Disclosure and Collaboration in Dynamic R&D Races

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Abstract

Recent models of multi-stage R&D have shown that a system of weak intellectual property rights may lead to faster innovation by inducing firms to share intermediate technological knowledge. However, I argue that the positive effect of weak intellectual property regimes on the sharing of intermediate technological knowledge vanishes when technology is complex, as is likely to be the case in many high-tech industries. Under this condition, technologically complex discoveries that are disclosed by a technological leader can be ignored by the follower, who does so when it can free ride off the research of the rival, since ignoring the disclosure encourages more research by the leader.

Keywords: dynamic patent races; intellectual property rights; imitation; knowledge sharing; cumulative innovation; not invented here syndrome.

JEL classification: L5; L2; D9.

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"One first step toward incorporating difficulty in the analysis of knowledge transfer is to recognize that a transfer is not an act, as typically modeled, but a process."

– G. Szulanski, "Sticky Knowledge"

1. Introduction

In the presence of technological spillovers, a profit-maximizing firm may decide to voluntarily reveal its intermediate innovative knowledge to rivals so as to benefit from their "cooperation" in the subsequent stages of the research (see De Fraja, 1993). Building on this insight, Bessen and Maskin (2009) and Fershtman and Markovich (2010) developed models of multi-stage R&D in which a system of weak intellectual property rights may foster innovation by inducing firms to share intermediate technological knowledge.

In this article, I argue that this literature may have overestimated the potential benefits from a leaky system. In fact, most theoretical models consider *disclosure*, that is placing new findings in the public domain, as the first viable way to succeed in the spreading of innovative knowledge which promotes the rapid diffusion of new inventions.[1] However, although new methods, ideas, or products could be made available for copy and costless to imitate, a natural observation is that firms are not likely to learn them merely because they reside in the public domain. That is, the underlying technological knowledge can be so difficult to understand that they could comprehend most of the

[1]The role of disclosure in ensuring cumulative progress has also been studied, among others by Scotchmer and Green (1990), Scotchmer (1991), Gallini (1992), Anton and Yao (2004), Denicolò and Franzoni (2004), Bessen (2005) and Bhattacharya and Guriev (2006) in different frameworks.

ingredients but still fail to grasp the recipe. This suggests that technological leakages or spillovers are greatly facilitated by a more direct *collaboration* with agents who understood and can explain the new knowledge to other researchers who intend to acquire it.[2] Accordingly, in this article, I explore the effects of disclosure and collaboration in dynamic R&D races. It turns out that such distinction is crucial to understand how different legal systems set the incentives to innovate. Specifically, I show that a system with weak patent protection may fail to promote innovation since collaboration in research is not adequately encouraged.

To see this point, consider two research units, a and b, which are conducting research and development for a new drug. Innovation is achieved in two stages. The first consists of base research which is not profitable per se but is necessary to proceed to the drug development stage. Consider that a has achieved the first stage and can begin to undertake the next phase. Missing patent protection, a would imitate the eventual success of the rival and therefore it finds profitable to promote collaboration to share development costs with b. First, it decides to place its discovery in the public domain by means of written documents. Then, since new findings are difficult to codify, it becomes available to a more thorough sharing by organizing meetings or visits from b's researchers. However, for the same reason, b might avoid collaboration so as to let the rival count less on imitation and, as a result, to support the largest R&D effort. In other words, because of a problem of

[2] Examples include the case of "collective innovation" in 19th century English iron industry, Allen (1983), knowledge sharing among pharmaceutical firms, Henderson and Cockburn, (1998), the development of open source softwares, Henkel, (2003), users knowledge transmission, Harhoff et al. (2003), and technological districts, Antonelli, (2000).

free-riding, the absence of intellectual property rights protection undermines the incentives for the follower b to collaborate with the leader a.

The scenario discussed above highlights a general feature: followers may sometimes be reluctant to cooperate with leaders when the appropriability regime is weak. That is, followers sometimes want to strategically ignore spillovers when they can free ride off the research of the leader, since ignoring the new knowledge encourages more research by the leader.

To see this point, reconsider the reason why an inventor may want to disclose his superior intermediate technological knowledge to its competitors. This is to enable the rival to conduct research on an equal footing in the next stages of the race, and hence, to increase his research effort. Since under a weak property regime the final innovation is not fully appropriable, the leader may then benefit from the rival's eventual success. However, I show that when innovative knowledge is complex, the rival may prefer to remain ignorant and free-ride on the leader's R&D effort in the last stages of the patent race. This behaviour is prevented only if the rival can be "forced" to acquire the innovative knowledge, as when knowledge is plain to understand and collaboration is not needed. When knowledge is more sophisticated, however, collaborating becomes important and free riding is a feasible strategy. This means that knowledge acquisition will only take place if it is *incentive-compatible* for both firms.

To study this insight in greater depth, I develop a simple model of a multi-stage patent race among two firms. The innovation is commercialized only after all stages have been completed and a single parameter allows me to study all possible cases of knowledge ranging from sophisticated to plain to imitate. I compare two different patent regimes: the *strong* protection regime, where the first inventor alone can utilize the invention, and the *weak* protection, in which both firms can utilize the new technology irrespective of who achieved it.

I show that in the plain technology case, weak protection can be socially desirable in terms of both the pace of innovation and expected consumer surplus. However, in the case of sophisticated knowledge, this result is reversed and a strong patent protection is typically socially desirable.[3]

This article contributes to two related research questions: why do some innovators freely reveal their intellectual property? And in as much as this happens, is it at least possible that even without the current IPRs system, discovery might develop just as rapidly as it does with patent protection? Related to the first question, De Fraja (1993) shows that although a firm prefers to achieve a successful innovation by itself, it might find it profitable to help the rival to achieve such a success, if there are enough benefits from arriving "second" in the innovation race. Stein (2008) shows that a spontaneous collaboration will arise if firms have the opportunity to repeatedly share their own progress when facing multi-step sequential innovation. My contribution is complementary to theirs as I consider the incentives for the follower to acquire the new knowledge when inventors cannot exclude them from the final market and so they face a free-riding problem due to a lack

[3]This result might explain why, in some industries, we sometimes observe a positive effect from weak protection, e.g., open source softwares, whereas we observe a negative effect in other industries with more complexity to imitate products.

of property rights protection. Related to the second issue, the possible advantage of a weak system was also considered by Fershtman and Markovich (2010) and Bessen and Maskin (2009). With respect to their analysis, I add the distinction between plain and sophisticated knowledge which allows followers to sometimes refuse to collaborate in research, even when it would be cost-less for both, preferring to free-ride on the leader's effort instead.

Although it may be surprising, the reluctance of recipients to implement intermediate innovations that they didn't create themselves is well documented, e.g., Katz and Allen (1982). In many cases this is a puzzling situation as there are no other factors that dictate an internally developed solution would be superior – or with the words of Agrawal et al. (2010): "firms tend to draw disproportionately from their firms own prior inventions relative to what would be expected given the underlying distribution of innovative activity across all inventing firms in a particular technology field". This is often called "Not Invented Here" syndrome and seems to be wide spread across both individual researchers and entire organizations.

Within this empirical literature it is important to note that, as recently shown by Hussinger and Wastyn (2011), the NIH syndrome depends on the source of external knowledge and that it occurs when knowledge is acquired by competitors, but not if knowledge is acquired by suppliers and customers. Although many different explanations could be given to this phenomenon, this piece of evidence is certainly consistent with the story told in this paper that such reluctance can be driven by free riding motives across rivals.

