STAT2003/STAT3102 In Course Assessment

Tuesday 4th March 2014, 12 noon

- This is an open book test which is subject to UCL exam regulations.
- Your solutions should be **your own work**. Any copying will normally result in zero marks for all students involved, and may mean that your overall examination mark is recorded as "non-complete", i.e. you might not obtain a pass for the course.
- Non-submission of in-course assessment may mean that your overall examination mark is recorded as non-complete.
- Your work should be handed in by yourself to the invigilator.
- Your work will be returned to you for feedback and you will receive a provisional grade - *grades are provisional until confirmed by the Statistics Examiners' Meeting in June 2014*. You should keep your work until after the Statistics Examiners' Meeting - your work may be required for perusal by the examiners.
- A total of 50 marks are available. Answer all questions. The numbers in square brackets indicate the relative weight attached to each part question.
- Please write your name at the top of the front page of your answer booklet(s).

Answer ALL questions. Time allowed 45 minutes.

1. **(36 marks)**

   Consider the Markov chain $\{X_n : n = 0, 1, 2 \ldots\}$ with state space $S = \{1, 2, 3, 4, 5, 6\}$ and transition matrix

$$P_S = \begin{pmatrix} 0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0.7 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.8 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

   (a) Find the irreducible classes of intercommunicating states of $\{X_n\}$ and classify them in terms of positive or null recurrence, transience, periodicity and ergodicity. Also state whether each class is closed or not. [8]

   (b) Given that $X_0 = 6$:

   i. Write down the row vector $\boldsymbol{p}^{(2)}$. [2]

   ii. Calculate $\mathrm{P}(X_5 = 1)$. [2]

   iii. Calculate all possible invariant distributions for the chain. [6]

   iv. Is there an equilibrium distribution? Why? If there is, name it. [4]

   v. Calculate the expected number of times the chain passes through state 3 before it enters a closed class. [8]

   (c) A seventh state, 7, is added to the state space S to form T, with transition matrix $\boldsymbol{P}_T$, for Markov Chain $\{W_n : n = 0, 1, 2 \ldots\}$. Write down one possible matrix $\boldsymbol{P}_T$ such that the new chain is irreducible and its first 6 rows and columns have as few changes from $\boldsymbol{P}_S$ as possible. [6]

2. **(14 marks)**

   Consider a sequence of iid random variables Y_n, $n = 0, 1, 2, \ldots$, each of which takes the values 0, 3 and 6 with equal probability.

   (a) Explain why $\{Y_n : n = 0, 1, 2, \ldots\}$ is a Markov Chain. [2]

   (b) Write down $\mathrm{P}(Y_{16} = 3 \mid Y_0 = 6)$. [2]

   (c) $Z_n = Y_n + 2Y_{n-1}$. Write down the state space for $\{Z_n\}$ and find an example that shows it is not a Markov Chain, calculating the two required probabilities. [10]