Revision Excersises Week 8

Excercise 1

Let I_A and I_B the indicator variables for the event A and B. That is,

$$I_A = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$$
$$I_B = \begin{cases} 1 & \text{if } B \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$$

Find $\mathbb{E}(I_A), \mathbb{E}(I_B), \mathbb{E}(I_A \cdot I_B), \mathrm{Cov}(I_A, I_B)$ and determine when the indicator variables for A and B are positively correlated, uncorrelated or negatively correlated.

Excercise 2

Let $X_1, \ldots, X_n$ be independent and identically distributed random variables having variance σ^2. Show that

$$\mathrm{Cov}(X_i - \bar{X}, \bar{X}) = 0$$

Excercise 3

A plane has gone missing and it is assumed that it is equally likely to have gone down in any of 3 possible regions. Let $1 - \alpha_i, i = 1, 2, 3$ be the probability that the plane will be found upon a search in region i. What is the conditional probability that the plane is in the ith region, given that a search of region 1 is unsuccessful?