

# Cosmological N-Body Simulations

Nemanja Martinović

*Petnica School of Cosmology*

August 7, 2013

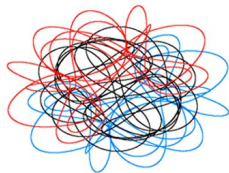
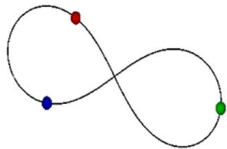
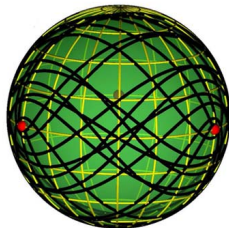
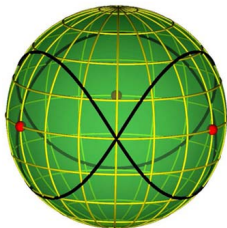
# Numerical (computational) astrophysics

- ▶ 3 body and more  $\rightarrow$  numerical;
- ▶ Different domains of applicability;
- ▶ Spans many orders of magnitude;
- ▶ Large part of numerical astrophysics are N-body simulations;



# Physicists Discover a Whopping 13 New Solutions to Three-Body Problem

by Jon Cartwright on 8 March 2013, 4:30 PM | [36 Comments](#)



# Domains of applicability

- ▶ Celestial mechanics
  - ▶ single body  $\rightarrow$  dominates field;
  - ▶ very high accuracy for perturbative terms;
- ▶ Dense stellar systems
  - ▶ collisional systems  $\rightarrow$  rich dynamics;
- ▶ Sphere of influence of massive a BH
  - ▶ interesting combination of two previous cases;
- ▶ Galaxy dynamics and cosmology
  - ▶ AREA OF INTEREST TO US;

# Cosmological simulations

- ▶ They come down to gravity;
- ▶ Gravity is by far weakest force;

Interaction	Present Theory	Relative Strength	Range (m)	Predicted Mediators
Strong	Quantum chromodynamics	1.00E+39	1.00E-14	gluons
Electromagnetic	Quantum electrodynamics	1.00E+37	$\infty$	photons
Weak	Electroweak theory	1.00E+26	1.00E-17	W & Z bosons
Gravitation	General Relativity	1	$\infty$	gravitons <sup>1</sup>

# Cosmological simulations

- ▶ They come down to gravity;
- ▶ Gravity is by far weakest force;

Interaction	Present Theory	Relative Strength	Range (m)	Predicted Mediators
Strong	Quantum chromodynamics	1.00E+39	1.00E-14	gluons
<b>Electromagnetic</b>	Quantum electrodynamics	<b>1.00E+37</b>	$\infty$	photons
Weak	Electroweak theory	1.00E+26	1.00E-17	W & Z bosons
<b>Gravitation</b>	General Relativity	<b>1</b>	$\infty$	gravitons <sup>1</sup>

- ▶ yet it dominates at large scales!

# Cosmological simulations

- ▶ Most systems  $\rightarrow$  well approximated by ensembles of point masses under mutual attraction;
- ▶ Systems dominated by long-range forces are not well treatable by statistical mechanical methods:
  - ▶ Energy is not extensive;
  - ▶ Canonical and micro-canonical ensembles do not exist;
  - ▶ Heat capacity is negative;
- ▶ Also, gravitational encounters are inefficient for re-distributing kinetic energy, such that many such encounters are required for relaxation, i.e. equipartition of kinetic energy.

# Cosmological simulations

- ▶ Collisional vs Collisionless simulations:

- ▶ for collisional the process from previous slide is important over their lifetime;
- ▶ timescale for that two-body relaxation is:

- ▶ 
$$t_{relax} \simeq \frac{N}{8\ln\Lambda} t_{dyn}$$





## Little bit of a history

- ▶ van Hoerner  $\rightarrow N = 16$ , Aarseth (1963)  $\rightarrow N = 100$ ;
- ▶ Peebles (1970),  $N = 300$

