Saathi

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COMPUTER ORIENTED OPTIMIZATION

Graphical Method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$
$$x_i \geq 0 \quad ; i = 1,2,3,4$$

no. of solutions $= {}^4C_2 = \frac{4!}{2!2!} = 6$

value that satisfies the equation

- Feasible solution - all $x_i \geq 0$
- Basic solution - involves basic variables
- Basic Feasible solution - value of all basic variables ≥ 0
- Optimal Basic Feasible solution - sol^n which gives Maximum / Minimum value based on Maximization / Minimization problem.

* Basic variables - the variables which we use, other than the ones which we put equal to '0'.

Optimization Function

* Max / Min $\quad z = c_1x_1 + c_2x_2$

Maximization / Minimization Problem

$$a_{11}x_1 + a_{12}x_2 \leq b_1 \quad \text{—} (1)$$
$$a_{21}x_1 + a_{22}x_2 \leq b_2 \quad \text{—} (2)$$
$$a_{31}x_1 + a_{32}x_2 \leq b_3 \quad \text{—} (3)$$

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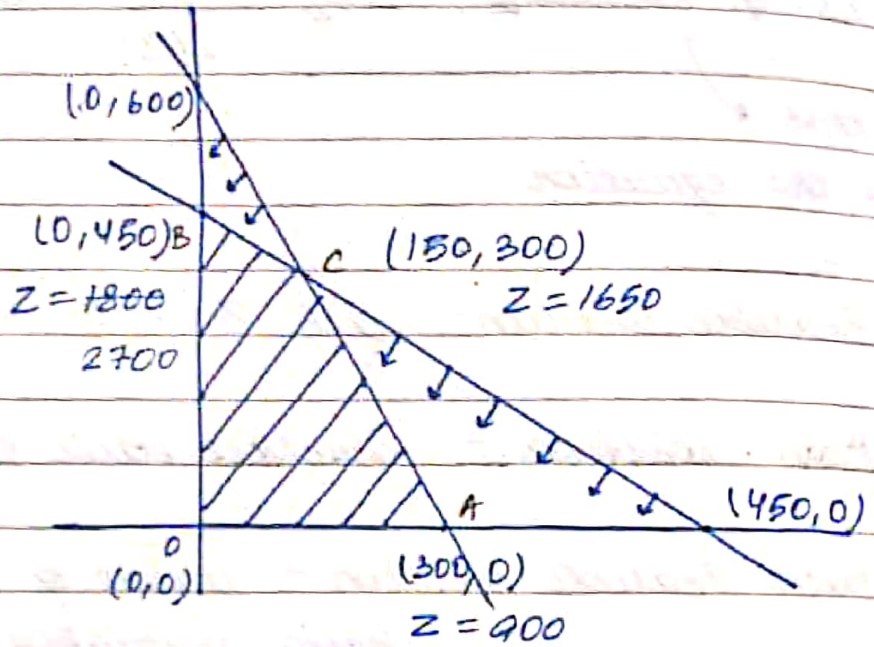
8) Max $Z = 3x_1 + 6x_2$

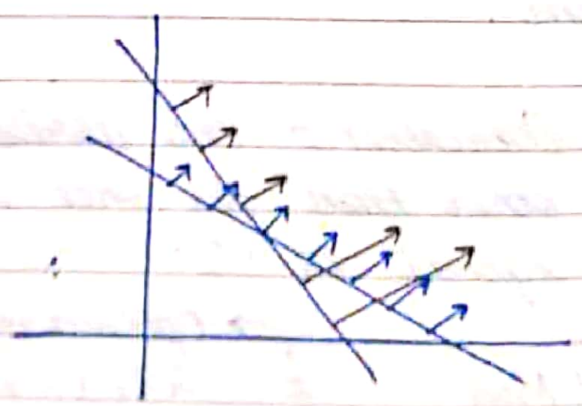
s.t. $x_1 + x_2 \le 450$

$2x_1 + x_2 \le 600$; x_1, x_2



Max $Z = \cancel{1800}$ 2700 at (0,450)

1. Unbounded solution



2. No feasible solution

3 Unique solution

4. Infinite solution

eg. Max $z = \boxed{2x_1 + x_2}$

$x_1 + 2x_2 \leq 450$

$\boxed{2x_1 + x_2} \leq 600$ $\quad x_1, x_2 \geq 0$

1 Degenerate solution - if value of any basic variable is '0'

2. non - degenerate solution - if value of all basic variable is not '0'

D) Max $z = 12x_1 + 8x_2 + 14x_3 + 10x_4$

St. $5x_1 + 4x_2 + 2x_3 + x_4 = 100$

$2x_1 + 3x_2 + 8x_3 + x_4 = 75$

$x_i \geq 0$

$X_1 = (0, 25, 0, 0)$ $\quad X_2 = (\frac{325}{18}, 0, \frac{175}{36}, 0)$

$X_3 = (25/3, 0, 0, 175/3)$ $\quad X_4 = (0, 25, 0, 0)$

$X_5 = (0, 25, 0, 0)$ $\quad X_6 = (0, 0, -25/6, 325/3)$

$5x_1 + 4x_2 = 100$

$2x_1 + 3x_2 = 75$

$\Rightarrow x_1 = 0 \quad x_2 = 25$

- X_1, X_2, X_3, X_4, X_5 ↳ basic feasible Solⁿ
- X_6 - not feasible as -ve value
- X_1, X_4, X_5 - Degenerate

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$$5x_1 + 2x_3 = 100$$
$$2x_1 + 8x_3 = 75$$

$$18x_1 = 325$$
$$\Rightarrow x_1 = \frac{325}{18}$$

$$\Rightarrow x_3 = \frac{1}{2}\left(100 - 5 \times \frac{325}{18}\right) = \frac{175}{36}$$

→ for finding maximum value of 'z', we need not use value of x_6 as it is not a Basic Feasible Solution.

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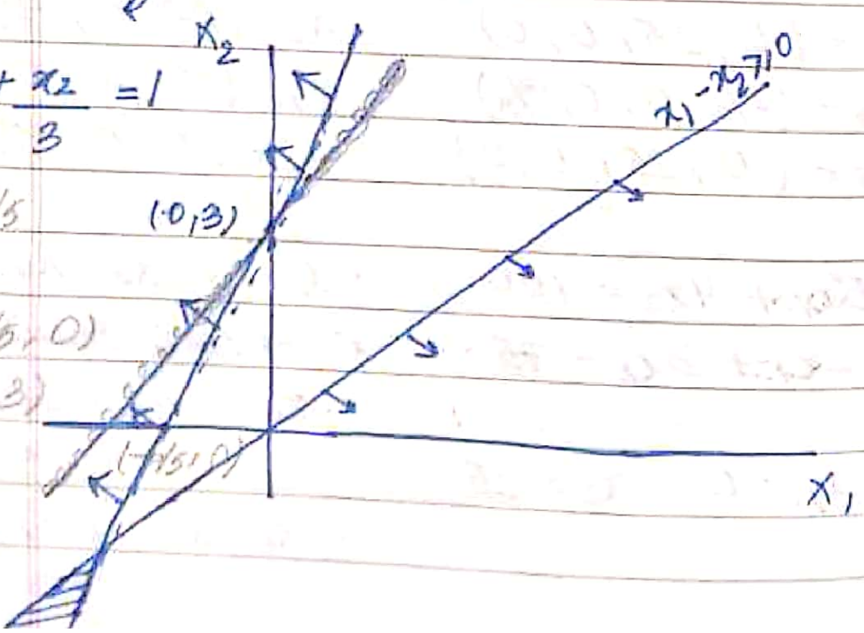
8) Max $Z = 3x_1 + 4x_2$

s.t. $x_1 - x_2 \geq 0$

$\frac{5}{2}x_1 - x_2 \leq -3$ $\quad x_1, x_2 \geq 0$

$$\frac{x_1}{-6/5} + \frac{x_2}{3} = 1$$

- $a = -6/5$
- $b = 3$
- $A(-6/5, 0)$
- $B(0, 3)$



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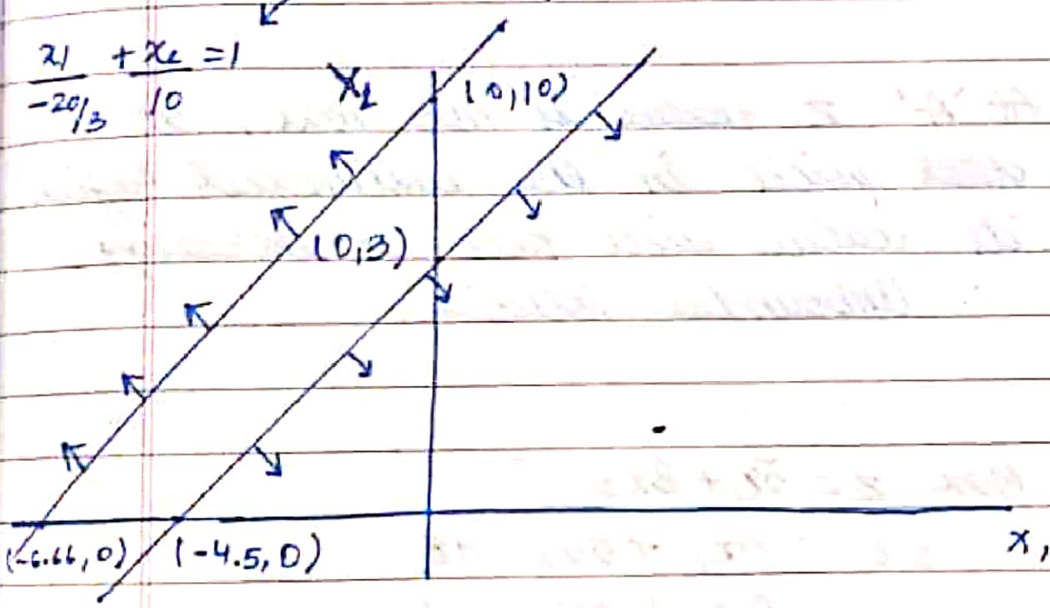
The given problem does not have any feasible solution, as possible region lies in III quad. but it is given that value of x_1 & $x_2 \geq 0$. i.e only possible in I quad.

