29/10/15

Example 1.29

Urn urn-models

5 red balls, 5 black balls, 5 yellow balls

$P(1 \text{ red}, 1 \text{ black}, 1 \text{ yellow})$

$P_{red} = \frac{5}{15} = \frac{1}{3}$, $P_{black} = \frac{1}{3}$, $P_{yellow} = \frac{1}{3}$

$n = 3$

$$P(N_R = 1, N_B = 1, N_Y = 1) = \frac{n!}{n_R!\, n_B!\, n_Y!} P_{red}^{n_R} \cdot P_{black}^{n_{black}} \cdot P_Y^{n_Y}$$

$$= \frac{3!}{1!\,1!\,1!} \left(\frac{1}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^1 = \frac{6}{27} = \frac{2}{9}$$

Example 1.30

$P_A = 0.05$, $P_B = 0.3$, $P_F = 0.05$, $P_{other} = 0.6$

$n = 100$

$$P(N_A = n_A, N_B = n_B, N_F = n_F) =$$

$$\frac{100!}{n_A!\, n_B!\, n_F!\, n_{other}!} P_A^{n_A} P_B^{n_B} P_F^{n_F} P_{other}^{100 - n_A - n_B - n_F}$$

$$n_{other} = 100 - n_A - n_B - n_F.$$

Marginal $P(N_A = n_A) = \binom{n}{n_A} P_A^{n_A} (1 - P_A)^{n - n_A}$

$$= \frac{100!}{n_A!\,(100 - n_A)!} 0{,}05^{n_A}\, 0.95^{100 - n_A}$$