Please put in your tutor's postbox in Room 117 by 12pm Thursday January 21st.

1. A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel which returns him to his cell after two days travel. The second door leads to a tunnel that returns him to his cell after three days travel. The third door leads immediately to freedom.
   (a) Assuming the prisoner will always select doors 1, 2 and 3 with probabilities 0.5, 0.3 and 0.2 respectively, what is the expected number of days until he reaches freedom?
   (b) Calculate the variance of the number of days until the prisoner reaches freedom.
2. Now assume that the probabilities in question 1 are changed, so that at each stage the prisoner is equally likely to choose any of the doors *that he has not yet used.* For example, initially there are three doors with each chosen with probability 1/3, and once he has chosen an incorrect door, he will not choose it again. What is the expected number of days until he reaches freedom?
3. The number of fish that a fisherman catches in a day is a Poisson random variable with mean 30. On average, the fisherman throws back two out of every three fish he catches.
   (a) Explain (in no more than two sentences) why the following argument is **NOT** complete in computing the probability that, on a given day, the fisherman takes home k fish.

> Since the fisherman throws back 2/3 of the fish, assuming that each fish is thrown back independently of all other fish, the distribution of the number of fish he takes home is Poisson with parameter $30 \times 1/3 = 10$. Therefore,
>
> $$P(X = k) = \frac{\exp(-10)10^k}{k!}$$

   (b) Write out a correct version of this argument, stating any independence assumptions that you make. **Hint**: it is quickest to use probability generating functions. You may find it useful to look through the course notes for similar examples.

Challenges

Questions in this section will not be covered in the tutorial. If you think you have solved the questions or would like some help, do post to the discussion forum.

1. If a random variable has a standard geometric distribution (ie $X \sim \text{Geom}(p)$), then it takes values $\{1, 2, \dots\}$, so that $P(X = k) = (1-p)^{k-1}p$, and has mean $1/p$. Later in the course we will see a random variable, Y, with a shifted geometric distribution which takes values $\{0, 1, 2, \dots\}$ so that $P(Y = k) = (1-p)^k p$. Show that the mean of this shifted geometric distribution is $(1-p)/p$. What is its variance?
2. Let $X_1, X_2, \dots, X_N$ be iid random variables with common moment generating function $M_X(s) = E[e^{sX}]$, where N is a random variable with probability generating function $G_N(s)$. Define $Y = X_1 + X_2 + \dots + X_N$. Show that $M_Y(s) = G_N(M_X(s))$.