UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **COMPGI08**

ASSESSMENT PATTERN : **COMPGI08A**

MODULE NAME : **Graphical Models**

DATE : **09-May-11**

TIME : **10:00**

TIME ALLOWED : **2 Hours 30 Minutes**

2010/11-COMPGI08A-001-EXAM-28

Graphical Models, GI08, 2010

Answer THREE of FIVE questions.

Marks for each part of each question are indicated in square brackets

Calculators are NOT permitted

1. a. Consider the distribution

$$p(x) = \phi(x_1, x_2, x_3, x_4)\phi(x_1, x_2)\phi(x_2, x_3)\phi(x_3, x_4)\phi(x_4, x_1)$$

i. Draw the Markov network representation of $p(x)$. [3 marks]

ii. Draw the Factor Graph representation of $p(x)$. [3 marks]

iii. Write down a set of conditional independence statements that hold for $p(x)$. [2 marks]

b. Consider the distribution

$$p(x) = \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)\phi(x_1, x_2)\phi(x_2, x_3)\phi(x_3, x_4)\phi(x_4, x_1)$$

i. Draw the Markov network representation of $p(x)$. [2 marks]

ii. Draw the Factor Graph representation of $p(x)$. [2 marks]

iii. Write down a set of conditional independence statements that hold for $p(x)$. [2 marks]

c. Consider the distributions

$$p_1(x_1, x_2, x_3) = \phi(x_1, x_2)\phi(x_2, x_3), \qquad p_2(x_1, x_3, x_4) = \phi(x_1, x_4)\phi(x_1, x_3)\phi(x_3, x_4),$$

i. Draw the Markov network representation of $p_1(x_1, x_2, x_3)$. [2 marks]

ii. Draw the Markov network representation of $p_2(x_1, x_3, x_4)$. [2 marks]

iii. Write down a set of conditional independence statements that hold for $p_1(x_1, x_2, x_3)$. [2 marks]

iv. Write down a set of conditional independence statements that hold for $p_2(x_1, x_3, x_4)$. [2 marks]

d. Consider the distribution

$$p_3(x_1, x_2, x_3, x_4) \propto p_1(x_1, x_2, x_3) p_2(x_1, x_3, x_4)$$

where p_1 and p_2 are defined in part (c) of the question above.

i. Draw the Markov network representation of $p_3(x_1, x_2, x_3, x_4)$. [3 marks]

ii. Write down a set of conditional independence statements that hold for $p_3(x_1, x_2, x_3, x_4)$. [3 marks]

e. Consider a distribution $p_1(\mathcal{X}_1)$ defined on a set of variables $\mathcal{X}_1$ with an associated set of conditional independence statements I_1. Similarly, consider a distribution $p_2(\mathcal{X}_2)$ defined on a set of variables $\mathcal{X}_2$ with an associated set of conditional independence statements I_2. With reference to the Markov networks of p_1 and p_2, describe the structure of the Markov network representation of $p_3(\mathcal{X}_1 \cup \mathcal{X}_2) \propto p_1(\mathcal{X}_1) p_2(\mathcal{X}_2)$ and the set of conditional independence statements I_3 that holds for p_3.

[5 marks]

[Total 33 marks]

2. A priori, a light is on with probability $p(x=1)$ or off with probability $p(x=0)$. The probability that person i detects the light is given by $p(y_i|x)$, with $y_i=1$ if person i detects the light, and $y_i=0$ otherwise. Explicitly, $p(y_i=1|x=1)=\theta_{i1}$ and $p(y_i=1|x=0)=\theta_{i0}$. There are a set of 100 people, $i=1,\ldots,100$, each independently attempting to detect the light with associated parameters $\theta_{i1},\theta_{i0}, i=1,\ldots,100$.

a. Draw a belief network that represents the distribution $p(x,y_1,\ldots,y_{100})$.

[3 marks]

b. Explain how to compute $p(x=1|y_1,\ldots,y_{100})$ and discuss the computational complexity of performing this inference.

