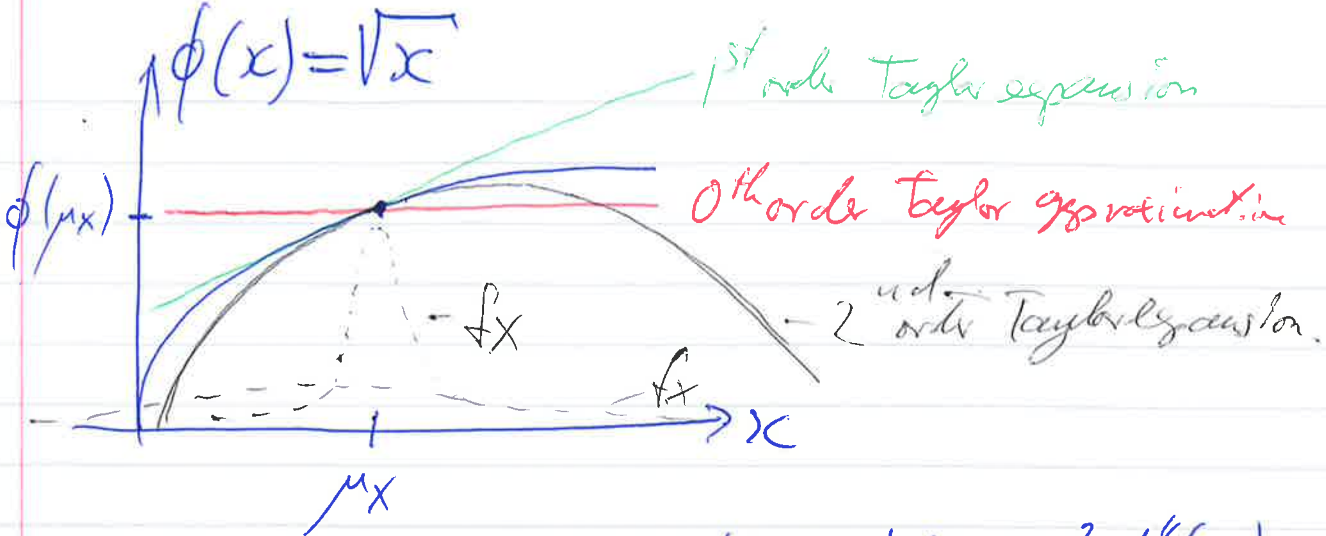

$$\phi(x) \approx \underbrace{\underbrace{\phi(\mu_X)}_{} + (x-\mu_X)\,\phi'(\mu_X)}_{} + \frac{1}{2}(x-\mu_X)^2\phi''(\mu_X)$$

$$\phi'(x) = \frac{1}{2\sqrt{x}}\,, \quad \phi''(x) = \frac{-1}{4}x^{-3/2}$$

Example 2.8 $X \sim Poi(\mu)$, $\mathbb{E}X=\mu$, $\mathrm{Var}\,X=\mu$

$$Y = \phi(X) = \sqrt{X}\,, \quad \phi'(x) = \frac{1}{2}x^{-\frac{1}{2}}\,, \quad \phi''(x) = -\frac{1}{4}x^{-3/2}$$

$$\begin{aligned}\mathbb{E}[Y] &\approx \phi(\mu_X) + \frac{1}{2}\phi''(\mu_X)\cdot\sigma_X^2 \\ &= \sqrt{\mu} + \frac{1}{2}\cdot\left(-\frac{1}{4}\mu^{-3/2}\right)\cdot\mu \\ &= \sqrt{\mu} - \frac{1}{8\sqrt{\mu}}\end{aligned}$$

$$\mathrm{Var}(Y) \approx (\phi'(\mu_X))^2\cdot\sigma_X^2 = \left(\frac{1}{2\sqrt{\mu}}\right)^2\cdot\mu = \frac{1}{4}$$

Does not depend on μ:
we call ϕ a variance-stabilizing transformation.