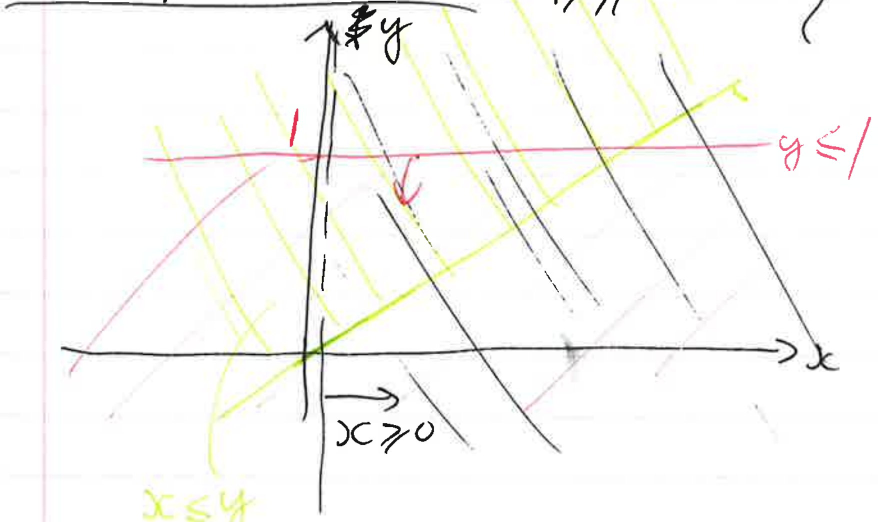
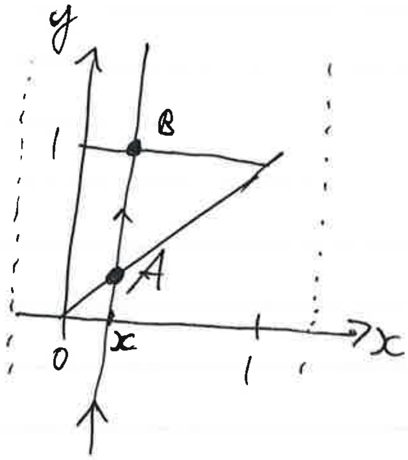
Example 1.22

$$f_{X,Y}(x,y) = \begin{cases} 8xy & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$





The area is a triangle with corners $(0,0)$, $(0,1)$, $(1,1)$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy = \int_x^1 8xy\, dy$$

if $x > 0$, $x < 1$

NB: integration for y, x is constant, starting at point $A = (x,x)$ stopping at point $B = (x,1)$

$$= \left[4xy^2\right]_x^1$$

$$= 4x(1-x^2) \text{ if } 0 < x < 1$$

0 otherwise.

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx = \int_0^y 8xy\, dx = 4y^3 \text{ if } y \in (0,1)$$

0 otherwise.

$$\int_{-\infty}^{\infty} f_X(x)\, dx = \int_0^1 4x(1-x^2)\, dx = \left[2x^2 - x^4\right]_0^1 = 1 \checkmark$$

Example 1.23

$$f_X(x) \cdot f_Y(y) = 4x(1-x^2) \cdot 4y^3 = 16x(1-x^2)y^3$$

if $x, y \in (0,1)$

0 otherwise.

$\ne f_{X,Y}(x,y)$, so X and Y are not independent.