

BIT MANIPULATION

(1) $80x$ general

(2) 101 \oplus

Bit-wise Operators

→ Binary AND ($\&$)

→ Binary OR ($|$)

→ Binary XOR (\wedge)

→ Binary One's Complement (\sim)

→ Binary Left Shift (\ll)

→ Binary Right Shift (\gg)

Binary AND ($\&$)

A	B	$A \& B$
0	0	0
0	1	0
1	0	0
1	1	1

Eg. 101 (5) $\&$ 110 (6) $\Rightarrow 5 \& 6 = 4$

$\frac{101}{110}$ $\underline{\quad}$ 100 (4)

Binary OR ($|$)

A	B	$A B$
0	0	0
0	1	1
1	0	1
1	1	1

Eg. 101 (5) $|$ 110 (6) $\Rightarrow 5 | 6 = 7$

$\frac{101}{110}$ $\underline{\quad}$ 111 (7)

Binary XOR (^)

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Eg. $101_2 (5) \quad 110_2 (6)$

$$\begin{array}{r} 101_2 \\ 110_2 \\ \hline 011_2 (2) \end{array}$$

Binary One's Complement (\sim)

Eg. $\sim 5 = -6$

Explanation $\rightarrow (5)_10 = (101)_2 = (00000101)_2$

1's complement = $(1111010)_2$

$P = 222$ \leftarrow msB i.e. 1 (which means it would be a negative number)

Now find the magnitude of this negative number

so $\sim 5 = -6$ \leftarrow calculate 1's complement = $(00000101)_2$

Now add 1 $\rightarrow 00000101 + 1$

$$\begin{array}{r} 00000101 \\ + 1 \\ \hline 00000110 = (6)_10 \end{array}$$

That's why $\sim 5 = -6$

Binary Left Shift ($<<$)

Eg. $5 << 2 = 10$

$$(5)_{10} = (101)_2 = \begin{array}{r} 00000101 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 00010100 \\ = (20)_{10} \end{array}$$

Discarded Filled

Imp.

$$a << b = \cancel{a \times 2^b} = a \times 2^b$$

Binary Right Shift ($>>$)

Eg. $5 >> 2 = 1$

$$(5)_{10} = (101)_2 = \begin{array}{r} 00000101 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 00000001 \\ = (1)_{10} \end{array}$$

Discarded Filled

Imp.

$$a >> b = \frac{a}{2^b}$$

$$= 0 >> 1$$

$$0001 = 2 >> 1 \rightarrow \text{from MSB to LSB}$$

$$\begin{array}{r} 0101 \\ 0001 \\ \hline 1 \leftarrow 0001 \end{array}$$

Check if Odd or Even

- In binary, Even numbers have 0 at LSB and Odd numbers have 1 at LSB.

```
if (n & 1)
    print ("Odd")
```

```
else
    print ("Even")
```

$$\text{Eq. } (5)_{10} = (101)_2$$

$$\begin{array}{r} 101 \\ \times 0 \\ \hline 001 \end{array}$$

(true)

$$(6)_{10} = (110)_2$$

$$\begin{array}{r} 110 \\ \times 0 \\ \hline 000 \end{array}$$

(false)

Get ith Bit

- To get the ith Bit from right side of a number, create a bit mask.

Bit mask = $1 \ll i$

- if (number & Bit mask == 0)

```
    print ("ith Bit is 0");
```

```
else
    print ("ith Bit is 1");
```

Eg. 1010

$$\begin{array}{r} 0 \\ \times 1010 \\ \hline 0 \end{array}$$

Result

get 0th \rightarrow Bit mask = $1 \ll 0 = 1$

$$\begin{array}{r} 1010 \\ \times 0001 \\ \hline 0000 \Rightarrow 0 \end{array}$$

get 3rd \rightarrow Bit mask = $1 \ll 3 = 1000$

$$\begin{array}{r} 1010 \\ \times 1000 \\ \hline 1000 \Rightarrow 1 \end{array}$$

Set i^{th} Bit

Set i^{th} Bit means, make i^{th} bit 1.

Bit mask = $1 \ll i$

number = number | bit mask

Clear i^{th} Bit

Clear i^{th} Bit means, make i^{th} bit 0.

Bit mask = $(1 \ll i) \&$

number = number & bit mask

Clear range of bits

Need to clear the bits from i to j .

a = $(\text{~}0) \ll (j+1)$

b = $(\text{~}1 \ll i) - 1$

Bit mask = a | b

number = number & bit mask

Count set bits in a number

set bits: 1's bit

count = 0

while ($n > 0$)

{
 if ($n \& 1 == 0$)
 count++;

$n = n \gg 1$;

}

Fast Exponentiation

Take an example -

find 3^5

Now $(5)_{10} = (101)_2$

create a variable ans = 1; a = 3; n = 5

while($n > 0$)

{

if ($((n \& 1) != 0)$

ans = ans * a;

a = a * a

n = n >> 1;

}

Dry Run:- 3^5

ans = 1, a = 3

$(5 > 0)$ true

$((5 \& 1) != 0)$ true

ans = 3;

a = $3 * 3 = 9$

n = 2;

$(2 > 0)$ true

$((2 \& 1) != 0)$ false

a = $9 * 9 = 81$

n = 1;

$(1 > 0)$ true

$((1 \& 1) != 0)$ true

ans = $3 * 81 = 243$

a = 6561

ans = 243

ans = 1, a = 3^5

$(2 > 0)$ true

$((2 \& 1) != 0)$ false

a = $3 * 3 = 2^5$

n = 1

$(1 > 0)$ true

$((1 \& 1) != 0)$ true

ans = $1 * 2^5$

ans = 2^5

n = 0

ans = 2^5