Example 4.4

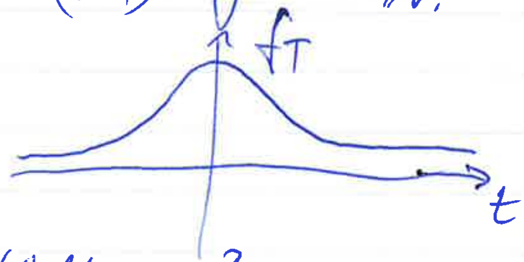
$$T = \frac{Z}{\sqrt{U/\nu}} \qquad Z \sim \mathcal{N}(0,1), \quad U \sim \chi^2_\nu, \quad Z \perp\!\!\!\perp U.$$

$$f_T(t) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)} \frac{1}{\sqrt{\nu\pi}} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad t \in \mathbb{R}.$$

For $\nu = 1$: $$f_T(t) = \frac{\Gamma(1)}{\Gamma(\frac{1}{2})} \cdot \frac{1}{\sqrt{\pi}} \left(1 + t^2\right)^{-1}$$

Note $\Gamma(1) = 1$ because $\Gamma(n) = (n-1)!$ for $n \in \mathbb{N}$.
$\Gamma(\frac{1}{2}) = \sqrt{\pi}$ (last week)

$$f_T(t) = \frac{1}{\pi(1+t^2)}$$



$$\mathbb{E}\,T = \underbrace{\int_{-\infty}^{0} t \cdot f_T(t)\,dt}_{= -\infty} + \underbrace{\int_{0}^{\infty} t\, f_T(t)\,dt}_{= \infty} = ?$$

even symmetry $f_T(t) = f_T(-t)$

So $t \cdot f_T(t)$ will have odd symmetry.

Note $\lim_{q \to \infty} \int_{-q}^{q} t\, f_T(t)\,dt = 0$ as expected.

But $\lim_{q \to \infty} \int_{-q}^{2q} t\, f_T(t)\,dt \neq 0$.

[In fact, $\lim_{q \to \infty} \int_{-q}^{2q} t\, f_T(t)\,dt = \frac{\log 2}{\pi}$]

So the result depends on how quickly you send the integration limits to infinity. Therefore the integral is said to not exist.