Revision Excersises Week 6

Excercise 1

Let $x_1, \ldots, x_n$ be any numbers and $\bar{x} = \sum_{i=1}^n x_i/n$. Show that

(a) $\min_a \sum_{i=1}^n (x_i - a)^2 = \sum_{i=1}^n (x_i - \bar{x})^2$

(b) $(n-1)s^2 = \sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$

Excercise 2

Let $X_1, \ldots, X_n$ be independent and identically distributed random variables and let $g(x)$ be a function such that $\mathbb{E}\{g(X_1)\}$ and $\mathrm{Var}\{g(X_1)\}$ exist. Show that

(a) $\mathbb{E}\{\sum_{i=1}^n g(X_i)\} = n\mathbb{E}\{g(X_1)\}$

(b) $\mathrm{Var}\{\sum_{i=1}^n g(X_i)\} = n\,\mathrm{Var}\{g(X_1)\}$

Excercise 3

Let $X_1, \ldots, X_n$ be a random sample from a population wuth mean μ and variance $\sigma^2 < \infty$. Show that

(a) $\mathbb{E}\{\bar{X}\} = \mu$

(b) $\mathrm{Var}\{\bar{X}\} = \frac{\sigma^2}{n}$

(c) $\mathbb{E}(S^2) = \sigma^2$

where

$$\begin{aligned} \bar{X} &= \frac{\sum_i X_i}{n} \\ S^2 &= \frac{\sum_i (X_i - \bar{X})^2}{n-1} \end{aligned}$$