Example 5.11 $\quad X_i \underset{\text{i.i.d.}}{\sim} \operatorname{Exp}\left(\frac{1}{\mu}\right), \quad \mu > 0$

$\Theta = (0, \infty)$

$$\mathcal{L}(\mu) = \prod_{i=1}^{n} f_X\left(x_i; \frac{1}{\mu}\right) = \left(\frac{1}{\mu}\right)^n \cdot \exp\left(-\frac{1}{\mu}\sum_{i=1}^{n} x_i\right)$$

same algebra as example 5.8.

$$\ell(\mu) = -n \log \mu - \frac{1}{\mu}\sum_{i=1}^{n} x_i$$

$$\frac{d\ell}{d\mu} = \frac{-n}{\mu} + \frac{1}{\mu^2}\sum_{i=1}^{n} x_i \overset{!}{=} 0 \Rightarrow \hat{\mu}_{ML} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(omit $\frac{d^2\ell}{d\mu^2}$ & boundary). $\quad \lambda = \frac{1}{\mu}$.

$\hat{\lambda} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^{-1}$ from Example 5.8

$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$ from Example 5.11

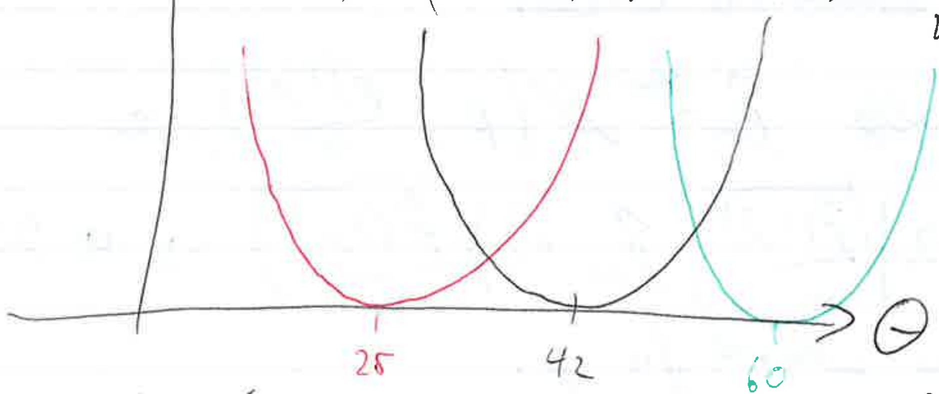
we find $\quad \hat{\lambda} = \frac{1}{\hat{\mu}}$ in this case.

$T_n(x_1, \dots, x_n) = 42. \quad \operatorname{var}(T_n) = 0$

$\operatorname{bias}(T_n; \theta) = \theta - 42$

$\operatorname{mse}(T_n) = (\operatorname{bias}(T_n;\theta))^2 + \operatorname{var}(T_n;\theta)$

$\operatorname{mse}(T_n; \theta) = (\theta - 42)^2$



So, the hypothetical "perfect" estimator would have to have

$\operatorname{mse}(T;\theta) = 0$ for <u>all</u> values θ.

$\Rightarrow \operatorname{var}(T;\theta) = 0 \Rightarrow T = \text{const.}$ $\lightning$

The perfect estimator does not exist.