(b) ctd. $T = \sum Y_i$, Y_i i.i.d.

$$M_T(s) = \prod_{i=1}^{n} M_{Y_i}(s) = \left(\frac{\exp\left(\frac{\alpha^2 s}{1-2s}\right)}{\sqrt{1-2s}} \right)^n$$

$$= (1-2s)^{-\frac{n}{2}} \exp\left(\frac{n\alpha^2 s}{1-2s}\right), \quad s < \frac{1}{2}$$

(c) $M \sim \text{Poi}(n\alpha^2/2)$, $\quad Z := \sum_{i=1}^{n+2M} X_i^2$

$Z \mid M = m \sim \chi^2_{n+2M}$ $\quad M_{Z|M}(s) \underset{(b),\ \alpha=0}{=} (1-2s)^{-\frac{n+2M}{2}}, \quad s < \frac{1}{2}$

$$M_Z(s) = \mathbb{E}[\exp(sZ)]$$

$$= \mathbb{E}_M\left[\mathbb{E}_{X_1, X_2, \ldots | M}\left[\exp\left(s \sum_{i=1}^{n+2M} X_i^2\right) \Big| M \right]\right]$$

$$= \mathbb{E}_M\left[(1-2s)^{-\frac{n+2M}{2}} \right]$$

$$= \sum_{k=0}^{\infty} (1-2s)^{-\frac{n+2k}{2}} \cdot \left(\frac{n\alpha^2}{2}\right)^k \cdot \frac{e^{-n\alpha^2/2}}{k!}$$

$$= (1-2s)^{-n/2} \sum_{k=0}^{\infty} (1-2s)^{-k} \left(\frac{n\alpha^2}{2}\right)^k e^{-n\alpha^2/2} / k!$$

$$= (1-2s)^{-n/2} \cdot e^{-n\alpha^2/2} \cdot \sum_{k=0}^{\infty} \left([1-2s]^{-1} \frac{n\alpha^2}{2} \right)^k / k!$$

$$= (1-2s)^{-n/2} e^{-n\alpha^2/2} \exp\left(\frac{n\alpha^2}{2(1-2s)}\right) \qquad \sum_{l=0}^{\infty} \frac{a^l}{l!} = e^a$$

$$= (1-2s)^{-n/2} \exp\left(n\alpha^2 s / (1-2s)\right), \quad s < \frac{1}{2}$$

which is the mgf of T. The mgf is defined on an open interval including 0, namely $(-\infty, \frac{1}{2}) \ni 0$.

Therefore Z and T follow the same distribution.