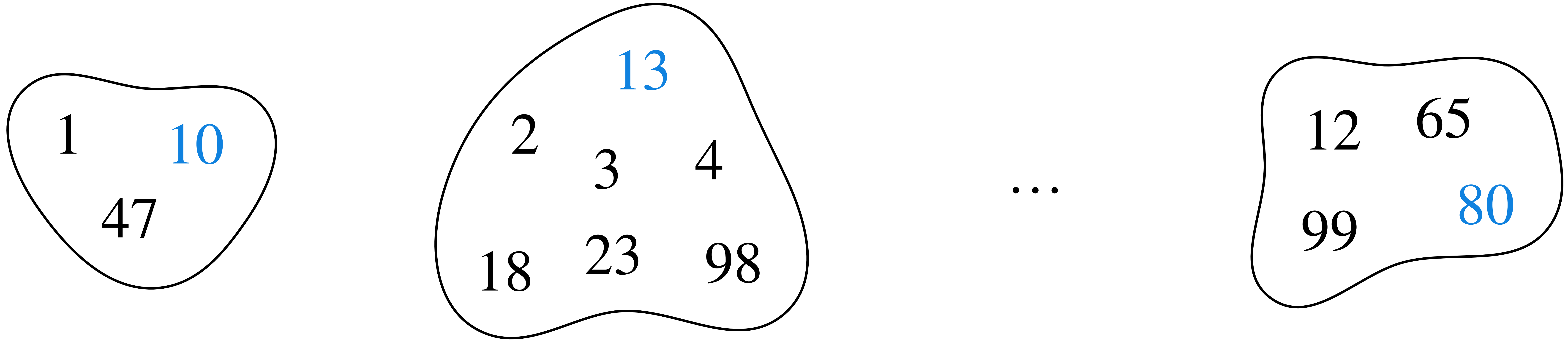

Goal: Design a data-structure so that:

1. Merging is fast.
2. Finding representative is fast.

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- **Find-Set**(x): Gives the representative of the unique set that contains x.

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- Finding connected components in graphs.
- Kruskal’s algorithms to find minimum spanning tree.
- Finding systems on the same network, etc.

Disjoint-Sets as Linked Lists

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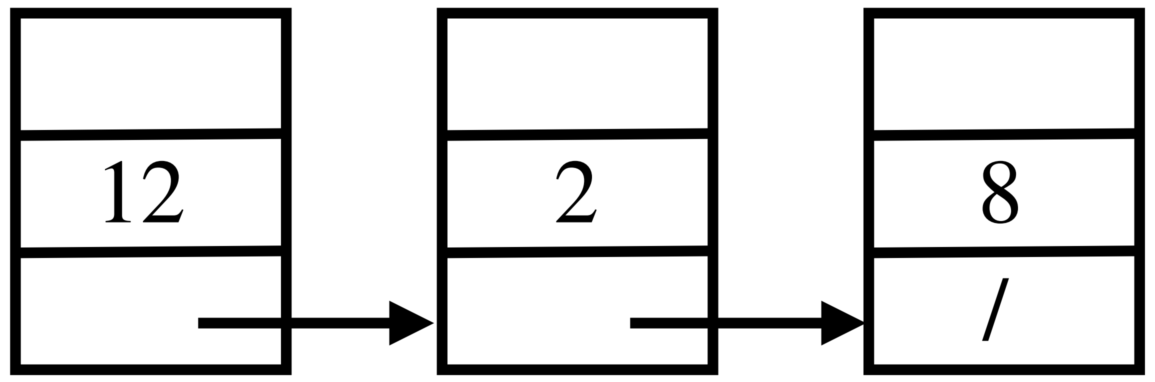
S_1



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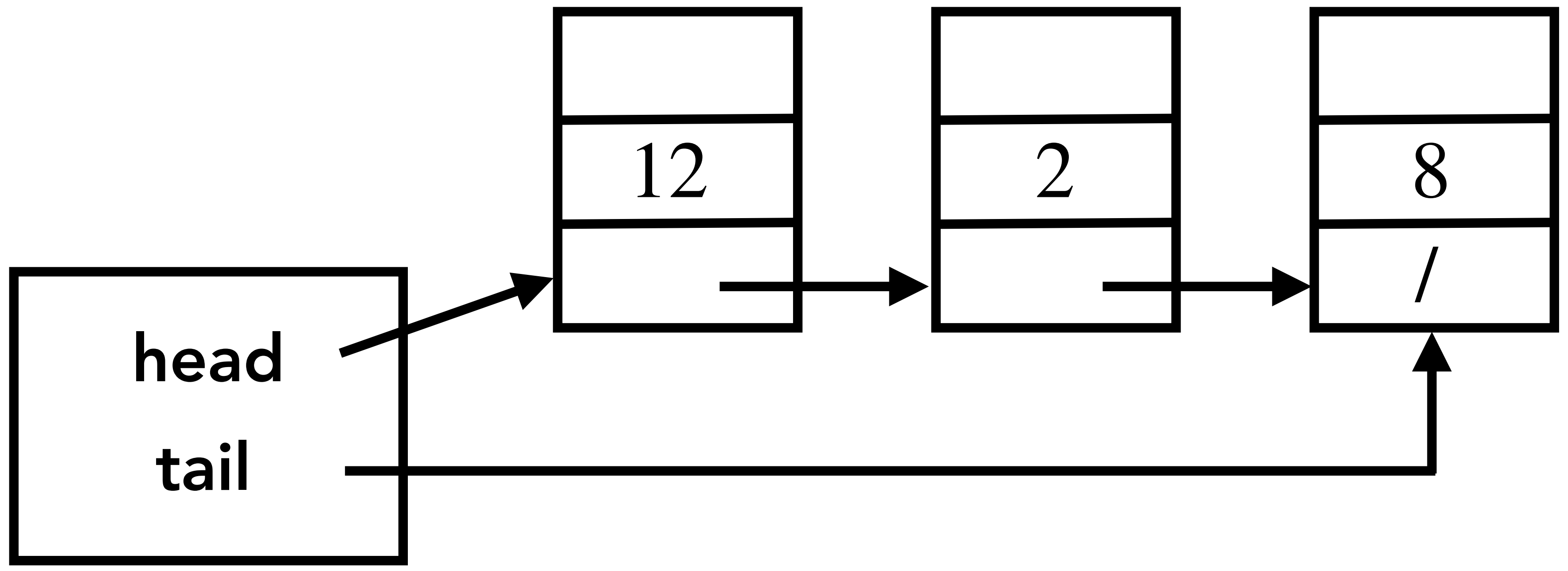

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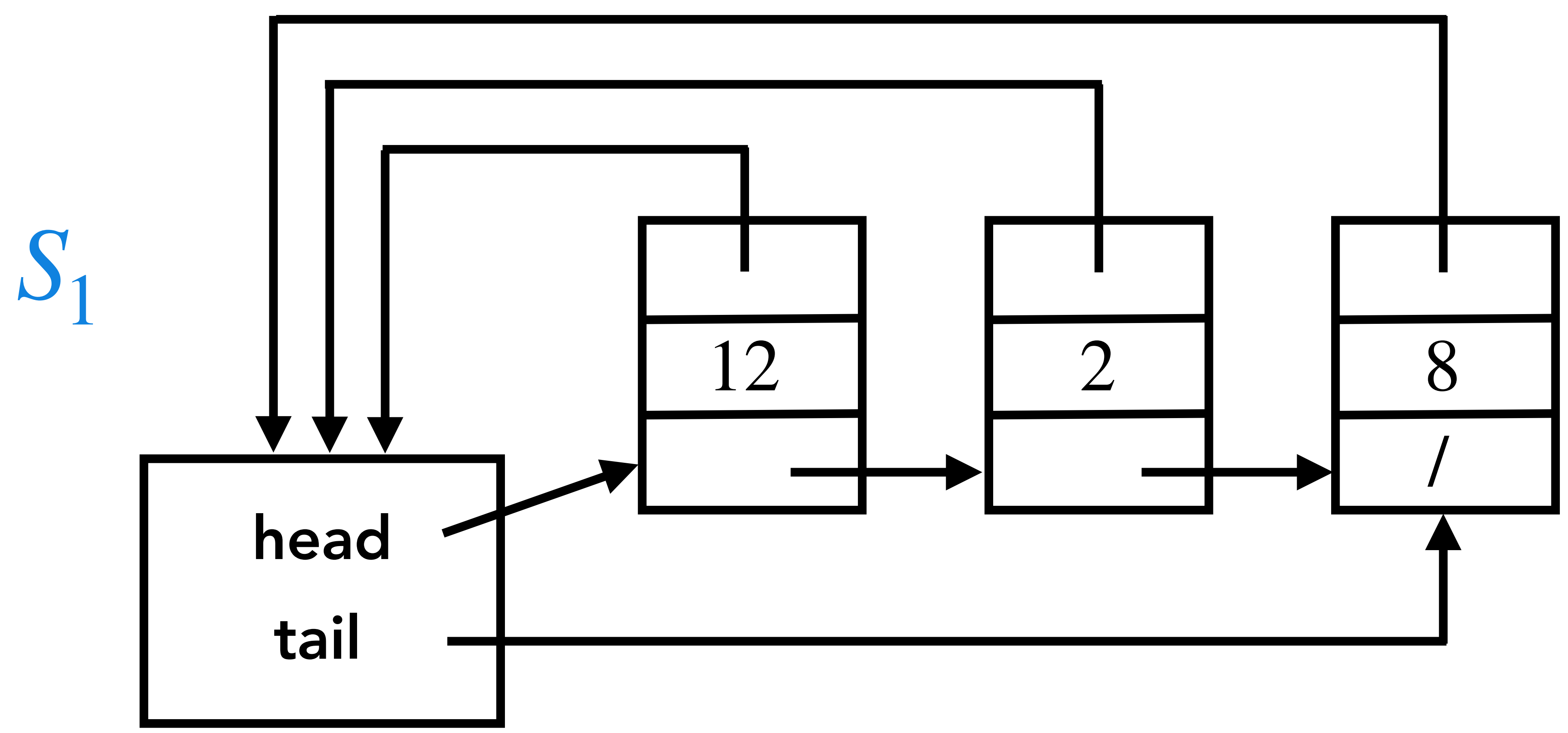