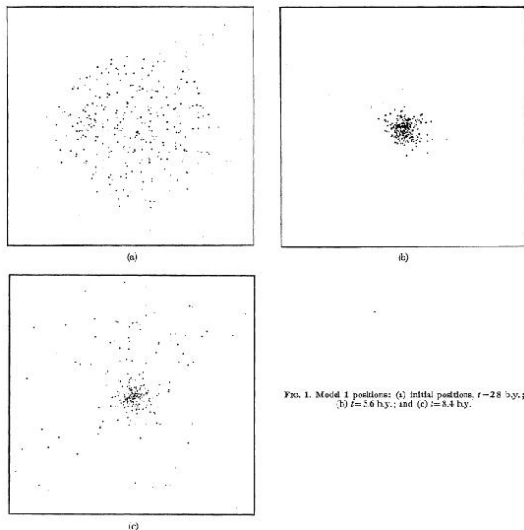
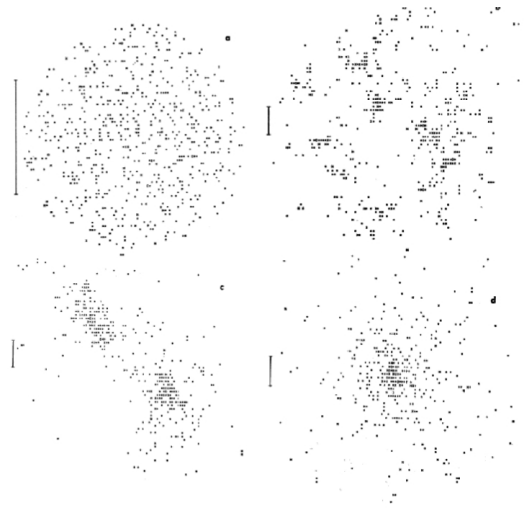


FIG. 1. Model 1 positions: (a) initial positions,  $t = 28$  byr; (b)  $t = 5.6$  byr; and (c)  $t = 8.4$  byr.

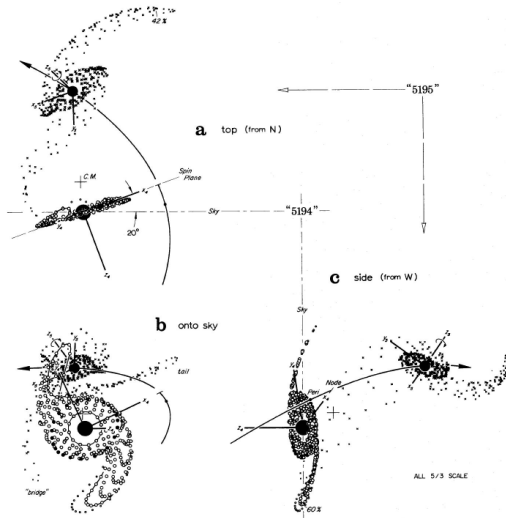
# Little bit of a history

- White (1970),  $N = 700$

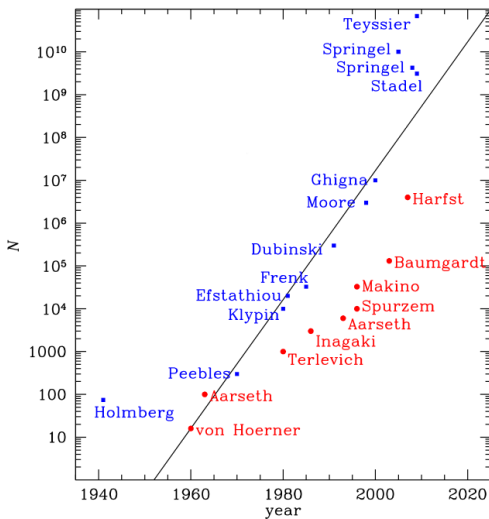


# Little bit of a history

## ► Toomre & Toomre (1972)



# Little bit of history



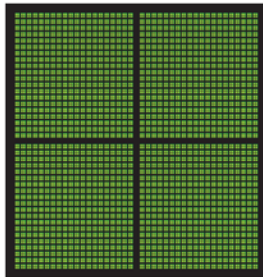
Little bit of future



# Little bit of future



CPU  
MULTIPLE CORES



GPU  
THOUSANDS OF CORES

# Simulation breakdown

Every simulation can be broken down into following parts:

