What you may not realise is that the first question on this exercise sheet was a whole question from last year's exam. It would also be typical of an ICA question, too. There were a few common errors this week, some of which are very important.

- Don't forget that the Markov property states that:

$$\mathrm{P}(X_{n+1} = j \mid X_n = i_n, \cdots, X_0 = i_0) = \mathrm{P}(X_{n+1} = j \mid X_n = i_n),$$

  Notice that we must have the process at time n, X_n, equal to a particular value in order to drop the information given by $X_{n-2}, ..., X_0$. Check your lecture notes/ slides again!

- REVISE CONDITIONAL PROBABILITY! It seems that many of you are making fundamental errors in calculating conditional probabilities. Think back to the in-class quiz in the very first lecture on conditional probability (the questions were made available via a Moodle forum post).

  In particular, for mutually exclusive events B and C,

$$P(A|B \text{ or } C) = \frac{P(A,B)}{P(B \text{ or } C)} + \frac{P(A,C)}{P(B \text{ or } C)}.$$

  In particular note that:

$$P(A|B \text{ or } C) \neq P(A|B) + P(A|C)$$

1. (a) You could enter the information required neatly in a table. It is then easy to check that you have given all the required information and that each state is allocated to one class, and one class only.

Class	Type	Periodicity
$\{1,2,3\}$	Transient	3
$\{4,5\}$	Transient	1
$\{6\}$	Transient	1
$\{7\}$	Positive recurrent	1

   (b) Using the Markov property and time homogeneity, the probability is 1/64.

   (c) Let T be the amount of time that the chain stays in state 5, given it starts there. Then $T \sim \text{Geometric}(3/4)$. This is basic book-work, but some of you failed to answer this.

   (d) When it leaves state 5, will either go to state 4 or 7.

$$P(X_{n+1} = 4|X_n = 5, X_{n+1} \neq 5) = 2/3$$
$$P(X_{n+1} = 7|X_n = 5, X_{n+1} \neq 5) = 1/3.$$

   For this question some of you wrote out the formulae from the notes for these probabilities without using the actual probabilities from the transition matrix. This makes me wonder whether you understand what they are and how to use them. Look at the formulae again, make sure you can calculate the above probabilities, and make sure you understand *why* the answer makes sense intuitively.

2. For parts (a) and (b), make sure you understand when to 'add 1'.

Please don't guess at it and hope for the best. If you are still struggling with this, here are some recommendations:

(i) There is a section in Chapter 1 of the notes dedicated to explaining the concept of first step decomposition, with an explanation of the different types of questions you can answer using this method. There's also a worked example here.

(ii) There are now two interactive videos on Moodle, both with fully worked examples, which you can use to cement your understanding of this technique. The videos are 'Basic skills storyline' (choose the 'first step decomposition' option when you first enter this video), and the other is the video on the 'winning topic' from the first student vote.

(iii) There are more examples in the lecture slides (and therefore on Lecturecast!) using first step decomposition in the section on closed classes and absorption.

(iv) Come and see me during office hours!

Note for both (a) and (b) below, the first thing I do is to define appropriate notation!

(a) Let $L_i = E[\text{Time to reach state } 3|X_0 = i]$. We want L_0. Calculating each of L_0, L_1, L_2 and L_3 and making the appropriate substitutions gives $L_0 = 17/6$.

(b) Let $L_i = E[\text{Number of visits to state 1 before entering state } 3|X_0 = i]$. We want L_0. Calculating each of L_0, L_1, L_2 and L_3 and making the appropriate substitutions gives $L_0 = 5/6$.

For the remaining parts of the question, it's probably worth tabulating Z_n for various values of X_n. That way you will spot instantly that if $Z_n = 0$ we have uncertainty about the value of X_n.

(c) For this first conditional probability, we cannot have both $X_{n-1} = 2$ and $X_n = 0$ in the conditioning set. By spotting this, the calculation is easy and

$$P(Z_{n+1} = 2|Z_n = 0, Z_{n-1} = 2) = 0.$$

(d) Things to watch out for: because there is ambiguity in the value of X_n in this case, we cannot use the Markov property to justify removing $X_{n-1} = 3$. By working through the calculation carefully (apply Bayes's theorem!), we get:

$$P(Z_{n+1} = 2|Z_n = 0, Z_{n-1} = 3) = 1/4.$$

(e) Comparing our answer to (c) and (d), it is clear that Z_{n+1} depends upon Z_{n-1} and so we don't have a MC. Some of you didn't use your answer to (c) and (d) here and tried to concoct a separate argument which was unnecessary.

AS ALWAYS, YOU ARE WELCOME TO COME TO SEE ME DURING OFFICE HOURS TO DISCUSS ANYTHING YOU'RE FINDING TRICKY.