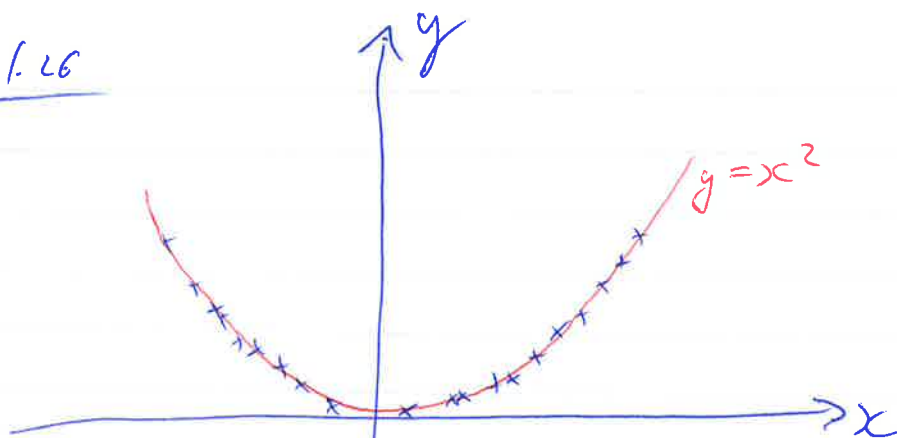


Example 1.16



$$\begin{aligned} \text{cov}(X, Y) &= E[XY] - E[X] \cdot E[Y] \\ &= E[X^3] - E[X] \cdot E[X^2] = 0 \end{aligned}$$

$$E[X] = 0, \quad E[X^3] = \int_{-1}^1 x^3 \cdot \frac{1}{2} dx = 0$$

odd

$$\text{cov}(X, Y) = 0 \Rightarrow \text{corr}(X, Y) = 0$$

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

$$\begin{aligned} \text{Var}(X|Y) &= E_{X|Y}[(X - E_{X|Y}[X|Y])^2 | Y] \quad \text{4(y)} \\ &= E_{X|Y}[X^2 | Y] - 2 \underbrace{E_{X|Y}[X \cdot E_{X|Y}[X|Y] | Y]}_{= 4(y)} + E_{X|Y}[E_{X|Y}[X|Y]^2 | Y] \\ &\stackrel{\text{Tok}}{=} E[X^2 | Y] - 2 E[X|Y] \cdot E[X|Y] + (E[X|Y])^2 \\ &= E[X^2 | Y] - (E[X|Y])^2 \end{aligned}$$

$$E_Y[\text{Var}_{X|Y}[X|Y]] + \text{Var}_Y[E[X|Y]]$$

$$= E_Y[E[X^2 | Y] - (E[X|Y])^2] + E_Y[(E[X|Y])^2]$$

$$= E_X[X^2] - (E[X])^2 - (E_Y[E[X|Y]])^2$$

$$= \text{Var}(X) \quad \square$$