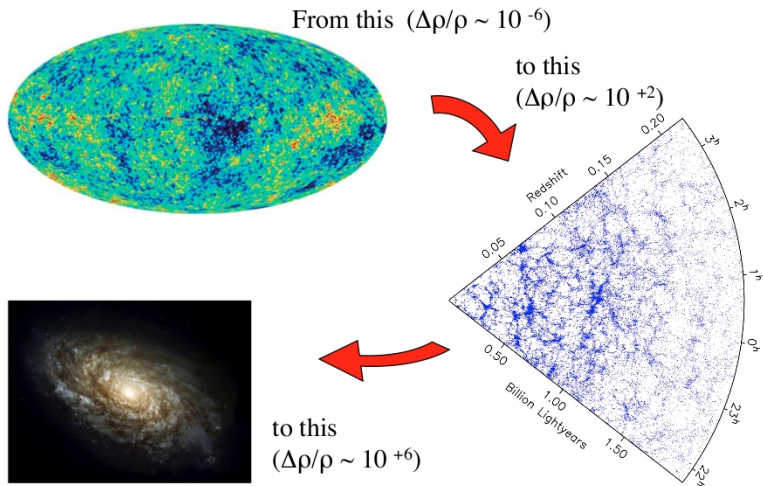
- ▶ Initial conditions;
- ▶ Integration (running the simulation);
- ▶ Analysis;



Considering that simulation has to cover period from Big Bang to today, first part can be summarized as follows:

1. Quantum fluctuations in the primordial era were created during inflationary phase;
2. Fluctuations enter the horizon of the radiation-dominated universe and grow linearly until recombination epoch;
3. Post-recombination era  $\rightarrow$  growth is strongly affected by the presence and nature of dark matter;
4. Fluctuations grow large enough so that non-linear processes become important  $\rightarrow$  N-Body comes into play and end results are large-scale structures preferably ones like we see today.

# The Problem



# Linear growth of density perturbations

Fluctuations in density evolution in time?

Lets consider flat, matter-dominated universe:

Friedmann equation for it is:

$$\blacktriangleright H^2 - \frac{8}{3}\pi G\rho = -\frac{kc}{a^2} = 0$$

and an area of slight overdensity of matter:

$$\blacktriangleright H^2 - \frac{8}{3}\pi G\rho' = -\frac{kc}{a^2}$$

$\blacktriangleright \Downarrow$

$$\blacktriangleright -\frac{8}{3}\pi G(\rho' - \rho) = -\frac{kc}{a^2}$$

# Linear growth of density perturbations

Lets just rewrite it as:

$$\blacktriangleright (\rho' - \rho) = -\frac{3k}{8\pi G a^2}$$

introducing fractional overdensity  $\delta$  as:

$$\blacktriangleright \delta = \frac{\rho' - \rho}{\rho} = -\frac{3k}{8\pi G a^2 \rho}$$

# Linear growth of density perturbations

So we have:

$$\blacktriangleright \delta \sim \frac{1}{a^2 \rho} \sim \frac{1}{a^2 a^{-3}} \sim a$$

And finally:

$$\blacktriangleright a \sim (1+z)^{-1} \quad \rightarrow \quad \delta \sim (1+z)^{-1}$$

# Linear growth of density perturbations

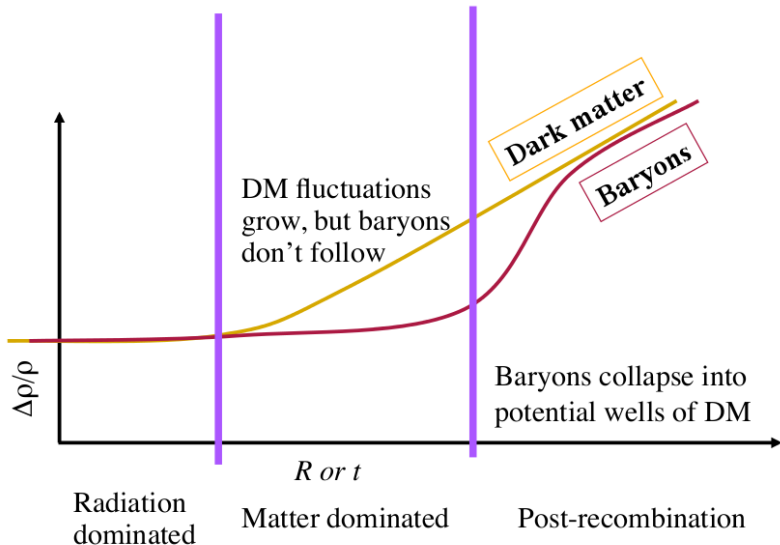
Lets say that we are interested in overdensity ratio:

$$\blacktriangleright \frac{\delta_f}{\delta_i} = \frac{1 + z_i}{1 + z_f}$$

And lets calculate difference from recombination to  $z \sim 5$ :

$$\blacktriangleright \delta_f = 10^{-5} \left( \frac{1 + 1000}{1 + 5} \right) \sim 0.002$$

# The Evolution of Fluctuations



# Evolution of spherical top-hat model

Now lets see how that sphere would evolve.

