Please put in your tutor's postbox in Room 117 by 12pm Thursday February 11th.

1. A Markov chain with statespace $\{1, 2, 3, 4\}$ has transition matrix

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/4 & 3/4 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & \alpha & 1-\alpha \end{pmatrix}.$$

   Find the irreducible classes and state whether they are recurrent or transient:

   (a) if $\alpha = 0$,

   (b) if $\alpha > 0$.

2. A fair die is thrown repeatedly and independently. The process is said to be in state j at time n if j is the highest integer obtained in the first n throws.

   (a) Explain why a Markov model is appropriate.

   (b) Write down the transition matrix for the chain.

   (c) Find the invariant distribution for the chain.

3. Consider the Markov chain with $S = \{1, 2, 3\}$ and

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}.$$

   (a) Find all the invariant distributions for this chain.

   (b) Can this chain have an equilibrium distribution?

4. Consider the Markov chain with $S = \{0, 1, 2, \ldots\}$ and, for i in S, transition probabilities $p_{ii} = 1/2$, $p_{i,i+1} = 1/2$ and $p_{ij} = 0$ for $j \neq i$, $j \neq i + 1$. Show that there is no invariant distribution for this Markov chain.

Challenges

Questions in this section will not be covered in the tutorial. If you think you have solved the questions or would like some help, do post to the discussion forum.

Individual equations from $\underline{\pi} = \underline{\pi}P$ will always have one equation that is redundant, i.e. at least one equation is a linear combination of the other equations.

(a) Prove this statement.

(b) Show that P will always have a left eigenvalue 1. What is the corresponding eigenvector?