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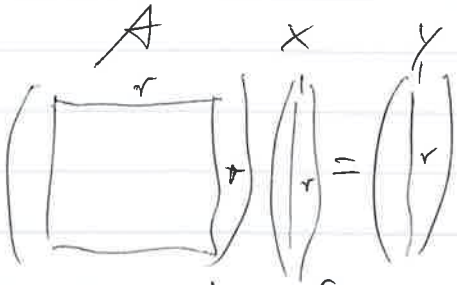
Example 2.7 $Y = AX$, $X = A^{-1}Y$

$A = (a_{ij})_{i,j=1}^{r} \in \mathbb{R}^{r \times r}$

$$y_j = \sum_{k=1}^{r} a_{jk} x_k$$



$$\frac{\partial y_j}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\sum_{k=1}^{r} a_{jk} x_k\right) = \sum_{k=1}^{r} \frac{\partial}{\partial x_i}\left(a_{jk} x_k\right) = \sum_{k=1}^{r} a_{jk} \frac{\partial x_k}{\partial x_i}$$

$\delta_{i,k}$: Kronecker Delta, $\delta_{i,k} := \begin{cases} 0 & \text{if } i \neq k \\ 1 & \text{if } i = k \end{cases}$

$$= \sum_{k=1}^{r} a_{jk} \delta_{ik} = a_{ji} \cdot 1 = a_{ji}$$

$$J(x) = \det\left(\frac{\partial x}{\partial y}\right) = \left(\det\left(\frac{\partial y}{\partial x}\right)\right)^{-1} = \left(\det A\right)^{-1}$$

$$f_Y(y) = f_X(x(y)) \cdot \left|\det \frac{\partial x}{\partial y}\right| = f_X(A^{-1}y) \cdot |\det A|^{-1}$$