We start with virial theorem and energy conservation:

- ▶  $U + 2T = 0$
- ▶  $U + T = \text{const.}$

Lets define turnaround time, and then we have:

- ▶  $-\frac{GM^2}{R_{turn}} = -\frac{GM^2}{R_{virial}} + T = -\frac{GM^2}{2R_{virial}}$
- ▶  $R_{virial} = \frac{1}{2}R_{turn}$



# Evolution of spherical top-hat model

Lets now calculate final density.

We start from Friedmann equation and its solution:

- ▶  $\dot{a}^2 = \frac{A^2}{a} - kc^2$
- ▶  $A^2 = \frac{8\pi G}{3}\rho_0 a_0^3$
- ▶  $a = \left(\frac{3A}{2}\right)^{\frac{2}{3}} t^{\frac{2}{3}}$  for  $k = 0$

So we can get parametric solution for a closed,  $k = 1$  universe:

- ▶  $a = \frac{1}{2} \frac{A^2}{c^2} (1 - \cos(\Psi))$
- ▶  $t = \frac{1}{2} \frac{A^2}{c^3} (\Psi - \sin(\Psi))$

# Evolution of spherical top-hat model

So now we can calculate desired quantities (time and size):

- ▶  $\frac{da}{d\Psi} = \frac{1}{2} \frac{A^2}{c^2} \sin\Psi = 0 \rightarrow \Psi = 0, \pi, \dots$

At turnaround we then have:

- ▶ For  $\Psi = \pi$  time is  $t_{turn} = \frac{1}{2} \frac{A^2}{c^3} \pi$
- ▶  $a_{sphere} = \frac{1}{2} \frac{A^2}{c^2} (1 - \cos\pi) = \frac{A^2}{c^2}$
- ▶  $a_{background} = \left(\frac{3A}{2}\right)^{\frac{2}{3}} t_{turn}^{\frac{2}{3}} = \left(\frac{3\pi}{4}\right)^{\frac{2}{3}} \frac{A^2}{c^2}$

# Evolution of spherical top-hat model

So we get that density contrast at turnaround is:

$$\blacktriangleright \frac{\rho_{sphere}}{\rho_{background}} = \left( \frac{a_{sphere}}{a_{background}} \right)^{-3} = \frac{9\pi^2}{16}$$

At the time when the collapsing sphere virialized:

- ▶  $t = 2t_{turn}$
- ▶  $R_{turn} = 2R_{virial}$  density increases 8 times;
- ▶ Background density decreases:  $(2^{\frac{2}{3}})^3 = 4$  times;

**A collapsing object virializes when its density is greater than the mean density of the universe by a factor of  $18\pi^2 \sim 180$ .**

# Quantifying density field

Density fluctuation field:

$$\blacktriangleright \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

Fourier Transform of density field:

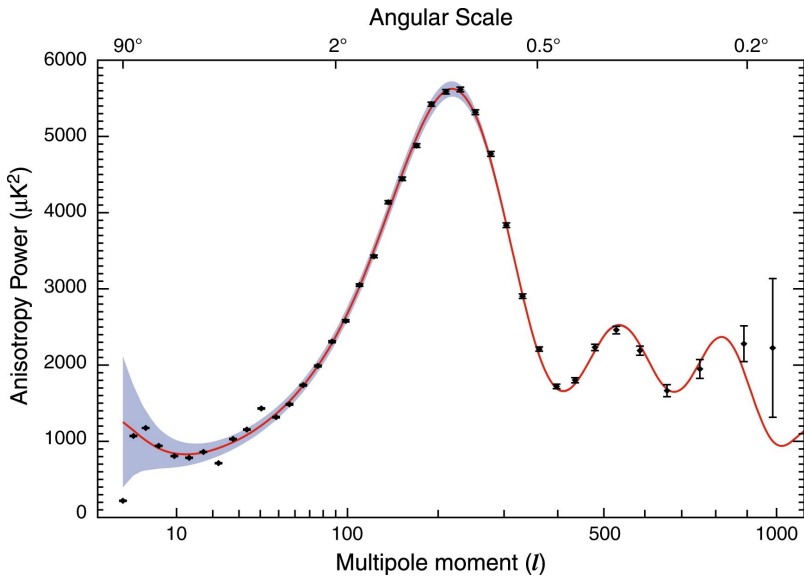
$$\blacktriangleright \delta_k = \sum \delta e^{-i \vec{k} \cdot \vec{r}}$$

Its Power Spectrum:

$$\blacktriangleright P(k) = \langle |\delta_k|^2 \rangle$$

$$P(k) = \langle |\delta_k|^2 \rangle \propto k^n$$

For Harrison - Zel'dovich  $n = 1$



# Zel'dovich approximation

Zel'dovich developed a kinematical approach to the formation of the structure.

