Image Enhancement

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1 Spatial Filtering

Spatial Filtering

- Use of spatial masks for image processing
- Linar and Nonlinear

Different types

- Low-pass filters
- High-pass filters
- Band-pass filters

Spatial Filtering

- **Low-pass** filters eliminate or attenuate high frequency component (sharp image details) in the frequency domain, and result in image blurring.
- **High-pass** filters eliminate and attenuate the low frequency components and result in sharpening edges and other sharp details.
- **Band-pass** filter, remove selected frequency regions between low and high frequencies.

Spatial Filtering
Linear filtering

Linear filtering of an image f of size $M \times N$ with a filter mask of size $m \times n$:

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

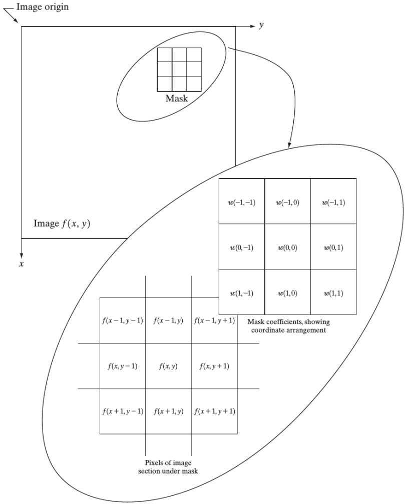
$a = (m - 1)/2$ and $b = (n - 1)/2$
for $x = 0, 1, ..., M - 1$ and $y = 0, 1, ..., N - 1$
Also called convolution (primarily in the frequency domain)

Spatial filter
Linear filtering

The basic approach is to sum products between the mask coefficients and the intensities of the pixels under the mask at a specific location in the image.
for 3×3 filter:

$$R = w_1 z_1 + w_2 z_2 + ... + w_9 z_9$$

Spatial Filtering
Linear filtering



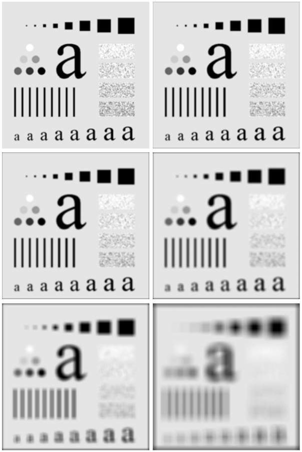
Spatial Filtering
Smoothing linear filter

- Replacing the value of every pixel in an image by the average of the gray levels in the neighborhood defined by the filter mask.
- Averaging filter
- Blurring the edges
- Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

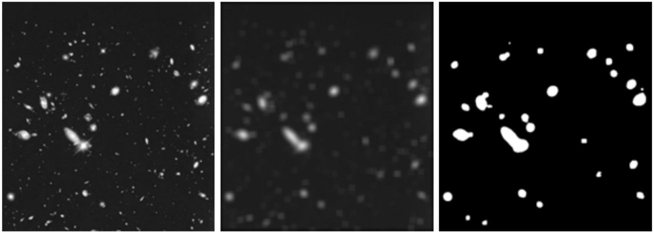
Spatial Filtering
Smoothing linear filter

Results of smoothing with square averaging filter masks of sizes n=3, 5, 9, 15, and 35, respectively.



Spatial Filtering
Smoothing linear filter

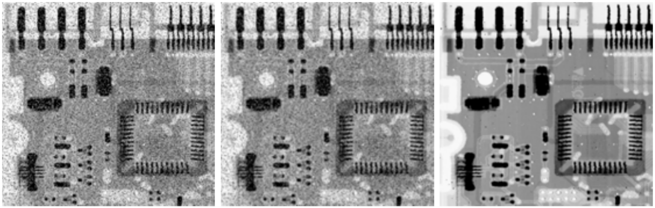
(b) Image processed by 15×15 average mask. (c) Result of thresholding



Spatial Filtering
Smoothing nonlinear filter

Median filtering also used for noise elimination.

- The gray level of each pixel is replaced by the median gray levels in the neighborhood of the pixel instead of the average.
- X-ray image of the circuit board corrupted by salt-and-pepper noise. Noise reduction with 3×3 average and median filter, respectively.



Spatial Filtering
Sharpening filter

- To highlight fine detail or to enhance blurred detail.
  - smoothing integration
  - sharpening differentiation

Categories of sharpening filters:

- Derivative operator
- Basic high-pass spatial filter
- High-boost filtering

Spatial Filtering
Sharpening filter - Derivative filter

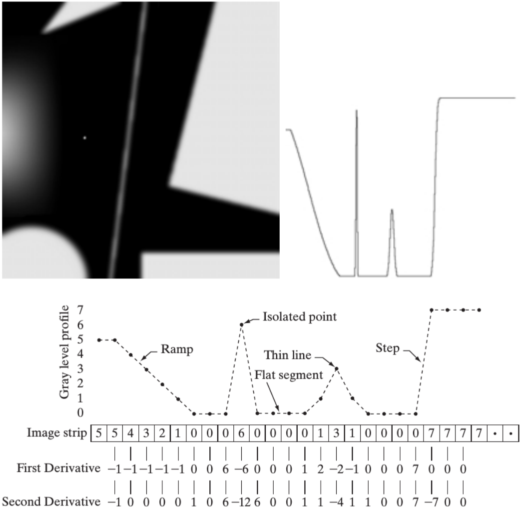
- First-order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Spatial Filtering
Sharpening filter - Derivative filter