Q) Max $z = 3x_1 + 2x_2$

s.t. $-2x_1 + 3x_2 \leq 9$

$3x_1 - 2x_2 \leq -20$ $\qquad x_1, x_2 \geq 0$



no feasible solution - as no common region in the I quadrant.

Q) Max $z = -x_1 + 4x_2$

s.t. $3x_1 - x_2 \geq -3$

$-0.3x_1 + 1.2x_2 \leq 3$

$x_1, x_2 \geq 0$

$\frac{x_1}{-1} + \frac{x_2}{3} = 1$ $\qquad \frac{x_1}{-10} + \frac{x_2}{5/2} = 1$

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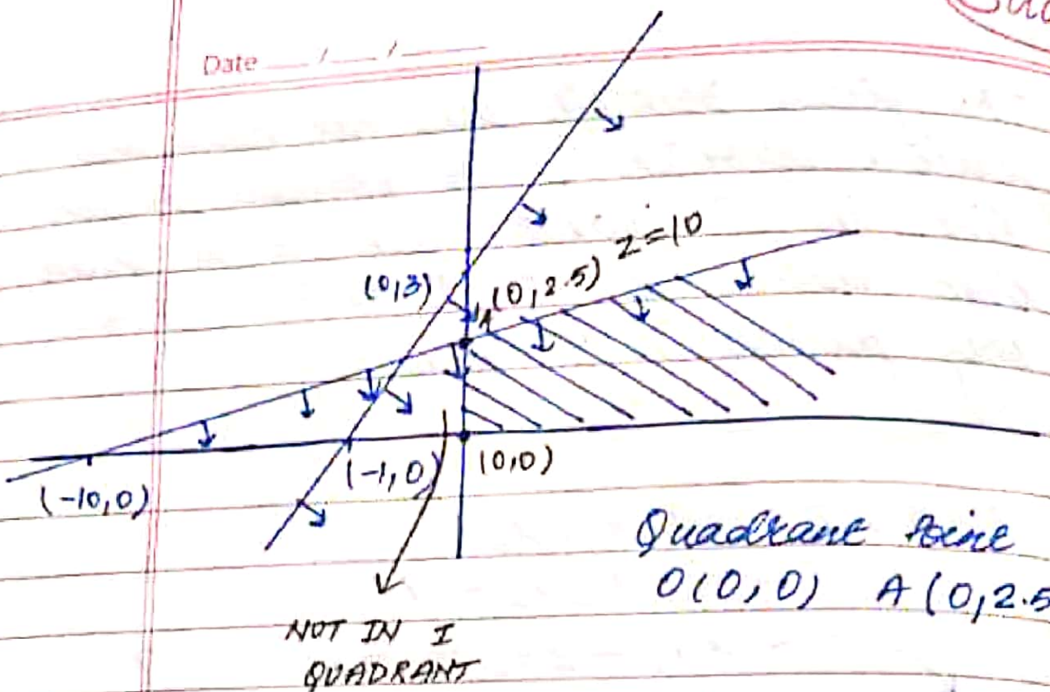


Quadrant Point
O(0,0) A(0,2.5)

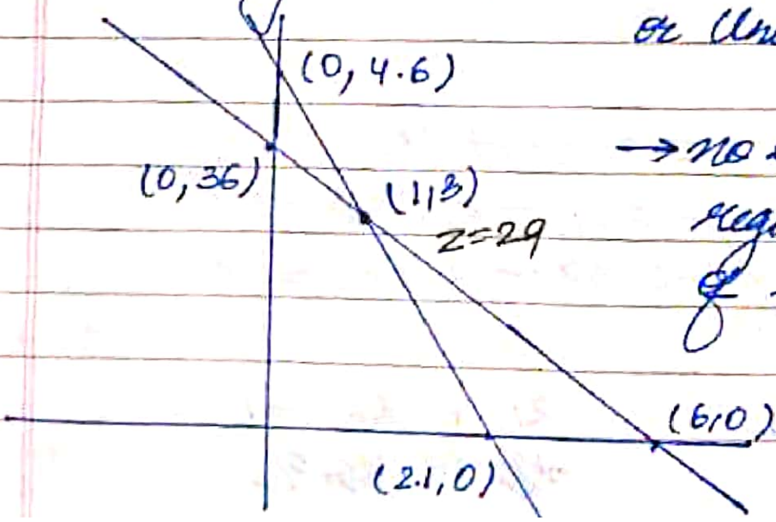
At 'A' z value is not max, at other points in the unbounded region its value will keep on increasing
∴ Unbounded Solution.

8) Max $z = 5x_1 + 8x_2$

s.t. $3x_1 + 5x_2 = 18$

$5x_1 + 3x_2 = 14$ $\qquad x_1, x_2 \geq 0$

as this has '=' sign, no inequality so only two possibilities – No solution or Unique solution



→ no upper or lower region - only point of intersection

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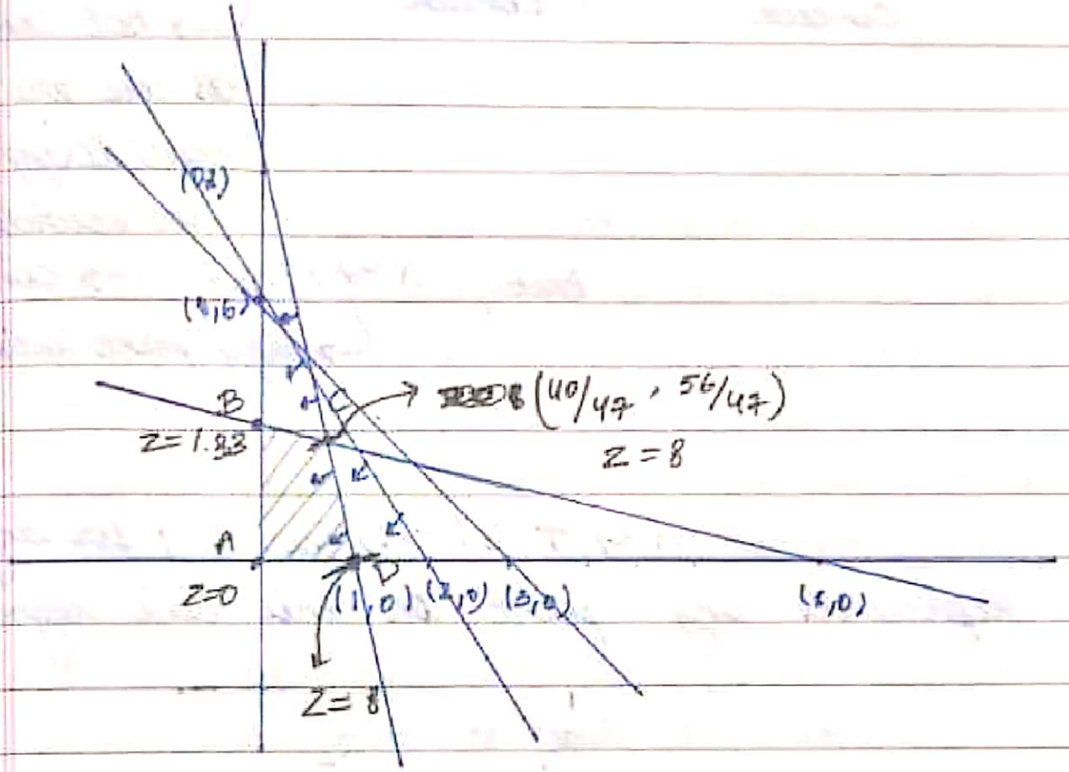
B) Max $z = 8x_1 + x_2$

s.t. $8x_1 + x_2 \le 8$

$2x_1 + x_2 \le 6$

$3x_1 + x_2 \le 6$

$x_1 + 6x_2 \le 8$ $\qquad x_1, x_2 \geq 0$



Max value at two points 'C' & 'D', i.e $z=8$

for Convex Set, given x_1 & x_2

$\lambda x_1 + (1-\lambda)x_2 \qquad 0 \le \lambda \le 1$

here any point on line CD will given maximum value, so infinite points are there.

$\therefore$ Infinite Points.

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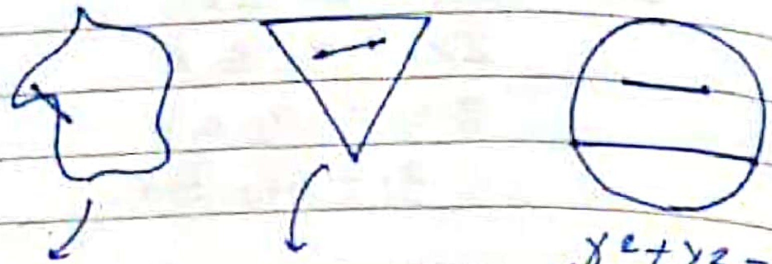
Convex Set



not Convex

Convex

$x^2+y^2=1$

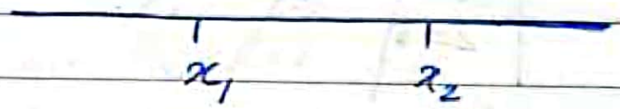
→ not convex, as we are only considering points on boundary

but, $x^2+y^2 \le 1$ → convex

→ any point inside the

$$\lambda x_1 + (1-\lambda)x_2 \quad ; \text{ for any } \lambda$$

represents any point on this line segment



$$\lambda x_1 + (1-\lambda)x_2 \quad ; \quad 0 \le \lambda \le 1$$

represents any point b/w point x_1 & x_2

here,

D (1, 0) C $\left(\frac{40}{47}, \frac{56}{47}\right)$

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So. $\left(1.\lambda + (1-\lambda)\frac{40}{47},\ 0.\lambda + (1-\lambda)\frac{56}{47}\right)$

→ this will give us all the points b/w CD.

⇒ solving this we will get two points, say x_1^* & x_2^*, putting them in z, we get 8.

