Example 4.5

$X \sim N(2,9)$, $Y \sim t_4$, $U \sim \chi^2_3$

X, Y, U independent.

i) $\left(\frac{X-2}{3}\right)^2 \sim \chi^2_1$

ii) $U + \left(\frac{X-2}{3}\right)^2 \sim \chi^2_4$ (NB: $U \perp\!\!\!\perp X$)

iii) $\dfrac{\left(\frac{X-2}{3}\right)}{U/3} \sim t_3$ (NB: $X \perp\!\!\!\perp U$)

iv) $Y^2 \sim F_{1,4}$

because $Y \sim t_4$, i.e. $Y = \dfrac{W}{\sqrt{Z/4}}$

$W \sim N(0,1)$, $Z \sim \chi^2_4$

$\Downarrow$

$W^2 \sim \chi^2_1$

$$Y^2 = \frac{W^2}{\left(\sqrt{Z/4}\right)^2} = \frac{W^2/1}{Z/4}$$

So Y^2 is quotient of two independent χ^2 random variables with 1 and 4 degrees of freedom, both normalised to expectation 1, i.e. of W^2 and $Z/4$.