The rest of the article is organized as follows. The next section introduces the model. Sections 3 and 4 characterize firms' equilibrium R&D investment under a strong and weak policy regime, respectively. The pace of innovation under either systems is studied in Section 5, and the corresponding expected consumer surplus in Section 6. Section 7 analyzes the effects due of the introduction of licensing contracts in the strong regime. Finally, Section 8 summarizes the main results and concludes.

2. The Model

Consider the classic model of a multi-stage patent race as pioneered by Grossman and Shapiro (1987) and Harris and Vickers (1987); there are two risk-neutral firms $i = A, B$ undertaking research and development to create a new product. Innovation is sequential and involves two stages. A first stage has no value to consumers but it produces an intermediate technological knowledge that is necessary to develop and commercialize the next stage.

For simplicity, there are three periods $t = 0, 1, 2$. In the first two periods $t = 0, 1$, firms compete in research and development. Research outcomes are uncertain and depend on the effort exerted in R&D by each contender. Specifically, each step of innovation requires one unit of time to be developed and any chosen level of R&D produces a given probability to successfully complete the current stage at the end of each period. Then, at $t = 2$, the process of innovation is brought to end and successful firms will compete in the product market. Zero payoffs are obtained by firms if the process has stopped at some periods before.[4]

[4] Note that, as a consequence of the described staggered research process, firms obtain zero payoffs when there is even a single period without improvements in research. Hence, they have incentives to speed up the process even without the spur of competition, i.e. waiting one period is very costly.

Research & development.. At each period $t = 0, 1$ firms compete in research. They select simultaneously and non-cooperatively the intensity of their R&D effort $x_t^i \in [0, \bar{x}_t]$ whose upper-limit $\bar{x}_t$ is uncertain;[5] suppose it is drawn from a continuous probability distribution $\mathcal{F}$ on the interval $[0, 1]$, independently and identically at the beginning of each period.

Let us denote firm i's current level of technological knowledge by s_t^i. All firms share the same initial level of knowledge which is standardized to zero, i.e., $s_0^i = 0$, $\forall i$. At the end of each period, knowledge may increase by one unit as the outcome of firm's R&D effort. Specifically,

$$s_{t+1}^i = \begin{cases} s_t^i + 1 & \text{with probability} \quad x_t^i \\ s_t^i & \text{with probability} \quad 1 - x_t^i \ . \end{cases} \tag{1}$$

Hence, even if both firms are initially symmetric, their level of knowledge can differ over time. The firm that leads the race is called (technological) "leader" while the laggard is the "follower". That is, a firm i is defined a leader if $s_t^i \geq t$.

Beyond costly research efforts, firms' knowledge may increase also because of imitation from the rival which is assumed to be costless. More specifically, it is assumed that, disclosure leads follower's technological knowledge to reach leader's status with probability $\beta \in (0, 1]$.[6] Thus, even a lagged behind firm

[5] Uncertainty on the upper limit is not necessary for the results, but it captures the idea that difficulties in research are unknown at the beginning of the race, and it is analytically convenient.

[6] Assuming an exogenous spillover rate is without loss of generality as one can always imagine a more complex model in which the spillover rate is endogenous but costly, e.g.,

might be able to do research but with an additional uncertainty on imitation at the current stage. Hence, $1-\beta$ is a measure of the degree of *complexity to imitate* for the new intermediate knowledge.[7]

As imitation could harm the leader's expected profits, at the beginning of each period it must decide whether to disclose its findings or not. Indeed, a leader's only sure way to protect intermediate inventions from imitation is to keep them secret from rivals. However, as discussed before, it might sometimes have an incentive to encourage imitation by a broad dissemination of results in the public domain.

When all working details of the intermediate technology are in the public domain, firms can collaborate to achieve a better knowledge sharing. More specifically, the leader can offer to explain the new technological knowledge to the follower, who must decide whether to accept collaboration or not.[8] If he accepts, collaboration leads to a probability of imitation $\hat{\beta} > \beta$ and without a loss of generality $\hat{\beta} = 1$.

Finally, I assume that secrecy cannot be attained when the good is put on the marketplace; one can imagine that a costless process of reverse-engineering may take place so as to get a severe problem of appropriability of inventors'

costs to codify the new findings.

[7]For example, some source code lines of a new software might have a β close to one as you can learn it by reading it. Vice versa, a new theorem and/or a new bio-technological technique may require a more intense and direct contact with the author to be fully transferred and acquired by a reader.

[8]For instance, receiving visits from the rival company's workers or organizing meetings of researchers can be seen as outcomes of such bilateral decision of cooperation.

rents.

Product market competition.. At the last period, $t = 2$, if all stages of research have been completed, a new product can be brought to the market. For simplicity, the demand function for this product is assumed to be linear

$$P = \alpha - Q \tag{2}$$

where $\alpha \in (0, \infty)$ measures the size of the market.

Firms' payoffs depend on patent policy, which determines whether imitation is lawful or not. As in Fershtman and Markovich (2010), I assume that only the final innovation is patentable. Thus, intermediate discoveries are left with no legal protection during the race, but they may be kept secret.[9]

As for the final innovation, I study two alternative regimes. The first, called "strong", prevents imitation, thereby creating a barrier to enter the final market. This guarantees monopoly profits to the first inventor

$$\Pi^m = \frac{\alpha^2}{4} \ , \tag{3}$$

whereas the laggard obtains nothing.[10]

The alternative regime, called "weak", allows perfect, costless imitation. In

[9] In other words, only the new good which encompasses all previous inventions, meets the patentability requirements, specifically, the non-obviousness requirement, which specifies the size of the innovative step needed to qualify for patent protection.

[10] Since time is discrete, when both firms reach innovation at the same period, it is assumed that only one patent protection could be granted and each inventor has the same 1/2 probability to obtain this.

this case, as soon as the new good is developed by one firm, it can be produced and commercialized by both. Thus, the market is always a duopoly, and firms equally share profits

$$\Pi^d = (1-\delta)\frac{\Pi^m}{2} \tag{4}$$

where the parameter $\delta \in [0,1)$ captures the intensity of product market competition. This allows me to study in a reduced form all possible competitive configurations ranging from Bertrand ($\Pi^d = 0$) to perfect collusion ($\Pi^d = \Pi^m/2$).

Timing of the game.. To summarize, the timing of the game is as follows:

- At $t = 0$, firms are symmetric, a value $\bar{x}_0$ is drawn, and firms set their R&D efforts simultaneously. Nature then determines which firm succeeds. Firms observe the progress in research of each contender.

- At $t = 1$, a value $\bar{x}_1$ is drawn, and the leader may decide to disclose or keep its superior knowledge secret. The follower decides whether to ignore or accept collaboration from the leader when it is available and the new technology is in the public domain. Finally, R&D efforts are again chosen simultaneously. Nature determines whether imitation occurs or not and which firm succeeds at this stage.

- At $t = 2$, if at $t = 1$ at least one firm succeeded, the good is commercialized and firms earn profits.

3. Strong Patent Regime

In this benchmark case, imitation is prohibited by law, inventors can patent the new product and exclude competitors in the final market and monopoly profits are granted to one firm only: the first inventor.

