Ex. 1.11

x	0	1	2
$P(X=x)$	$1/4$	$\frac{1}{2}$.	$1/4$.

$$Var(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$
$$= \sum_{x_i=0}^{2} (x_i - \mathbb{E}[X])^2 \cdot p_X(x_i)$$
$$= (0-1)^2 \cdot \frac{1}{4} + (1-1)^2 \cdot \frac{1}{2} + (2-1)^2 \cdot \frac{1}{4}$$

$\uparrow$ $\mathbb{E}X = 1$ in Example 1.10

$$= \frac{1}{2}$$

$y=\phi(x)$	0	1
$P(Y=y)$	$\frac{1}{2}$	$\frac{1}{2}$

(Ex. 1.10)

$$Var(\phi(X)) = \mathbb{E}\left[(\phi(X) - \mathbb{E}[\phi(X)])^2\right] = \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

Reminder: $Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$.

Here, since $\phi(X) = \phi^2(X)$ for all possible values of X,

$$\mathbb{E}[(\phi(X))^2] = \mathbb{E}[\phi(X)] = \frac{1}{2}$$

So $Var(\phi(X)) = \frac{1}{4}$.

Example 1.12

$$Y = X - (2 - X) = 2X - 2$$

$$\mathbb{E}Y = \mathbb{E}[2X - 2] = 2\mathbb{E}X - 2 = 2 \cdot 1 - 2 = 0$$

$$Var\,Y = Var[2X-2] = 2^2\, Var(X) = 4 \cdot \frac{1}{2} = 2.$$

$$P(x_1 < X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$
$$= \int_{-\infty}^{x_2} f_X(x)\,dx - \int_{-\infty}^{x_1} f_X(x)\,dx$$
$$= F_X(x_2) - F_X(x_1) =$$

$$P(x_1 < X \leq x_2) = P(X \leq x_2) - P(X \leq x_1)$$