STAT2001/3101 Peer Learning Session 3:

Challenge Question

1. Let X follow a univariate standard normal distribution. Let $Y = \exp(X+1)$ and compute the pdf of Y. Also compute the moments $\mathbb{E}[Y^k]$ for $k \in \mathbb{N}$. `Hint:` The solution for the moments requires only one or two lines of algebra. [6]

2. Define a continuous random variable Z with pdf

$$f_Z(z) = f_Y(z) \cdot (1 + \sin(2\pi(-1 + \log z))), \qquad z \in (0, \infty)$$

   and show that its moments $\mathbb{E}[Z^k]$ are the same as those of Y.
   `Hint:` the integral of an even function f multiplied by an odd function g is zero, i.e. $\int_{-\infty}^{\infty} f(x)g(x)dx = 0$, if f is absolutely integrable, i.e. $\int_{-\infty}^{\infty} |f(x)|dx < \infty$, and $|g|$ is bounded, i.e. there exists a $C > 0$ such that $|g(x)| < C$ for all $x \in \mathbb{R}$. [7]

3. Explain how identical sequences of moments would normally imply that the moment generating functions M_Y and M_Z are identical. Explain how this implies that the moment generating function M_Y cannot be defined on an open interval including zero. [3]