Solutions to Exercise Sheet 2

A: Warm-up questions

1. (a) $P(A) = P(\{1,2\}) = P(\{1\} \cup \{2\}) = P(\{1\}) + P(\{2\}) = 1/2 + 1/4 = 3/4$.
   (b) $P(X = 1|X \in A) = P(X = 1|A) = \frac{P(X=1 \text{ and } X \in A)}{P(X \in A)} = \frac{1/2}{3/4} = 2/3$

2. (a) It's given directly in the table : $P(X = 2, Y = 1) = 1/12$.
   (b) We need to sum up all entries in columns one and two of the two-way table: $P(X = 1 \text{ or } X = 2) = 1/8 + 1/8 + 1/4 + 1/12 + 1/6 = 18/24 = 3/4$.
   (c) Dividing the marginal probability mass function for X by the probability of $X \in A$: we obtain

x	1	2	3
$P(X = x\|X \in A)$	2/3	1/3	undefined

   Note that $P(X = 1|X \in A) = 2/3$ from this table just as calculated above.
   (d) Summing over the first row yields the maringal pmf for Y as $P(Y = 1) = 1/8 + 1/12 + 0 = 5/24$.
   (e) Similarly to (c) above, we divide the joint probability mass function by the marginal probability mass function for Y evaluated at $y = 1$:

x	1	2	3
$P(X = x\|Y = 1)$	3/5	2/5	0

3. Joint probability table

		X_2			
		0	1	2	3
	0	0	10/36	14/36	1/36
X_1	1	2/36	8/36	0	0
	2	1/36	0	0	0

   Marginals: $p_1(x_1) = 25/36, 10/36, 1/36$ for $x_1 = 0, 1, 2$
   $p_2(x_2) = 3/36, 18/36, 14/36, 1/36$ for $x_2 = 0, 1, 2, 3$
   Conditionals: $p_{1|2}(x_1|x_2 = 1) = 10/18, 8/18, 0$ for $x_1 = 0, 1, 2$
   $p_{2|1}(x_2|x_1 = 1) = 2/10, 8/10, 0, 0$ for $x_2 = 0, 1, 2, 3$
   $p(x_1, x_2) = p_1(x_1)p_2(x_2)$ fails for $(x_1, x_2) = (1, 1)$ for example, so not independent.
   Means $\mu_1 = 12/36, \mu_2 = 49/36$; variances $\sigma_1^2 = 360/36^2, \sigma_2^2 = 587/36^2$; covariance $\sigma_{12} = -300/36^2$; correlation $\rho = -0.65 \neq 0$, so X_1 and X_2 are correlated.

B: Questions to hand in

1. (a) Let D denote the event that the widget is defective and let A denote the event that is was made in factory A.

$$P(D) = ... = 0.15$$

   (b) This asks about a probability conditioned "the other way around" relative to the given probabilities and this was emphasized in lectures as the main use of

Bayes' theorem.

$$P(\bar{A}|D) = ... = 1/9$$

(c) Let C denote the event that the widget was made using the new type of production line.

$$P(\bar{D}) = ... = \frac{523}{600} > (1 - 3/20) = \frac{510}{600}$$

2. (a) Students can use the appendix of the lecture notes where the expectation of a Gam(α, β) is given as $\frac{\alpha}{\beta}$. Hence, $\mathbb{E}[Y|X = x] = 10(x + 1)$.

   (b)

$$\mathbb{E}[Y] = ... = 160$$

   (c)

$$\mathbb{E}[2X^3 + 2X^2 - \frac{1}{5}YX^2] = ... = 0$$

3. (a) as discusssed in letcures...

   Explanations need to include English sentences to attract marks. Writing down nothing but a formula does not constitute an explanation.

   (b)

$$\mathbb{E}[S|C = 0] = ... = 406$$
$$\mathbb{E}[S|C = 1] = ... = 609$$
$$\mathbb{E}[S|C \geq 2] = ... = 2015$$

   (c)

$$\mathbb{E}[S] = ... = 537.20$$

C: Exam Practice Questions

later