Question Ex1B3 $VaR_\alpha(L) = \inf\{\ell \in \mathbb{R} \mid F_L(\ell) \geq \alpha\}$

(a) $f_{L_1}(\ell) = \begin{cases} (\ell+1)^{-2} & \text{if } \ell \geq 0 \\ 0 & \text{otherwise} \end{cases}$

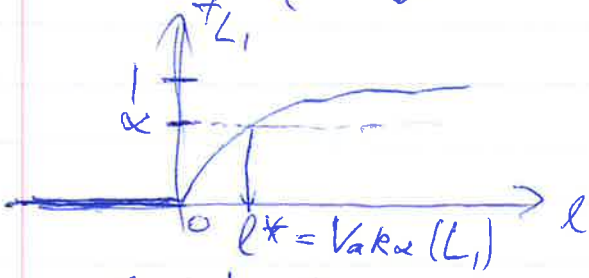
$$F_{L_1}(\ell) = \int_{-\infty}^{\ell} f_{L_1}(x)\,dx \overset{\ell \geq 0}{=} \int_0^{\ell} (x+1)^{-2}\,dx$$

$$= \left[-(x+1)^{-1}\right]_0^{\ell} = 1 - (1+\ell)^{-1}$$

$$F_{L_1}(\ell) = \begin{cases} 1-(\ell+1)^{-1} & \text{if } \ell \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$\inf\{(0,1)\} = 0$

min $(0,1)$ does not exist!



$$\alpha = 1-(\ell+1)^{-1} \Rightarrow \ell = \frac{\alpha}{1-\alpha}$$

So $VaR_\alpha(L_1) = \frac{\alpha}{1-\alpha}$

(b) $$F_{L_2}(\ell) = \int_{-\infty}^{\ell} f_{L_2}(x)\,dx \overset{\ell \geq 0}{=} \int_0^{\ell} 2xe^{-x^2}\,dx \overset{z=x^2}{=} \int_0^{\ell^2} e^{-z}\,dz$$

$$= 1 - e^{-\ell^2}$$

$$\alpha = 1 - e^{-\ell^2} \Rightarrow e^{-\ell^2} = 1-\alpha \Rightarrow -\ell^2 = \log(1-\alpha)$$

$$\ell = \sqrt{-\log(1-\alpha)}$$

(c) $Var(aX+b) = a^2 Var(X)$

$L_3 := L_2 + 42 \Rightarrow Var(L_3) = Var(L_2)$

$VaR_\alpha(L_3) = VaR_\alpha(L_2+42) = VaR_\alpha(L_2) + 42$

$$\mathbb{E}L_1 = \int_0^{\infty} x \cdot (x+1)^{-2}\,dx = \infty.$$