Please put in your tutor's postbox in Room 117 by 12pm Thursday January 28th.

1. Let $X_0, X_1, X_2, \ldots$ be a Markov chain with statespace $\{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/3 & 1/6 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   (a) What happens if the chain starts in state 4?

   (b) If the chain starts in state 1, can it ever reach state 4?

   (c) Find the three-step transition matrix, $P^{(3)}$. Hence, knowing that $X_0 = 1$, write down the probability that $X_3 = 1$.

2. Suppose that coin 1 has probability 0.4 of coming up heads, and coin 2 has probability 0.8 of coming up heads. The coin to be flipped initially is equally likely to be coin 1 or coin 2. Thereafter, if the flipped coin shows heads then coin 1 is chosen for the next flip, and if the flipped coin shows tails then coin 2 is chosen for the next flip.

   Let X_0 be the coin chosen for the initial flip, and, for $n \geq 1$, let X_n be the coin chosen for the nth flip after the initial flip.

   (a) Explain why $X_0, X_1, X_2, \ldots$ is a Markov chain. Write down its statespace and find its transition matrix.

   (b) Let $\underline{p}^{(n)}$ be the probability row vector giving the distribution of X_n (as in lectures). Find $\underline{p}^{(0)}$, $\underline{p}^{(1)}$, and $\underline{p}^{(2)}$.

   (c) Write down the probability that coin 1 is chosen for the second flip after the initial flip. [HINT: Use your answer to part (b), noting that there is no need to calculate anything new here.]

   (d) Find the probability that coin 1 is chosen for the second and third flip after the initial flip.

3. Let $X_0, X_1, X_2, \ldots$ be a Markov chain, with statespace $\{0, 1\}$ and

$$P(X_0 = 1) = p \qquad \text{and} \qquad P(X_{n+1} = 1 | X_n = 1) = \alpha \quad \text{for } n = 0, 1, 2, \ldots.$$

   Let $Z_n = X_0 X_1 \ldots X_n$.

   (a) Explain briefly why $Z_0, Z_1, Z_2, \ldots$ is a Markov chain [no mathematics required here!], and find its transition matrix.

   (b) Find the probability that $Z_n = 1$. [HINT: if $Z_n = 1$, deduce what $X_0, X_1, ...X_n$ must be.]

Challenges

Questions in this section will not be covered in the tutorial. If you think you have solved the questions or would like some help, do post to the discussion forum.

1. Consider the game of Monopoly (apologies if you do not know the game) and the sequence of your locations at the end of each turn (eg at the end of turn 1, you are in location X_1). Is the sequence $X_1, X_2, \ldots$ a Markov Chain? Why?