16/10/15

Example 1.16 $N \sim Poi(\lambda)$, $R|N=n \sim Bin(n, \pi)$

$\pi \in [0,1]$, $\lambda > 0$.

$$p_R(r) = \sum_{n=0}^{\infty} p_{R,N}(r,n) = \sum_{n=0}^{\infty} p_{R|N}(r|n) \cdot p_N(n)$$

$$= \sum_{n=0}^{\infty} \binom{n}{r} \pi^r (1-\pi)^{n-r} e^{-\lambda} \frac{\lambda^n}{n!}$$

$$= \sum_{n=r}^{\infty} \frac{\cancel{n!}}{r!(n-r)!} \pi^r (1-\pi)^{n-r} e^{-\lambda} \frac{\lambda^n}{\cancel{n!}}$$

$$= \frac{\pi^r e^{-\lambda}}{r!} \sum_{n=r}^{\infty} \frac{(1-\pi)^{n-r}}{(n-r)!} \lambda^n$$

$$\underset{k=n-r}{=} \frac{\pi^r e^{-\lambda}}{r!} \lambda^r \sum_{k=0}^{\infty} \frac{(1-\pi)^k}{k!} \lambda^{k\cancel{+r}}$$

$$= \frac{(\pi\lambda)^r e^{-\lambda}}{r!} e^{\lambda(1-\pi)} \qquad e^a = \sum_{n=0}^{\infty} \frac{a^n}{n!}$$

$$= \frac{(\pi\lambda)^r}{r!} e^{-\pi\lambda}$$ which is the pmf of a $Poi(\pi\lambda)$.