Quasinormal Modes and Electromagnetic Eigenvalue Problems – Single Dipole

$$\mathbf{p}(\omega) = \overleftrightarrow{\alpha}(\omega) \cdot \mathbf{E} \qquad\qquad \overleftrightarrow{\alpha}^{-1}(\omega) \cdot \mathbf{p}(\omega) = \mathbf{E}$$

The <u>self-sustaining</u> solutions are the solution of the <u>homogeneous</u> problem:

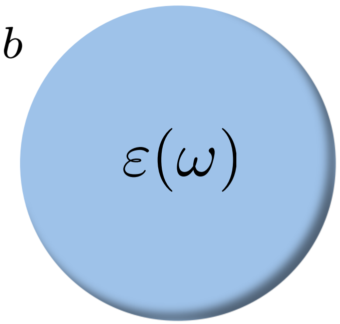
$$\overleftrightarrow{\alpha}^{-1}(\omega) \cdot \mathbf{p}(\omega) = \mathbf{0}$$

$$\det\left[\overleftrightarrow{\alpha}^{-1}(\omega)\right] = 0$$

Eigen frequencies are determined by the complex poles of the NP polarizability

The Electromagnetic Eigenvalue Problem – Single Dipole

ε_b



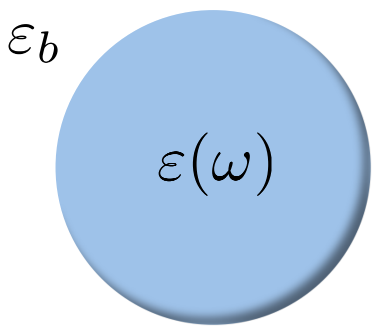
$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_{\mathrm{p}}^2}{\omega^2 + i\gamma\omega}$$

$$\alpha(\omega) = \xi \frac{\varepsilon(\omega) - \varepsilon_b}{\varepsilon(\omega) + 2\varepsilon_b} \quad = \xi \left(1 - \frac{3\varepsilon_b}{\varepsilon(\omega) + 2\varepsilon_b} \right)$$

Poles are the roots of the denominator:

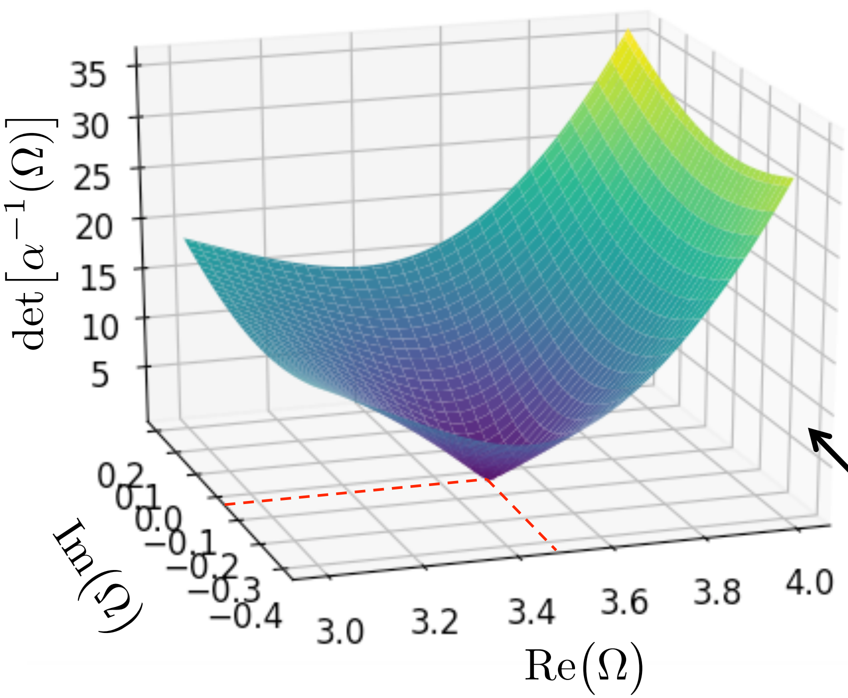
$$\varepsilon_\infty - \frac{\omega_{\mathrm{p}}^2}{\omega^2 + i\gamma\omega} + 2\varepsilon_b = 0 \qquad \Omega = \sqrt{\frac{\omega_{\mathrm{p}}^2}{\varepsilon_\infty + 2\varepsilon_b} - \left(\frac{\gamma}{2}\right)^2} - i\frac{\gamma}{2}$$

