Ex 281 $D := $ widget is defective

$A := $ widget comes from factory A.

(a)
$$P(D|A) = 0.2 \quad P(D|\bar{A}) = 0.05$$
$$P(A) = 2P(\bar{A}) \quad P(A) + P(\bar{A}) = 1 \quad \Rightarrow P(A) = \tfrac{2}{3} \quad P(\bar{A}) = \tfrac{1}{3}$$

$$P(D) = P(D \cap A) + P(D \cap \bar{A})$$
$$= P(D|A) \cdot P(A) + P(D|\bar{A}) \cdot P(\bar{A}) = 0.2 \cdot \tfrac{2}{3} + 0.05 \cdot \tfrac{1}{3} = \underline{\underline{0.15}}$$

(b) $P(\bar{A}|D) = \dfrac{P(\bar{A} \cap D)}{P(D)} = \dfrac{P(D|\bar{A}) \cdot P(\bar{A})}{P(D)} = \dfrac{0.05 \cdot \tfrac{1}{3}}{0.15} = \underline{\underline{\tfrac{1}{9}}}$

(c) $\quad 2P(A) = P(\bar{A}) \quad P(A) + P(\bar{A}) = 1 \Rightarrow P(A) = \tfrac{1}{3}, \quad P(\bar{A}) = \tfrac{2}{3}$

$$P(\bar{D}) = P(\bar{D}|A) \cdot P(A) + P(\bar{D}|\bar{A}) \cdot P(\bar{A})$$

$C := $ widget was made on the new type of production line

$$= \tfrac{1}{3} \left(P(\bar{D}|A \cap C) \cdot \tfrac{1}{10} + P(\bar{D}|A \cap \bar{C}) \cdot \tfrac{9}{10} \right)$$
$$\tfrac{2}{3} \left(P(\bar{D}|\bar{A} \cap C) \cdot \tfrac{1}{5} + P(\bar{D}|\bar{A} \cap \bar{C}) \cdot \tfrac{4}{5} \right)$$
$$= \tfrac{1}{3} \left(\tfrac{1}{10} \cdot \tfrac{3}{4} + \tfrac{9}{10} \cdot \tfrac{4}{5} \right) + \tfrac{2}{3} \left(\tfrac{1}{5} \cdot \tfrac{3}{4} + \tfrac{4}{5} \cdot \tfrac{19}{20} \right)$$
$$= \tfrac{523}{600} > \left(1 - \tfrac{3}{20} \right) = \tfrac{510}{600}$$