Spatial Filtering
Sharpening filter - Derivative filter

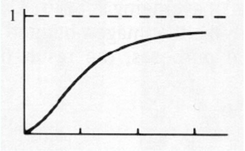
- First derivative:
  - 0 in constant gray segments
  - Non-zero at the onset of steps or ramps
  - Non-zero along ramps
- Second derivative:
  - 0 in constant gray segments
  - Non-zero at the onset and end of steps or ramps
  - 0 along ramps of constant slope.

Spatial Filtering
Sharpening filter - Derivative filter

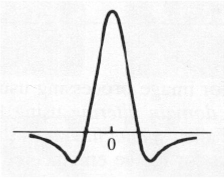
- First-order derivatives generally produce thicker edges in an image
- Second-order derivatives have a stronger response to fine detail, such as thin lines and isolated points
- First-order derivatives generally have a stronger response to a gray-level step
- Second-order derivatives produce a double response at step changes in gray level
- Second-order derivatives have stronger response to a line than to a step and to a point than to a line

Spatial Filtering
Sharpening filter - Basic Highpass Spatial filter

- Cross section of frequency domain filter:



- Cross section of spatial domain filter:



Spatial Filtering
Sharpening filter - Basic Highpass Spatial filter

- The filter should have positive coefficients near the center and negative in the outer periphery

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & 8 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

- The sum of the coefficients is 0, indicating that when the filter is passing over regions of almost stable gray levels, the output of the mask is 0 or very small.
- Some scaling and/or clipping is involved to compensate for possible negative gray levels after filtering.

Spatial Filtering
Sharpening filter - 2D, Second order derivatives

- Isotropic filters, rotation invariant
- Laplacian (linear operator)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete version:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Spatial Filtering
Sharpening filter - 2D, Second order derivative

- Digital implementation

$$\nabla^2 f = [f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]-4f(x,y)$$

- Two definitions, one is negative of the other

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{Center of the mask is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{Center of the mask is positive} \end{cases}$$

Spatial Filtering
Sharpening filter - 2D, Second order derivative

- Filtering and recovering the original part:

$$g(x,y) = f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] + 4f(x,y)$$

$$g(x,y) = 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] + 4f(x,y)$$

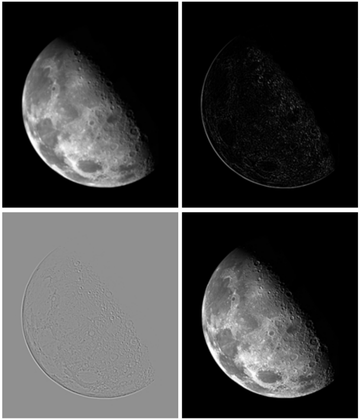
Spatial Filtering
Sharpening filter - 2D, Second order derivative

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

Spatial Filtering
Sharpening filter - 2D, Second order derivative

Image of the north pole of the moon, laplacian filtered image, laplacian image scaled for display and image enhanced by laplacian, respectively.



Spatial Filtering
Sharpening filter - High-boost filter

- Unsharp masking : $f_s(x,y) = f(x,y) - \bar{f}(x,y)$
- High-pass filtered image = original - low-pass filtered image.
- Consider A as an amplification factor

$$High - pass = A.original - low - pass(blurred)$$

$$= (A-1).original + original - low - pass$$

$$= (A-1).original + high - pass$$

- A =1 –> Standard high-pass filter
- A >1 –> Enhanced image depending on the value of A

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

Spatial Filtering
Gradient filter - first derivative

- the most common method of differentiation in image processing

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- It is non-isotropic
- Its magnitude (often call the gradient) is rotation invariant

$$\nabla f = |G_x| + |G_y| = [(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial x})^2]^{1/2}$$

Spatial Filtering

Gradient filter - first derivative

Different masks use to calculate the gradient of a region of interest with z_5 as a central pixel, Robert cross gradient masks middle row and Sobel filters in the last row.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Spatial Filtering
Gradient filter - first derivative

- Computation:
- Cross differences as used in early development of digital image processing: $G_x = (z_9 - z_5), \quad G_y = (z_8 - z_6)$
- Robert cross gradient:

$$\nabla f \approx [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2}$$

- Sobel filter

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_9 + z_6) - (z_1 + 2z_4 + z_7)|$$

Spatial Filtering

Whole body bone scane, laplacian image, sharpened image using the laplacian and sobel filtered image, respectively from right to left and first row to second.