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8) A dairy firm has two milk plants with daily milk production of 6 million liters and 9 million litres respectively. Each day the firm must fulfill the needs of its three distribution centers, which have milk requirement of 7,5 and 3 million liters respectively.

Cost of shipping one million liter of milk from each plant to each distribution center is given, in hundreds of rupees below. Formulate the linear Problem model to minimize the transportation cost.

		Distribution Centers			
		1	2	3	Supply
Plants	1	2 x_1	3 x_2	11 $6-x_1-x_2$	6
	2	1 $7-x_1$	9 $5-x_2$	6 x_1+x_2-3	9
Demand		7	5	3	

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$$\text{Min } z = 2x_1 + 3x_2 + 11(6 - x_1 - x_2) + 7 - x_1 + 9(5 - x_2) + 6(x_1 + x_2 -$$

$$\Rightarrow -4x_1 - 11x_2 + 100$$

$$\Rightarrow \boxed{100 - 4x_1 - 11x_2 = Z}$$

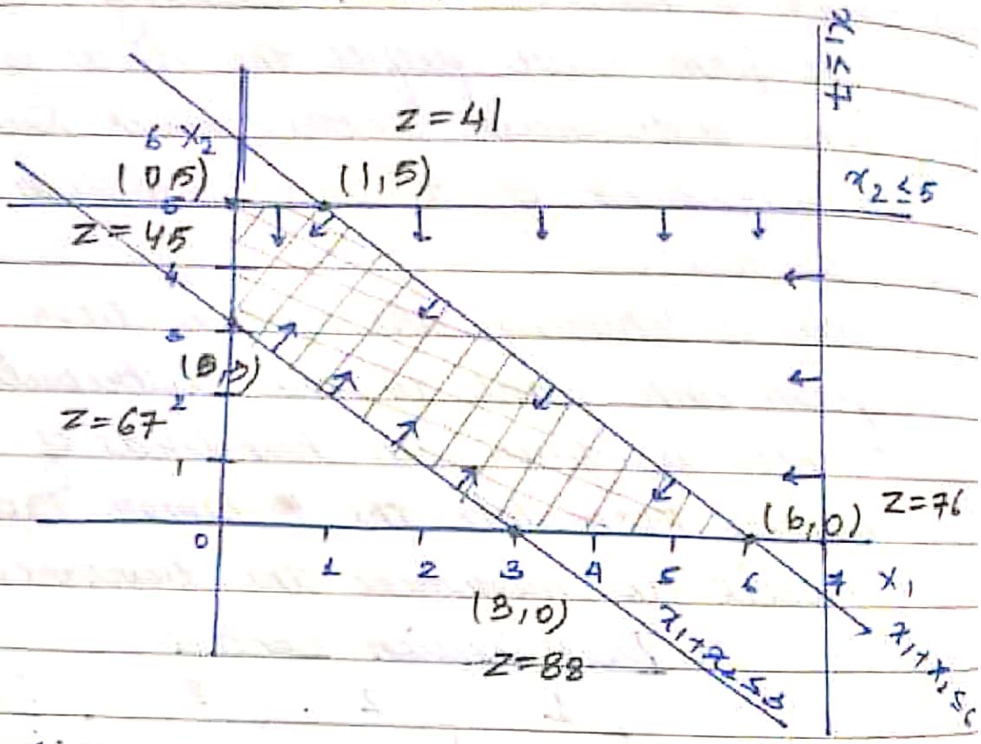
s.t. $\cdot\; x_1 + x_2 \le 6$ $\qquad x_1, x_2 \geq 0$

$(6 - x_1 - x_2 \geq 0)$

$\cdot\; x_1 \le 7$

$\cdot\; x_2 \le 5$

$\cdot\; x_1 + x_2 \geq 3$



Minimum value at $(1,5)$ with $z = 41$

$x_1 = 1 \qquad x_2 = 5.$

		1	2	3	Supply
Plants	1	1	5	0	6
	2	6	0	3	9
		7	5	3	

Demand

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Q) A manufacturer of . . . .

A — B — Profit - max.

x_1 — x_2

$$\text{Max} \quad z = 8x_1 + 7x_2$$

Subject To (S.t.)

$$x_1 \le 20{,}000$$
$$x_2 \le 40{,}000$$
$$x_1 + x_2 \le 45{,}000$$

1000 bottles $\rightarrow$ 3 hours

x_1 bottles $\rightarrow \frac{3}{1000} x_1$ hours

x_2 bottles $\rightarrow \frac{\cancel{1}}{1000} x_2$ hours

$$\frac{3x_1}{1000} + \frac{x_2}{1000} \le 66 \qquad x_1, x_2 \geq 0$$

9/1/23 Simplex Method

$$\text{Max/Min} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

S.t. when $\le b_1$, added ← slack variable

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + s_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + s_2 = b_2$$

artificial variable

$$\vdots$$
$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + a_1 = b_i$$
$$\vdots$$

$x_i \geq 0$

$s_i \geq 0$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n - s_m = b_m$$

when $\geq b_1$, subtracted surplus variable ← surplus variable

Date ___/___/___ Unique Solution.

Q) Max $z = 2x_1 + 5x_2$

s.t. $x_1 + 4x_2 \leq 24$

$3x_1 + x_2 \leq 21$

$x_1 + x_2 \leq 9$ $\quad x_1, x_2 \geq 0$

I Standard form is:

Max $z = 2x_1 + 5x_2 + 0.s_1 + 0.s_2 + 0.s_3$

s.t. ~~$x_1 + 4x_2 + s_1 + 0.s_2 + 0.s_3 = 24$~~

$x_1 + 4x_2 + s_1 = 24$

$3x_1 + x_2 + s_2 = 21$

Cost of Basic Var. $x_1 + x_2 + s_3 = 9$

coeff of Basic Var. in Objective Function (Basic Variable) $x_1, x_2, s_1, s_2, s_3 \geq 0$

→ Cost → Co.eff of Objective Function → Minimum Ratio

		C_j	2	5	0	0	0	Min $\left(\frac{X_B}{B_{ik}}\right)$
B.V.	C_B	X_B (b)	X_1	X_2	S_1	S_2	S_3	
s_1	0	24	1	4	1	0	0	6 ←
s_2	0	21	3	1	0	1	0	21
s_3	0	9	1	1	0	0	1	9
$z_j - c_j$			-2	-5 ↑	0	0	0	

Key column ← Key Row ← always '0'

Key Element

$z_j = C_{B1} X_1 + C_{B2} X_2 + C_{B3} X_3$

$c_j = '2'$ → make all '0' below Key Ele.

↳ We have to make '1' at Key Element

$24/4 = 6 \quad 21/1 = 21 \quad 9/1 = 1$

[Key Row divided by key element]

x_2	5	6	1/4	1	1/4	0	0	24
s_2	0	15	11/4	0	-1/4	1	0	60/11
s_3	0	3	3/4 ↑	0	-1/4	0	1	4 ←

For filling rest of the rows :-
{key − key ratio} × corr. element in key row

eg. Row 2 column 3

$$3 - \frac{1}{4} \times 4 = \frac{11}{4}$$

continue ...

$z_j - c_j$		30	$-[illegible]/4$	0	5/4	0	0
* x_2	5	* 5	0	1	1/3	0	−1/3
s_2	0	4	~~1~~ 0	0	2/3	1	−11/3
* x_1	2	* 4	~~0~~ 1	0	−1/3	0	4/3
$z_j - c_j$		33	0	0	1	0	1

→ Optimal Value

Stop when all the values become ≥ 0

Max $Z = 33$

$x_1 = 4$ $x_2 = 5$

* if any value of $\text{Min}\left(\frac{x_B}{B_{ik}}\right)$ becomes −ve or '0', then -
Unbounded Solution

* if value of Basic Variable becomes '0' - Infinite Solution (alias Alternative Solution)

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Q) Max $z = 2x_1 + x_2$

S.t. $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

x_1, x_2

The standard form is :

Max $z = 2x_1 + x_2 + 0.s_1 + 0.s_2$

s.t.

$x_1 - x_2 + s_1 = 10$

$2x_1 - x_2 + s_2 = 40$ x_1, x_2, s_1, s_2

	C_j		2	1	0	0	Min $\left(\frac{X_B}{B_{ik}}\right)$
B.V.	C_B	X_B	x_1	x_2	s_1	s_2	
s_1	0	10	1	-1	1	0	10 →
s_2	0	40	2	-1	0	1	20
$z_j - c_j$		0	$-2\uparrow$	-1	0	0	
x_1	2	10	1	-1	1	0	-10
s_2	0	20	0	1	-2	1	20
$z_j - c_j$		20	0	$-3\uparrow$	2	0	
x_1	2	30	1	0	-1	1	
x_2	1	20	0	1	-2	1	
$z_j - c_j$		80	0	0	$-4\uparrow$	3	

We have -4, but all the elements of the column are also negative.

∴ Unbounded solution

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8) Min $z = -40x_1 - 100x_2$ $\quad x(A+B)$

s.t.

$10x_1 + 5x_2 \le 250$ $\quad A+B \le C$

$2x_1 + 5x_2 \le 100$ $\quad Max = x \cdot C$

$2x_1 + 3x_2 \le 90$ $\quad x_1, x_2 \ge 0$

Convert to Maximization Problem

The Standard form is

Max $z^* = 40x_1 + 100x_2 + 0.s_1 + 0.s_2 + 0.s_3$

s.t.

