

Example 1.27

$$P(X=1|Y=0) = P(X=2|Y=0) = \frac{1}{2}$$

$$P(X=0|Y=1) = P(X=1|Y=1) = P(X=2|Y=1) = \frac{1}{3}$$

$$\begin{aligned} \text{Var}[X|Y=0] &= \sum_{x_i=1}^2 (x_i - E[X|Y=0])^2 \cdot P_{X|Y}(x_i|0) \\ &= \left(2 - \frac{3}{2}\right)^2 \cdot \frac{1}{2} + \left(1 - \frac{3}{2}\right)^2 \cdot \frac{1}{2} \\ &= \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{4} \end{aligned}$$

$$\text{Var}[X|Y=1] = \frac{2}{3}$$

$$\begin{aligned} \text{Var}[X] &= E_Y[\text{Var}[X|Y]] + \text{Var}_Y(E_{X|Y}[X|Y]) \\ &= \sum_{y_j=0}^1 \text{Var}[X|Y=y_j] \cdot P_Y(y_j) + \sum_{y_j=0}^1 (E_{X|Y}[X|Y=y_j] - EX)^2 \cdot P_Y(y_j) \\ &= \frac{1}{4} \cdot \frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4} + \left(\frac{3}{2} - \frac{11}{8}\right)^2 \cdot \frac{3}{4} + \left(1 - \frac{11}{8}\right)^2 \cdot \frac{1}{4} \\ &= \frac{53}{128} \end{aligned}$$

$$\text{Var} X = \sum_{x_i} (x_i - EX)^2 \cdot P(x_i)$$

$$= \frac{77}{192}$$

as it should be to agree with previously quoted results!