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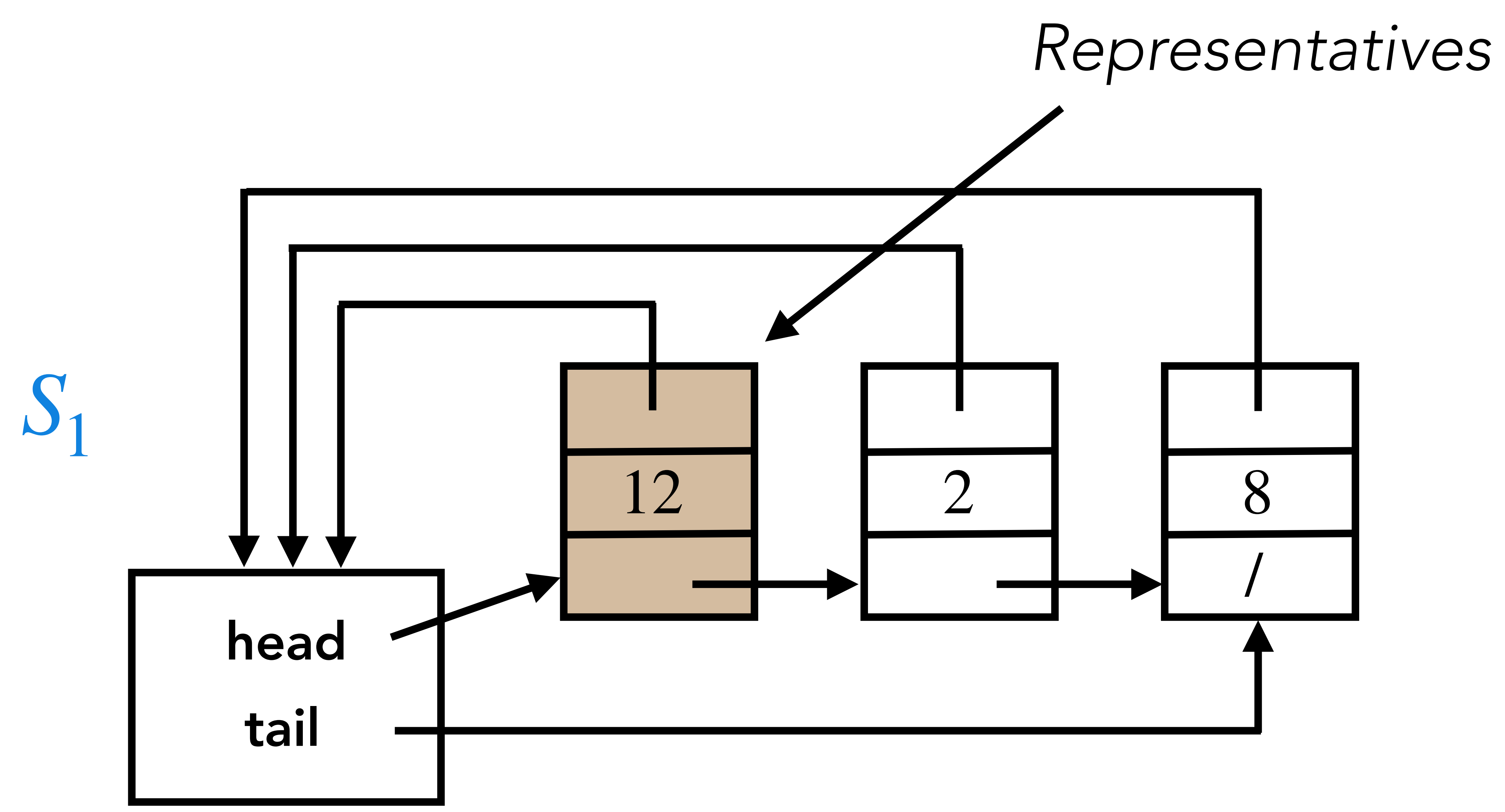

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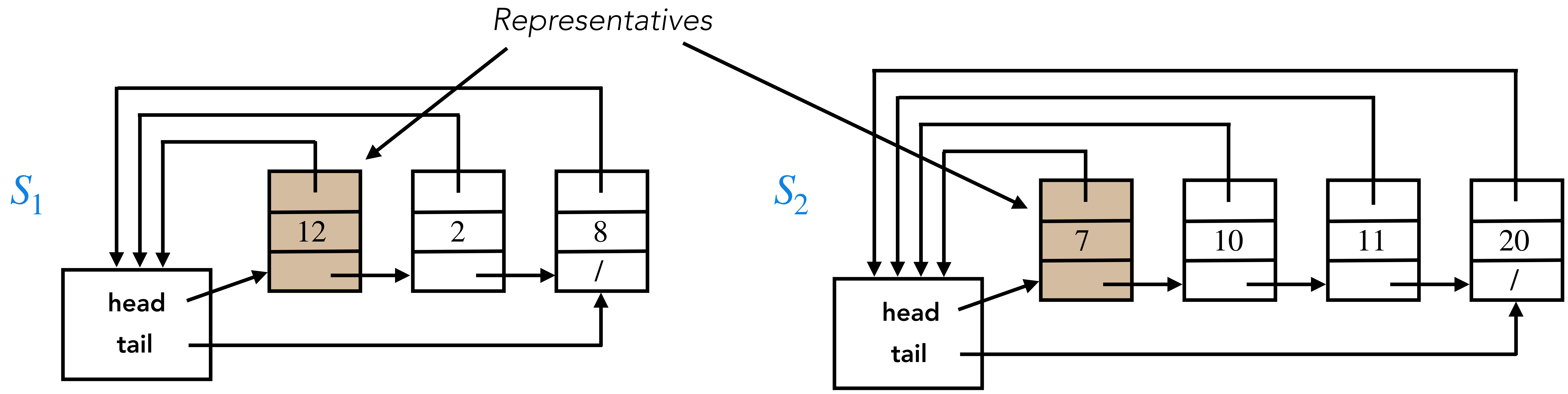

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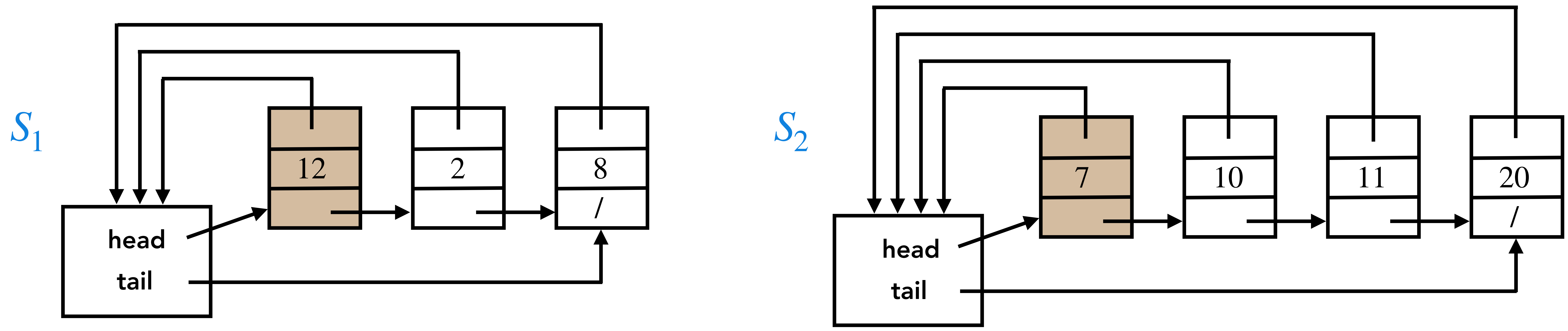