$10x_1 + 5x_2 + s_1 = 250$

$2x_1 + 5x_2 + s_2 = 100$

$2x_1 + 3x_2 + s_3 = 90$

$x_1, x_2, s_1, s_2, s_3 \ge 0$

	c_j		40	100	0	0	0	Min $\left(\frac{x_B}{a_{ie}}, a_{ie} > 0\right)$
B.V.	c_B	x_B	x_1	x_2	s_1	s_2	s_3	
s_1	0	250	10	5	1	0	0	50
s_2	0	100	2	5	0	1	0	20 →
s_3	0	90	2	3	0	0	1	30
$z_j - c_j$		0	-40	-100	0	0	0	
s_1	0	150	8	0	1	-1	0	75/4 →
x_2	100	20	2/5	1	0	1/5	0	50
s_3	0	30	4/5	0	0	-3/5	1	75/2
$z_j - c_j$		2000	0^* ↑	0	0	20	0	

all ≥ 0 → more than one Min value

Basic variables - s_1, s_2, s_3 $\quad x_1 = 0$

Non Basic variables - x_1, x_2 $\quad x_2 = 200$

Min $z = -2000$

Here, $x_1 = 0$, but x_1 is a non-basic variable

$\therefore$ Alternative Solution / Infinite

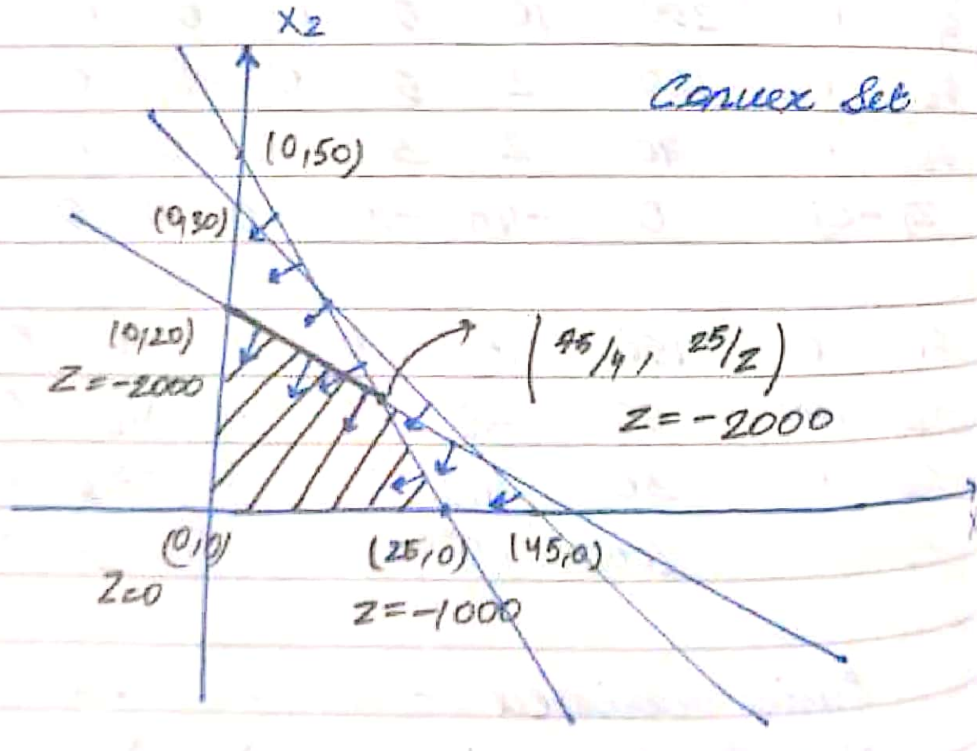
x_1	40	75/4	1	0	1/8	-1/8	0
x_2	100	25/2	0	1	-1/20	1/4	0
S_3	0	15	0	0	-1/10	-1/2	1
$Z_j - C_j$		2000	0	0	0	20	0

Value is not increasing, as there is ~~an~~ Alternative Solution (we stop here)

$\therefore$ Infinite Solution

$x_1 = \frac{75}{4}$ $\quad x_2 = \frac{25}{2}$ $\quad$ Min $z = -20$

~~Q) Min z = $-40x_1 - 100x_2$~~ Graphical M



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$$x_1^* = \lambda.0 + (1-\lambda)\,75/4$$
$$= 75/4\,(1-\lambda)$$

$$x_2^* = \lambda.20 + (1-\lambda)\,25/2$$

$$0 \le \lambda \le 1$$

H.W. 8)

$$\text{Max } Z = x_1 + 2x_2$$ (Alternative solution)

s.t.

$$-x_1 + 2x_2 \le 8$$
$$x_1 + 2x_2 \le 12$$
$$x_1 - 2x_2 \le 3 \qquad x_1, x_2 \geq 0$$

<u>Big M Method</u> <u>no feasible sol^n</u>

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$$\text{Max } Z = 5x_1 + 3x_2$$
$$2x_1 + x_2 \le 1$$
$$x_1 + 4x_2 \geq 6 \qquad x_1, x_2 \geq 0$$

<u>The standard form is</u> use very large possible numbers

$$\text{Max } Z = 5x_1 + 3x_2 + 0.s_1 + 0.s_2 - MA_1$$
$$2x_1 + x_2 + S_1 \cancel{+ a_1} \cancel{= 2} = 1$$
$$x_1 + 4x_2 - s_2 + a_1 = 6$$

introduced as there is no positive added term ← artificial var.

$$x_1, x_2, s_1, s_2, a_1 \geq 0$$

* We add 'artificial variable' to form unit matrix, when $\geq b_i$, i.e when 'surplus variable' is added

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their two form unit matrix

B.V.	C_B	X_B	C_j: 5 x_1	3 x_2	0 S_1	0 S_2	-M A_1	Min X_B/a_{ik}
s_1	0	1	2	$\boxed{1}$	1	0	0	1
a_1	-M	6	1	4	0	-1	1	1.5
$z_j - c_j$			-M-5	-4M-3	0	M	-M	

x_2	3	1	2	1	1	0	0
a_1	-M	2	-7	0	-4	-1	1
$z_j - c_j$			7M+1	0	4M-3	M	0

* all the values are greater than 0 now, so we stop.

* Since 'a_1' appears in the basic at positive level so the given problem has No Feasible solution

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Two Phase Method

* artificial variable comes for $\geq$ eqn only (A_1, A_2)

$$\text{Min } z = x_1 + x_2$$

$$\text{s.t. } 2x_1 + x_2 \geq 4$$

$$x_1 + 7x_2 \geq 7$$

$$x_1, x_2 \geq 0$$

The standard form for phase I is

$$\text{Max } z = -a_1 - a_2$$

$$\text{s.t. } 2x_1 + x_2 - s_1 + a_1 = 4$$

$$x_1 + 7x_2 - s_2 + a_2 = 7$$

$$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$$

Optimization function will only contain artificial variables

Phase I

B.V.	C_B	X_B	X_1	X_2	S_1	S_2	A_1	A_2	Min ratio X_B / a_{ij}
C_j			0	0	0	0	-1	-1	
a_1	-1	4	2	1	-1	0	1	0	4
a_2	-1	7	1	7	0	-1	0	1	1
$z_j - c_j$		-11	-3	-8	1	1	0	0	
a_1	-1	3	13/7	0	-1	1/7	1	~~-1/7~~	21/13
x_2	0	1	1/7	1	0	-1/7	0	~~1/7~~	7
$z_j - c_j$		-3	-13/7	0	1	-1/7	0		
x_1	0	21/13	1	0	-7/13	1/13			
x_2	0	10/13	0	1	1/13	-2/13			
$z_j - c_j$			0	0	0	0			

won't calculate for this

$z_j - c_j \geq 0$, then we stop

C_j			-1	-1	0	0
B.V.	C_B	X_B	X_1	X_2	S_1	S_2
x_1	-1	21/13	1	0	-7/13	1/13
x_2	-1	10/13	0	1	1/13	-2/13
$Z_j - C_j$		-31/13	0	0	6/13	1/13

Here $Z_j - C_j \geq 0$ so we stop here

$$x_1 = \frac{21}{13} \qquad x_2 = \frac{10}{13} \qquad \text{Min } Z = \frac{31}{13}$$

8) $\text{Max } Z = 5x_1 + 3x_2$

s.t. $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6 \qquad x_1, x_2 \geq 0$

Using Big - M

C_j			0	0	0	0	-1	Min ( )
B.V.	C_B	X_B	X_1	X_2	S_1	S_2	A_1	
S_1	0	1	2	1	1	0	0	1 →
a_1	-1	6	1	4	0	-1	1	1.5
$Z_j - C_j$			-1	-4 ↑	0	1	0	
x_2	0	1	2	1	1	0	0	
a_1	-1	2	-7	0	-4	-1	1	
$Z_j - C_j$			7	0	4	1	0	

Here $Z_j - C_j \geq 0$, so we stop.

Next checking value of a_1 → it is greater than '0' so no solution.

If $a_1 = 0$ → go to next phase

a_1 cannot be negative

16/1/22

Q)

$$\text{Max } z = 2x_1 + x_2$$

$$\text{s.t. } 4x_1 + 3x_2 \le 12$$
$$4x_1 + x_2 \le 8$$
$$4x_1 - x_2 \le 8$$

$x_1, x_2 \ge 0$

		c_j	2	1	0	0	0	
B.V.	C_B	X_B	X_1	X_2	S_1	S_2	S_3	Min( )
s_1	0	12	4	3	1	0	0	3
s_2	0	8	4	1	0	1	0	2
s_3	0	8	$\boxed{4}$	-1	0	0	1	2
	$z_j - c_j$		-2 ↑	-1	0	0	0	

s_1	0	4	0	4	1	0	-1	1
s_2	0	0	0	$\boxed{2}$	0	$\cancel{0}$ 1	-1	0 →
x_1	2	2	1	-1/4	0	0	1/4	—
	$z_j - c_j$		0	-3/2 ↑	0	0	1/2	

s_1	0	4	0	0	1	$\cancel{0}$ -2	$\boxed{1}$	4 →
x_2	1	0	0	1	0	$\cancel{0}$ 1/2	-1/2	—
x_1	2	2	1	0	0	$\cancel{0}$ 1/8	1/8	16
	$z_j - c_j$	4	0	0	0	-3/4	-1/4 ↑	

* <u>Degeneration</u>: Basic variable '0' in X_B column.

It indicates some slack variable will leave and then come back.