[3 marks]

c. Given a set of states $y_1,\ldots,y_{100}$, with each $y_i \in \{0,1\}$ a form of summary is made using

$$p(z=1|y_1,\ldots,y_{100}) = \sigma\left(\sum_{i=1}^{100} w_i y_i\right)$$

where the $w_i, i=1,\ldots,100$ are given weights and

$$\sigma(u) = \frac{e^u}{1+e^u}$$

i. Draw a belief network of the distribution

$$p(x,y_1,\ldots,y_{100},z) = p(z|y_1,\ldots,y_{100})p(x,y_1,\ldots,y_{100})$$

where $p(x,y_1,\ldots,y_{100})$ is defined in part(a) above.

[3 marks]

ii. Explain how to compute the conditional marginal $p(x=1|z)$ based on the joint distribution $p(x,y_1,\ldots,y_{100},z)$ and discuss the computational complexity of performing this inference.

[3 marks]

iii. Draw a junction tree for the distribution $p(x,y_1,\ldots,y_{100},z)$.

[5 marks]

iv. Using your above junction tree, explain how to compute the marginal $p(y_i = 1)$. Explain whether or not the junction tree approach is computationally efficient compared to more direct ways to compute $p(y_i = 1)$.

[5 marks]

v. Given a state of z we wish to estimate the state of x and we wish to find a procedure that will be accurate when repeated many times. Explain why the probability of making a correct estimate of the light state may be taken to be

$$p(x=1,z=1)+p(x=0,z=0)$$

[3 marks]

vi. Describe a procedure to adjust the weights $w_1,\ldots,w_{100}$ that maximises the probability of making a correct decision as to the light state and discuss the computational issues which may arise for this procedure.

[8 marks]

[Total 33 marks]

3. a. Consider the distribution

$$p(a,b,c,d,e,f,g) = p(b)p(a)p(g|a,b)p(c|b)p(f|a)p(e|f,d)p(d|c)$$

i. Draw the Directed Acyclic Graph that represents this distribution. [4 marks]

ii. Draw the moralised graph. [2 marks]

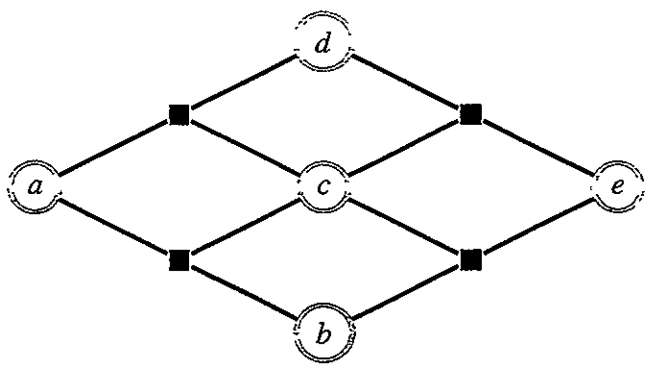
iii. Draw the triangulated graph. Your triangulated graph should contain cliques of the smallest size possible. [3 marks]

iv. Draw a Junction Tree for the above graph and verify that it satisfies the running intersection property. [4 marks]

v. Describe a suitable initialisation of clique potentials. [4 marks]

vi. Describe the Absorption procedure and an appropriate message updating schedule. [4 marks]

b. Consider the factor graph

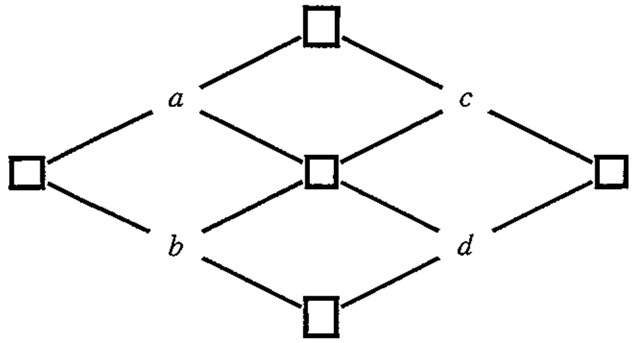


i. Explain if the factor graph is singly connected. [2 marks]

ii. Draw a Markov network that corresponds to this factor graph. [4 marks]

iii. Draw a junction tree that corresponds to the above Markov network. [3 marks]

c. Consider the factor graph



Draw a junction tree that corresponds to this factor graph.

