

Example 1.1 Two Coins

assign 8/10/2015

$$\Omega = \{HH, TT, HT, TH\}$$

$$P(HH) = \frac{1}{4} \left(= \frac{1}{2} \cdot \frac{1}{2} \right)$$

$$P(HT) = \frac{1}{4} = P(TH) = P(TT)$$

$$A = \{HH, HT\}, B = \{HH, TH\}, C = \{HT, TH\}$$

$$P(A) = P(HH) + P(HT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(B) = P(HH) + P(TH) = \frac{1}{2}, \quad P(C) = \frac{1}{2}$$

Are A, B, C independent?

No, because $A \cap B \cap C = \emptyset$ ← empty set.

$$\text{So } P(A \cap B \cap C) = P(\emptyset) = 0$$

$$\text{But } P(A) \cdot P(B) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \neq P(A \cap B \cap C)$$

So, A, B and C are not independent.

Note:

$$HH \in \Omega$$

$$\{HH\} \subseteq \Omega$$

not the same!