The Electromagnetic Eigenvalue Problem – Single Dipole

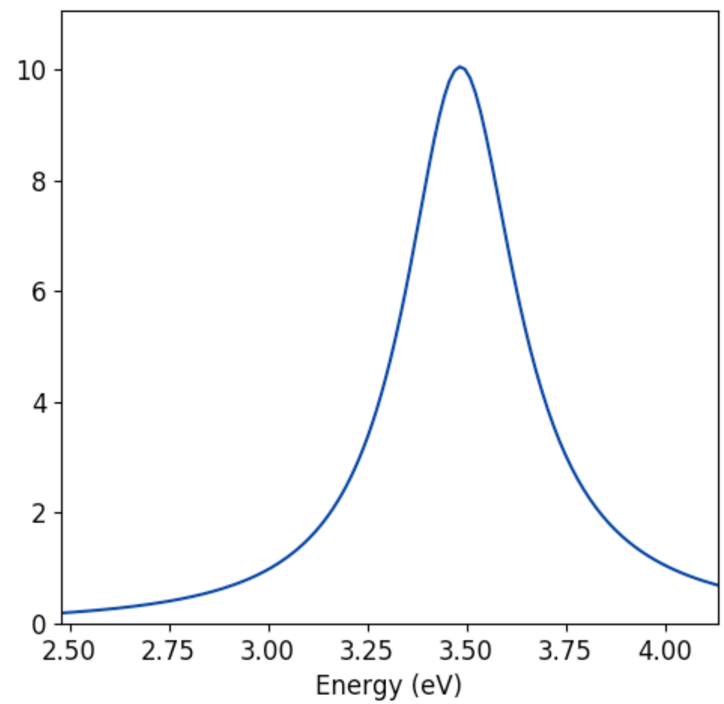


$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_{\mathrm{p}}^2}{\omega^2 + i\gamma\omega}$$

