I was very pleased to see that many of you answered these questions very well indeed. I was particularly impressed with your explanations of why the process in question 2(a) is a Markov process - this is a vast improvement from exercise sheet 2. If this were an exam or ICA question, just remember to check how many marks it is worth. A full page explanation would be overkill for one or two marks only.

There were some common slip-ups, however, which I make reference to in the solutions below.

1. (a) $\{1, 2, 3\}$ transient while $\{4\}$ is (positive) recurrent.

   (b) $\{1, 2, 3, 4\}$ (positive) recurrent. Remember that a Markov chain with finite state space cannot have all states transient!

2. (a) Let Y_n be the nth throw and X_n the highest number thrown in n throws, being the chain of interest. Then $X_{n+1} = \max(Y_{n+1}, X_n)$ which is not affected by X_{n-1} or earlier. So this is a Markov process.

   (b) State space is $\{1, 2, 3, 4, 5, 6\}$ and

$$P = \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 2/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 3/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 4/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 6/6 \end{pmatrix}.$$

   (c) The (unique) invariant distribution here is $\underline{\pi} = (0, 0, 0, 0, 0, 1)$. Many of you went to the effort of solving the simultaneous equations, which was not necessary. Remember that if state i is transient, then $\pi_i = 0$ (think about why this makes sense!), and for this particular Markov chain, states 1,2,3,4,5 are all transient. Therefore, since the sum of the elements of $\underline{\pi}$ must be one, the only possibility is $\underline{\pi} = (0, 0, 0, 0, 0, 1)$.

3. (a) Any vector of the form $(1 - 2a, a, a)$ where $0 \le a \le 1/2$, is an invariant distribution.

   (b) There are an infinite number of invariant distributions, so there cannot be an equilibrium distribution.

4. The main error here was to assume that the state space was finite, though the question clearly indicates that this is not the case. The transition matrix has countable (**but infinite**) dimensions. As usual, we solve $\underline{\pi} = \underline{\pi}P$ for $\underline{\pi} = (\pi_1, \pi_2, \dots)$, an infinite vector. This gives, progressively $\pi_i = 0$, $i = 0, 1, 2, \dots$, which is not a proper probability distribution. Therefore no invariant distribution exists.