STAT2003/STAT3102 In-course assessment

Tuesday 4th of March 2014

1. (a) $\{1, 2, 4\}$: closed, positive recurrent, aperiodic, ergodic.
   $\{3, 5, 6\}$: not closed, transient, period=3, not ergodic.

   (b) i. $\boldsymbol{p}^{(2)} = (0, 0.3, 0, 0, 0.7, 0)$
   ii. $\mathrm{P}(X_5 = 1) = 0.044$
   iii. $\boldsymbol{\pi} = (5, 8, 0, 8, 0, 0)/21$
   iv. Yes
   v. 1 / 0.44

   (c) eg

$$P_T = \begin{pmatrix} 0.1 & 0.8 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0.7 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.8 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

2. (a)

   (b) 1/3

   (c) eg

$$\mathrm{P}(Z_{n+1} = 3 \mid Z_n = 6, Z_{n-1} = 9) = \frac{1}{3}$$

   and

$$\mathrm{P}(Z_{n+1} = 3 \mid Z_n = 6, Z_{n-1} = 12) = 0.$$