We work out initial displacement of particles and assume that they continue to move in initial direction.

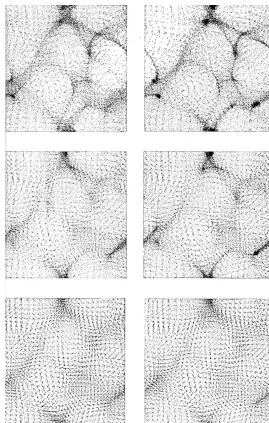
- ▶  $x(t) = a(t)q + b(t)f(q)$
- ▶  $x$  is Eulerian position;  $q$  is Lagrangian position;

# Zel'dovich approximation

To get the Eulerian density  $\rightarrow$  use the Jacobian of the transformation between  $\mathbf{x}$  and  $\mathbf{q}$ , so that  $\rho$  is constant:

- ▶  $\frac{\rho}{\rho_0} = [(1 - \frac{b}{a}\alpha)(1 - \frac{b}{a}\beta)(1 - \frac{b}{a}\gamma)]^{-1}$
- ▶  $(-\alpha, -\beta, -\gamma)$  are eigenvalues of the strain tensor or deformation tensor;
- ▶ Collapse takes place along axis defined by the largest negative eigenvalue  $\rightarrow$  Zel'dovich pancakes.

# Zel'dovich approximation





# Zel'dovich approximation

Lets assume tensor symmetry and that the density perturbation originated from a growing mode;

Displacement field is then irrotational, and we write it in terms of a potential:

$$\blacktriangleright f(q) = \nabla\psi(q) \Rightarrow \frac{\partial f_i}{\partial q_j} = \frac{\partial^2 \psi}{\partial q_i \partial q_j}.$$

If we linearize the density relation we have:

$$\blacktriangleright \delta = -\frac{b}{a}(\alpha + \beta + \gamma) = -\frac{b}{a}\nabla \cdot f;$$

$$\blacktriangleright u = \frac{1}{a}(\dot{x} - \frac{\dot{a}}{a}x) = (\frac{\dot{b}}{a} - \frac{\dot{a}b}{a^2})f$$

# Zel'dovich approximation

Since the linearized density relation is:

$$\blacktriangleright \delta = \frac{b}{a} \nabla \cdot f$$

We can tell immediately that:

$$\begin{aligned} \blacktriangleright \frac{b(t)}{a(t)} &= D(t) \\ \blacktriangleright x(t) &= a(t)[q + D(t)f(q)]; \end{aligned}$$

# N-Body ICs

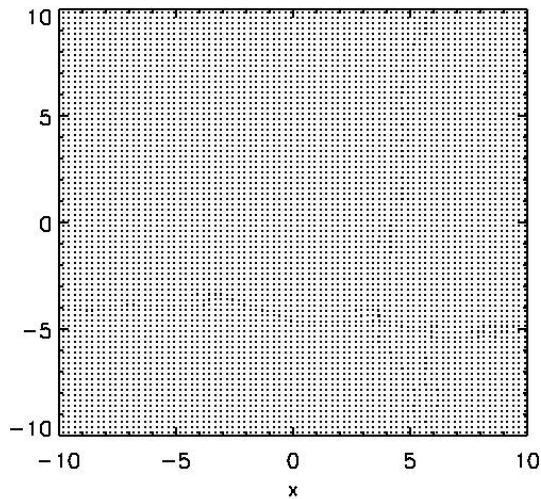
First thing before setting initial conditions, we have pre-initial condition preparation, and that is setting up uniform distribution of particles.

