Exercise Sheet 1

A: Warm-up questions: Sections 1.1 – 1.2

next week.

B: Final answers only

1. (a)

$$\Omega = \{(d_1, d_2, d_3) : d_1 \in \{1,2,3,4,5,6\},\ d_2 \in \{1,2,3,4,5,6\},\ d_3 \in \{1,2,3,4,5,6\}\}$$
$$\Omega = \{(d_1, d_2, d_3) : \forall i \in \{1,2,3\} : d_i \in \{1,2,3,4,5,6\}\}$$
$$\Omega = \{1,2,3,4,5,6\}^3$$

These are all acceptable with the first one being the most likely variant following lecture content.

(b)

$$P(A) = ... = \frac{6}{216}$$
$$P(B_1) = \frac{1}{6}$$
$$...$$

Therefore, A and B_1 are independent.

(c) For a selection of just two events, independence is straightforward following the model $P(B_1 \cap B_2) = ...$ and $P(B_1) \cdot P(B_2) = ...$ For all three events, it is $P(B_1 \cap B_2 \cap B_3) = ... = \frac{1}{216}$ which agrees with $P(B_1) \cdot P(B_2) \cdot P(B_3) = ... = \frac{1}{216}$ establishing independence.

(d)

$$P(A \cap B_1 \cap B_2) = ...$$
$$P(A) \cdot P(B_1) \cdot P(B_2) = ... = \frac{1}{1296}$$

2. (a)

$$p_Y(n) = \begin{cases} e^{-\mu}(1+\mu) & \text{if } n = 1 \\ e^{-\mu}\frac{\mu^k}{n} & \text{if } k! = n \text{ and } n > 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) The sketch should show ...
the values at which the jumps happen should be marked for the first three jumps at 1,2 and 6 and the three values $P(Y \leq 1) = e^{-\mu}(1+\mu)$, $P(Y \leq 2) = e^{-\mu}(1+\mu+\mu^2/2)$ and $P(Y \leq 6) = e^{-\mu}(1+\mu+\mu^2/2+\mu^3/6)$ should be written down.

(c)

$$\mathbb{E}[Y] = ... = e^{-\mu}\frac{1}{1-\mu}$$

3. (a)

$$\mathrm{VaR}_\alpha(L_1) = \frac{\alpha}{1-\alpha}$$

(b)

$$\mathrm{VaR}_\alpha(L_2) = \sqrt{-\log(1-\alpha)}$$

(c) ... so $L_3 = L_2 + 42$ does the job.

C: Exam Practice Questions

later.