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Opgave 1

a)

Totalmatricen:

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 2 \\ 1 & 1 & 1 & 1 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r1 \leftrightarrow r2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 2 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r2 \rightarrow r2 - 2r1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r3 \rightarrow r3 - 4r1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & -5 & a-4 & 0 \end{array}\right]$$

$$r3 \rightarrow r3 + 5r2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & a-19 & 0 \end{array}\right]$$

Matricen er på reduceret rækkeechelonform men ikke reduceret da der er flere konstanter i hver række.

b)

$$a = 19$$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & 0 & 4 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$x1 = 1-4t$
$x2 = 3t$
$x3 = t$

c

$$a = 20$$

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\0 & 1 & -3 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r1 \to r1 - r2$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 4 & 1 & -1 & 0\\0 & 1 & -3 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r2 \to r2 + 3r3$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 4 & 1 & -1 & 0\\0 & 1 & 0 & 0 & 1 & 3\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r1 \to r1 - 4r3$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & -1 & -4\\0 & 1 & 0 & 0 & 1 & 3\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

Den inverse er dermed

$$\begin{bmatrix} 1 & -1 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Opgave 2

a)

$$\mathbf{E}_1 = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : r_2 \to r_2 - 4r_1$$

$$\mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} : r_2 \leftrightarrow r_3$$

$$\mathbf{E}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} : r_3 \to \frac{1}{5} r_3$$

$$\mathbf{E}_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : r_1 \to r_1 + r_3$$

B

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{5} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ -\frac{4}{5} & \frac{1}{5} & 0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix}1 & 0 & 0\\4 & 0 & 0\\0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 5\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=$$

$$\begin{bmatrix}1 & 0 & 0\\4 & 0 & 0\\0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & -1\\4 & 0 & -4\\0 & 0 & 1\end{bmatrix}$$

C

$AF = I_m \leftrightarrow A = F^{-1} I_m$

$F^{-1} = \begin{bmatrix}1 & 0 & -1\\4 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

Opgave 3

a

Nabomatricen for grafen ser således ud

$$\begin{bmatrix}0 & 1 & 1 & 0 & 1\\1 & 0 & 0 & 0 & 1\\0 & 1 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}$$

Det aflæses fra matricen at antallet af veje er 12.

b

For god orden opskrives antallet af udgående links fra webbet.

$N_{web_1} = 3$
$N_{web_2} = 2$
$N_{web_3} = 2$
$N_{web_4} = 1$
$N_{web_5} = 1$

Linkmatricen opskrives:

$$\begin{bmatrix}0 & \frac{1}{5} & 0 & 0 & 1\\\frac{1}{3} & 0 & \frac{1}{5} & 0 & 0\\\frac{1}{3} & 0 & 0 & 1 & 0\\0 & 0 & \frac{1}{5} & 0 & 0\\\frac{1}{3} & \frac{1}{5} & 0 & 0 & 0\end{bmatrix}$$

c

Ligningssystem opskrives på formlen $\mathbf{Ax} = \mathbf{x}$

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{1}{5} & 0 & 0 & 1 \\ \frac{1}{3} & 0 & \frac{1}{5} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} & 0 & 0 \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & 0 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix}$$

Vi omskriver systemet til $\mathbf{Ax} - \mathbf{x} = 0$ og opstiller den tilsvarende totalmatrix

$$\mathbf{A} = \left[\begin{array}{ccccc|c} 0 & \frac{1}{5} & 0 & 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{5} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & 0 & 0 \end{array}\right]$$

Vi laver rækkeoperationer og får identitetsmatricen, hvilket betyder at der alle sider er lige vigtige (nok en fejl et sted)

Opgave 4
Se koden i "src/".

Eksempel på kørsel

A:

[1.0 4.0 7.0]

[2.0 5.0 8.0]
[3.0 6.0 9.0]

B:
[-9.0 6.0 -3.0]
[8.0 -5.0 2.0]
[-7.0 4.0 -1.0]

A new:
[1.0 4.0 -4.0]
[2.0 5.0 8.0]
[3.0 6.0 9.0]

A after mul and add:
[-7.0 14.0 11.0]
[12.0 5.0 18.0]
[-1.0 16.0 17.0]