The (sub game perfect) equilibria of this game are straightforward and are obtained by backward induction.

After one period, suppose that only one firm grabbed intermediate technological knowledge, i.e., $\{s_1^i > s_1^j\}$. Clearly, since the legal system grants profits to the first inventor only, the optimal disclosure strategy for a leader is to keep its knowledge secret, forcing the rival to quit research at this stage.[11] In this case, the leader maximizes expected profits

$$V^{1,0} = x_1^i \cdot (\Pi^m - c) \tag{5}$$

and to avoid a proliferation of cases, I will consider throughout that $\Pi^m > c$, meaning that there is a unique equilibrium R&D investment by which the firm will approach its maximum R&D capacity, i.e., $x_1^{i^*} = \bar{x}_1$.

Because of secrecy, broad patents imply that competition in research on the final step occurs only when both firms have innovated during the first period, i.e., $\{s_1^i \wedge s_1^j = 1\}$. In this case they simultaneously choose a level of R&D to

[11] In a more general case, however, the leader could reach an agreement with the follower seeking royalty fees for the legal permission to use technology. That is, firms can sign licensing contracts. Here, I assume that any form of such agreements is forbidden for anti-trust reasons or, is simply impossible. This assumption will be removed in Section 7.

maximize the following expected profits

$$V^{1,1} \quad = \quad x_1^i \cdot \left(\Pi^m - x_1^j \cdot \frac{\Pi^m}{2} \right) - c \cdot x_1^i \ . \tag{6}$$

Note that by (6) an increase in one rival's R&D decreases the marginal revenue of the other, thereby decisions are strategic substitutes. The following lemma characterizes all the equilibria in this case.

Lemma 1. *Suppose that both firms have succeeded in the first period. Then, there exists a threshold*

$$\hat{x}_1 = 2 - \frac{8c}{\alpha^2} \tag{7}$$

such that

(i) if $\bar{x}_1 < \hat{x}_1$, the unique equilibrium investment in R&D is $x_1^{i^*} = \bar{x}_1$, $\forall i$, maximum effort,

(ii) if $\bar{x}_1 \geq \hat{x}_1$, the unique (symmetric) equilibrium investment in R&D is $x_1^{i^*} = \hat{x}_1$, $\forall i$, limited effort.

Proof. See the Appendix. □

That is, when the R&D capacity for each firm is sufficiently low ($\bar{x}_1 < \hat{x}_1$), innovation is difficult to achieve but the eventual inventor is also unlikely to be excluded from patent protection when that occurs. This increases incentives to innovate and the constraint is binding. On the contrary, when both have large R&D capacity ($\bar{x}_1 \geq \hat{x}_1$), investment decisions mutually offset one another. At this point, multiple equilibria arise. To focus on the most favorable situation for research, firms behave symmetrically, each exerting a positive R&D effort that is strictly below its upper limit so as to let them break even.

It is important to note that the threshold (7) is decreasing in the ratio c/α. Thus, the larger is the market size, the higher is the probability that both firms will exert the maximum R&D effort at the intermediate step.

Proceeding backwards to the first period, firms are necessarily symmetric and each firm i maximizes the following function

$$V^{0,0} = x_0^i(1 - x_0^j)EV^{1,0} + x_0^i \cdot x_0^j EV^{1,1} - c \cdot x_0^i \ , \tag{8}$$

where EV denotes the expected profits for a generic continuous distribution $\mathcal{F}$. The next lemma characterizes all equilibria.

Lemma 2. *In the first period, there exists a threshold*

$$\hat{x}_0 = \frac{EV^{1,0} - c}{EV^{1,0} - EV^{1,1}} \tag{9}$$

such that

(i) if $\hat{x}_0 < 0$, both firms do not invest in R&D,

(ii) if $\bar{x}_0 < \hat{x}_0$, the unique equilibrium investment in R&D is $x_0^{i^*} = \bar{x}_0 \ \forall i$, maximum effort,

(iii) if $\bar{x}_0 \geq \hat{x}_0$, the unique (symmetric) equilibrium investment in R&D is $x_0^{i^*} = \hat{x}_0 \ \forall i$, limited-effort.

Proof. See the Appendix. □

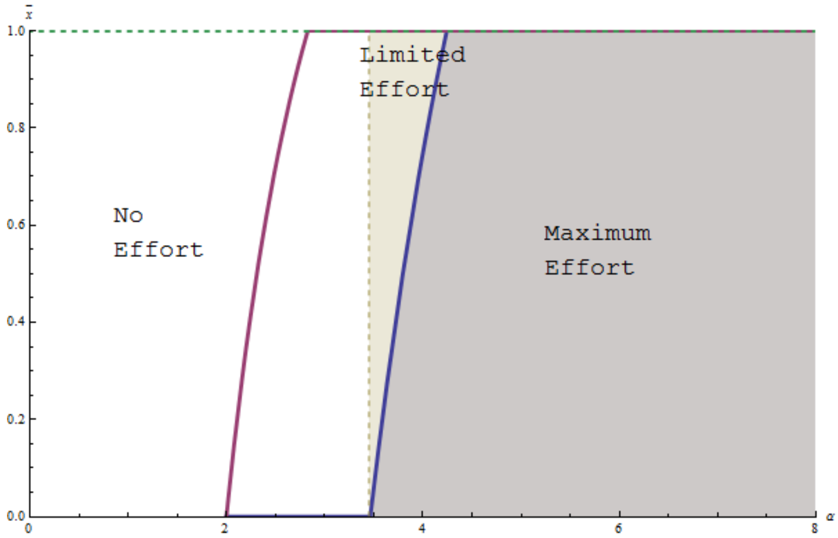

Figure 1: Strong patent protection regime: equilibrium cut-off functions $\hat{x}_1$ red curve and $\hat{x}_0$ blue curve, for $c = 1$.

Of course, given the recursive structure of the race, the equilibria described above are analogous to the previous lemma. Again, the risk of facing tough competition in the future can reduce investment at the current stage.

Taking stock of all the results obtained so far, Figure 1 depicts the two thresholds $\hat{x}_0$ and $\hat{x}_1$ as functions of the market size. We observe various patterns of R&D corresponding to three different parameter regions: *no effort, limited effort,* and *maximum effort* region. More specifically, when $\alpha < 2\sqrt{3c \cdot (1-\delta)}$, the first period threshold is negative and all firms do not invest in R&D. As market size increases, firms invest at the first stage but may exert a limited effort because of competition. Such reduced R&D is confined to a small area of values, otherwise investments reach the maximum R&D capacity at every period.

4. Weak Patent regime

In a regime of "weak" protection, there is no legal shield available for first inventors, imitation is costless, and trade secret protection is not feasible at the final step of the race, i.e., $t = 2$. Hence, irrespective of who invented first, both rivals will be competing on equal footing in the product market and obtain half duopoly profits $\Pi^d = (1 - \delta) \cdot \alpha^2/8$ from achieving all steps of innovation.

Proceeding backwards to $t = 1$, there are three possible alternative scenarios. First, the initial innovation has not been attained and both firms quit research obtaining zero profits. In the second scenario, both firms have completed the first stage and must decide how much to invest in product development. In the third case, one firm has lagged behind the rival and must choose whether to disclose its findings (being available for collaboration) or not. Then, if the leader has settled to cooperate, the follower must decide to accept collaboration or not. Finally, both simultaneously decide the level of R&D.

