Example 3.13 $X_i \sim Ber(\pi)$ i.i.d.

$\mathbb{E}X_i = \pi, \quad Var X_i = \pi(1-\pi)$

$$\sum_{i=1}^{n} X_i \sim Bin(n, \pi)$$

CLT says $P\left(\frac{(\bar{X}_n - \mu)}{\sigma/\sqrt{n}} \le x\right) \approx \Phi(x)$.

$$= P\left(\frac{\left(\frac{1}{n}\sum_{i=1}^{n} X_i - \pi\right)}{\sqrt{\pi(1-\pi)}/\sqrt{n}} \le x\right) = P\Big(\sum_{i=1}^{n} X_i \le \underbrace{n\pi + x\sqrt{n\pi(1-\pi)}}_{=:a}\Big)$$

$$P\left(\sum_{i=1}^{n} X_i \le a\right) \approx \Phi\left(\frac{a - n\pi}{\sqrt{n\pi(1-\pi)}}\right)$$

So, approximately $S := \sum_{i=1}^{n} X_i \overset{approx}{\sim} \mathcal{N}(n\pi, n\pi(1-\pi))$.

But, observe $P(S \le k) = P(S \le k + c)$ for any $c \in [0,1)$

Choose $c = \frac{1}{2}$ as a "happy medium".

$$P\left(\sum_{i=1}^{n} X_i \le k\right) = P\left(\sum_{i=1}^{n} X_i < k + \tfrac{1}{2}\right) \approx \Phi\left(\frac{k + \frac{1}{2} - n\pi}{\sqrt{n\pi(1-\pi)}}\right).$$

where $k \in \mathbb{N}$

This is known as "continuity correction".

CLT works from $n \ge ?$ onwards:

$n = 30$.

$n = 36$.

$n = 40$

$n = 25$