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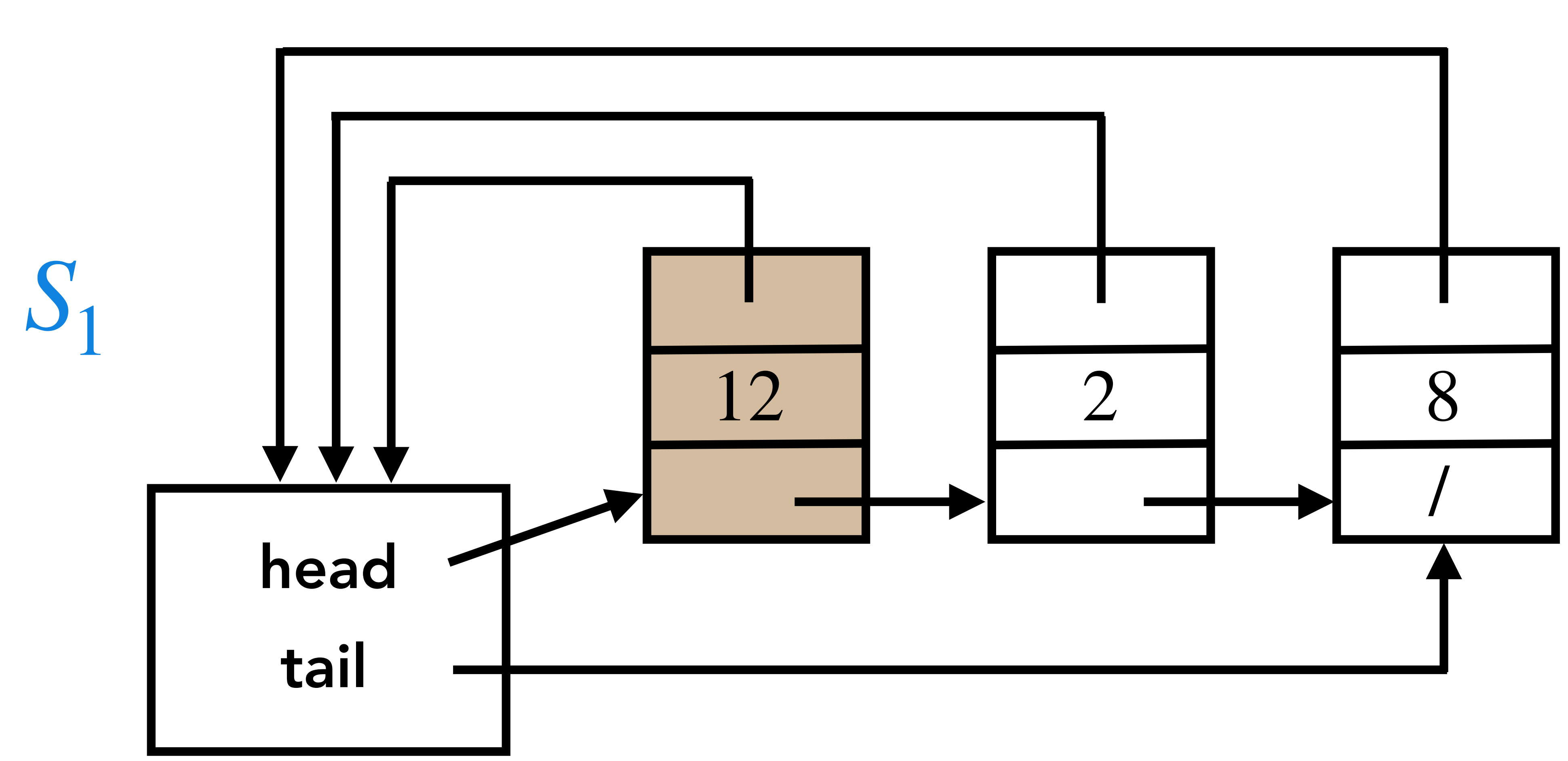