Let us analyze first the second scenario: firms are symmetric as both innovated in-house during the first period, and each firm i maximizes the following expected payoff function

$$\hat{V}^{1,1} = \left[1 - (1 - x_1^i)(1 - x_1^j)\right] \cdot \Pi^d - c \cdot x_t^i \tag{10}$$

with $x_1^i \leq \bar{x}_1$. Unlike in the strong protection case, an increase in the rival's R&D raises the marginal revenue of the other, thereby making decisions as strategic complements. The following lemma characterizes all equilibrium outcomes in this case.

Lemma 3. *In the second period, if all firms have succeeded in the initial innovation, there exists a threshold*

$$\hat{x}^{w_1} = 1 - \frac{8c}{(1-\delta)\cdot\alpha^2} \tag{11}$$

such that

(i) if $\hat{x}^{w_1} < 0$, neither firm invests in R&D,

(ii) if $\hat{x}^{w_1} > \bar{x}_1$, the unique equilibrium investment in R&D is $x_1^{i^*} = \bar{x}_1\ \forall i$, maximum effort,

(iii) if $\hat{x}^{w_1} \leq \bar{x}_1$ the unique (symmetric) equilibrium investment in R&D is $x_1^{i^*} = \hat{x}^{w_1}\ \forall i$, limited effort.

Proof. See the Appendix. □

As for the strong patent case, the equilibrium R&D investment depends on a cut-off value (11). If the realized R&D capacity is above this value, firms jointly moderate their R&D efforts; otherwise, both exert the maximum effort in research. Unlike before, firms are now playing a game of private provision of a public good (from the viewpoint of the firms), i.e., the innovative technological knowledge. In this game, both firms may end up reducing the costs of research as the only symmetric non-cooperative equilibrium because of free-riding.

To see how the presence of such free-riding problem can hinder or promote knowledge transmission among rivals, let's turn our attention to when one firm lagged behind the other. It is clear that the first inventor is willing to share its innovative knowledge with the laggard to let him contribute to the provision of the public good. However, disclosure does not allow imitation per se, and the follower wants sometimes to reject collaboration to better free ride on the leader's effort.

To better understand this point, let's first consider that technological knowledge is plain, i.e., $\beta = 1$. In this case, transmission of knowledge is achieved by disclosure alone. That is, the follower cannot pretend to be ignorant of the last advancements in research quitting the race. In this case, the equilibrium effort is described as before by Lemma 3.

Now, consider that $\beta < 1$ and the follower must decide whether to accept the offer to cooperate with the leader or not. Clearly, accepting the offer produces the same equilibrium actions as in the $\beta = 1$ case. Rejecting makes the leader i maximize the following expected payoffs

$$V^{1,0} = (x_1^i + \beta x_1^j - \beta x_1^j \cdot x_1^i)\Pi^d - c \cdot x_1^i \tag{12}$$

whereas the follower j will maximize

$$V^{0,1} = (x_1^i + \beta x_1^j - \beta x_1^j \cdot x_1^i)\Pi^d - c \cdot x_1^j \ . \tag{13}$$

Lemma 5 in the appendix shows all the corresponding equilibrium outcomes. Here, it is important to point out the difference between the decision to collaborate or not when firms have large R&D capacity, i.e., $\bar{x}_1 > \frac{\beta\Pi^d - c}{\beta\Pi^d}$. In this situation, free-riding is more intense among rivals. Therefore, when the follower rejects collaboration and the effects of disclosure on the sharing of knowledge are small, i.e., $\beta << 1$, there exists a unique equilibrium at which the leader sets R&D to maximum capacity and the follower quits research. At this point, we can compare this result with lemma 3 and the next proposition shows under what conditions there exists collaboration or not.

Proposition 1. *In the second stage, a technological leader always settles to collaborate in research with the rival; then,*

i) if $\bar{x} \leq \frac{\Pi^d - c}{\Pi^d}$, the follower accepts collaboration and both firms exert the maximum effort;

ii) if $\frac{\Pi^d - c}{\beta\Pi^d} > \bar{x} > \frac{\Pi^d - c}{\Pi^d}$, the follower rejects collaboration; the leader sets maximum effort whereas the followers quits research, i.e., no effort.

iii) if $\frac{\Pi^d - c}{\beta\Pi^d} \geq \bar{x}$ the follower accepts collaboration and both firms exert limited effort.

Proof. First suppose that $\bar{x} < \frac{\Pi^d - c}{\Pi^d}$. If the follower accepts cooperation, technological knowledge is symmetric and both firms exert the maximum effort (Lemma 3). By rejecting, however, the leader's equilibrium effort would not change and a follower receives no gains in terms of free riding from tying its own hands on research: collaboration arises in this case. Suppose instead that $\bar{x} > \frac{\Pi^d - c}{\beta\Pi^d}$. Now, without collaboration, both firms exert asymmetric limited levels of effort (Lemma 5). With an easy comparison with the symmetric case, collaboration turns out to be more advantageous for the follower. In the last case we have $\frac{\Pi^d - c}{\beta \Pi^d} > \bar{x} > \frac{\Pi^d - c}{\Pi^d}$. The follower's decision to reject the offer implies that the leader will switch from a limited effort (Lemma 3) to its maximum capacity level (Lemma 5). Moreover, the asymmetric equilibrium implies that, as a best reply, the follower quits research setting zero R&D effort. Therefore, as it gives a better possibility to free ride, the follower will reject collaboration. □

To summarize, there is a clear connection between the uncertainty about imitation and the degree to which the follower can free ride on the leader's effort. Indeed, because of the absence of a strong protection, a technological leader may want to turn the rival into active research so as to share the cost of effort of the last stage. However, when β is sufficiently low, this will not arise because of the follower's decision to avoid cooperation. Such reluctance is strategic as it forces the leader to increase its effort, reducing the role of any cost sharing among rivals. In this setting, due to the linearity of payoffs, the leader sets a maximum level of R&D effort, whereas the follower quits research. By contrast, as considered in most standard models, when β is sufficiently high, this situation never happens as cooperation is always better for both rivals.

Going back to the first period $t = 0$, firms are again symmetric. Each firm i maximizes the following value function

$$\hat{V}^{0,0} = x_i \cdot x_j \widehat{EV}^{1,1} + x_i \cdot (1 - x_j) \widehat{EV}^{1,0} + (1 - x_i) \cdot x_j \widehat{EV}^{0,1} - c \cdot x_i \quad (14)$$

where the equilibrium expected payoffs of the second stage are averaged over all possible values of $\bar{x}_1$ according to a generic continuous distribution $\mathcal{F}$ and maximization is constrained as $x_i \leq \bar{x}_0$. The next lemma characterizes equilibria.

Lemma 4. *At the first period there exists a threshold*

$$\hat{x}^{w_0} = \frac{\widehat{EV}^{1,0} - c}{\widehat{EV}^{1,0} + \widehat{EV}^{0,1} - \widehat{EV}^{1,1}} \tag{15}$$

such that

(i) if $\hat{x}^{w_0} < 0$, neither firm invest in R&D,

(ii) if $\hat{x}^{w_0} > \bar{x}_0$, the unique equilibrium investment in R&D is $x_i^* = \bar{x}_0$ $\forall i$, maximum effort,

(iii) if $\hat{x}^{w_0} \leq \bar{x}_0$, the unique (symmetric) equilibrium investment in R&D is $x_i^* = \hat{x}^{w_0}$ $\forall i$, limited effort.