- ▶ Poisson distribution (not good);
- ▶ Glass distribution;
- ▶ Lattice distribution;

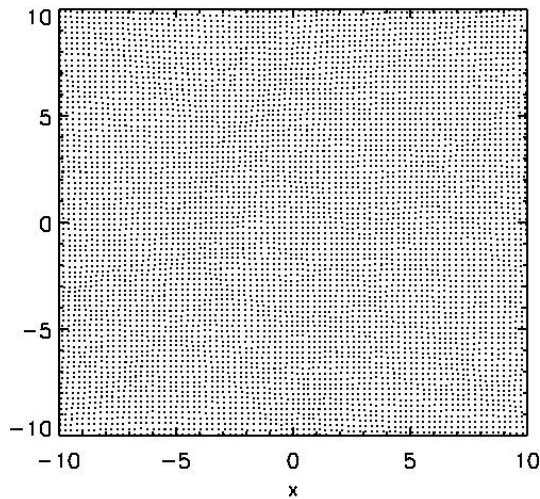
# N-Body ICs

```
27  
28 read_parameterfile(argv[1]);  
29  
30 set_units();  
31  
32 initialize_powerspectrum();  
33  
34 initialize_ffts();  
35  
36 read_glass(GlassFile);  
37  
38 displacement_fields();  
39  
40 write_particle_data();  
41
```

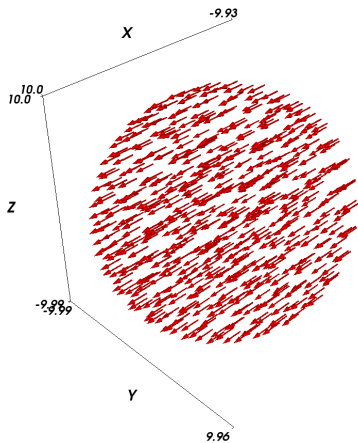
# N-Body ICs



# N-Body ICs



# N-Body ICs



SHORT BREAK!



# N-Body

We will concentrate on collisionless simulations. There are several things that comprise the simulation:

- ▶ Time integration (both choosing most appropriate method and choosing  $\Delta t$ ;
- ▶ Determining softening length;
- ▶ Choosing a method for calculating gravitational force;

# Time integration

Several popular time integration methods in use:

- ▶ Euler method;
- ▶ second-order leapfrog integrator;
- ▶ fourth-order Hermite scheme;

For Euler method we have:

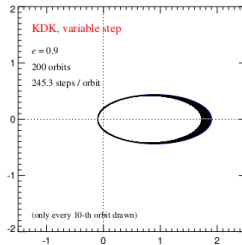
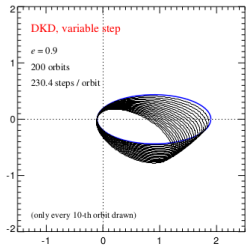
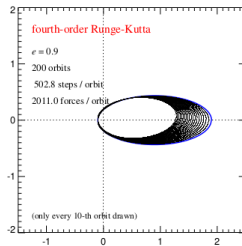
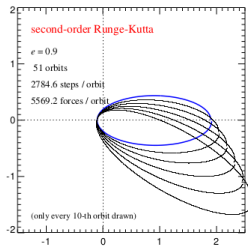
- ▶  $x(t + \Delta t) = x(t) + \dot{x}\Delta t;$
- ▶  $\dot{x}(t + \Delta t) = \dot{x}(t) + a\Delta t;$

# Leapfrog method

For leapfrog method we have:

- ▶  $x' = \dot{x}_0 + \frac{1}{2}a_0\Delta t;$
- ▶  $x_1 = x_0 + \dot{x}'\Delta t;$
- ▶  $\dot{x}_1 = \dot{x}' + \frac{1}{2}a_1\Delta t;$

# Leapfrog method



## Criterion for time step

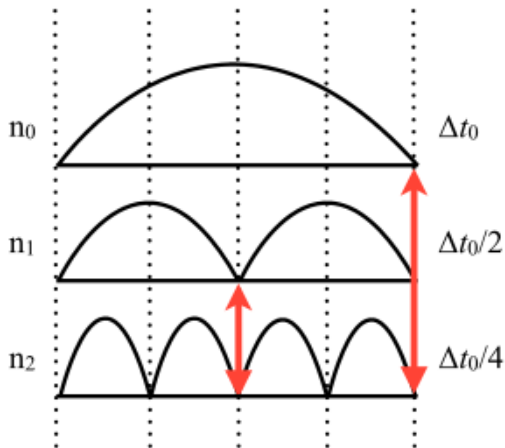
We now need some criteria to decide where a particle should be placed on. For low-order integrators like the leapfrog, we have only the acceleration to play with. In this case, a possible timestep criterion can be found by analogy with the Kepler problem:

$$\blacktriangleright \Delta t_i = \eta \sqrt{|\Phi|/|a_i|};$$

However, a timestep criteria that depends on the potential is worrisome since the transformation  $\Phi \rightarrow \Phi + \text{const}$  has no dynamical effect, but would alter the timesteps. In applications like cosmological N -body simulations, where the local potential has significant external contributions, simulators have typically employed:

$$\blacktriangleright \Delta t_i = \eta \sqrt{\epsilon/|a_i|};$$

# Timesteps



# Softening

$$\vec{F}_i = \sum \frac{Gm_i m_j (\vec{r}_i - \vec{r}_j)}{(|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2)^{\frac{3}{2}}}$$

The softening of gravity has two important aspects:

- ▶ First, at close distances  $r = |x_i x|$  the force no longer diverges as  $r^2$ , but for  $r \leq \epsilon$  actually decays to zero.
- ▶ The second aspect of force softening is a systematic reduction of gravity at close distances;
- ▶ example.