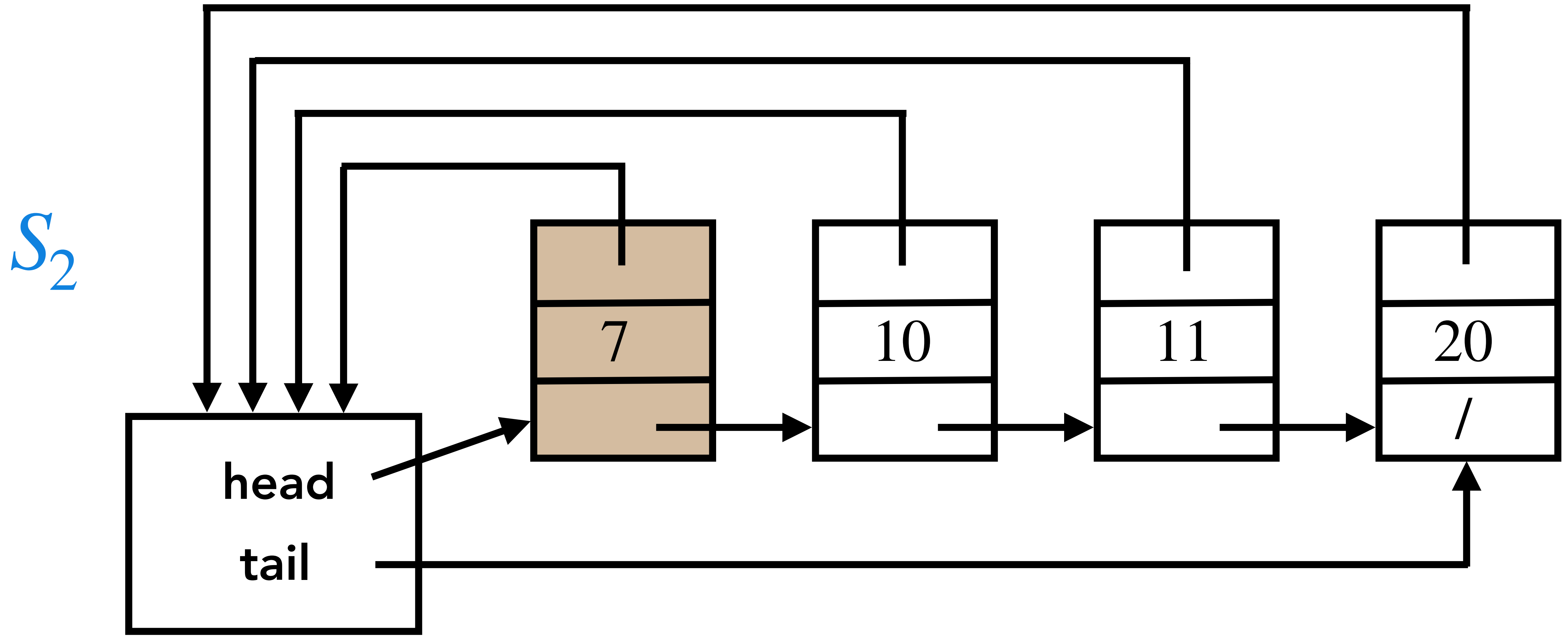

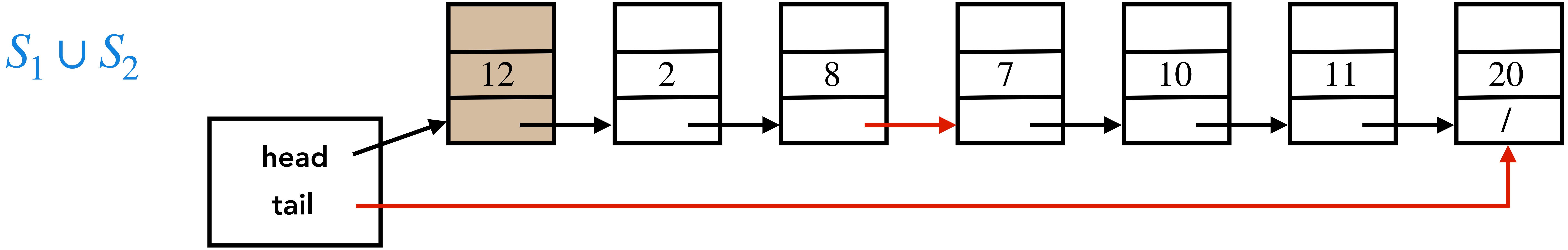

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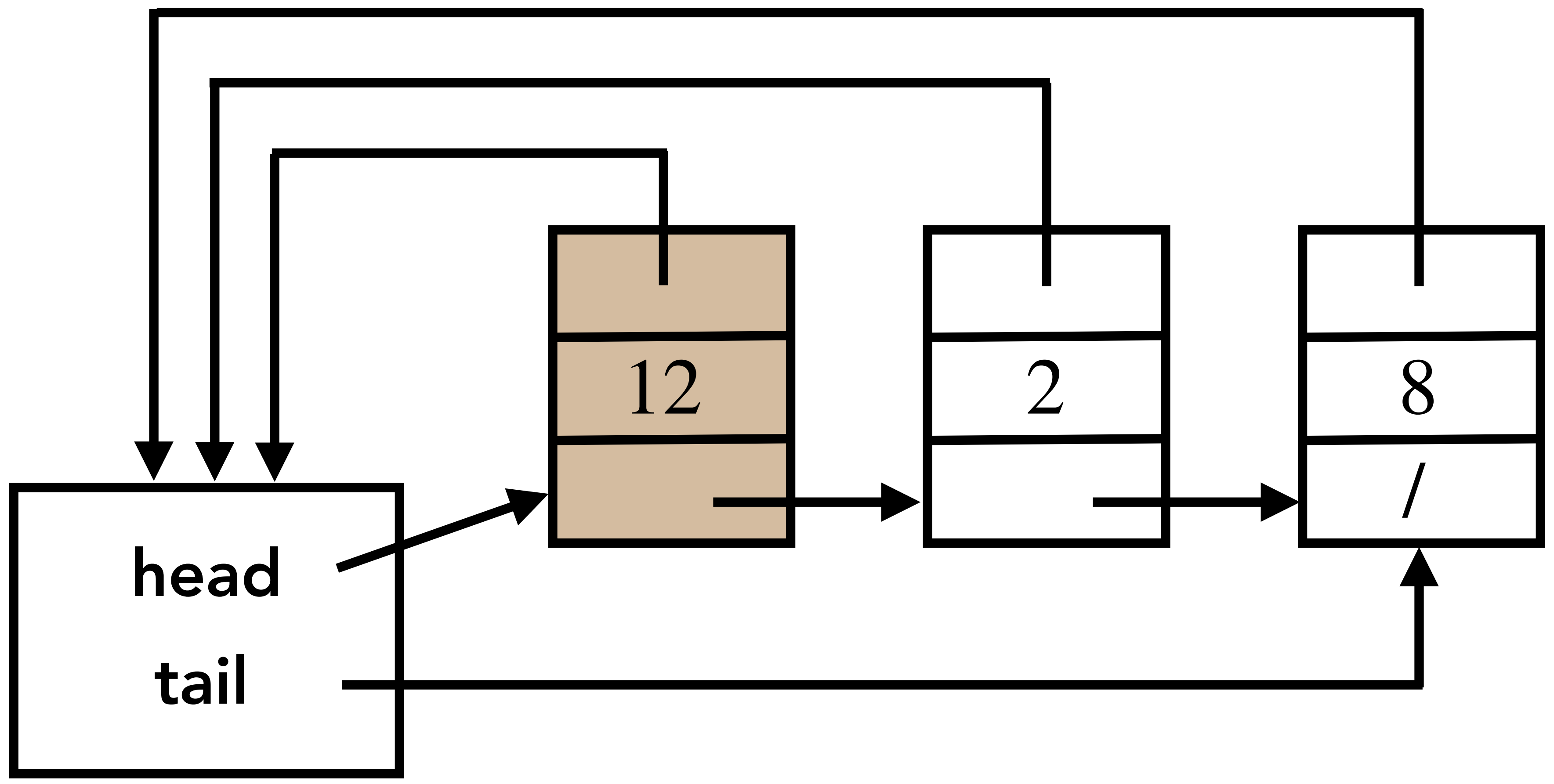

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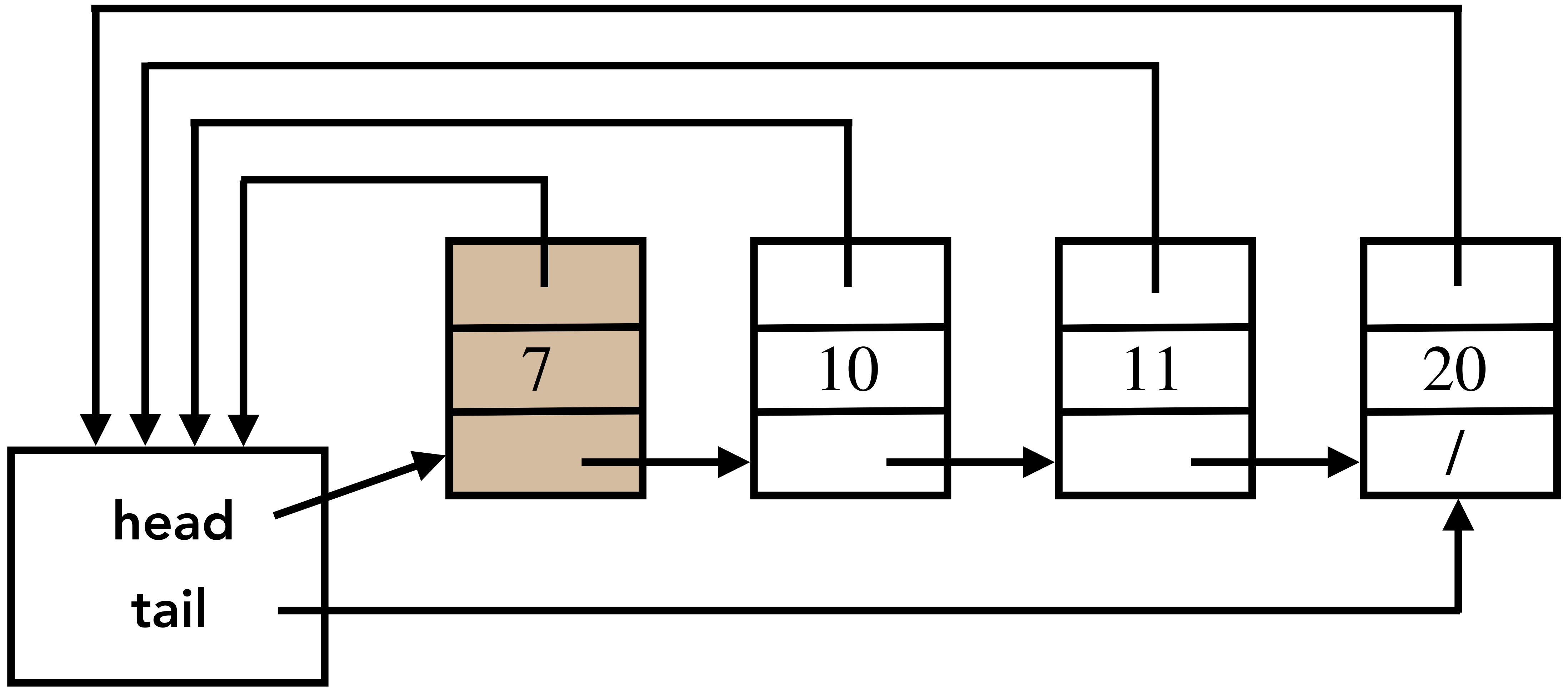

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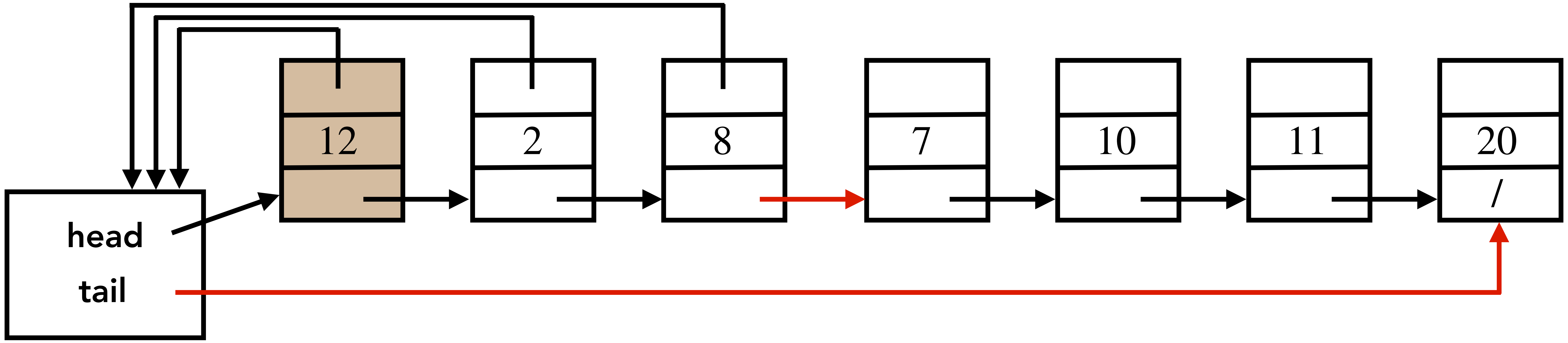

