Example 3.8 $X_1, \dots, X_n$ independent, $X = (X_1, X_2, \dots, X_n)^T$

$X_i \sim Bin(m_i, p)$

Q5 ... 27/11/15

From Example 3.2, $G_{X_i}(z) = (1-p+pz)^{m_i}$

$$Y = \sum_{i=1}^{n} X_i = a^T X$$

$$a = (1, 1, \dots, 1)^T \in \mathbb{R}^n$$

$$G_Y(z) = \prod_{i=1}^{n} G_{X_i}(z^{a_i}) = \prod_{i=1}^{n} G_{X_i}(z) = \prod_{i=1}^{n} (1-p+pz)^{m_i}$$

$a^n \cdot a^m = a^{n+m}$

$$= (1-p+pz)^{\sum_{i=1}^{n} m_i}$$

which is the pgf of a $Bin(\sum_{i=1}^{n} m_i, p)$.

Therefore, $Y \sim Bin(\sum_{i=1}^{n} m_i, p)$

Example 3.9 X_i indep., $X_i \sim Gam(\alpha_i, \lambda)$

$Y = \sum_{i=1}^{n} X_i$, $a = (1, \dots, 1)^T \in \mathbb{R}^n$ $\alpha_i > 0$, $\lambda > 0$

$$M_Y(s) = \prod_{i=1}^{n} M_{X_i}(s a_i) = \prod_{i=1}^{n} \left(1 - \frac{s}{\lambda}\right)^{-\alpha_i}$$

Example 3.6 $M_{X_i}(s) = \left(1 - \frac{s}{\lambda}\right)^{-\alpha_i}$

$= \left(1 - \frac{s}{\lambda}\right)^{-\sum_{i=1}^{n} \alpha_i}$ which is the mgf of $Gam(\sum_{i=1}^{n} \alpha_i, \lambda)$ defined on $(-\infty, \lambda) \ni s$, which is an open interval containing zero.

Therefore, $Y \sim Gam(\sum_{i=1}^{n} \alpha_i, \lambda)$