SEQUENCE

A sequence is just a list of elements usually written in a row.

EXAMPLES

1. 1, 2, 3, 4, 5, …
2. 4, 8, 12, 16, 20,…
3. 2, 4, 8, 16, 32, …
4. 1, 1/2, 1/3, 1/4, 1/5, …
5. 1, 4, 9, 16, 25, …
6. 1, -1, 1, -1, 1, -1, …

NOTE:

The symbol "…" is called ellipsis, and reads "and so forth"

FORMAL DEFINITION

A sequence is a function whose domain is the set of integers greater than or equal to a particular integer n_0.

Usually this set is the set of Natural numbers {1, 2, 3, …} or the set of whole numbers {0, 1, 2, 3, …}.

NOTATION

We use the notation a_n to denote the image of the integer n, and call it a term of the sequence. Thus

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

represent the terms of a sequence defined on the set of natural numbers N.

Note that a sequence is described by listing the terms of the sequence in order of increasing subscripts.

FINDING TERMS OF A SEQUENCE GIVEN BY AN EXPLICIT FORMULA

An explicit formula or general formula for a sequence is a rule that shows how the values of a_k depends on k.

EXAMPLE

$$a_k = \frac{k}{k+1} \qquad \text{for all integers } k \geq 1$$

Define a sequence a_1, a_2, a_3, … by the explicit formula

The first four terms of the sequence are:

$$a_1 = \frac{1}{1+1} = \frac{1}{2}, a_2 = \frac{2}{2+1} = \frac{2}{3}, a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$and \quad fourth\ term\ is a_4 = \frac{4}{4+1} = \frac{4}{5}$$

EXAMPLE

Write the first four terms of the sequence defined by the formula

$b_j = 1 + 2^j$, for all integers $j \geq 0$

SOLUTION

$$b_0 = 1 + 2^0 = 1 + 1 = 2$$
$$b_1 = 1 + 2^1 = 1 + 2 = 3$$
$$b_2 = 1 + 2^2 = 1 + 4 = 5$$
$$b_3 = 1 + 2^3 = 1 + 8 = 9$$

REMARK

The formula $b_j = 1 + 2^j$, for all integers $j \geq 0$ defines an infinite sequence having infinite number of values.

EXERCISE

Compute the first six terms of the sequence defined by the formula

$$C_n = 1 + (-1)^n \text{ for all integers } n \geq 0$$

SOLUTION

REMARK:

(1)If n is even, then $C_n = 2$ and if n is odd, then $C_n = 0$

Hence, the sequence oscillates endlessly between 2 and 0.

(2) An infinite sequence may have only a finite number of values.

EXAMPLE

Write the first four terms of the sequence defined by

$$C_n = \frac{(-1)^n n}{n+1} \qquad \text{for all integers } n \geq 1$$

SOLUTION

$$C_1 = \frac{(-1)^1(1)}{1+1} = \frac{-1}{2}, C_2 = \frac{(-1)^2(2)}{2+1} = \frac{2}{3}, C_3 = \frac{(-1)^3(3)}{3+1} = \frac{-3}{4}$$

$$And \quad fourth \;\; erm \; is C_4 = \frac{(-1)^4(4)}{4+1} = \frac{4}{5}$$

REMARK A sequence whose terms alternate in sign is called an alternating sequence.

EXERCISE

Find explicit formulas for sequences with the initial terms given:

1. 0, 1, -2, 3, -4, 5, …

SOLUTION

$a_n = (-1)^{n+1} n$ for all integers $n \geq 0$

2. $$1-\frac{1}{2}, \frac{1}{2}-\frac{1}{3}, \frac{1}{3}-\frac{1}{4}, \frac{1}{4}-\frac{1}{5}, \ldots$$

SOLUTION

$$b_k = \frac{1}{k} - \frac{1}{k+1} \quad \text{for all integers } n \geq 1$$

3. 2, 6, 12, 20, 30, 42, 56, …

SOLUTION

$C_n = n\,(n + 1)$ for all integers $n \geq 1$

4. 1/4, 2/9, 3/16, 4/25, 5/36, 6/49, …

SOLUTION

$$d_i = \frac{i}{(i+1)^2} \qquad \text{for all integers} \quad i \geq 1$$

OR

$$d_j = \frac{j+1}{(j+2)^2} \quad \text{for all integers} \quad j \geq 0$$

ARITHMETIC SEQUENCE

A sequence in which every term after the first is obtained from the preceding term by adding a constant number is called an arithmetic sequence or arithmetic progression (A.P.)