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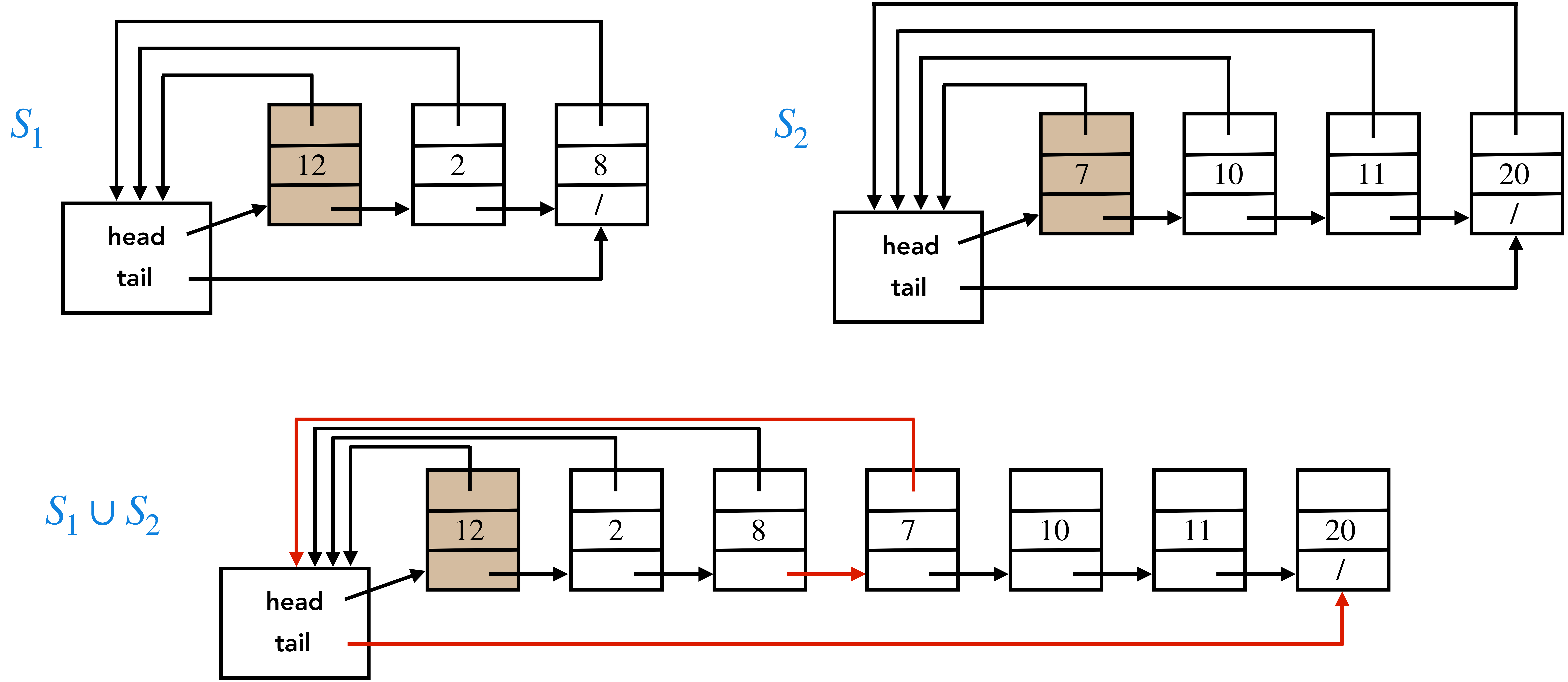

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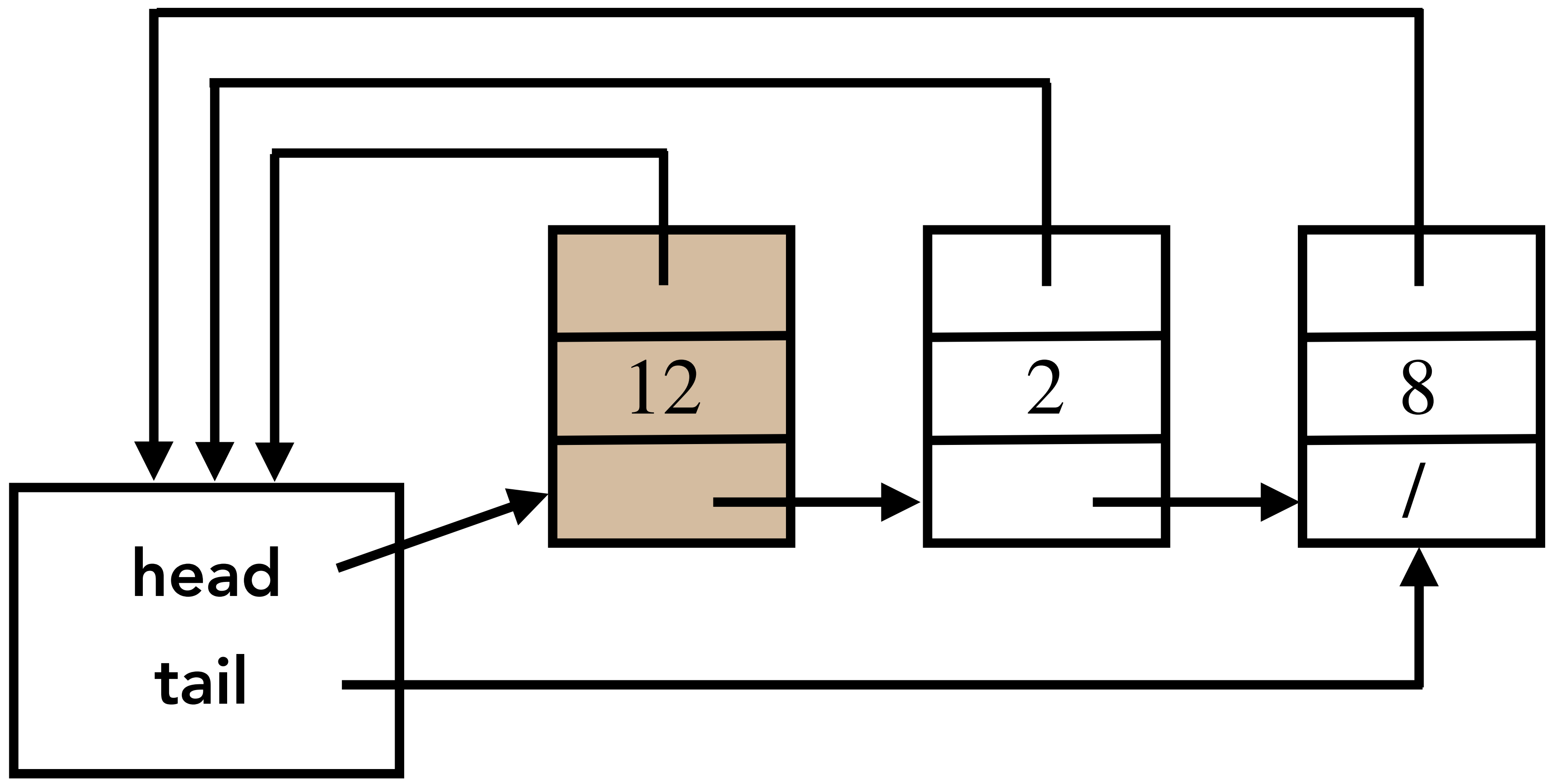

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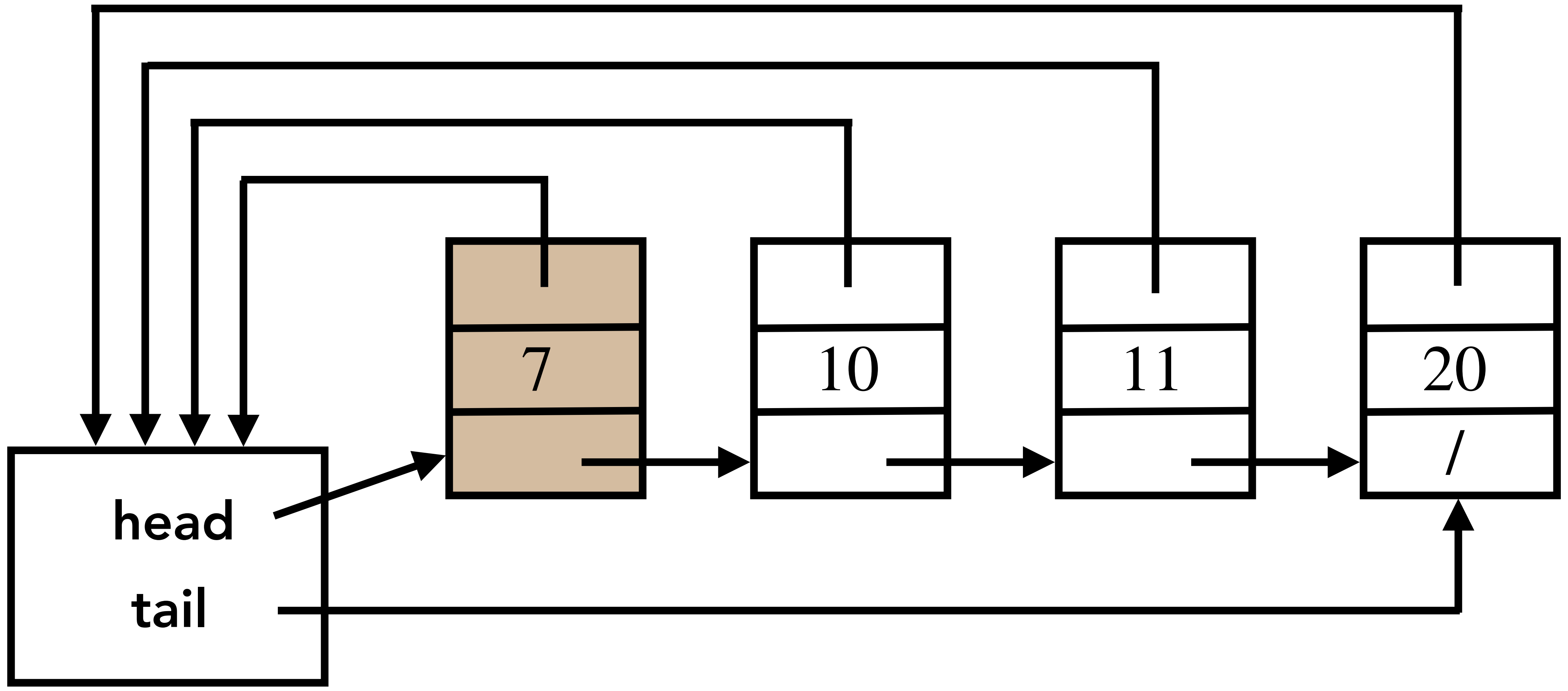

