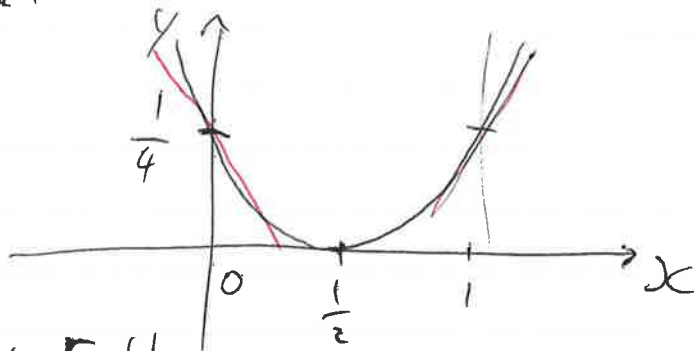Example 2·3 $f_X(x) = 2x$ on $(0,1)$

$Y = \left(x - \frac{1}{2}\right)^2$

$X = \frac{1}{2} \pm \sqrt{Y}$

$X \in (0,1) \Rightarrow Y \in \left[0, \frac{1}{4}\right)$

$$f_Y(y) = \sum_{\{x : \phi(x) = y\}} f_X(x) \left|\frac{dx}{dy}\right| \qquad \frac{dx}{dy} = \frac{\pm 1}{2\sqrt{y}}$$

$$= f_X\left(\frac{1}{2} + \sqrt{y}\right) \cdot \frac{1}{2\sqrt{y}} + f_X\left(\frac{1}{2} - \sqrt{y}\right) \cdot \frac{1}{2\sqrt{y}}$$

$$= \frac{2 \cdot \left(\frac{1}{2} + \sqrt{y}\right)}{2\sqrt{y}} + \frac{2 \cdot \left(\frac{1}{2} - \sqrt{y}\right)}{2\sqrt{y}} = \frac{1}{\sqrt{y}} \text{ on } \left(0, \frac{1}{4}\right)$$

Check: $\displaystyle\int_{-\infty}^{\infty} f_Y(y)\, dy = \int_0^{1/4} \frac{1}{\sqrt{y}}\, dy = \left[2\sqrt{y}\right]_0^{1/4} = 1 \checkmark$