Example 5.13 $X_i \sim Exp(\lambda)$, i.i.d.

from example 5.8. $\left\{ \hat{\lambda}_{ML} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^{-1}. \quad \frac{d^2 l}{d\lambda^2} = -\frac{n}{\lambda^2} \right.$

$$I(\lambda) = \mathbb{E}\left[-\frac{\partial^2}{\partial\lambda^2} l(\lambda; X)\right] = \mathbb{E}\left[-\left(-\frac{n}{\lambda^2}\right)\right] = \frac{n}{\lambda^2}.$$

So $\hat{\lambda}_{ML} \overset{\text{approx.}}{\sim} \mathcal{N}\left(\lambda, (I(\lambda))^{-1}\right) = \mathcal{N}\left(\lambda, \frac{\lambda^2}{n}\right)$

So, the 95% confidence interval will be

$$\left(\hat{\lambda} - \frac{2\hat{\lambda}}{\sqrt{n}}, \hat{\lambda} + \frac{2\hat{\lambda}}{\sqrt{n}}\right) = \left(\hat{\lambda}\left(1 - \frac{2}{\sqrt{n}}\right), \hat{\lambda}\left(1 + \frac{2}{\sqrt{n}}\right)\right).$$

Alternatively: $\sum X_i \sim Gam(n, \lambda)$

$$\frac{1}{n}\sum X_i \sim Gam(n, \lambda n)$$

$$\left(\frac{1}{n}\sum X_i\right)^{-1} \sim IGam(\dots)$$