$$\Omega_m = \omega_m - i\frac{\gamma_m}{2}$$

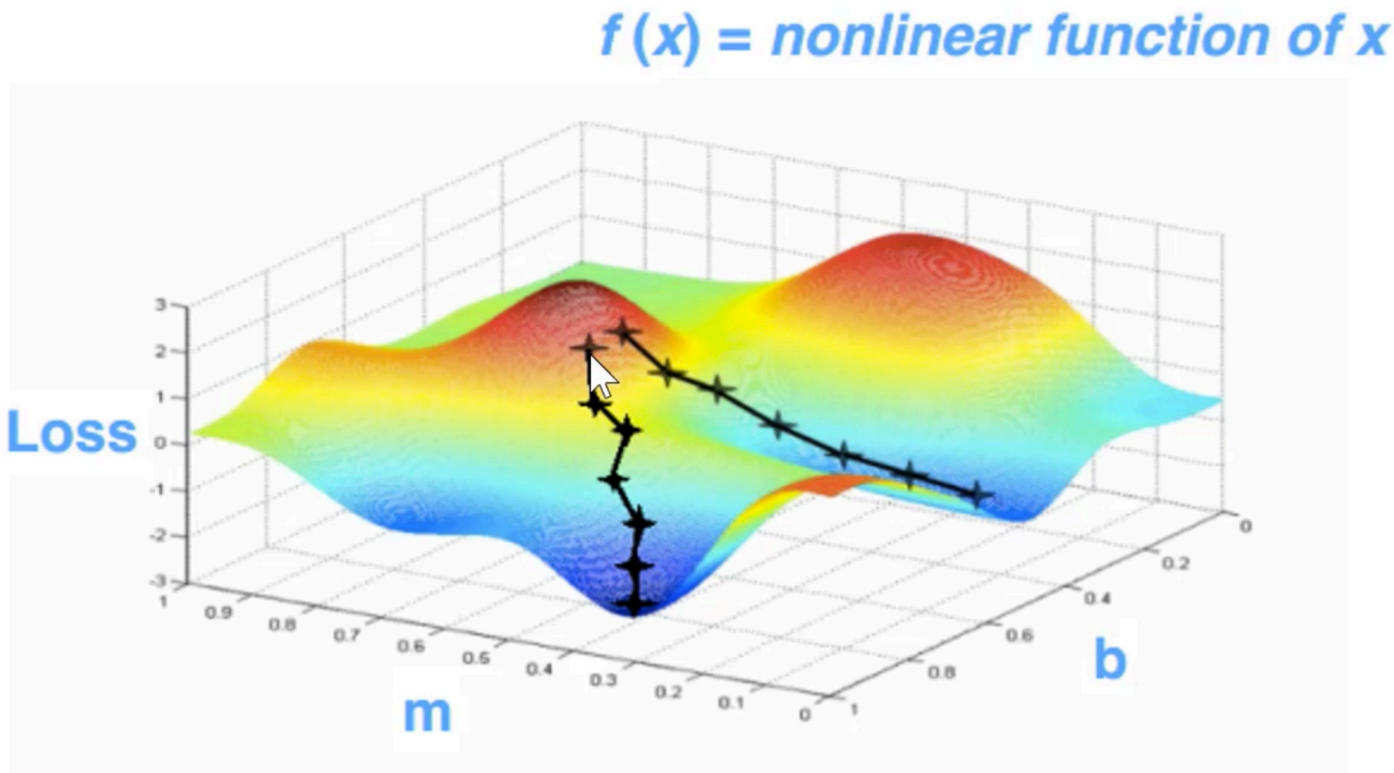


R = 25 nm; n = 1.0



SLRpy_1d/singleNP.py (run_type = “grid_search”)

Minimization Problem = Gradient Descent?



In our case, *f (x)* is:

- Nonlinear (sensitive to initial guess)
- In General it is challenging/impossible to compute gradients of *f*(*x*)

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Global Complex Roots and Poles Finding Algorithm Based on Phase Analysis for Propagation and Radiation Problems

Piotr Kowalczyk

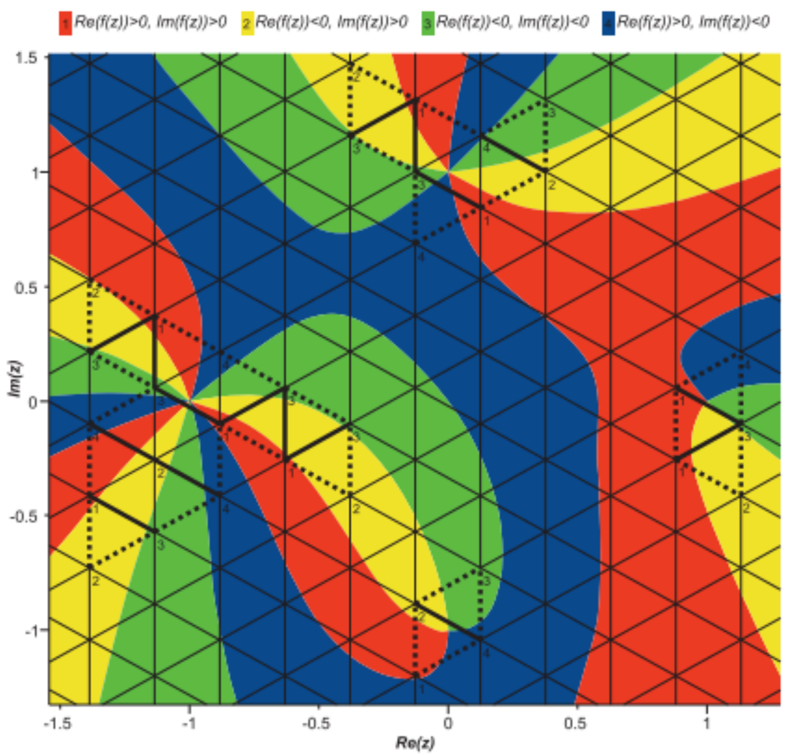

Fig. 1. Preliminary estimation algorithm applied for function $f(z) = (z-1)(z-i)^2(z+1)^3/(z+i)$. The numbers (colors): 1 (red), 2 (yellow), 3 (green), and 4 (blue) represent the quadrants in which the function values lie. Thick black lines: candidate edges. Black dotted lines: boundaries of the candidate regions.