* s_3 leaves and then comes back again.

s_3	0	4	0	0	1	-2	-1
x_2	1	2	0	1	1/2	-1/2	0
x_1	2	3/2	1	0	-1/8	3/8	0
$z_j - c_j$			0	0	1/4	1/4	0

$x_1 = \frac{3}{2}$ $x_2 = 2$ Max $z = 5$

* here 2 and 2 are same, so for only these two values we calculate ~~x_B / s_1~~ s_1 / x_2 to get to sol^n.

* then again 0 == 0, so we again calculate ~~$x_B /$~~ ~~s_2 / x_2~~ s_2 / x_1, for only these two values.

Duality

* $\geq$ to $\leq$ for all values
* $\ldots = y_i$; y_i - unrestricted variable

Primal

Max $z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots$

Dual

Min $w = b_1 y_1 + b_2 y_2$

$a_{11} x_1 + a_{12} x_2 + \ldots a_n x_n \leq b_1$

$\leq b_2$

$= y_i$

* If Max, all should be $\leq$
* If Min. all should be $\geq$

Primal Problem

Q) $$\text{Min } z = 2x_1 + 9x_2 + x_3$$
$$\text{s.t. } x_1 + 4x_2 + 2x_3 \geq 5$$
$$3x_1 + x_2 + 2x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

Dual Problem

$$\text{Max } w = 5y_1 + 4y_2$$
$$\text{s.t. } y_1 + 3y_2 \leq 2$$
$$4y_1 + y_2 \leq 9$$
$$2y_1 + 2y_2 \leq 1$$
$$y_1, y_2 \geq 0$$

if Primal Problem

$$\text{s.t. } x_1 + 4x_2 + 2x_3 \leq 5$$

$\Rightarrow$

$$-x_1 - 4x_2 - 2x_3 \geq -5$$

ALSO $\text{Max } Z = -5y_1 \ldots (\text{not } y_1')$

if $\text{s.t. } 3x_1 + x_2 + 2x_3 = 4$

$\Rightarrow$

$$3x_1 + x_2 + 2x_3 \leq 4$$

and

$$3x_1 + x_2 + 2x_3 \geq 4$$

$\Rightarrow$

$$-3x_1 - x_2 - 2x_3 \geq -4$$

and

$$3x_1 + x_2 + 2x_3 \geq 4$$

ALSO. $\text{Max } z = 5y_1 - 4y_2' + 4y_2''$

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s.t. $y_1 - 3y_2' + 3y_2'' \leq 2$
$4y_1 - y_2' + y_2'' \leq 9$
$2y_1 - 2y_2' + 2y_2'' \leq 1$

$y_1, y_2', y_2'' \geq 0$

18/1/23 Primal problem

Q) $\text{Max } z = 3x_1 + 17x_2 + 9x_3$

s.t. $x_1 - x_2 + x_3 \geq 3$
$-3x_1 + 2x_3 \leq 1$

$x_i \geq 0 \quad i = 1, 2, 3$

$\rightarrow -x_1 + x_2 - x_3 \leq -3$

Dual problem

$\text{Min } w = -3y_1 + y_2$

s.t.

$-y_1 - 3y_2 \geq 3$
$y_1 + 0 \cdot y_2 \geq 17$
$-y_1 + 2y_2 \geq 9$

$y_1, y_2 \geq 0$

Primal problem

If $x_1 - x_2 + x_3 = 3$

$\rightarrow x_1 - x_2 + x_3 \leq 3$

AND

$x_1 - x_2 + x_3 \geq 3$
$\rightarrow -x_1 + x_2 - x_3 \leq -3$

$-3x_1 + 2x_3 \leq 1$

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Dual Problem

$$\text{Min } w = 3y_1 - 3y_2 + y_3$$

s.t. $3(y_1 - y_2) + y_3 \Rightarrow 3y_4 + y_3$

$(y_1 - y_2) - 3y_3 \geq 3 \Rightarrow y_4 - 3y_3 \geq 3$

$-(y_1 - y_2) \geq 17$ $\quad -y_4 \geq 17$

$(y_1 - y_2) + 2y_3 \geq 9$ $\quad y_4 + 2y_3 \geq 9$

$y_1, y_2, y_3 \geq 0$ $\quad$ where $y_4 = y_1$

$y_1 = 1 \quad y_2 = 2$ $\quad y_4$ is unrestricted

$$\text{Max } z = 3x_1 + 17x_2 + 9x_3$$

s.t. $x_1 - x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$x_i \geq 0,\ i = 1, 2$ and x_3 is unrestricted

Let $x_3 = x_3' - x_3''$, where $x_3', x_3'' \geq 0$

$$\text{Max } z = 3x_1 + 17x_2 + 9(x_3' - x_3'')$$

s.t. $-x_1 + x_2 - x_3' + x_3'' \leq -3$

$-3x_1 + 2(x_3' - x_3'') \leq 1$

x_1, x_2, x_3', x_3''

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Dual problem

$$\text{Min } w = -3y_1 + y_2$$

s.t. $-y_1 - 2y_2 \geq 3$

$y_1 \geq 17$

$-y_1 + 2y_2 \geq 9$

$y_1 - 2y_2 \geq -9$ $\quad -y_1 + 2y_2 = 9$

$\rightarrow -y_1 + 2y_2 \leq 9$

* If one problem has feasible solⁿ then other problem has feasible solⁿ
* If one problem has unbounded solⁿ, then other one has no feasible solⁿ
* value of obj. fⁿ of Primal problem = value of obj. fⁿ of Dual problem

$\therefore$ $\boxed{\text{Max } z = \text{Min } w}$

Q)

$$\text{Max } z = 5x_1 + 8x_2$$

s.t. $x_1 + x_2 \leq 2$

$x_1 - 2x_2 \geq 0$

$-x_1 + 4x_2 \leq 1$

$x_1, x_2 \geq 0$

The dual of the given:

$$\text{Min } w = 2y_1 + y_3$$

s.t. $y_1 - y_2 - y_3 \geq 5$

$y_1 + 2y_2 + 4y_3 \geq 8$

$y_1, y_2, y_3 \geq 0$

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		C_j	-2	0	-1	0	0	$-M$	$-M$	
B.V.	C_B	Y_B	Y_1	Y_2	Y_3	S_1	S_2	A_1	A_2	Min R
a_1	$-M$	5	1	-1	-1	-1	0	1	0	
a_2	$-M$	8	1	2	$\boxed{4}$	0	-1	0	1	
$z_j - c_j$			$-2M+2$	$-M$	$-3M+1$	M	M	0	0	
a_1	$-M$	7	$5/4$	$-1/2$	0	-1	$-1/4$	1	$1/4$	$28/5$
y_3	-1	2	$1/4$	$1/2$	1	0	$-1/4$	0	$1/4$	8
$z_j - c_j$			$\frac{-5M}{4}+\frac{7}{4}$	$\frac{M}{2}-\frac{1}{2}$	0	M	$\frac{M}{4}+\frac{1}{4}$	0	$\frac{3M}{4}-\frac{1}{4}$	
y_1	-2	$28/5$	1	$-2/5$	0	$-4/5$	$-1/5$	$4/5$	$1/5$	
y_3	-1	$3/5$	0	$3/5$	1	$1/5$	$-1/5$	$-1/5$	$-1/5$	
$z_j - c_j$		$-59/5$	0	$1/5$	0	$7/5$	$3/5$	$M-7/5$	$M-3/5$	

Dual sol^n

$$\text{Min } w = \frac{59}{5} \qquad y_1 = \frac{28}{5} \qquad y_2 = 0 \qquad y_3 = \frac{3}{5}$$

same value

Primal sol^n

$$\text{Max } z = \frac{59}{5} \qquad x_1 = \frac{7}{5} \qquad x_2 = \frac{3}{5}$$

23/1/23 MODI Method

$\boxed{u_i + v_j = c_{ij}}$
(for Basic Cell)

cost (c_{ij}) — Weight

	W_1	W_2	W_3	W_4	W_5	u_i
F_1	7 $\boxed{5}$	6	4 $\boxed{15}$	5 $\boxed{20}$	9	0 → starting here
F_2	8	5 $\boxed{30}$	6 $\boxed{0}$	7	8	2
F_3	6 $\boxed{15}$	8	9	6	5 $\boxed{5}$	−1
F_4	5 $\boxed{10}$	7	7	8	6	−2
v_j	7	3	4	5	6	

No. of allocations $= m + n - 1$
F's ↙ ↳ W's
$= 4 + 5 - 1$
$= 8.$

But here we have only '7' allocations, so we select the smallest cost and make quantity '0'.
- here we selected one of the '6' and made it '0'.

Basic cell - the ones which are already allocated

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If multiple cells have the smallest cost, then we choose a cell which will give us the minimum value.

Q)

					u_i
7 $\boxed{5}$	6 $\boxed{0}$	4 $\boxed{15}$	5 $\boxed{20}$	9 (3)	7
8 (2)	5 $\boxed{30}$	6 (8)	7 (5)	8 (8)	6
6 $\boxed{15}$	8 (8)	9 (6)	6 (2)	5 $\boxed{5}$	6
5 $\boxed{10}$	7 (8)	7 (5)	8 (5)	6 (2)	5
v_j: 0	-1	-3	-2	-1	

starting here

$$\boxed{C_{ij} - (u_i + v_j)}$$

(for Non-Basic cell)

<u>Answer</u>

$x_{11} = 5$, $x_{12} = 0$, $x_{13} = 15$, $x_{14} = 20$

$x_{22} = 30$, $x_{31} = 15$, $x_{35} = 5$

$x_{41} = 10$

<u>Min z</u> = 510 (Multiplication of Basic Cells)

$\hookrightarrow (7 \times 5) + (6 \times 0) + (4 \times 15) + \ldots$

Non-Basic Cells – the ones which are not allocated.

