asgive 8/10/2015

Example 1.1 Two Coins

$\Omega = \{HH, TT, HT, TH\}$

Note:
$HH \in \Omega$
$\{HH\} \subseteq \Omega$
not the same!

$P(HH) = \frac{1}{4}$ $\left(= \frac{1}{2} \cdot \frac{1}{2}\right)$

$P(HT) = \frac{1}{4} = P(TH) = P(TT)$

$A = \{HH, HT\}$, $B = \{HH, TH\}$, $C = \{HT, TH\}$.

$P(A) = P(HH) + P(HT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

$P(B) = P(HH) + P(TH) = \frac{1}{2}$, $P(C) = \frac{1}{2}$.

Are A, B, C independent?

No, because $A \cap B \cap C = \phi$ ← empty set.

So $P(A \cap B \cap C) = P(\phi) = 0$.

But $P(A) \cdot P(B) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \neq P(A \cap B \cap C)$

So, A, B and C are not independent.

Example 1.2

$\bar{C} = \{HH, TT\}$,

$P(A|\bar{C}) = \frac{P(A \cap \bar{C})}{P(\bar{C})} = \frac{1/4}{1/2} = \frac{1}{2}$ $P(B|\bar{C}) = \frac{1}{2}$

$P(A \cap B|\bar{C}) = \frac{P(A \cap B \cap \bar{C})}{P(\bar{C})} = \frac{1/4}{1/2} = \frac{1}{2}$

$P(A \cap B|\bar{C}) = \frac{1}{2} \neq \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = P(A|\bar{C}) \cdot P(B|\bar{C})$

Thus, A and B are not conditionally independent given $\bar{C}$.