Proof. See the Appendix. $\square$

Corollary 1. *The threshold $\hat{x}^{w_0}$ is strictly increasing in β if $\beta > \frac{\Pi^d - c}{\Pi^d}$. Otherwise, if $\beta < \frac{\Pi^d - c}{\Pi^d}$, the threshold is constant with respect to β.*

To summarize the results, Figure 2 shows the two thresholds $\hat{x}^{w_1}$ and $\hat{x}^{w_0}$ for the case of perfect collusion in the product market where intermediate knowledge is plain. Because collusion ensures the highest possible reward to inventors under a weak regime and plain knowledge limits the free-riding problem, fixing $(\delta = 0, \beta = 1)$ constitutes the "upper bound" case.

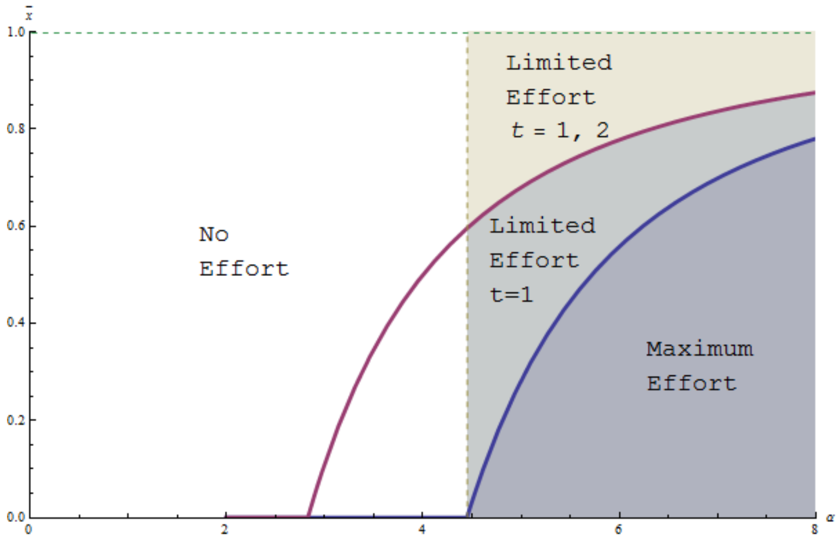

Figure 2: Weak patent protection regime: equilibrium cut-off functions, $\hat{x}^{w_1}$ red curve and $\hat{x}^{w_0}$ blue curve, for $\beta = 1$, $\delta = 0$ and $c = 1$.

As we may conclude from a first comparison with Figure 1, firms are now more likely to exert lower individual levels of effort than in the strong protection regime. Broad patents, however, do not necessarily produce higher aggregate levels of R&D. The reason is that, in a strong patent protection regime, a leader does not share technological knowledge with rivals. By contrast, in a weak regime, firms eagerly reveal information and cooperate in research, ensuring a higher overall number of firms active in research at the intermediate stage.

5. The Pace of Innovation

In this section, I examine some of the implications of different patent protection regimes. Specifically, I look at the aggregated probability to innovate as a measure of the market pace of innovation. That is, I follow the preceding equilibrium analysis to pin down the probability that all steps of innovation are accomplished by at least one firm. This is for the purpose of building a benchmark solution which can provide a basis for a comparison between strong and weak patent protection.

From this perspective, studying first the probability to innovate also permits the effects of a legal system on the innovation patterns to be studied separately from those on consumers' well-being. Then, in the next section I examine how consumer surplus is affected by both regimes.

5.1. Strong Patent Regime

In a system ensuring strong patent protection, cooperation in research is never spontaneous. The leader will not disclose for free the new intermediate knowledge to the follower and therefore, only successful innovators will move forward in the race.

To compute the pace of innovation, I proceed backwards using results obtained from Lemma 1 and Lemma 2. At the second period, innovation depends on the number of firms advancing in the race and on the realized R&D capacity, i.e., $\bar{x}_1$. Hence, the aggregate probability to innovate will take only four alternative values,

$$\mu_1^{s^i,s^j} = \begin{cases} 0 & \text{if } \{s_1^i = s_1^j = 0\} \\ \bar{x}_1 & \text{if } \{s_1^i \vee s_1^j = 1\} \\ \hat{x}_1(2-\hat{x}_1) & \text{if } \{s_1^i \wedge s_1^j = 1\} \text{ and } \bar{x}_1 > \hat{x}_1 \\ \bar{x}_1(2-\bar{x}_1) & \text{if } \{s_1^i \wedge s_1^j = 1\} \text{ and } \bar{x}_1 \leq \hat{x}_1 \ . \end{cases} \tag{16}$$

At the first period, the $\bar{x}_1$ has not been realized yet. To ease notation, suppose that the upper limit is uniformly drawn. Hence, it is a matter of algebra to compute the probability that all steps of innovation are accomplished at the initial period. First, let's compute this measure for a given initial upper bound,

$$\mu_0 = \begin{cases} (\hat{x}_0)^2 \cdot \int_0^1 \mu_1^{1,1}\ d\bar{x} + 2 \cdot \hat{x}_0(1-\hat{x}_0) \cdot \int_0^1 \mu_1^{1,0}\ d\bar{x} & \text{if } \bar{x}_0 > \hat{x}_0 \\ (\bar{x}_0)^2 \cdot \int_0^1 \mu_1^{1,1}\ d\bar{x} + 2 \cdot \bar{x}_0(1-\bar{x}_0) \cdot \int_0^1 \mu_1^{1,0}\ d\bar{x} & \text{if } \bar{x}_0 \leq \hat{x}_0 \end{cases} \tag{17}$$

and then average over all possible realizations of $\bar{x}_0$,

$$\mu = \int_0^1 \mu_0\ d\bar{x} \tag{18}$$

The pace of innovation (18) is illustrated in Figure 3 as a function of the market size, i.e., α, and for a marginal cost of one, i.e., $c = 1$. As expected, this probability exhibits an s-shaped curve, because there are three different parameter regions in which R&D can occur with various intensities. More specifically, there is zero R&D investment for low-demand inventions, positive investment but of limited intensity for inventions of intermediate size, and finally, investment of maximum intensity in the remaining cases. At this point, the curve touches an upper-bound $\bar{\mu} \approx 0.4$ and is constant.

5.2. Weak Patent Regime

Consider a legal regime such that patents are too narrow to protect revenues in the product market, thereby leaders are incentivized to disclose intermediate technological knowledge voluntarily.

