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**Opgave 1**

**a)**

Totalmatricen:

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 2 \\ 1 & 1 & 1 & 1 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r1 \leftrightarrow r2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 2 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r2 \rightarrow r2 - 2r1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 4 & -1 & a & 4 \end{array}\right]$$

$$r3 \rightarrow r3 - 4r1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & -5 & a-4 & 0 \end{array}\right]$$

$$r3 \rightarrow r3 + 5r2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & a-19 & 0 \end{array}\right]$$

Matricen er på reduceret rækkeechelonform men ikke reduceret da der er flere konstanter i hver række.

**b)**

$$a = 19$$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & 0 & 4 & 1\\0 & 1 & -3 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$x1 = 1 - 4t$
$x2 = 3t$
$x3 = t$

**c**

$$a = 20$$

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\0 & 1 & -3 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r1 \to r1 - r2$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 4 & 1 & -1 & 0\\0 & 1 & -3 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r2 \to r2 + 3r3$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 4 & 1 & -1 & 0\\0 & 1 & 0 & 0 & 1 & 3\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$r1 \to r1 - 4r3$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & -1 & -4\\0 & 1 & 0 & 0 & 1 & 3\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

Den inverse er dermed

$$\begin{bmatrix} 1 & -1 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

**Opgave 2**

**a)**

$$\mathbf{E}_1 = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : r_2 \to r_2 - 4r_1$$

$$\mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} : r_2 \leftrightarrow r_3$$

$$\mathbf{E}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} : r_3 \to \frac{1}{5} r_3$$

$$\mathbf{E}_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : r_1 \to r_1 + r_3$$

**B**

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{5} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \\ -\frac{4}{5} & \frac{1}{5} & 0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix}1 & 0 & 0\\4 & 0 & 0\\0 & 1 & 0\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 5\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} =$$

$$\begin{bmatrix}1 & 0 & 0\\4 & 0 & 0\\0 & 1 & 0\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & -1\\4 & 0 & -4\\0 & 0 & 1\end{bmatrix}$$

**C**

$AF = I_m \leftrightarrow A = F^{-1}I_m$

$$F^{-1} = \begin{bmatrix}1 & 0 & -1\\4 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

**Opgave 4**

Se koden i "src/".