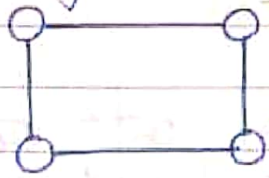
Q)

	W_1	W_2	W_3	W_4	W_5	u_j
F_1	7 (−) $\boxed{5}$	6, 3	4 (+) $\boxed{15}$	5 $\boxed{20}$	9, 3	0
F_2	8 (+) −1	5 $\boxed{30}$	6 (−) $\boxed{10}$	7, 0	8, 0	2
F_3	6 $\boxed{15}$	8, 6	9, 6	6, 2	5 $\boxed{5}$	−1
F_4	5 $\boxed{10}$	7, 6	7, 5	8, 5	6, 2	−2
v_j	7	3	4	5	6	

Calculated

If a non-basic cell is coming negative, then we do not have optimal solution. We have to go for another step.

Take most negative as Basic cell, and form



(from the point you start, you should end there)

Then,

allocate alternate sign

* The non-basic cell which have −ve ~~sign~~ value, is given '+ve' sign and then use alternate values.

						u_j
	7 $\boxed{5}$	6, 2	9 $\boxed{15}$	5 $\boxed{20}$	9, 3	7
	8 $\boxed{0}$	5 $\boxed{30}$	6, 1	7, 1	8, 1	8
	6 $\boxed{15}$	8, 5	9, 6	6, 2	5 $\boxed{5}$	6
	5 $\boxed{10}$	7, 5	7, 5	6, 5	6, 2	5
v_j	0	−3	−3	−2	−1	

start here

- allocate the min. value of '−ve' sign to the '−ve' value, and then solve.

i.e here '0' of '6' is allocated to '8'

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Q)

	1	2	3	4	5	Supply
A	4 [10]	3	1 [30]	2 [40]	6	80 (1)
B	5	2 [60]	3	4	5	60 (1)
C	3 [30]	5	6	3	2 [10]	40 (1)
D	2 [20]	4	4	5	3	20 (1)
Demand	60	60	30	40	10	200
	(1)	(1)	(2)	(1)	(1)	
	(1)	(1)	x	(1)	(1)	
	(1)	x		(1)	(1)	
	(1)			x	(1)	
					x	

						u_i
	4 [10] (−)	3 (3)	1 [30] (+)	2 [40]	6 (3)	0
	5 (+) (−1)	2 [60]	3 [0] (−)	4 (0)	5 (0)	2
	3 [30]	5 (6)	6 (6)	3 (2)	2 [10]	−1
	2 [20]	4 (5)	4 (5)	5 (5)	3 (2)	−2
v_j	4	0	1	2	3	

						u_i
	4 [10]	3 (2)	1 [30]	2 [40]	6 (3)	4
	5 [0]	2 [60]	3 (1)	4 (1)	5 (1)	5
	3 [30]	5 (5)	6 (6)	3 (2)	2 [10]	3
	2 [20]	4 (5)	4 (5)	5 (5)	3 (2)	2
v_j	0	−3	−3	−2	−1	

$x_{11} = 10$ $x_{13} = 30$ $x_{14} = 40$ $x_{21} = 0$

$x_{22} = 60$ $x_{31} = 30$ $x_{35} = 10$ $x_{41} = 20$

Min Z = 420

Q)

$x_{12} = 30$ $x_{14} = 40$

$x_{22} = 5$ $x_{23} = 50$

$x_{31} = 35$ $x_{34} = 5$

Min Z = 1160

(Multiply only Basic cells)

8)

(2) ← (8) ← x

x

(1) (1)

(1) (1)

					u_i
	6 (0)	1 (−7)	9 ⊖ [50]	3 ⊕ [20]	−4
	11 ⊖ [55]	5 (−8)	2 ⊕ (−12)	8 (0)	1
	10 ⊕ [30]	12 [35]	4 (−9)	7 ⊖ [15]	0
v_j	10	12	13	7	

					u_i
	6 (−12)	1 ⊕ (−19)	9 ⊖ [25]	3 [45]	8
	11 ⊖ [30]	5 (−8)	2 ⊕ [25]	8 (12)	1
	10 ⊕ [55]	12 ⊖ [35]	4 (3)	7 (12)	0
v_j	10	12	1	−5	

					u_i
	6 (7)	1 [25]	9 (19)	3 [45]	0
	11 ⊖ [5]	5 ⊕ (−8)	2 [50]	8 (−7)	12
	10 ⊕ [80]	12 ⊖ [10]	4 (3)	7 (−7)	11
v_j	−1	1	−10	3	

					u_i
	6 (7)	1 ⊕ [25]	9 (11)	3 ⊖ [45]	1
	11 (8)	5 [5]	2 [50]	8 (1)	5
	10 [85]	12 ⊖ [5]	4 (−5)	7 ⊕ (−7)	~~10~~ 12
v_j	−2~~0~~	0	−3	2	

$x_{12} = 30$ $x_{14} = 40$

$x_{22} = 5$ $x_{23} = 50$

$x_{31} = 85$ $x_{34} = 5$

Min z = 1160

(Multiply only Basic cells)

					u_i
	6 (0)	1 [30]	9 (11)	3 [40]	0
	11 (1)	5 [5]	2 [50]	8 (1)	4
	10 [85]	12 (7)	4 (2)	7 [5]	4
v_j	6	1	−2	3	

C_{ij}

Basic cell

	W_1	W_2	W_3	W_4	W_5	u_i
f_1	7 − $\boxed{5}$	6 3	4 + $\boxed{15}$	5 $\boxed{20}$	9 1	0
f_2	8 −1	5 $\boxed{30}$	6 − $\boxed{0}$	7 0	8 0	6−4 =2
f_3	6 $\boxed{15}$	8 6	9 6	6 2	5 $\boxed{5}$	−1
f_4	5 $\boxed{10}$	7 6	7 5	8 5	6 2	−2
v_j	7	5−2 =3	4	5	6	

most −ve

MODI METHOD

$\boxed{u_i + v_j = C_{ij}}$ — Basic Cell

$x +\ \ = 5$

$\boxed{C_{ij} - (u_i + v_j) \Rightarrow}$ Non-Basic Cell

$u_1 + v_1 = C_{11} = 7$

$u_1 + v_3 = C_{13} = 4$

$u_1 + v_4 = C_{14} = 5$

$u_2 + v_2 = C_{22} = 5$

$u_3 + v_1 = C_{31} = 6$

$u_3 + v_5 = C_{35} = 5$

						u_i
	7 $\boxed{5}$	6 $\boxed{0}$	4 $\boxed{15}$	5 $\boxed{20}$	9 3	7
	8 2	5 $\boxed{30}$	6 3	7 3	8 3	6
	6 $\boxed{15}$	8 3	9 6	6 2	5 $\boxed{5}$	6
	6 $\boxed{10}$	7 3	7 5	8 5	6 2	5
v_j	0	−1	−3	−2	−1	

$x_{11} = 5 \quad x_{12} = 0 \quad x_{13} = 15$

$x_{14} = 20 \quad x_{22} = 30$

$x_{31} = 15, \quad x_{35} = 5, \quad x_{41} = 10$

Min $z = 510.\ (35 + 60 + 100 + 150 + 25 + 50 + 90)$

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Q

					Supply	
4 $\boxed{10}$	3 $\boxed{\cancel{30}}$	1 $\boxed{30}$	2 $\boxed{40}$	6	80	(1) (1) (2) (2) — x
5	2 $\boxed{60}$	3	4	5	60	(1) (2) x
3 $\boxed{30}$	5	6	3	2 $\boxed{10}$	40	(1) (1) (1)
2 $\boxed{20}$	4	4	5	3	20	(1) (1) (1)
Demand → 60	60	30	40	10	$\boxed{200}$	

(1)	(1)	(2) ↑	(1)	(1)
(1)	(1)	x	(1)	(1)
(1)	x		(1)	(1)
(1)			x	(1)

$m+n-1 \Rightarrow 4+5-1 = 8$

Here, 7 basic cell $7<8$ so it is degenerated sol^n.

most -ve →

					u_i
4 − $\boxed{10}$	3 (3)	1 + $\boxed{30}$	2 $\boxed{40}$	6 (3)	0 (Initial)
5 + (−1)	2 $\boxed{60}$	3 − $\boxed{0}$	4 (0)	5 (0)	2 (3−1)
3 $\boxed{30}$	5 (6)	6 (6)	3 (2)	2 $\boxed{10}$	−1 (3−4)
2 $\boxed{20}$	4 (6)	4 (5)	5 (5)	3 (2)	−2 (2−4)
$v_j \Rightarrow$ 4 (4−0)	0 (2−2)	1 (1−0)	2 (2−0)	3	

					u_i
4 $\boxed{10}$	3 (2)	1 $\boxed{30}$	2 $\boxed{40}$	6 (3)	4 (4−0)
5 $\boxed{0}$	2 $\boxed{60}$	3 (1)	4 (1)	5 (1)	5 (5−0)
3 $\boxed{30}$	5 (5)	6 (6)	3 (2)	2 $\boxed{10}$	3 (3−0)
2 $\boxed{20}$	4 (5)	4 (5)	5 (5)	3 (2)	2 (2−0)
v_j 0 (Initial)	−3 (2−5)	−3 (1−4)	−2 (2−4)	−1 (2−3)	

All will be +ve so,

Min $\Rightarrow 4\times10 + 1\times30 + 2\times40 + 5\times0 + 2\times60 + 3\times30 + 2\times10 + 2\times20$

$\Rightarrow 40 + 30 + 80 + 0 + 120 + 90 + 20 + 40$

$\Rightarrow \boxed{420}$

Scanned with CamScanner

Q

				u_i
6 (0)	1 (-7)	9 [80]	3 [20]	-4
11 [55]	5 (-8)	2 (-12)	8 (0)	1
10 [30]	12 [25]	4 (-9)	7 [25]	0
v_j: 10	12	13	7	

				u_i
6 (-12)	1 + (-19)	9 − [25]	3 [45]	8
11 − [30]	5 (-8)	2 + [25]	8 (12)	1
10 + [55]	12 − [25]	4 (3)	7 (12)	0
$v_j \to$ 10	12	1	-5	

				u_i	
6 (7)	1 [25]	9 (19)	3 [45]	0	
11 − [8]	5 + (-8)	2 [80]	8 (-7)	12	(11-(-1))
10 + [80]	12 − [10]	4 (3)	7 (-7)	11	(12-1)
$v_j \to$ -1 (10-11)	1 (1-0)	-10 (2-12)	3 (3-0)		

				u_i
6 (7)	1 + [25]	9 (11)	3 − [45]	1
11 (8)	5 [5]	2 [80]	8 (1)	5
10 [85]	12 − [8]	4 (-8)	7 + (-7)	12
$v_j \to$ -2	0	-3	2	

				u_i
6 (6)	1 [30]	9 (11)	3 [40]	0
11 (1)	5 [5]	2 [50]	8 (1)	4
10 [85]	12 (7)	4 (2)	7 [5]	4
v_j: 6	1	-2	3	

$1 \times 30 + 3 \times 40 + 5 \times 5 + 2 \times 50 + 10 \times 85 + 7 \times 5$

$30 + 120 + 25 + 100 + 850 + 35$

$\Rightarrow \underline{1160}$

160
120
850
130
1160

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*) Unbalanced Transportation problem.

