STAT3001/STATM012/STATG012
Statistical Inference

Week 8: Problem Sheet 2

1. Consider a population consisting of measurements 0,3 and 12 and described by the following probability distribution

x	0	3	12
$p(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

   (a) Calculate the mean μ for this distribution.

   (b) Find the sampling distribution of the sample median M for random sample of $n = 3$ measurements.

   (c) Show that M is a biased estimator of μ in this situation.

2. The following values were obtained as a simple random sample from the Poisson distribution with mean θ: 7, 4, 8, 8, 3, 2, 5. Before observing any data, it was believed that uncertainty about θ could be expressed by a gamma distribution with mean 4 and standard deviation 2. What are the parameters of this gamma distribution?

   (a) Write down the likelihood function for θ.

   (b) Show that the posterior distribution of θ is again a gamma distribution and find its mean and standard deviation.

3. A coin has an unknown probability θ of landing head upwards (H) when it is tossed. Before observing any data, there is no reason to believe that any value of θ is more likely than any other, and therefore uncertainty about θ is expressed by a uniform distribution on [0,1]. In 20 tosses, 13 Hs are observed.

   (a) Write down the likelihood function for θ.

   (b) Find the posterior density of θ and find its mean and standard deviation.

4. Let $X_1, \ldots, X_n$ be independent variables, uniformly distributed on $[0, \theta]$. Define $S = \max\{X_1, \ldots, X_n\}$. Write down $P(S \leq x)$ and hence find the density, mean and variance of S. Let $T_1 = (n+1)S/n$ and $T_2 = 2\overline{X}$. Show that both are unbiased estimators of θ. Which estimator would you prefer?