as given 20/11/15

Example 3.1

$X \sim Poi(\mu)$, $G(z) = \mathbb{E}[z^X]$, $\mathbb{E}[X] = \sum_{k=0}^{\infty} k \cdot e^{-\mu} \frac{\mu^k}{k!}$

$$G_X(z) = \mathbb{E}[z^X] = \sum_{k=0}^{\infty} z^k \cdot e^{-\mu} \frac{\mu^k}{k!}$$

$$= \underbrace{\sum_{k=0}^{\infty} e^{-z\mu} \frac{(z\mu)^k}{k!}}_{=1} \cdot e^{-\mu} \cdot e^{z\mu} = e^{\mu(z-1)}$$

$$\boxed{\sum_{k=0}^{\infty} \frac{a^k}{k!} = e^a \quad (\text{any } a \in \mathbb{R})}$$

$G_X'(z) = \mu e^{\mu(z-1)}$ $\quad G_X'(1) = \mu = \mathbb{E}[X]$.

Example 3.2

$$G_X(z) = (1-p+pz)^n$$

Want to write this as $G_X(z) = \sum_{k=0}^{\infty} p_k z^k$

$n=1$: $\underbrace{1-p}_{p_0} + \underbrace{p}_{p_1}z$ $\quad$ ~~p_2~~ $p_2, p_3, p_4 \dots = 0$.

$n=2$: $(1-p+pz)^2 = q^2 + 2pqz + p^2z^2$

$q := 1-p$

$p_0 = q^2$, $p_1 = 2pq$, $p_2 = p^2$, $p_3 = p_4 = p_5 = \dots = 0$

$$G_X(z) = (q+pz)^n = \sum_{k=0}^{n} \binom{n}{k} (p\cancel{z})^k q^{n-k} z^k$$

$p_r = \binom{n}{r} p^r q^{n-r}$ which is $Bin(n,p)$.