Again, by proceeding backwards, the aggregate probabilities to innovate at

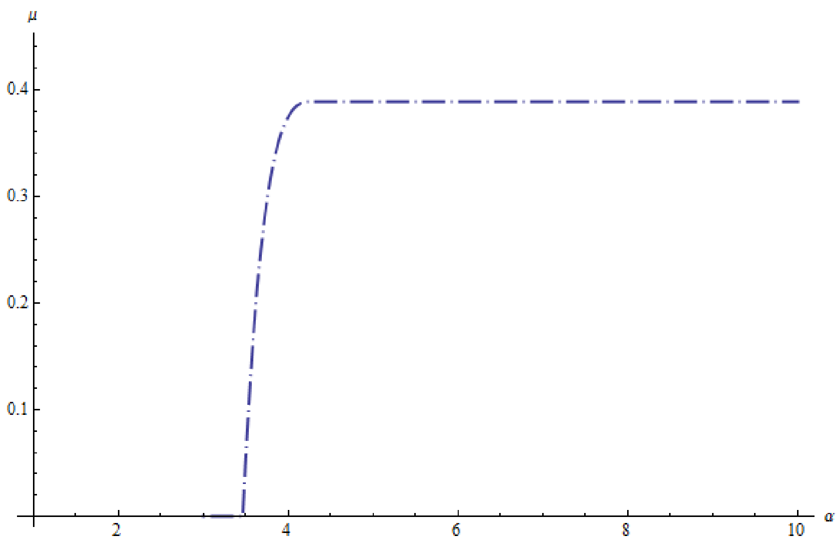

Figure 3: Pace of innovation under a strong patent protection regime.

each stage are as follow,

$$\hat{\mu}_1^{s^i,s^j} = \begin{cases} 0 & \text{if } \{s_1^i = s_1^j = 0\} \text{ or } \hat{x}^{w_1} < 0 \\ \bar{x}_1 & \text{if } \{s_1^i \neq s_1^j\} \text{ and } \frac{\hat{x}^{w_1}}{\beta} > \bar{x}_1 > \hat{x}^{w_1} \\ \hat{x}^{s_1}(2-\hat{x}^{s_1}) & \text{if } \{s_1^i = s_1^j = 1\} \text{ and } \bar{x}_1 < \hat{x}^{w_1} \\ \bar{x}_1(2-\bar{x}_1) & \text{if } \{s_1^i \wedge s_1^j = 1\} \text{ and } \bar{x}_1 \leq \hat{x}^{w_1} \text{ ;} \end{cases} \tag{19}$$

$$\hat{\mu}_0 = \begin{cases} (\hat{x}_0)^2 \cdot \int_0^1 \hat{\mu}_1^{1,1} \, d\bar{x} + 2 \cdot \hat{x}_0(1-\hat{x}_0) \cdot \int_0^1 \hat{\mu}_1^{1,0} \, d\bar{x} & \text{if } \bar{x}_0 > \hat{x}_0 \\ (\bar{x}_0)^2 \cdot \int_0^1 \hat{\mu}_1^{1,1} \, d\bar{x} + 2 \cdot \bar{x}_0(1-\bar{x}_0) \cdot \int_0^1 \hat{\mu}_1^{1,0} \, d\bar{x} & \text{if } \bar{x}_0 \leq \hat{x}_0 \end{cases} \tag{20}$$

and the pace of innovation under a weak regime is

$$\hat{\mu} = \int_0^1 \hat{\mu}_0 \, d\bar{x} \tag{21}$$

To better understand the difference between the strong and the weak regimes,

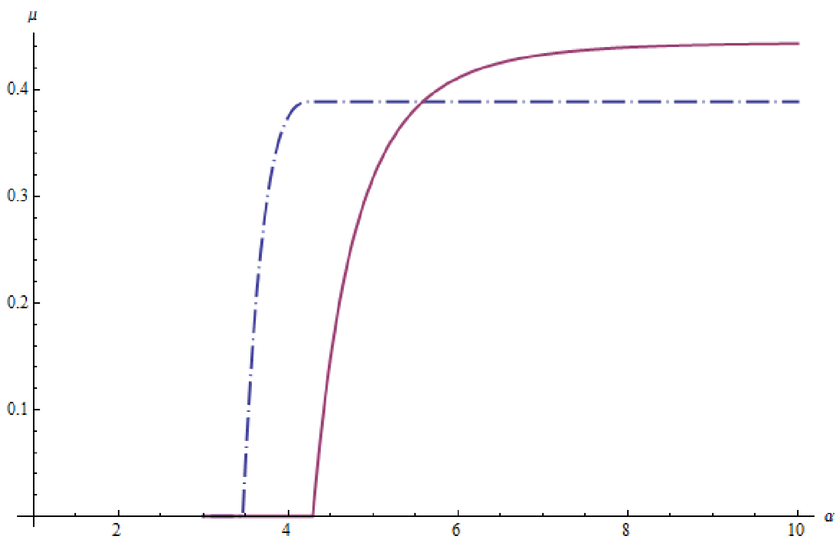

Figure 4: Comparison between the pace of innovation under a strong regime (dot-dashed line) and a weak regime (solid line) with $\beta = 1$, $c = 1$ and $\delta = 0$.

Figure 4 plots both probability curves as a function of market size.

As the figure illustrates, the (solid) curve representing the probability of success under a weak regime is positive for higher levels of market size and increases at a slower rate with respect to the (dot-dashed) curve representing the same probability under a strong regime.

Note that the two curves are monotonic and cross at $\alpha \approx 5.5$. Hence, although broad patents deliver a higher pace of innovation for intermediate values of market size, there exists a level of α above which the pace of innovation is enhanced by a weak system. Note that this must be true as long as the probability of success in a weak regime reaches an upper bound, here $\bar{\mu}^w \approx 0.44$, which is higher than the corresponding value for a strong patent regime.

The following proposition states the general form of this result:

Proposition 2. *For any continuous probability distribution $\mathcal{F}$, there exists* *a threshold value $\hat{\alpha}$ such that*

- *if $\alpha < \hat{\alpha}$, the pace of innovation under weak protection is smaller than that under strong protection,*
- *if $\alpha \geq \hat{\alpha}$, instead, the pace of innovation under weak protection is higher than that under strong protection.*

Proof. When $\alpha \to \infty$, firms' R&D reach the maximum capacity, since all thresholds tend towards either one or some value above. Thus, the only difference across regimes is that, by Proposition 1, firms will always collaborate to share their knowledge in a weak regime, whereas they don't with strong protection. So the pace of innovation in either regimes tends to differ and tends to be higher in a weak regime. Then, recall that the pace of innovation is a continuous function and that the strong regime requires smaller values of α to start the research process. Therefore there must be a finite value of α such that both curves cross. $\square$

The above result can be easily understood when taking into account the role of technological complementarity in multi-stage innovations, as emphasized among others by Bessen and Maskin (2009) and Fershtman and Markovich (2010). For instance, when complementarity is better exploited by technological transfers, relaxing competition through weak patent protection might increase the pace of innovation. Of course, a weak patent protection regime reduces the expected rewards for inventors. Thus, there exists a tension between inducing cooperation and incentivizing research with higher rewards. As the market size increases, however, such a trade-off disappears whereby a weak patent protection regime should prevail in a market for innovations with larger demand.

Corollary 2. *The threshold $\hat{\alpha}$ is non-decreasing in β.*

As discussed before, as soon as the intermediate knowledge becomes very complex to imitate and collaboration cannot be reached because of a free riding problem, the pace of innovation slows down. More specifically, this negative result for a weak system is based on two effects: first, unlike in most standard models, disclosure can produce poor effects on the sharing of sophisticated intermediate knowledge among firms. Only a costless process of collaboration will produce relevant spillovers, but the public nature of the innovative knowledge works against such collaboration. That is, because of a weak appropriability system, the lack of follower's incentives to share knowledge has the first negative impact on the speed of innovation. Second, while firms are indifferent between arriving "first" or "second" in innovating a plain intermediate invention, they expect collaboration to arise at that point. When knowledge is sophisticated, they prefer to arrive second and free ride on the first inventor. Therefore, this possibility to free ride at the second period also translates into a lower level of effort at the first period.

