Algorithms Assignment 1

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1 Master method

The master method can be applied to functions with form

$$T(n) = aT(\frac{n}{b}) + f(n)$$

Where

$$a \geq 1$$

$$b \geq 2$$

The master method is applied to the following recurrences

1. $p(n) = 8p(n/2) + n^2$

2. $p(n) = 8p(n/4) + n^3$

3. $p(n) = 10p(n/9) + n \log_2 n$

For the 1st recurrence we have $\begin{cases} a : 8 \\ b : 2 \end{cases}$

Since $f(n) = n^2$ and $n^{\log_b a} = n^{\log_2 8} = n^3$ there exists some constant $\epsilon = 4$ such that $n^2 = O(n^{\log_b a - \epsilon}) \Rightarrow \underline{\underline{p(n) \in \Theta(n^3)}}$

For the 2nd recurrence we have $\begin{cases} a : 8 \\ b : 4 \\ f(n) : n^3 \end{cases}$

There exists a constant $\epsilon = 56$ such that $n^3 = \Omega(n^{\log_4 8 + \epsilon})$

Since $af(n/b) \leq cf(n) \Leftrightarrow 8(n/4)^3 \leq cn^3$ we get $\underline{\underline{p(n) \in \Theta(n^3)}}$

For the 3rd recurrence we have $\begin{cases} a : 10 \\ b : 9 \\ f(n) : n \log_2 n \end{cases}$

There exists a constant $\epsilon > 1$ such that $n \log_2 n = \Omega(n^{\log_9 10 + \epsilon})$

Since $af(n/b) \leq cf(n) \Leftrightarrow 10\frac{n}{9} \log_2 \frac{n}{9} \leq cn \log_2 n$ we get $\underline{\underline{p(n) \in \Theta(n^{log_9 10})}}$

2 Substitution method

The substitution method is applied to the following recurrences

1. $p(n) = p(n/2) + p(n/3) + n$
2. $p(n) = \sqrt{n} \cdot p(\sqrt{n}) + \sqrt{n}$

We make a guess that the first recurrence has running time $O(n \log_2 n)$ This can be done by either just guessing or with a recurrence tree. The guess is done with a recurrence tree however the illustration is discarded in this report.