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Example 1.31

$N_A | N_F = 10$, $n = 100$

$p_A = 0.05$
$p_B = 0.3$
$p_F = 0.05$
$p_{Ora} = 0.6$

$$N_A | N_F = 10 \sim Bin\left(n-10, \frac{p_A}{1-p_F}\right)$$
$$= Bin\left(90, \frac{0.05}{0.95}\right)$$

$$corr(N_A, N_F) = -\sqrt{\frac{p_A \cdot p_F}{(1-p_A)(1-p_F)}} = -\sqrt{\frac{0.05^2}{0.95^2}} = -\frac{0.05}{0.95} = -\frac{1}{19}.$$

Example 1.32

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 7 \\ 8 \end{pmatrix}, \sigma_X = 1, \sigma_Y = 2, \rho = \frac{1}{2}\right)$$

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$$f_{X|Y}(x|9) \sim N\left(\mu_X + \rho \frac{\sigma_X}{\sigma_Y}(y - \mu_Y), \sigma_X^2(1-\rho^2)\right)$$

$$= N\left(7 + \frac{1}{2} \cdot \frac{1}{2}(9-8), 1^2 \cdot \left(1 - \left(\frac{1}{2}\right)^2\right)\right)$$

$$= N(7.25, 3/4)$$

compare with $X \sim N(7, 1)$

$$\underbrace{Corr(X,Y)}_{= \rho} = \frac{Cov(X,Y)}{\sqrt{\underbrace{Var(X)}_{\sigma_X^2} \underbrace{Var(Y)}_{\sigma_Y^2}}}$$

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$\Sigma = \begin{pmatrix} a & & \\ & b & \\ & & c \end{pmatrix} \quad 2\pi\Sigma = \begin{pmatrix} 2\pi a & & \\ & 2\pi b & \\ & & 2\pi c \end{pmatrix}$

Example 1·33

$$f_X(x) = \frac{1}{|2\pi\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

$$\overbrace{\underbrace{(x-\mu)^T}_{\in \mathbb{R}^3}\underbrace{\Sigma^{-1}}_{\in \mathbb{R}^{3\times 3}}\underbrace{(x-\mu)}_{\in \mathbb{R}^3}}^{\in \mathbb{R}} = \begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ x_3-\mu_3 \end{pmatrix}^T \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix} \begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ x_3-\mu_3 \end{pmatrix}$$

$$= \begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ x_3-\mu_3 \end{pmatrix}^T \begin{pmatrix} \frac{1}{a}(x_1-\mu_1) \\ \frac{1}{b}(x_2-\mu_2) \\ \frac{1}{c}(x_3-\mu_3) \end{pmatrix} = (x_1-\mu_1)\cdot\frac{1}{a}(x_1-\mu_1) + (x_2-\mu_2)^2/b + (x_3-\mu_3)^2/c$$

$$f_X(x) = \frac{1}{(2\pi)^{3/2}\sqrt{abc}} \exp\left(-\frac{1}{2a}(x_1-\mu_1)^2 - \frac{1}{2b}(x_2-\mu_2)^2 - \frac{1}{2c}(x_3-\mu_3)^2\right)$$

$$\exp(A+B+C) = \exp(A)\exp(B)\exp(C)$$

$$= \underbrace{\frac{1}{\sqrt{2\pi a}} \exp\left(-\frac{(x_1-\mu_1)^2}{2a}\right)}_{X_1 \sim N(\mu_1, a)} \underbrace{\frac{1}{\sqrt{2\pi b}} \exp\left(-\frac{(x_2-\mu_2)^2}{2b}\right)}_{X_2 \sim N(\mu_2, b)} \underbrace{\frac{1}{\sqrt{2\pi c}} \exp\left(-\frac{(x_3-\mu_3)^2}{2c}\right)}_{X_3 \sim N(\mu_3, c)}$$