6. Consumer Surplus

The analysis of the previous section has shown how different patent systems affect the pace of innovation. To better assess all the potential benefits from either systems, it is now convenient to explore the overall effect on consumers' well-being.

A regime of strong patent protection always results in a monopoly in the product market, and it carries relevant deadweight-losses and poor consumer surplus.[12] On the contrary, a weak system can count on competition to provide a larger range of alternatives that are more desirable from the viewpoint of consumers. However, consumers seek a higher surplus combined with a satisfactory probability that the innovation process is achieved. Hence, I shall consider a measure that puts together the pace of innovation and the surplus that is realized once all steps of innovation are accomplished.

Given the demand function (2), the consumer surplus can easily be computed (see the Appendix B) and represented in reduced form as a function of δ and α,

$$CS(\alpha, \delta) = \frac{\alpha^2}{8} \left(1 + \delta^{\frac{1}{2}}\right)^2 \quad . \tag{22}$$

This function encompasses both cases of duopoly and monopoly in the product market. For instance, in the presence of a duopoly with perfect colluding firms, i.e., $\delta = 0$, equation (22) takes the same value as that of a monopolistic market. Hence, let this value be denoted by $CS^m(\alpha) \equiv CS(\alpha, 0)$.

Thus, a function representing the *expected consumer surplus* is easily obtained and denoted by ECS. Clearly, this function depends on the legal

[12] I am implicitly considering that a patent's life is infinite. As we will discuss next, an "optimized" patent regime may allow for finite patent life and this may reduce the expected deadweight loss under a strong regime. However, restricting attention to this suboptimal case will bolster the argument against a weak patent system.

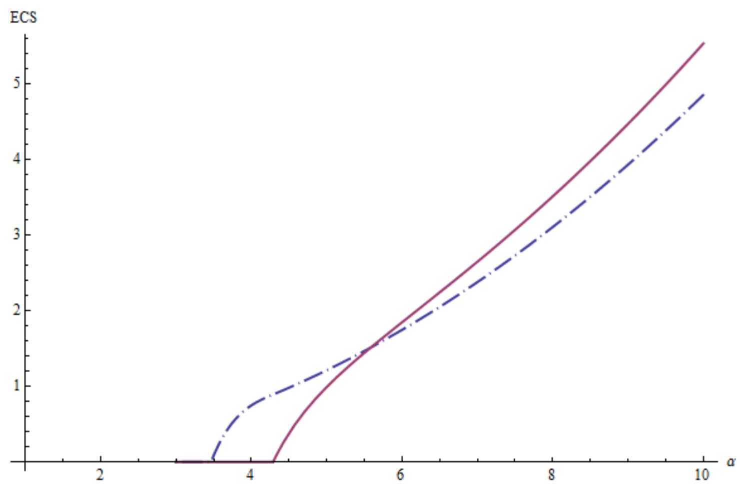

Figure 5: Comparison between the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime (solid line) with plain technological knowledge: $\beta = 1$, $c = 1$ and $\delta = 0$.

regime in place: if patent protection is strong, it is

$$ECS = \mu \cdot CS^m(\alpha) \ , \tag{23}$$

whereas, if patent protection is weak,

$$ECS^w = \hat{\mu} \cdot CS(\alpha, \delta) \ . \tag{24}$$

Fixing perfect collusion in the retail market, i.e., $\delta = 0$, Figure 5 shows both curves when technological knowledge is plain, i.e., $\beta = 1$. Recall that, for δ close to zero, both regimes offer roughly the same consumer surplus and the main difference is the pace of innovation. Again, a weak regime performs better with inventions of larger market size.

Suppose that there is a more intense competition in the product market, i.e., $\delta = 1/3$. By looking now at Figure 6, the curve for a weak regime rotates

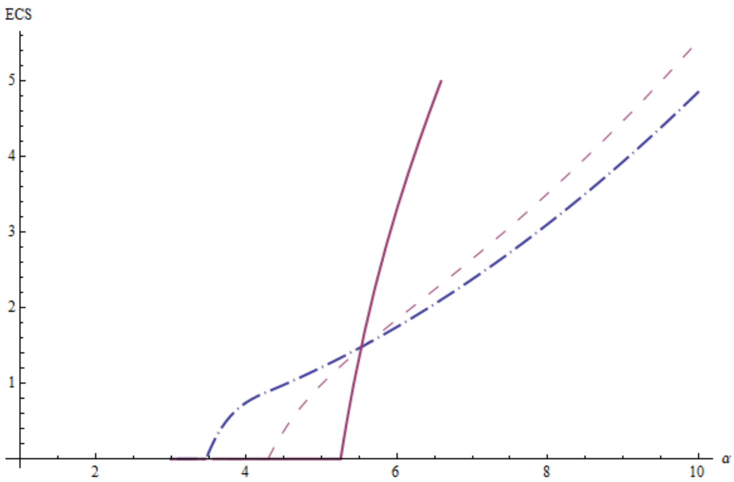

Figure 6: Comparison between the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime for $\delta = 1/3$ (solid line) and $\delta = 0$ (dashed line) with plain technological knowledge: $\beta = 1$ and $c = 1$.

when δ switches to positive values. This movement can easily be explained. On the one hand, firms' incentive to participate in the race are weakened, i.e., the initial value of market size must be larger. On the other hand, the difference between the expected consumer surplus in the two regimes grows exponentially larger as the market size increases.[13]

This comparatively static result adds to the growing literature and the common view that a weak patent system yields substantially larger social benefits than a strong regime does. Next, I am going to show an example of how the positive effects of a weak system could vanish if $\beta < 1$.

Figure 7 depicts the ECS curves as a function of the intensity of competition in the product market. The expected consumer surplus in a strong regime is obviously a constant function of the intensity of competition, whereas the

[13]Note that the expected consumer surplus (22) increases exponentially in α and it grows faster for $\delta > 0$.

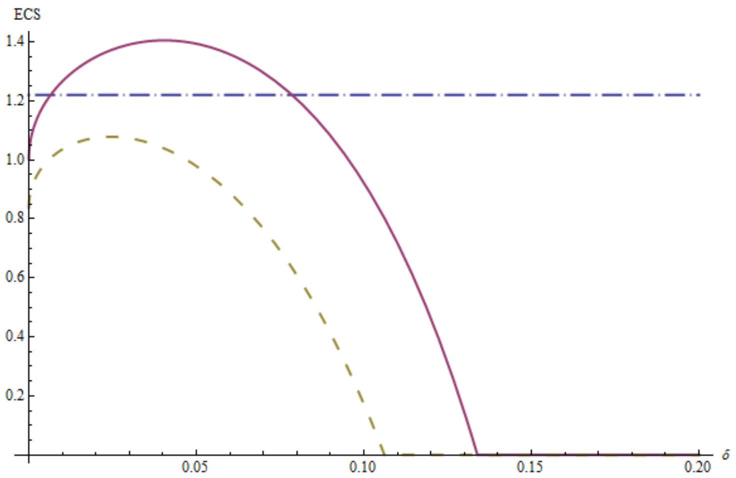

Figure 7: Comparison among the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime with $\beta = 1$ (solid line) and with $\beta = 2/3$ (dashed line); fixed $c = 1$ and $\alpha = 5$.