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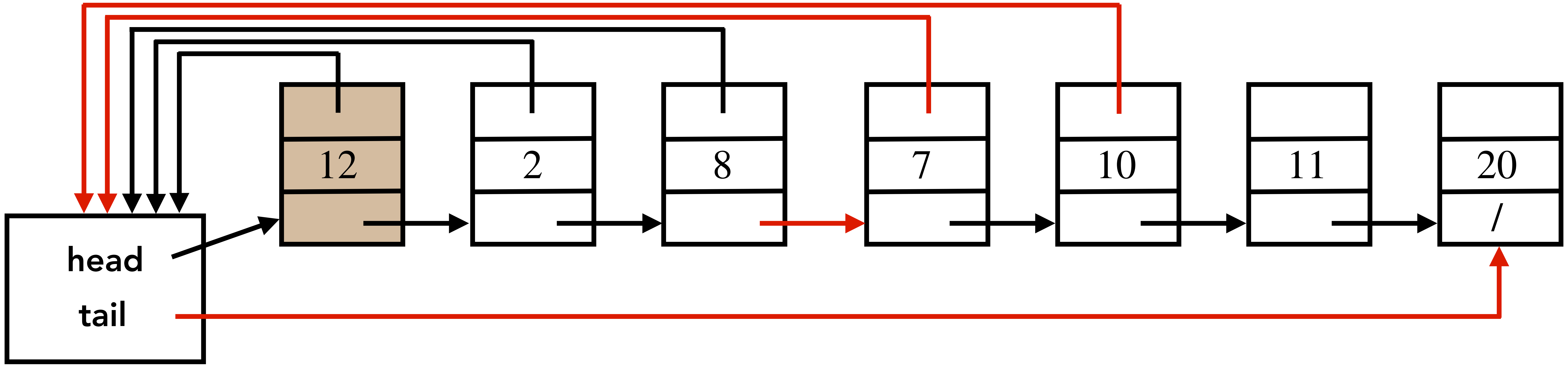

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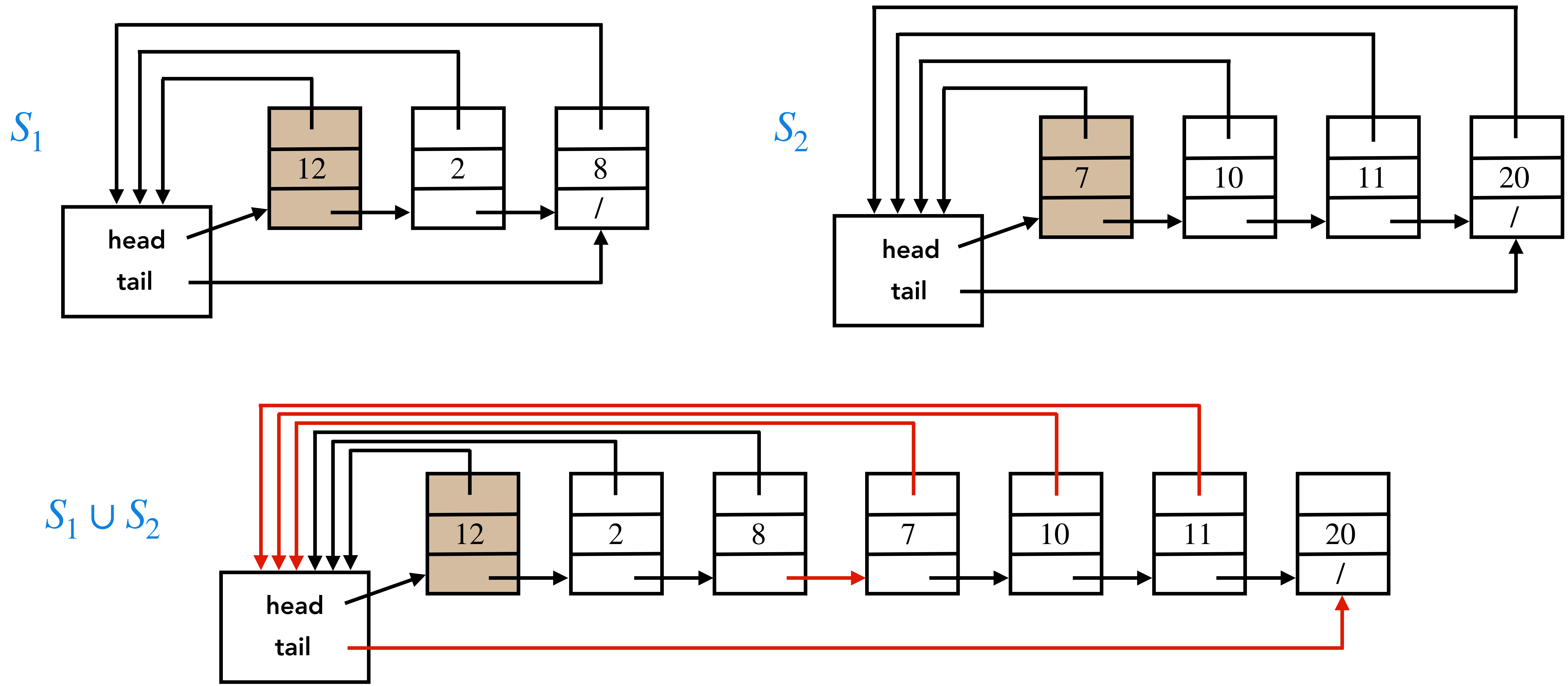

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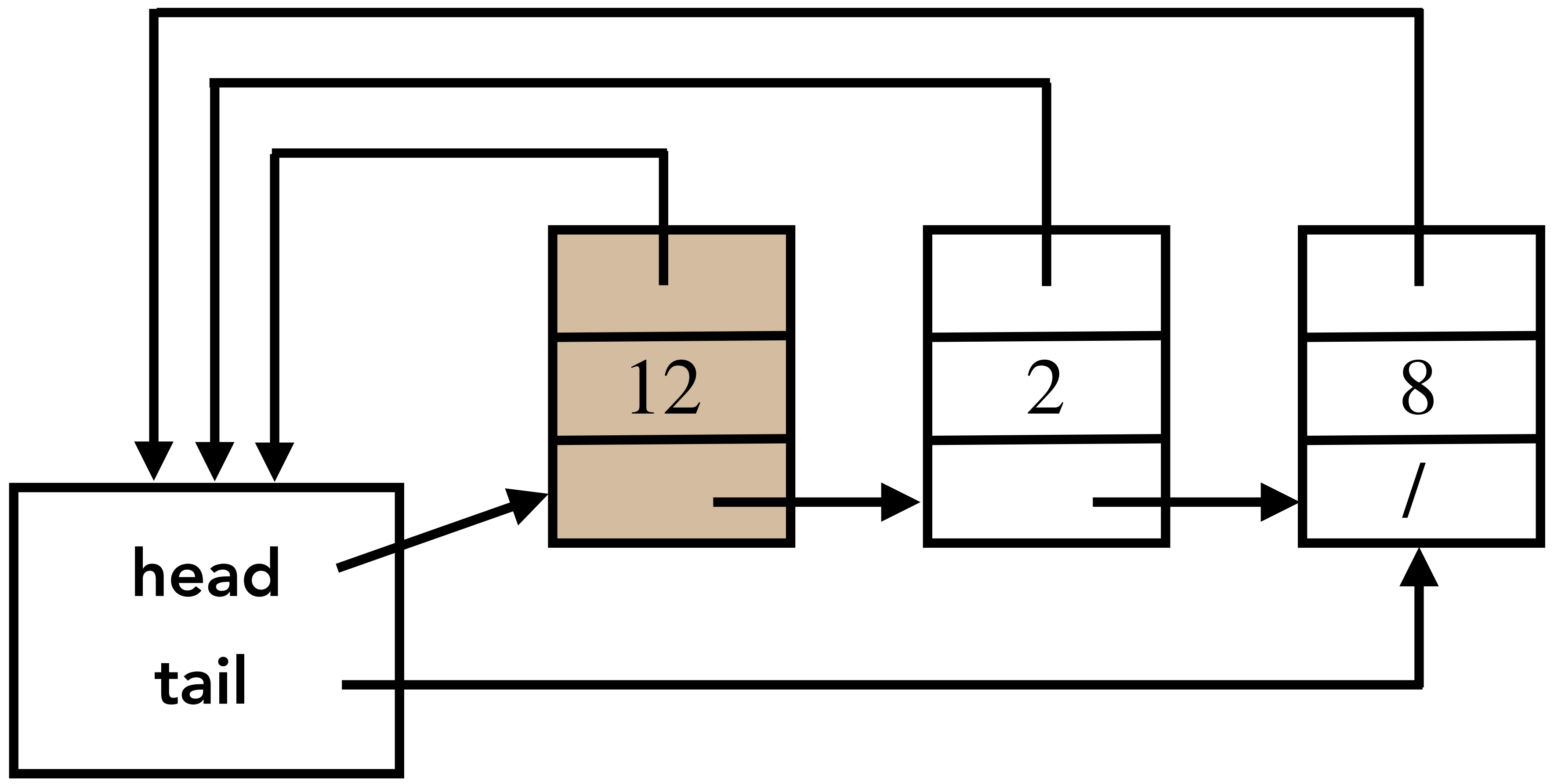

