Example 5.5 $X_i \sim N(\mu, \sigma^2)$ i.i.d., $\mu \in \mathbb{R}$, $\sigma \geq 0$

(i.e. random normal sample)

Find the method of moments estimators of μ and σ^2.

Population moments $\mathbb{E}[X_i] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \dots = \mu$

$$\mathbb{E}[X_i^2] = \int x^2 f_X(x)\,dx = \mu^2 + \sigma^2$$

$$= Var[X_i] + (\mathbb{E}[X_i])^2$$

Two sample moments:

$$\overline{X^1} = \sum_{i=1}^{n} \frac{X_i}{n}, \quad \overline{X^2} = \sum_{i=1}^{n} \frac{X_i^2}{n}$$

Equate sample moments & population moments:

$$\overline{X^1} = \mu \qquad \overline{X^2} = \mu^2 + \sigma^2$$

$$\mu = \overline{X^1} \qquad \sigma^2 = \overline{X^2} - (\overline{X^1})^2$$

$$\hat{\mu} = \overline{X^1}, \quad \hat{\sigma} = \sqrt{\overline{X^2} - (\overline{X^1})^2}$$