06/11/15

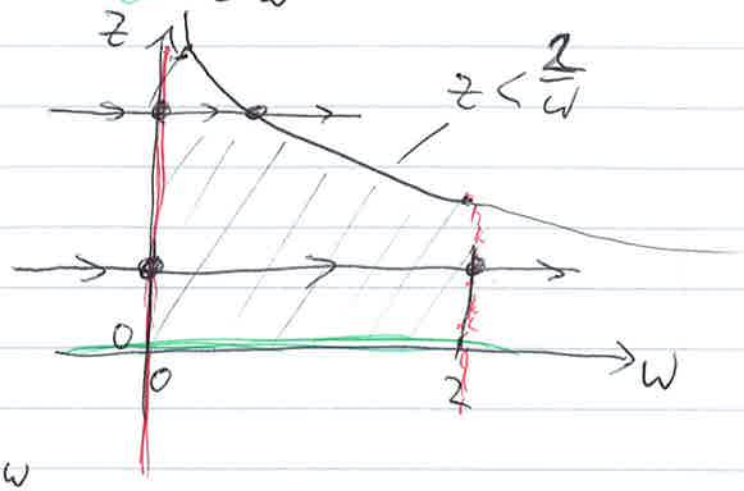
Example 2.5 $X, Y \sim \text{Unif}(0,2)$, $X \perp\!\!\!\perp Y$.

$\left(Z = \frac{X}{Y}, \quad W = Y \right) \Leftrightarrow X = ZW, \quad Y = W$

$0 < W < 2$ $\qquad 0 < Z < \frac{2}{W}$

$0 < ZW < 2 \mid : W$

$0 < Z < \frac{2}{W}$



$$\frac{\partial(x,y)}{\partial(w,z)} = \begin{pmatrix} z & w \\ 1 & 0 \end{pmatrix}$$

$$J(w,z) = \det\begin{pmatrix} z & w \\ 1 & 0 \end{pmatrix} = -w$$

$$f_{W,Z}(w,z) = f_{X,Y}(x(w,z), y(w,z)) \cdot |J(w,z)|$$

$$= \begin{cases} \frac{1}{2} \cdot \frac{1}{2} \cdot w & \text{if } 0 < w < 2,\ 0 < z < \frac{2}{w} \\ 0 & \end{cases}$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_{W,Z}(w,z)\,dw = \int_0^{\min\left(2, \frac{2}{z}\right)} \frac{w}{4}\,dw = \frac{1}{8}\left(\min\left(2, \frac{2}{z}\right)\right)^2$$

Hamise.

$z > 0$ $\qquad$ on $z > 0$.