Example 3.12 $X_i \sim Exp(\lambda)$, i.i.d.

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2},\ \sigma = \frac{1}{\lambda}$$

$$P(\bar{X}_n \le x) = P(\bar{X}_n - \mu \le x - \mu) \quad \text{(mean zero)}$$

$$= P\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \le \frac{x-\mu}{\sigma/\sqrt{n}}\right) \quad \text{(variance one)}$$

$$\overset{CLT}{\approx} \Phi\left(\frac{x-\mu}{\sigma/\sqrt{n}}\right)$$

$$= \Phi\left(\sqrt{n}(\lambda x - 1)\right)$$

$\cancel{P(\sum X_i}$

$$P\left(\sum_{i=1}^{n} X_i \le x\right) = P\left(\frac{1}{n}\sum_{i=1}^{n} X_i \le \frac{x}{n}\right) \approx \Phi\left(\sqrt{n}\left(\lambda \cdot \frac{x}{n} - 1\right)\right)$$

$$= \Phi\left(\frac{\lambda x}{\sqrt{n}} - \sqrt{n}\right)$$