05/11/15

Example 2.1

X : sum of scores on two dice

$Y = (X-7)^2$

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

y	0	1	4	9	16	25
$P(Y=y)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\cancel{8}\ \frac{2}{36}$

$P(Y=0) = P((X-7)^2=0) = P(X=7) = \frac{6}{36}$

$P(Y=1) = P((X-7)^2=1) = P(X=6 \text{ or } X=8) = \frac{10}{36}$

$P(X=6) = P(\{\omega : X(\omega)=6\})$

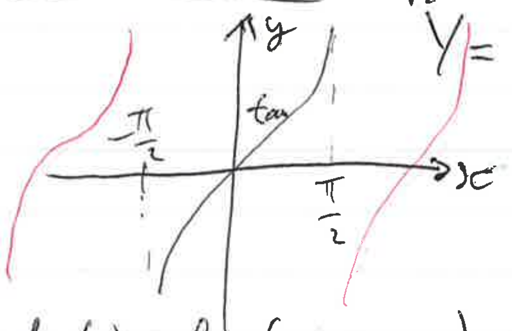
$P(X=6 \text{ or } X=8) = P(\{\omega : X(\omega)=6\} \cup \{\omega : X(\omega)=8\})$

"$P(X=6 \cup X=8)$" Problematic!

Example 2.2

$f_X(x) = \frac{1}{\pi}$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$Y = \tan X \Leftrightarrow X = \arctan Y$

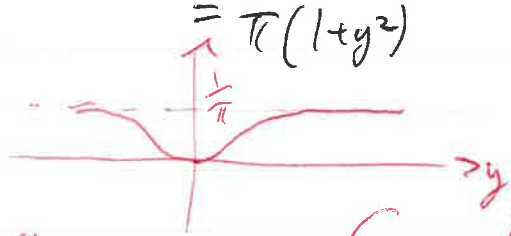


Aside:
$\tan(\arctan y) = y$

$f_Y(y) = f_X(\arctan y)\cdot\left|\frac{d}{dy}\arctan y\right| = \frac{1}{\pi}\left|\frac{1}{1+y^2}\right| = \frac{1}{\pi(1+y^2)}$

for all $y \in \mathbb{R}$

$\mathbb{E}Y^2 = \int_{-\infty}^{\infty} \frac{y^2}{\pi(1+y^2)}\,dy = \infty$



$\mathbb{E}|Y| = \int_{-\infty}^{\infty} \frac{|y|}{\pi(1+y^2)}\,dy = 2\int_{0}^{\infty} \frac{y}{\pi(1+y^2)}\,dy = \infty$ Cauchy