asgier 27/11/15

Example 3.11 $X \sim N\left(\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1 & 0{,}7\\0{,}7 & 1\end{pmatrix}\right)$

$$Y = X_1 - X_2 \quad Y = a^T X, \quad a = (1,-1)^T = \begin{pmatrix}1\\-1\end{pmatrix}$$

works because

$$a^T X = (1,-1)\cdot\begin{pmatrix}X_1\\X_2\end{pmatrix} = 1\cdot X_1 + (-1)\cdot X_2 = X_1 - X_2$$

$$Y \sim N(a^T\mu, \cancel{\tfrac{1}{2}} a^T \Sigma a)$$

$$= N\left((1,-1)\begin{pmatrix}2\\1\end{pmatrix}, \cancel{\tfrac{1}{2}}(1,-1)\begin{pmatrix}1 & 0{,}7\\0{,}7 & 1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right)$$

$$= N\left(1\cdot 2 + (-1)\cdot 1, \cancel{\tfrac{1}{2}}(1,-1)\begin{pmatrix}0{,}3\\-0{,}3\end{pmatrix}\right)$$

$$= N(1, 0{,}6)$$

Is $\begin{pmatrix}1 & 0{,}7\\0{,}7 & 1\end{pmatrix}$ positive definite?

$$a^T\begin{pmatrix}1 & 0{,}7\\0{,}7 & 1\end{pmatrix}a = a^T\begin{pmatrix}a_1 + 0{,}7a_2\\0{,}7a_1 + a_2\end{pmatrix}$$

$$= a_1\cdot(a_1 + 0{,}7a_2) + a_2\cdot(0{,}7a_1 + a_2)$$

$$= a_1^2 + 1{,}4a_1a_2 + a_2^2$$

$$= \underbrace{(a_1 + 0{,}7a_2)^2}_{\geq 0} + \underbrace{0{,}51a_2^2}_{\geq 0} \geq 0$$

if $a \neq 0 \Rightarrow a_2 \neq 0$ or else $a_1 \neq 0$

$$\Rightarrow 0{,}51a_2^2 > 0$$

So $a^T\begin{pmatrix}1 & 0{,}7\\0{,}7 & 1\end{pmatrix}a > 0$ as long as $a \neq \begin{pmatrix}0\\0\end{pmatrix}$.