# Force computations

In collisionless N -body methods the force is only ever an estimate, which unavoidably carries with it an estimation error. Therefore, we may as well use less accurate methods:

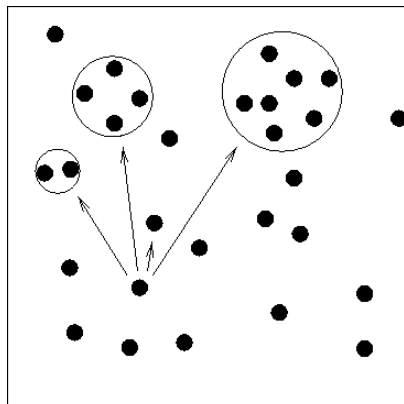
- ▶ Direct summation;
- ▶ Tree codes (Barnes & Hut 1986);
- ▶ Particle-mesh codes;
- ▶  $P^3M$  and PM-Tree codes;
- ▶ Adaptive Mesh Refinement method;
- ▶ Fast multiple methods;



# Direct summation

$$\vec{F}_i = \sum \frac{Gm_im_j(\vec{r}_i - \vec{r}_j)}{(|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2)^{\frac{3}{2}}}$$

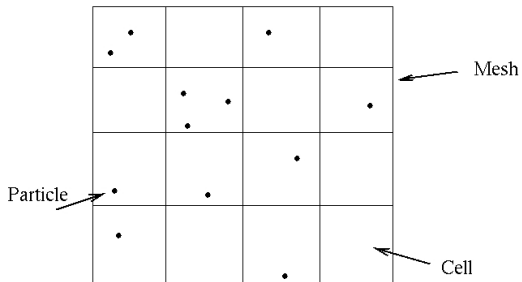
## Tree code



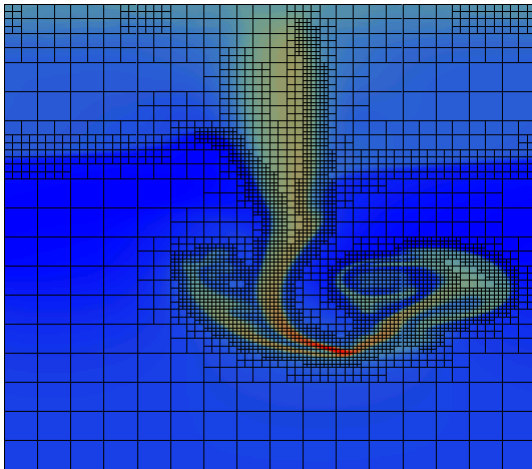
simulation box

SHOW GIF!

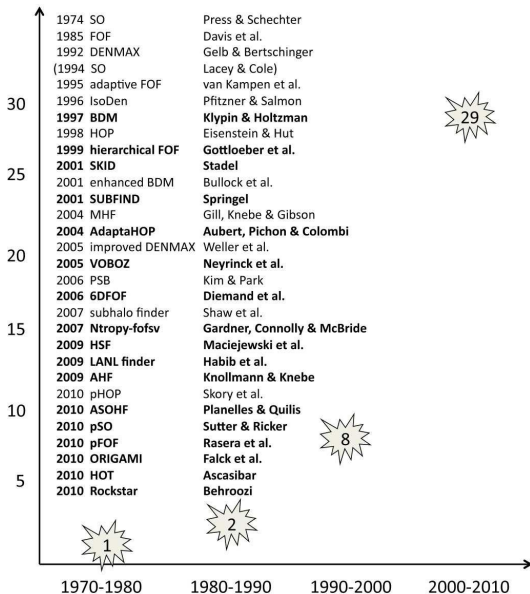
# Particle mesh



# AMR



# Analysis - halos



# Analysis - halos

Nearly all halo finding methods can be reduced to following two methods:

