Example 1.25

$P_Y = \left(\frac{3}{4}, \frac{1}{4}\right)$ (0, 1)

$P_{X|Y}(\cdot|Y=0) = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ (0, 1, 2)

$P_{X|Y}(\cdot|Y=1) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

From Example 1.21: $\mathbb{E}[XY] = \frac{1}{4}$.

$$\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y] = \frac{1}{4} - \frac{11}{8}\cdot\frac{1}{4} = \frac{-3}{32}$$

We need $\mathbb{E}Y$ and $\mathbb{E}X$

$$\mathbb{E}[Y] = 0\cdot\frac{3}{4} + 1\cdot\frac{1}{4} = \frac{1}{4}$$

$$\mathbb{E}[X|Y=0] = 0\cdot 0 + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{2} = \frac{3}{2}$$

$$\mathbb{E}[X|Y=1] = 0\cdot\frac{1}{3} + 1\cdot\frac{1}{3} + 2\cdot\frac{1}{3} = 1$$

$$\mathbb{E}[X] = \mathbb{E}[X|Y=0]\cdot P(Y=0) + \mathbb{E}[X|Y=1]\cdot P(Y=1)$$

$$= \frac{3}{2}\cdot\frac{3}{4} + 1\cdot\frac{1}{4}$$

$$= \frac{11}{8}$$

$$\mathrm{corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} = \frac{-3/32}{\sqrt{\frac{77}{192}\cdot\frac{3}{16}}} \simeq -0,483$$

$$\mathrm{Var}\,Y = \frac{3}{16} \qquad \mathrm{Var}\,X = \frac{77}{192}$$