$$p_X(k) = \frac{1}{e^\mu - 1} \frac{\mu^k}{k!}$$

$$\mathbb{E}[X] = \sum_{k=1}^{\infty} k \frac{\mu^k}{(e^\mu - 1) k!} = \sum_{k=1}^{\infty} \frac{\mu^k}{(e^\mu - 1)(k-1)!}$$

$$= \sum_{k=0}^{\infty} \frac{\mu^{k+1}}{(e^\mu - 1) k!} = \frac{\mu}{e^\mu - 1} \sum_{k=0}^{\infty} \frac{\mu^k}{k!}$$

$$= \frac{e^\mu}{e^\mu - 1} \cdot \mu .$$

Example 1.10 $\phi(0) = \phi(2) = 0$, $\phi(1) = 1$

$$Y = \phi(X)$$

$$\mathbb{E}[X] = \sum_{x_i=0}^{2} x_i p_X(x_i) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1$$

$$\mathbb{E}[\phi(X)] = \sum_{x_i=0}^{2} \phi(x_i) p_X(x_i) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{4} = \frac{1}{2}$$