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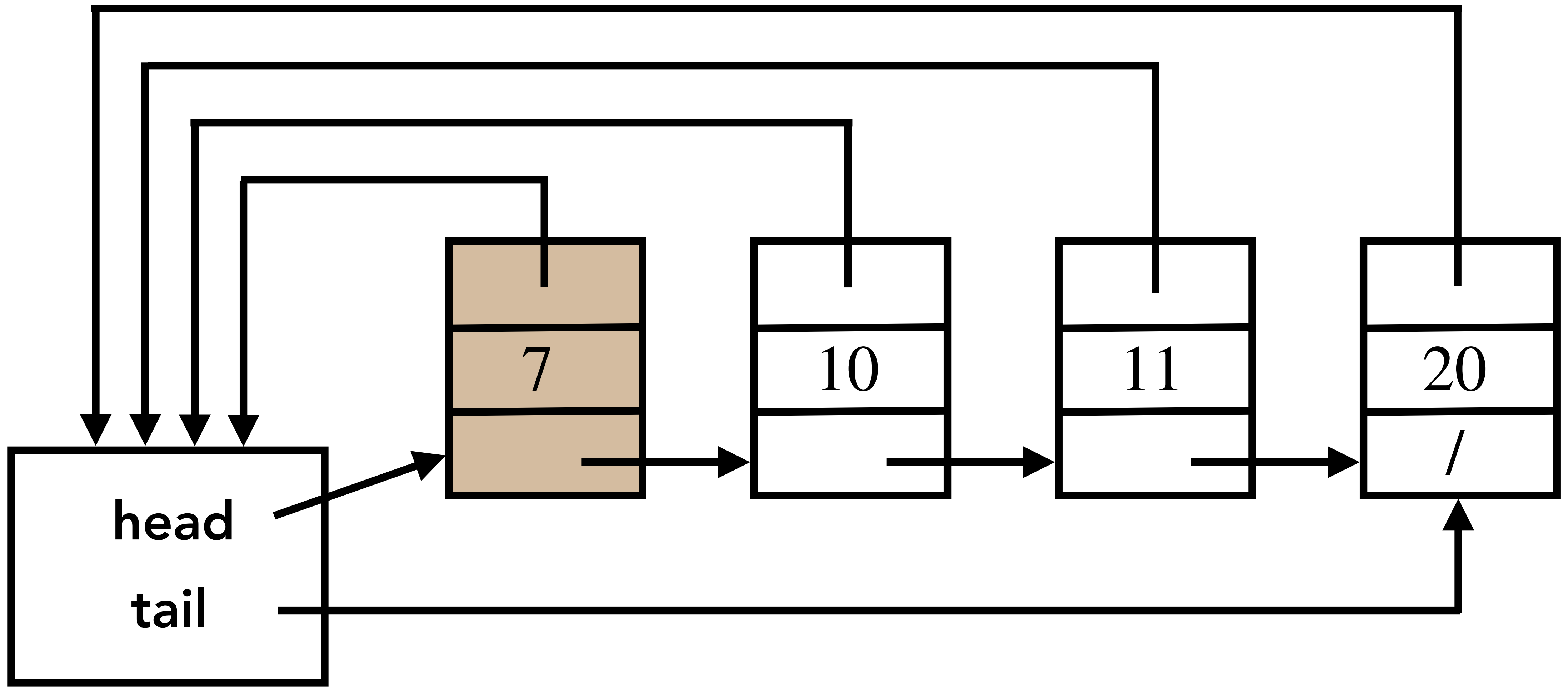

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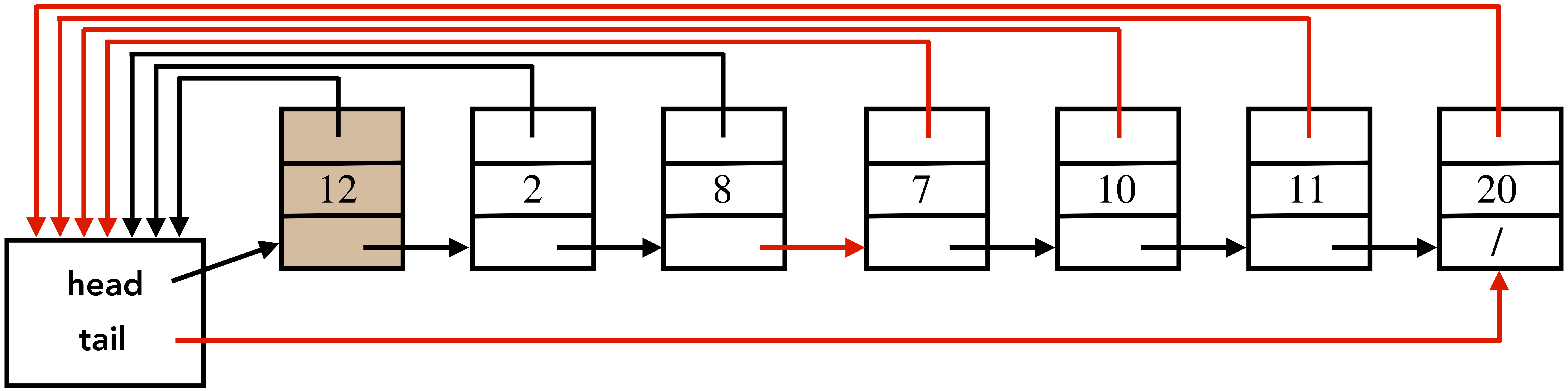

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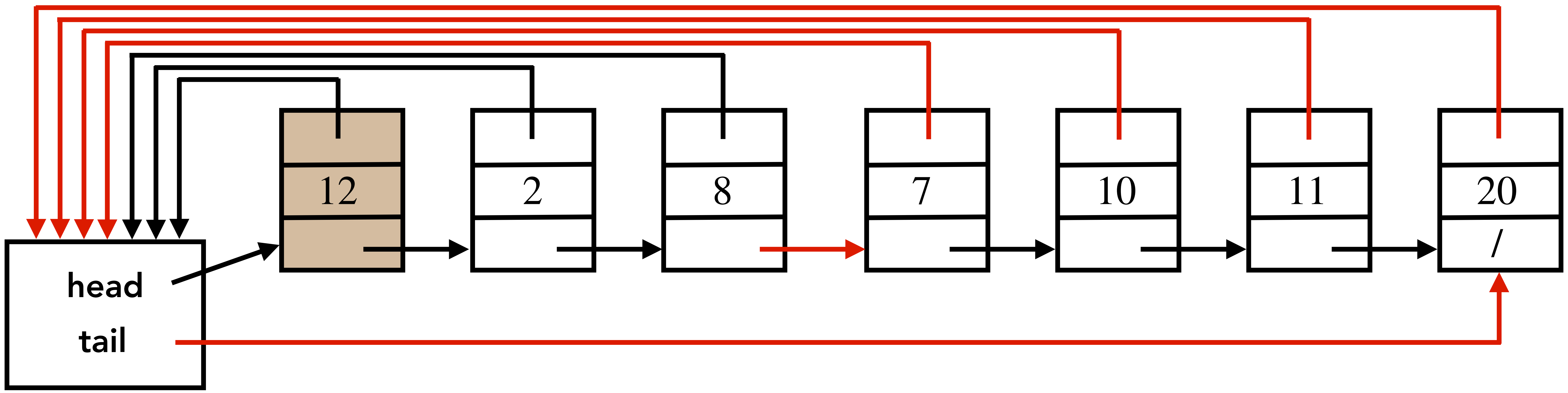

