Example 5.4 $f_X(x) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1, \\ 0 & \text{otherwise} \end{cases}$, where $\theta > 0$.

Find MME of θ. One unknown $\Rightarrow k = 1$.

Sample Moment: $\frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$ (1^{st} sample moment = sample mean)

Population Moment

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_0^1 x \cdot \theta x^{\theta-1}\,dx$$

$$= \left[\frac{1}{\theta+1} x^{\theta+1}\right]_0^1 \cdot \theta = \frac{\theta}{\theta+1}$$

Equate Sample Moment & Population Moment:

$$\bar{X} = \frac{\theta}{\theta+1}$$

Solve for θ:

$$(\theta+1)\bar{X} = \theta$$

$$\theta(\bar{X}-1) = -\bar{X}$$

$$\theta = \frac{-\bar{X}}{\bar{X}-1} = \frac{\bar{X}}{1-\bar{X}}.$$

So $\hat{\theta} = \frac{\bar{X}}{1-\bar{X}}$ is the method of moments estimator of θ.