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Example 2.7  $Y = AX, X = A^{-1}Y$

$$A = (a_{ij})_{i,j=1}^r \in \mathbb{R}^{r \times r}$$

$$y_j = \sum_{k=1}^r a_{jk} x_k$$

$$\begin{pmatrix} \boxed{r} & \boxed{r} \end{pmatrix} \begin{pmatrix} \boxed{r} \\ \boxed{1} \end{pmatrix} = \begin{pmatrix} \boxed{r} \\ \boxed{1} \end{pmatrix}$$

$$\frac{\partial y_j}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \sum_{k=1}^r a_{jk} x_k \right) = \sum_{k=1}^r \frac{\partial}{\partial x_i} (a_{jk} x_k) = \sum_{k=1}^r a_{jk} \frac{\partial x_k}{\partial x_i}$$

$$\begin{aligned} & \delta_{i,k} : \text{Kronecker Delta, } \delta_{i,k} := \begin{cases} 0 & \text{if } i \neq k \\ 1 & \text{if } i = k \end{cases} \\ & = \sum_{k=1}^r a_{jk} \delta_{ik} = a_{ji} \cdot 1 = a_{ji} \end{aligned}$$

$$J(x) = \det \left( \frac{\partial X}{\partial Y} \right) = \left( \det \left( \frac{\partial Y}{\partial X} \right) \right)^{-1} = \left( \det A \right)^{-1}$$

$$f_Y(y) = f_X(x(y)) \cdot \left| \det \frac{\partial X}{\partial Y} \right| = f_X(A^{-1}y) \cdot |\det A|^{-1}$$