[3 marks]

[Total 33 marks]

4. In this question, the notation $\mathcal{A} \perp\!\!\!\perp \mathcal{B} | \mathcal{C}$ means that, conditioned on the variables $\mathcal{C}$ the variables $\mathcal{A}$ and $\mathcal{B}$ are independent. If $\mathcal{C} = \emptyset$, we may write $\mathcal{A} \perp\!\!\!\perp \mathcal{B}$. Similarly, the symbol $\mathcal{A} \top\!\!\!\top \mathcal{B}$ denotes that variables $\mathcal{A}$ and $\mathcal{B}$ are dependent.

   a. Consider the distribution

   $$p(a,b,c,d,e,f,g,h) = p(a)p(b|c)p(c)p(d|a)p(e|a)p(f|d,e)p(g|b,a)p(h|c,e)$$

      i. Draw the belief network for the above distribution.

      [4 marks]

   For the following questions you must explain your reasoning:

      ii. Is $b \perp\!\!\!\perp e$?

      [2 marks]

      iii. Is $b \perp\!\!\!\perp e | g$?

      [2 marks]

      iv. Is $(b,c) \perp\!\!\!\perp (e,f)$?

      [2 marks]

      v. Is $(b,h) \perp\!\!\!\perp e | (c,a)$?

      [2 marks]

   b. If $a \perp\!\!\!\perp b$ and $b \perp\!\!\!\perp c$ is it necessarily true that $a \perp\!\!\!\perp c$? Explain your reasoning.

   [2 marks]

   c. If $a \top\!\!\!\top b$ and $b \top\!\!\!\top c$ is it necessarily true that $a \top\!\!\!\top c$? Explain your reasoning.

   [3 marks]

   d. What is the meaning of a perfect map?

   [2 marks]

   e. Explain the concept of Markov equivalence and explain what manipulations of two graphs need to be carried out to check Markov equivalence.

   [2 marks]

f. Give an example of a Belief network whose set of independence statements cannot be represented by a Markov network.

[4 marks]

g. Give an example of a Markov network whose set of independence statements cannot be represented by a Belief network.

[4 marks]

h. Give an example of a distribution whose set of conditional independence statements cannot be represented by either a Belief network nor a Markov network.

[4 marks]

[Total 33 marks]

5. a. With the help of a diagram, explain the concept of a decision tree.

[3 marks]

b. Explain how to solve a decision tree through the help of an example.

[4 marks]

c. With the help of a diagram, explain the concept of an influence diagram.

[3 marks]

d. Explain how to solve an influence diagram through the help of an example.

[4 marks]

e. Both decision trees and influence diagrams can be used to solve sequential decision problems. Explain the advantages and disadvantages of using a decision tree versus an influence diagram.

[3 marks]

f. Explain the concept of expected utility and, with reference to a betting example, explain why the 'psychological' value of money is not equal to the numerical value of money.

[4 marks]

g. Mr Benn is on a British TV game show. He's placed at the entrance to a maze and given a map of the maze. Each of the 100 positions in the maze is represented on the map. Mr Benn can move from a square to a neighbouring square to the north, south, east or west of his current position, provided that this is possible according to his map.

Mr Benn is told that every move he makes in the maze will cost him £100 and that he should try to get from the entrance to the exit of the maze by losing as little money as possible.

Explain how to formulate this problem using an influence diagram and explain the process by which the problem can be solved.

[7 marks]

h. Mr Kato is on a Japanese TV game show. The situation is the same as for Mr Benn above. except that three treasure objects are placed in the maze and shown on the map. Each object has a numerical prize value in Japanese yen and each move costs Mr Kato a certain number of yen. Each treasure object can only be picked up once and Mr Kato must get to the exit of the maze. Explain how to formulate a solution to this problem that maximises the number of yen for Mr Kato.

[5 marks]

[Total 33 marks]

END OF PAPER