Example 1.21

$P(Y=0) = \frac{3}{4} \quad P(Y=1) = \frac{1}{4}$

$P(X=1|Y=0) = P(X=2|Y=0) = \frac{1}{2}$

$P(X=0|Y=1) = P(X=1|Y=1) = P(X=2|Y=1) = \frac{1}{3}$

$\mathbb{E}[XY] = \mathbb{E}_Y\left[\mathbb{E}_{X|Y}[XY|Y]\right]$ $\quad \psi(x,y) = x$, $\phi(y) = y$

$\overset{\text{TOK}}{=} \mathbb{E}_Y\left[Y \cdot \mathbb{E}_{X|Y}[X|Y]\right]$

$= \sum_{y_j=0}^{1} y_j \cdot \mathbb{E}_{X|Y}[X|Y=y_j] \cdot p_Y(y_j)$

$= \mathbb{E}_{X|Y}[X|Y=1] \cdot \frac{1}{4}$

$= \left(0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3}\right) \cdot \frac{1}{4} = \underline{\underline{\frac{1}{4}}}$

as indeed 16/10/15