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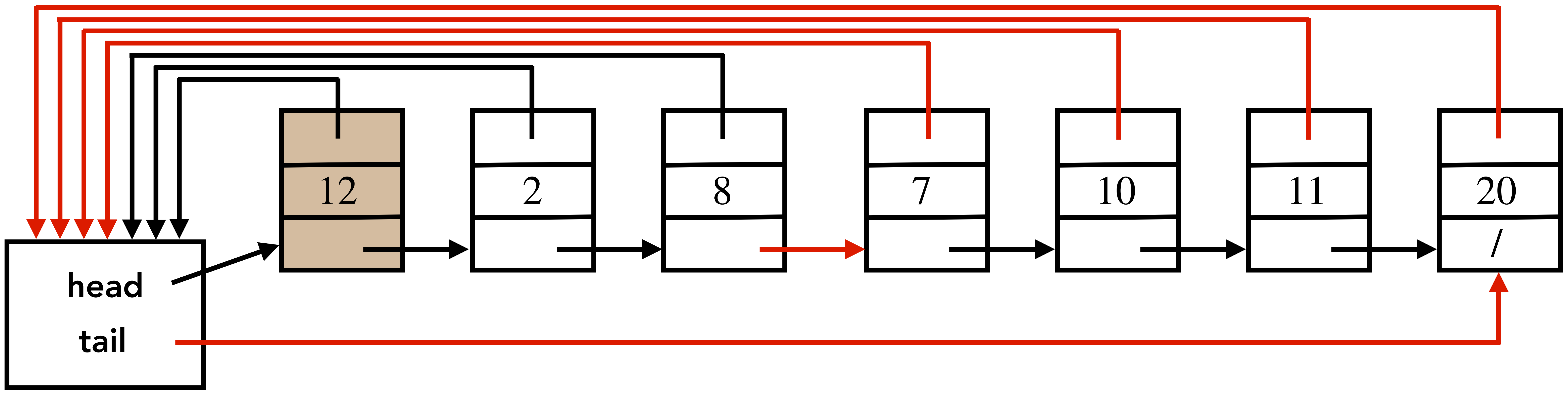

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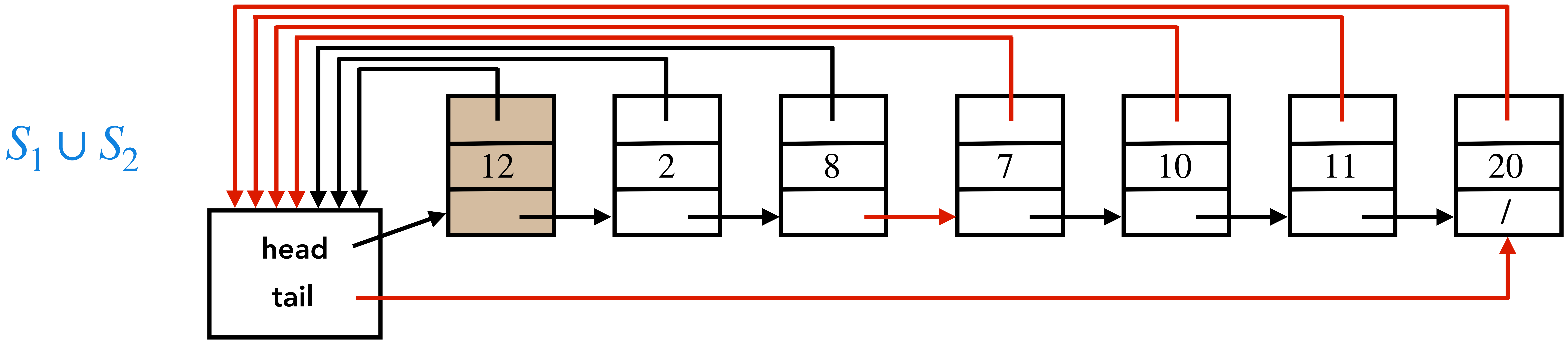

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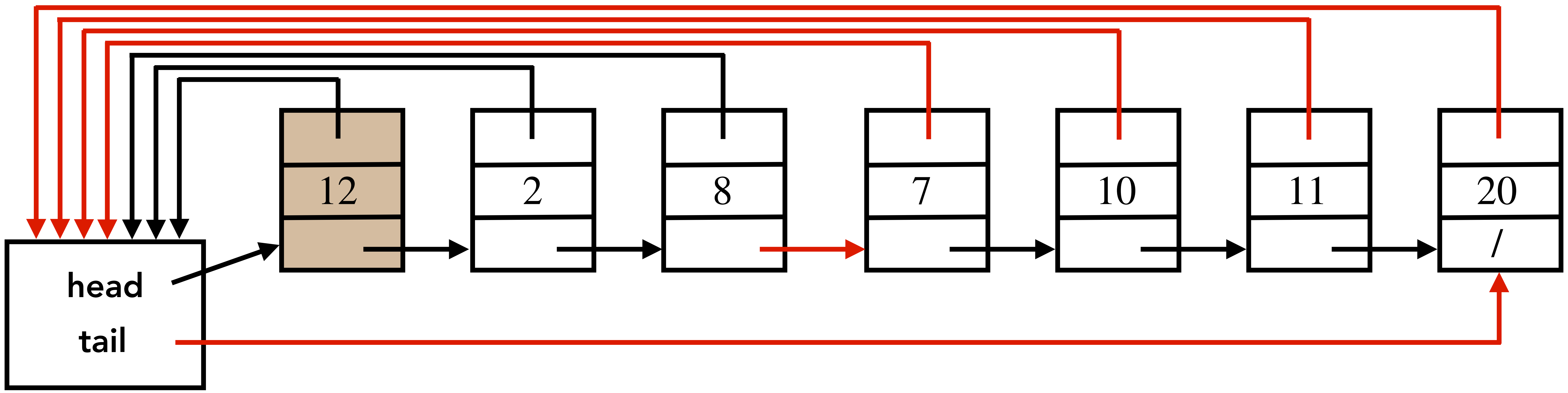

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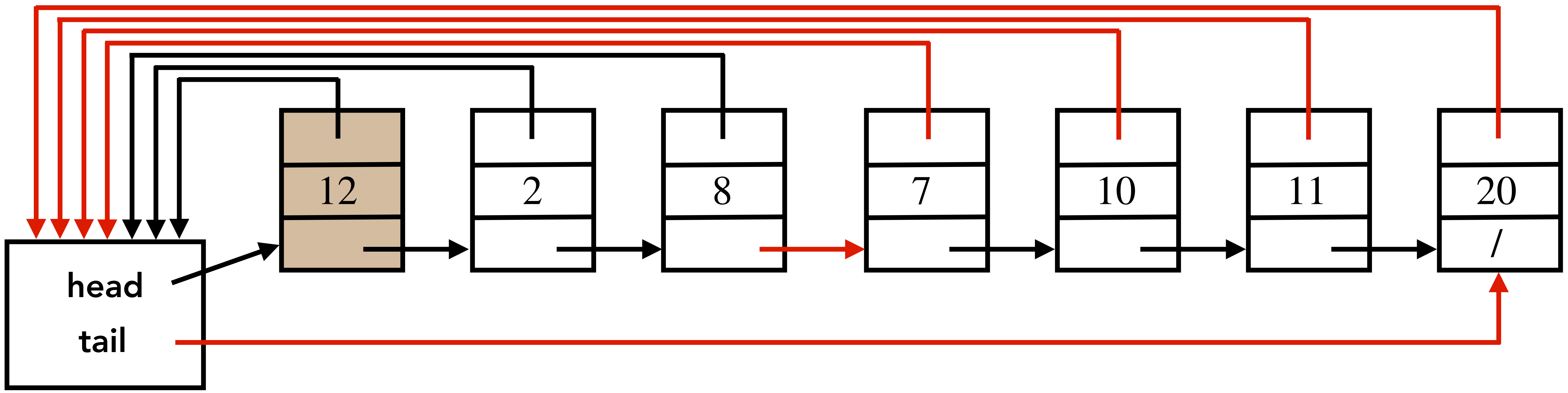

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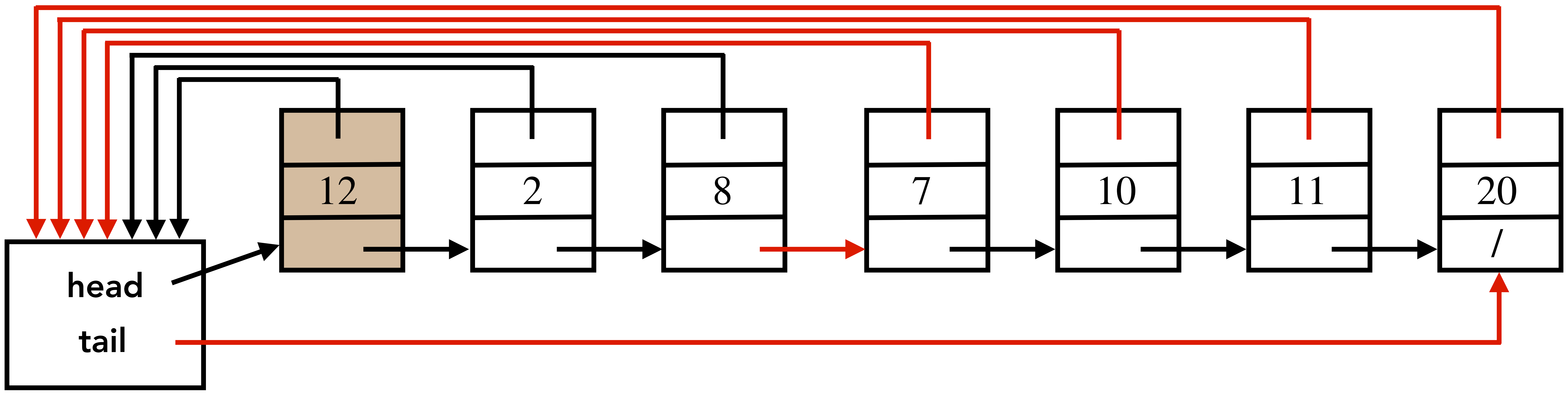

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Proof: DIY.