	1	2	3	4	Supply
A					50
B					70
C					30
D					50
Demand	25	35	105	20	200

Here, demand = 185 and supply = 200 so,
demand < supply, i.e. we add a column with 0 cost and.

	1	2	3	4	5	Supply
A	2 [25]	4	6 [25]	11	0	50
B	10	8	7 [55]	5	0 [15]	~~70~~ 55
C	13	3 [30]	9	12	0	30
D	4	6 [5]	8 [25]	3 [20]	0	50
Demand	25	35	105	20	15	200

find u_i & v_j then $\boxed{C_j - (u_i + v_j)}$ for non-basic cell

	1	2	3	4	5	u_i
A	2 [25]	4 (0)	6 [25]	11 (10)	0 (1)	6
B	10 (7)	8 (3)	7 + [55]	5 (3)	0 − [15]	7
C	13 (12)	3 [30]	9 (4)	12 (12)	0 (2)	(3−(−2)) = 5
D	4 (0)	6 [5]	8 − [25]	3 [20]	0 + (−1)	(8 − 0) = 8
v_j	(6−2) 4	(6−8) −2	(7−7) 0	(3−8) −5	(0−7) −7	

					$u_i \downarrow$
2 $\boxed{25}$	4 (0)	6 $\boxed{25}$	11 (10)	0 (2)	6
10 (7)	8 (3)	7 $\boxed{70}$	5 (2)	0 (1)	7
13 (12)	3 $\boxed{30}$	9 (4)	12 (2)	0 (3)	5
4 (0)	6 $\boxed{5}$	8 $\boxed{10}$	3 $\boxed{20}$	0 $\boxed{15}$	8
$v_j \rightarrow$ -4	-2	0	-5	-8	

$x_{11} = 25 \quad x_{13} = 25 \quad x_{23} = 70$

$x_{32} = 30 \quad x_{42} = 5 \quad x_{43} = 10$

$x_{44} = 20 \quad x_{45} = 15$

Min $Z = 950$

Q

Maximization

					Supply
4	2	3	2	6	8
5	4	5	2	1	12
6	5	4	7	3	14
Demand 4	4	6	~~8~~ 12	8	34

Convert into minimization.

Subtract each element from maximum element (7)

							Penalty		
A	3	5	4	5	1 $\boxed{8}$	~~8~~ 0	(2) [3-1]	(2)	X
B	2 $\boxed{2}$	3 $\boxed{4}$	2 $\boxed{6}$	5	6	~~12~~ ~~6~~	(0)	(0)	(0)
C	1 $\boxed{2}$	2	3	0 $\boxed{12}$	4	~~14~~ 2	(1)	(1)	(1)
	4	4	~~6~~ 0	~~12~~ 0	~~8~~ 0				
	(1)	(1)	(1)	(5) ↑	(3)				
	(1)	(1)	(1)	X	(3)				
	(1)	(1)	(1) ↑		X				

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*) Assignment Problem :-

Jobs →

Machine ↓

	J_1	J_2	J_3	J_4
M_1	5	7	11	6
M_2	8	5	9	6
M_3	4	7	10	7
M_4	10	4	8	3

Subtract with minimum value at rows/ columns. So, in every row/ column one zero should be present.

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

→

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

Subtract ~~with~~ ~~minimum~~ minimum number with uncover element. and add with intersecting element.

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

→

	J_1	J_2	J_3	J_4
M_1	0	0	0	$\boxed{0}$
M_2	5	$\boxed{0}$	~~0~~	2
M_3	$\boxed{0}$	1	~~0~~	2
M_4	8	~~0~~	$\boxed{0}$	~~0~~

→ $6+5+4+8 = 23$

Q

11	17	8	16	20
9	7	12	6	15
13	16	15	12	16
21	24	17	28	26
14	10	12	11	13

→

3	9	0	8	12
3	1	6	0	9
1	4	3	0	4
4	7	0	11	9
4	0	2	1	3

upto order of matrix

(Subtract with 1)

minimum element is 1 then subtract & add intersection

2	9	0	8	9
2	1	6	0	6
0	4	3	0	1
3	7	0	1	6
3	0	2	1	0

→

1	8	0	8	8
1	0	6	0	5
0	4	4	1	1
2	6	0	1	5
3	0	3	2	0

→

0	~~8~~ 7	0	7	7
1	0	7	0	5
0	4	5	1	1
1	5	0	10	4
3	0	4	2	0

→

	J_1	J_2	J_3	J_4	J_5
M_1	$\boxed{0}$	6	~~0~~	6	6
M_2	2	$\boxed{0}$	8	~~0~~	5
M_3	~~0~~	3	5	$\boxed{0}$	~~0~~
M_4	1	4	$\boxed{0}$	9	3
M_5	4	~~0~~	5	2	$\boxed{0}$

$\Rightarrow 11 + 7 + 12 + 17 + 13$

$\Rightarrow 60$

*) Replacement Policy :-

Capital Cost: (C):

$f(t)$ – maintenance Charge

(S). – ~~Resub~~ Scrape cost.

$$\boxed{T_n = C - S + \sum_{t=1}^{n} f(t)}$$

Let maintenance cost of an equipment is a function increasingly with time and whose scrap value is constant. The time value of money is not be considered, that

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Case 1: when time t is a continuous variable.

Let C = capital cost of an equipment.

S = scrap value of the equipment.

T = Average annual total cost of the equipment

n = number of years the equipment is to be in use.

$f(n)$ = operating and maintainance cost of the equipment at time t.

Total cost incured during n years, $T_n = C - S + \int_0^n f(t)\,dt$

Annual cost incured $T = \frac{1}{n}\left[C - S + \int_0^n f(t)\,dt\right]$

$\frac{dT}{dn} = 0 \qquad \frac{dT}{dn} = -\frac{1}{n^2}\left[C - S + \int_0^n f(t)\,dt\right] + \frac{1}{n} f(n)$

$$f(n) = \frac{1}{n}\left[C - S + \int_0^n f(t)\,dt\right]$$

$$\frac{d^2T}{dn^2} = \frac{2}{n^3}\left[C - S + \int_0^n f(t)\,dt\right] + \cancel{\frac{1}{n^2} f(n)} - \cancel{\frac{1}{n^2} f(n)} + \frac{1}{n} f'(n)$$

Here $\frac{d^2T}{dn^2} > 0 \Rightarrow$ doubtfull case

$\frac{d^3f}{dx^3} \neq 0$

$$T(n) = \frac{1}{n}\left[C - S + \sum_{t=1}^{n} f(t)\right]$$

$$T(n+1) - T(n) = \frac{1}{n+1}\left[C - S + \sum_{t=1}^{n+1} f(t)\right] - \frac{1}{n}\left[C + S - \sum_{t=1}^{n} f(t)\right]$$

$\cancel{+ f(n+1)}$.

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$$\Rightarrow \frac{n}{n+1}\left[\frac{1}{n}\left\{C-S+\sum_{t=1}^{n} f(t)\right\}+\frac{1}{n}f(n+1)\right]-\frac{1}{n}\left[C-S+\sum_{t=1}^{n} f(t)\right]$$

$$= \frac{n}{n+1}\left[\frac{1}{n}\left\{C-S+\sum_{t=1}^{n} f(t)\right\}\right]+\frac{1}{n+1}f(n+1)-\frac{1}{n}\left[C-S+\sum_{t=1}^{n} f(t)\right]$$

$$= \left(\frac{n}{n+1}-1\right)\frac{1}{n}\left[C-S+\sum_{t=1}^{n} f(t)\right]+\frac{1}{n+1}f(n+1)$$

$$= \frac{1}{n+1}f(n+1)-\frac{1}{n+1}T(n) > 0.$$

Year	Maintenance cost ($f(n)$)	$\sum_{t=1}^{n} f(t)$	$C-S+\sum_{t=1}^{n} f(t)$	$\frac{1}{n}\left[C-S+\sum_{t=1}^{n} f(t)\right]$
1	100	100	6100	6100
2	250	350	6350	3175 (6350/2)
3	400	750	6750	2250 (6750/3)
4	600	1350	7350	1837.50
5	900	2250	8250	1650
6	1200	3450	9450	1575
7	1600	5050	11,050	1578.57
8	2000	7050	13,050	1631.25

Q. A machine owner finds from his past records that the costs per year of maintenance of a machine whose purchase price is Rs. 6000/- are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance cost (Rs)	1000	1200	1400	1800	2300	2800	3400	4000
Resale price	3000	1500	750	375	200	200	200	200

Determine at what age is a replacement due?

*) Theorem 2: The maintenance cost increases with time and the money value decreases with constant rate. Then replacement policy will be

(i) Replace if the next period maintenance cost is greater than the weighted average of previous costs.

ii) Do not replace if the next period's cost is lesser than the weighted average of previous costs.

Proof: Let c be the purchase price of the items to be replaced. Let R_i be the maintenance cost incurred at the beginning of i^{th} year.

Let r be the rate of interest.

let $v = \frac{1}{1+r}$ be the present worth of a rupee to be spent a year.