The constant number, being the difference of any two consecutive terms is called the common difference of A.P., commonly denoted by "d".

EXAMPLES

1. 5, 9, 13, 17, … (common difference = 4)
2. 0, -5, -10, -15, … (common difference = -5)
3. x + a, x + 3a, x + 5a, … (common difference = 2a)

GENERAL TERM OF AN ARITHMETIC SEQUENCE

Let **a** be the first term and **d** be the common difference of an arithmetic sequence. Then the sequence is

$$a, a+d, a+2d, a+3d, \dots$$

If a_i, for $i \geq 1$, represents the terms of the sequence then

a_1 = first term = $a = a + (1-1)\,d$

a_2 = second term = $a + d = a + (2-1)\,d$

a_3 = third term = $a + 2d = a + (3-1)\,d$

By symmetry

a_n = nth term = $a + (n - 1)d$ for all integers $n \geq 1$.

EXAMPLE

Find the 20th term of the arithmetic sequence

3, 9, 15, 21, …

SOLUTION

Here a = first term = 3

d = common difference = 9 - 3 = 6

n = term number = 20

a_{20} = value of 20th term = ?

Since $a_n = a + (n - 1)\, d; \quad n \geq 1$

$\therefore \quad a_{20} = 3 + (20 - 1)\, 6$

$= 3 + 114$

$= 117$

EXAMPLE

Which term of the arithmetic sequence

4, 1, -2, …, is -77

SOLUTION

$$\frac{-81}{-3} = n - 1$$

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EXERCISE

Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8th term is 17.

SOLUTION

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GEOMETRIC SEQUENCE

A sequence in which every term after the first is obtained from the preceding term by multiplying it with a constant number is called a geometric sequence or geometric progression (G.P.)

The constant number, being the ratio of any two consecutive terms is called the common ratio of the G.P. commonly denoted by "r".

EXAMPLE

1. 1, 2, 4, 8, 16, … (common ratio = 2)
2. 3, - 3/2, 3/4, - 3/8, … (common ratio = - 1/2)
3. 0.1, 0.01, 0.001, 0.0001, … (common ratio = 0.1 = 1/10)

GENERAL TERM OF A GEOMETRIC SEQUENCE

Let **a** be the first tem and **r** be the common ratio of a geometric sequence. Then the sequence is a, ar, ar^2, ar^3, …

If a_i, for $i \geq 1$ represent the terms of the sequence, then

a_1 = first term = a = ar^{1-1}

a_2 = second term = ar = ar^{2-1}

a_3 = third term = ar^2 = ar^{3-1}

………………

………………

a_n = nth term = ar^{n-1};   for all integers $n \geq 1$

EXAMPLE

Find the 8th term of the following geometric sequence

4, 12, 36, 108, …

SOLUTION

Here a = first term = 4

r = common ratio =   = 3

n = term number = 8

a_8 = value of 8th term = ?

Since $a_n = ar^{n-1}$; $n \geq 1$

$\Rightarrow$ $a_8 = (4)(3)^{8-1}$

$$\frac{12}{4}$$

= 4 (2187)

= 8748

EXAMPLE

Which term of the geometric sequence is 1/8 if the first term is 4 and common ratio 1/2.

SOLUTION

Given a = first term = 4

r = common ratio = 1/2

a_n = value of the nth term = 1/8

n = term number = ?

Since $a_n = ar^{n-1}$ $n \geq 1$

$$\Rightarrow \quad \frac{1}{8} = 4\left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad \frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{5} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad n-1=5 \qquad \Rightarrow n=6$$

Hence 1/8 is the 6th term of the given G.P.

EXERCISE

Write the geometric sequence with positive terms whose second term is 9 and fourth term is 1.

SOLUTION

Let **a** be the first term and **r** be the common ratio of the geometric sequence. Then

$$a_n = ar^{n-1}; \qquad n \geq 1$$

Now $\quad a_2 = ar^{2-1}$

$\Rightarrow \quad 9 = ar$……………………(1)

Also $\quad a_4 = ar^{4-1}$

$1 = ar^3$ ……………………(2)

Dividing (2) by (1), we get,

$$\frac{1}{9} = \frac{ar^3}{ar}$$

$$\Rightarrow \quad \frac{1}{9} = r^2$$

$$\Rightarrow \quad r = \frac{1}{3} \qquad \left(\text{rejecting } r = -\frac{1}{3}\right)$$

Substituting r = 1/3 in (1), we get

$$9 = a\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad a = 9 \times 3 = 27$$

Hence the geometric sequence is

27, 9, 3, 1, 1/3, 1/9, …