Mesh Refinement Algorithm

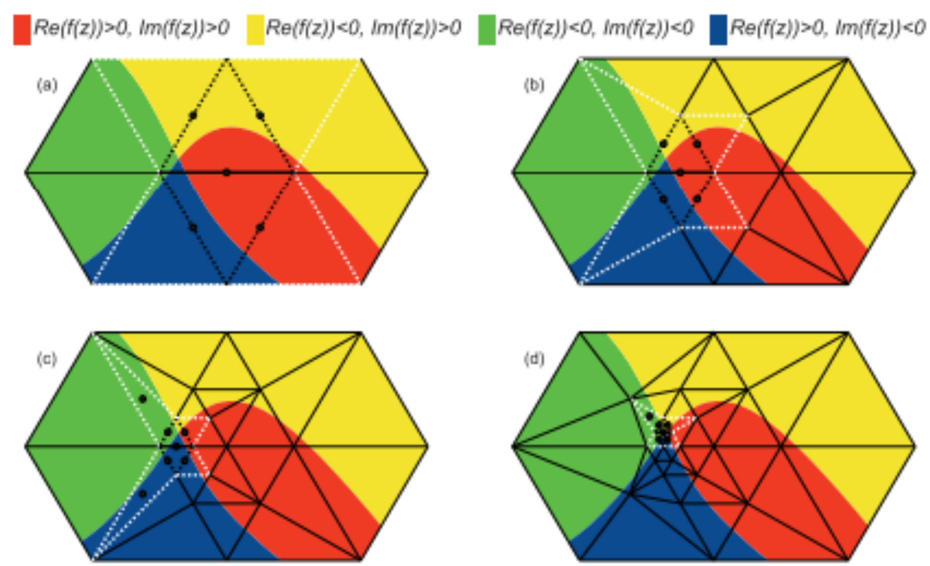

Fig. 2. Simple example of the mesh refinement process. (a)–(d) Four consecutive iterations. Thick black lines: candidate edges. Black dotted lines: boundary of the candidate regions. White dotted line: boundary of the extra zone.

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https://github.com/PioKow/GRPF

Modified GRPF – GRPF_python Overview

main.py

- set run parameters
- start Matlab engine
- for each k vector in sweep:
  - generate Matlab input files
  - run GRPF to find roots and poles of function f defined in pyfunc.py
  - write identified root and pole info to respective files
- stop Matlab engine
- exit

GRPF_python.m

- slightly modified version of GRPF code
- modified to define a Matlab function that could be run by the Matlab engine in main.py

pyfunc.py

- all definitions of python functions related to calculating things related to NP and QE properties/ coupling
- function with name "f" is what GRPF tries to locate the roots and poles of

see SLRpy_1d/GRPF_python/NOTES.txt for additional information

*Seems to ONLY Run with python 2.7

Some Important Notes:

```
% main program GRPF
%
close all;
clear;
clc;
format long;

%
% Set up ability to work with Python
%
clear classes;
if count(py.sys.path, '') == 0
    %insert(py.sys.path,int32(0),'/home/mrb179/Projects/EigenvalueProblem/GRPF/PYTHON_GRPF/');
    insert(py.sys.path,int32(0),'/home/mrb179/Projects/SLRpy_1D/GRPF_python/');
end
modu = py.importlib.import_module('pyfunc');
py.reload(modu);
```

Need to specify proper path!!

```
%% general loop
it=0;
while it<ItMax&&NrOfNodes<NodesMax
    it=it+1;

    NodesCoord=[NodesCoord ; NewNodesCoord];

    disp(['Evaluation of the function in ',num2str(size(NewNodesCoord,1)),' new points...'])

    for Node=NrOfNodes+1:NrOfNodes+size(NewNodesCoord,1)
        z=NodesCoord(Node,1)+1i*NodesCoord(Node,2);
        disp("Made it this far");
        FuntionValues(Node,1)= py.pyfunc.f(real(z), imag(z), RNP, Nind); % <------- If you change definition of f
in pyfunc.py, need to change argument list here!!

```

Changes in argument of pyfunc.f() must be reflected in GRPF_python.m

- Advantages of GRPF – locates all poles within a region
- It is not clear to me that GRPF is always better than simplex optimization (see singleNP.py)