Let the item be replaced at the end of every n^{th} year. The yearwise present worth of expenditure on the item in the successive cycles of n years can be calculated as follows —

Year	1	2	3	......	n	n+1	n+2	....	2n	2n+1	....
Present worth	$C+R_1$	R_2v	R_3v^2		R_nv^{n-1}	$(C+R_1)v^n$	R_2v^{n+1}		R_nv^{2n-1}	$(C+R_1)v^{2n}$	.....

Assuming that the item has no resale price at the time of replacement, the present worth of all the future discounted costs associated with the policy of replacing the item at the end of every n year will be given by

$$P(n) = [(C+R_1)+R_2v+ \dots + R_nv^{n-1}]$$
$$+ [(C+R_1)v^n + R_2v^{n+1} + \dots + R_nv^{2n-1}]$$
$$+ [(C+R_1)v^{2n} + R_2v^{2n+1} + \dots + R_nv^{3n-1}]$$
$$+ \dots\dots$$

$$\Rightarrow (C+R_1)\{1+v^n+v^{2n}+\dots\}$$
$$+ R_2v\{1+v^n+v^{2n}+\dots\}$$
$$+ R_3v^2\{1+v^n+v^{2n}+\dots\}$$
$$\vdots$$
$$+ R_nv^{n-1}\{1+v^n+v^{2n}+\dots\}$$

$$\Rightarrow (C+R_1+R_2v+R_3v^2+\dots+R_nv^{n-1})(1+v^n+v^{2n}+\dots)$$

$$\Rightarrow \frac{C+R_1+R_2v+R_3v^2+\dots+R_nv^{n-1}}{1-v^n} \quad \text{as} \quad v<1$$

$$= \frac{F(n)}{1-v^n}, \text{ where } F(n) = C+R_1+R_2v+\dots+R_nv^{n-1}$$

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Now $P(n)$ will be minimum if $P(n+1) > P(n)$ and $P(n-1) > P(n)$.

$$\therefore \quad P(n+1) - P(n) = \frac{F(n+1)}{1-v^{n+1}} - \frac{F(n)}{1-v^{n}}$$

$$= \frac{F(n) + R_{n+1} v^{n}}{1-v^{n+1}} - \frac{F(n)}{1-v^{n}}$$

$$= \frac{(1-v^{n})F(n) + R_{n+1} v^{n}(1-v^{n}) - F(n)(1-v^{n+1})}{(1-v^{n+1})(1-v^{n})}$$

$$= \frac{v^{n}(1-v^{n}) R_{n+1} - F(n)\, v^{n}(1-v)}{(1-v^{n+1})(1-v^{n})}$$

$$= \frac{v^{n}(1-v)\left[\frac{1-v^{n}}{1-v} R_{n+1} - F(n)\right]}{(1-v^{n+1})(1-v^{n})} > 0$$

$$\therefore \quad \frac{1-v^{n}}{1-v} R_{n+1} > F(n)$$

$$(1+v+v^{2}+\dots+v^{n-1}) R_{n+1} > F(n)$$

$$R_{n} < \frac{F(n)}{1+v+v^{2}+\dots+v^{n-1}} < R_{n+1} \quad \text{--- } (1)$$

$$\therefore \quad P(n-1) - P(n) = \frac{F(n-1)}{1-v^{n-1}} - \frac{F(n)}{1-v^{n}}$$

$$= \frac{F(n) - R_{n} v^{n-1}}{1-v^{n-1}} - \frac{F(n)}{1-v^{n}}$$

$$\Rightarrow \frac{F(n)(1-v^n) - R_n v^{n-1}(1-v^n) - F(n)(1-v^{n-1})}{(1-v^{n-1})(1-v^n)}$$

$$\Rightarrow \frac{\cancel{F(n)} - v^n F(n) - R_n v^{n-1} + R_n v^{2n-1} - \cancel{F(n)} + v^{n-1} F(n)}{(1-v^{n-1})(1-v^n)}$$

$$\Rightarrow \frac{R_n v^{n-1}(v^n - 1) + F(n) v^{n-1}(1-v)}{(1-v^{n-1})(1-v^n)}$$

$$\Rightarrow \frac{F(n) v^{n-1}(1-v) - R_n v^{n-1}(1-v^n)}{(1-v^{n-1})(1-v^n)}$$

$$\Rightarrow \frac{v^{n-1}(1-v)\left[F(n) - \frac{R_n(1-v^n)}{1-v}\right]}{(1-v^{n-1})(1-v^n)}$$

$$F(n) > \frac{R_n(1-v^n)}{1-v}$$

$$\therefore \boxed{\frac{F(n)}{1+v+\dots+v^{n-1}} > R_n} \quad \boxed{R_n < \frac{C+R_1+R_2 v+\dots+R_n v^{n-1}}{1+v+\dots+v^{n-1}} < R_{n+1}}$$

Q. A manufacturer is offered two machines A and B. A is priced at Rs 5000 and running cost are estimated at Rs 800 for each of the first 5 years, increasing by Rs 200 per year in the sixth and subsequent years. Machine B which has same capacity as A, costs Rs 2500 but will have running costs of Rs 1200 per year for six years, increasing by Rs 200 per year thereafter. If the money is worth 10% per year, which machine should be purchased?

Year (n)	R_n	$f(n)$	$1+v+\dots+v^{n-1}$	$\frac{f(n)}{1+v+\dots+v^{n-1}}$
1	800	5800	1	5800
2	800	6527	1.9091	3419
3	800	7188	2.7356	2628
4	800	7789	3.4868	2234
5	800	8335	4.1698	1999
6	1000	8956	4.7907	1896
7	1200	9633	5.3552	1799
8	1400	10351	5.8684	1764
9	(R_n) 1600	11097	6.3349	$\boxed{1752}$ → $\frac{f(n)}{1+v+v^2+\dots+v^{n-1}}$
10	(R_{n+1}) 1800	11860	6.7590	1788

$$R_n < \frac{C+R_1+R_2v+\dots\dots+R_nv^{n-1}}{1+v+v^2+\dots+v^{n-1}} < R_{n+1}$$

$$\boxed{1600 < 1752 < 1800}$$

*) Group Replacement Policy:

a) One should group replace at the end of the n-th period if the cost of individual replacement for the n-th period is greater than the average cost per period through the end of the n-th period.

b) One should not give group replace at the end of

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of nth period if the cost of individual replacements at the end of (n-1)th period is less than the average cost per period through the end of nth period.

Proof: Here it is proposed to replace all items at fixed interval 'n' whether they have failed or not, and continue replacing failed items as and when they fail.

Let N_n = number of items failed during time n
N = Total number of item in the system
$C(n)$ = cost of group replacement after time period n.
C_1 = individual replacement cost on failure.
C_2 = per unit cost of replacement in a group.

Then

$$C(n) = C_1 N_1 + C_1 N_2 + \cdots + C_1 N_{n-1} + C_2 N$$

Average cost per unit period is -

$$F(n) = \frac{C(n)}{n} = \frac{C_1(N_1 + N_2 + \cdots + N_{n-1}) + C_2 N}{n}$$

If $F(n)$ is minimum then

$$\boxed{F(n-1) < F(n) < F(n+1)}$$

$$\therefore \quad F(n+1) - F(n) = \frac{C_1(N_1 + N_2 + \cdots + N_n) + C_2 N}{n+1} - \frac{C_1(N_1 + N_2 + \cdots N_{n-1}) + C_2 N}{n}$$

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$$\Rightarrow \frac{C_1(N_1+\cdots+N_{n-1}+N_n)n + C_2Nn - C_1(N_1+\cdots+N_{n-1})(n+1) - C_2N(n+1)}{(n+1)n}$$

$$\Rightarrow \frac{C_1N_n n - C_1(N_1+N_2+\cdots+N_{n-1}) - C_2N}{n(n+1)} > 0$$

$$\Rightarrow C_1N_n n > C_1(N_1+N_2+\cdots+N_{n-1}) + C_2N$$

$$C_1N_n > \frac{C_1(N_1+N_2+\cdots+N_{n-1})+C_2N}{n}$$

n item failed. $\leftarrow C_1N_n > \underset{\text{Average cost}}{F(n)}$

$\because$ $F(n-1) - F(n) \geq$

$$\Rightarrow \frac{C_1(N_1+N_2+\cdots+N_{n-2})+C_2N}{(n-1)} - \frac{C_1(N_1+N_2+\cdots+N_{n-1})+C_2N}{n}$$

$$\Rightarrow \frac{C_1(N_1+\cdots+N_{n-3}+N_{n-2})+C_2Nn - C_1(N_1+N_2+\cdots+N_{n-2}+N_{n-1})(n-1) - C_2N(n-1)}{(n-1)n}$$

$$\Rightarrow \frac{C_2N - C_1N_{n-1}n + C_1(N_1+N_2+\cdots+N_{n-1})}{(n-1)n}$$

$$C_1N_{n-1}n < C_1(N_1+N_2+\cdots+N_{n-1}) + C_2N$$

$$C_1N_{n-1} < \frac{C_1(N_1+N_2+\cdots+N_{n-1})+C_2N}{n}$$

$$C_1N_{n-1} < F(n)$$

Q Following failure rates have been observed for a certain types of light bulbs:

$+C_2N$

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week:	1	2	3	4	5
failed item at end of a week (%)	10	25	50	80	100

Q There are 1000 bulbs in use, and it costs Rs 10 to replace an individual bulbs which has been burn out. If all bulbs were replaced simultaneously, it would cost Rs 4 per bulb. At a what interval all the bulbs should be replaced?

Solⁿ:- Let P_i be the probability that a light bulb fails during i^{th} week of its life.

Then, we have $P_1 = 0.1$, $P_2 = \frac{25-10}{100} = 0.15$, $P_3 = \frac{50-25}{100} = 0.25$, $P_4 = \frac{80-50}{100} = 0.3$, $P_5 = \frac{100-80}{100} = 0.2$

$P_n = 0, n \geq 6$

Let N_i be the number of replacements at the end of the i^{th} week.

Then, $N = 1000$