- ▶ Spherical overdensity method (Press & Schechter, 1974);
- ▶ friends-of-friends algorithm (Davies et al., 1985);

So more broadly they can be divided into:

- ▶ density peak locator (+ particle collection)
- ▶ particle collectors

# The Rockstar Phase-Space Halo Finder

Peter Behroozi\* (Stanford), Risa Wechsler, Hao-Yi Wu

(Robust Overdensity Calculator using K-Space Topologically Adaptive Refinement)

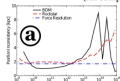
\*behroozi@stanford.edu

## Key Features:

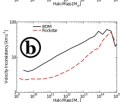
- 1 Class-leading recovery of halo properties in phase space (see Knebe et al. 2011 for a comparison with sixteen other halo finders).
- 2 Extremely reliable substructure tracking, even at halo centers and in major mergers.
- 3 Massively parallel (up to 10,000 CPUs), extremely efficient implementation (60 GB of memory, 10 CPU hours per billion particles analyzed).
- 4 Public codebase: <http://code.google.com/p/rockstar/>

## How it Performs:

(All comparisons made for the Bolshoi Simulation)



- (a) Rockstar can recover the position of most halos to within the force resolution of the simulation. Compared to position-space finders, it has fewer catastrophic errors in major mergers at high halo masses.



- (b) Using phase-space information, Rockstar can recover halo velocities much better than position space halo finders.

- (c) As Rockstar can distinguish halos in velocity space, it can recover the autocorrelation function for halos nearly all the way to the force resolution limit of the simulation.

- (d) Similarly, phase-space finding allows for lower  $v_{\max}$  completeness limits. For subhalos, this is especially true; Rockstar has substantially lower completeness limits in the central regions of host halos.



The simulation volume is divided into 3D Friends-of-Friends groups for easy parallelization.



For each group, particle positions and velocities are divided (normalized) by the group position and velocity dispersions, giving a natural phase-space metric.



A phase-space linking length is adaptively chosen such that 70% of the group's particles are linked together in subgroups.



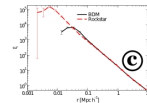
The process repeats for each subgroup: renormalization, a new linking-length, and a new level of substructure calculated.



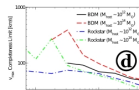
Once all levels of substructure are found, seed halos are placed at the lowest substructure levels and particles are assigned hierarchically to the closest seed halo center in phase space. (see Knebe et al. 2011 for specific details).



Once particles have been assigned to halos, unbound particles are removed and halo properties (positions, velocities, spherical masses, radii, spins, etc.) are calculated.

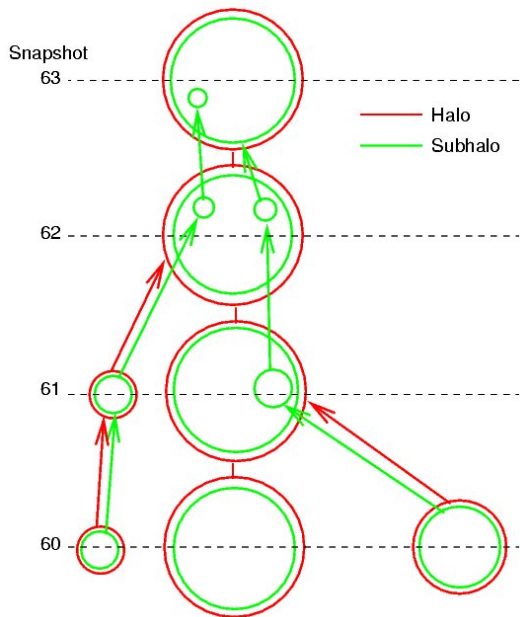


Autocorrelation function for all halos with  $v_{\max} > 150$  km/s in Bolshoi at  $z=0$ .



$v_{\max}$  completeness limit for satellites as a function of distance to the host halo center.

# Analysis - merger tree





# I za kraj...

Definišite da li i šta želite da analizirate kao rezultat modelovanja/simulacije.

**Define if and what do you want to analyse as an outcome of modeling/simulation.**

Bolje jednostavan model koji radi nego komplikovan koji ni radi ništa (ili proradi 2017).

**Better simple model that works than complicated that doesnt (or that becomes operational in 2017).**

Ne plašite se programiranja!

**Dont be scared of programming!**

Pišite svoje kodove! (Čak iako ćete na kraju koristiti tuđi)

**Write your own codes! (Even if you are going to end up using someone elses)**

**SREĆNO!!!  
GOOD LUCK!!!**