ECS from a weak system varies in a non-monotonic way with respect to δ. This is because the pace of innovation is monotonically decreasing while the consumer surplus is monotonically increasing in the intensity of competition. Consider two curves for the weak regime, one depicts the case of $\beta = 1$ and the second the case of $\beta = 2/3$. It can be noted that as far as the case with plain technological knowledge is considered, there exists a set of parameter values of δ such that expected consumer surplus is enhanced by a weak system. This result, however, is strikingly overturned in the sophisticated case, i.e., when $\beta = 2/3$.

In summary, as this simple example shows, a policy-maker should carefully take into account the appropriate definition of technological knowledge before adopting a weak system of patent protection.

7. Licensing

In general, firms are able to enter contracts to share technology and seek positive licensing fees. To capture the role of licensing contracts in a simple way, suppose that the follower could offer a contingent contract to induce the leader's disclosure. As long as offers leave the leader's expected profits untouched after disclosure, technological sharing can be achieved even in a strong patent protection regime.[14] By restricting attention to the upper bound case, i.e., $\delta = 0$, the following result ensues from licensing:

Proposition 3. *If $\delta = 0$, the expected consumer surplus of a strong regime is greater than or equal to that of a weak regime with plain technological knowledge, i.e., $\beta = 1$.*

As illustrated in Figure 8, licensing substantially improves social welfare in a strong regime, thus compensating some of the advantages highlighted for a weak regime. And yet, for $\delta > 0$, a weak regime could perform better. However, a full comparative statics on such other cases is beyond the scope of this article. A full comparison between the two regimes should consider a strong patent system that is "optimized." In this case, a strong system may compensate the improved performance of a weak patent regime by setting, for instance, adequate length and breadth of patents.

[14] For example, the contract could state that if the licensee develops the new invention, it keeps all profits. Otherwise, if both firms develop the last-stage invention at the same time, profits accrue only to the earlier innovator.

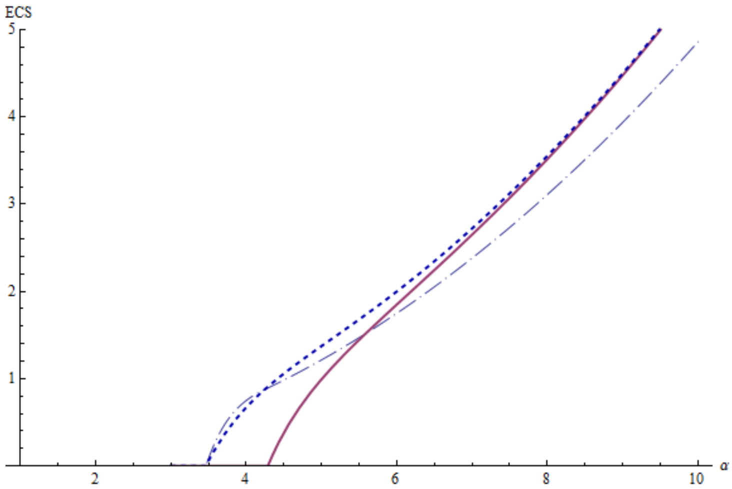

Figure 8: Comparison between the expected consumer surplus under a strong regime without licensing (dot-dashed line), with licensing (dashed line) and under a weak regime with plain technological knowledge (solid line). Fixed $c = 1$ and $\delta = 0$.

8. Conclusions

Recent models of multi-stage R&D have shown that a system of weak intellectual property rights may lead to faster innovation by inducing firms to share intermediate technological knowledge. In this article, I developed a simple model of multi-stage patent race between two firms. At each step, firms produce intermediate technological knowledge that they may decide to share with the rival. Then, I explored the possibility that technologically complex discoveries that are disclosed by a technological leader can be ignored by the follower. And it turns out that followers sometimes want to ignore disclosures when they can free ride off the research of the leader, since ignoring the disclosure encourages more research by the leader.

I have shown that this behaviour is prevented, and weak protection can be a spur to innovation, only when intermediate knowledge is plain to imitate[15]

[15] For example, releasing new lines of code of open source softwares or identifying a bug

In fact, since R&D investment in a weak regime can be seen as a public good from the viewpoint of firms, lagged-behind firms may want to commit to fully free ride on leader's effort. However, this strategy is feasible only when knowledge is sophisticated as spillover rates crucially depend on some direct collaboration among inventors, whereas it is prevented in the case with plain technology.

Although it may be surprising, the reluctance of recipients to implement intermediate innovations made freely available by other inventors is a well documented situation. As a recent literature shows (see the discussion in the introductive section), it is wide spread across both individuals and corporations. To that literature, this paper adds the possibility that innovative firms may end up approving any "Not Invented Here" attitudes from their employees as a consequence of strategic free-riding motives.

Finally, a straightforward policy implication comes out of this work. Since the effects of disclosure on the sharing of intermediate knowledge are likely to differ across industries and technologies, policy maker should carefully look at the distinctive nature of technological sharing when deciding to ease protection rules intended to foster collaboration.

produce knowledge that do not need any help from the first inventor to be understood.

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9. Appendix

Proof of Lemma 1.

When both firms succeed in the first period, i.e., $\{s_1^i = s_1^j = 1\}$, the payoff function of firm i (6) is linear in x_1^i, and its slope depends on rival's effort x_1^j.

As a first step, I define the level of equilibrium rival's effort such that the slope of (6) vanishes,

$$\hat{x}_1 \equiv 2 \cdot \left(1 - \frac{c}{\Pi^m}\right)$$

By the linearity of the demand function, this is equivalent to

$$\hat{x}_1 = 2 \cdot \left(1 - \frac{4c}{\alpha^2}\right) \tag{25}$$

Note that $\hat{x}_1$ is also the level of rival's effort such that leader's expected reward is zero, irrespective of its own effort. Hence, if this threshold exhibits negative values, the optimal action of both firms is to choose zero investments. By restricting attention to $\alpha > 2\sqrt{c}$, however, this case is ruled out. At $t = 1$ an upper limit $\bar{x}_1$ is drawn. Thus, two alternative scenarios are possible. First, consider that $\bar{x}_1 < \hat{x}_1$ and so expected profits are positive for any rival's level of R&D. Thereby a unique equilibrium exists in which both agents exert the highest level of effort, i.e., $x_1^{i^*} = \bar{x}_1 \ \forall i$.

Consider, next, the opposite case in which $\bar{x}_1 \geq \hat{x}_1$. Here, multiple equilibria arise. There are two possible asymmetric equilibria: either $x_1^{i^*} = \overline{x}_1$ and $y = 0$ or vice-versa. However there is also a unique symmetrical equilibrium in which effort is $x_1^{i^*} = \hat{x}_1, \forall i$. In this equilibrium, both firms earn zero expected profits. **Q.E.D.**

Proof of Lemma 2.

Recall first that firms hold some priors about $\overline{x}_1 \in (0, 1)$. Thus, according to the equilibrium payoffs described by lemma 1, expected payoffs at the next stage must be averaged over all possible upper-limit values. Thus,

$$EV^{1,0} = \int_0^1 \bar{x}(\Pi^m - c)f(\bar{x})d\bar{x} \tag{26}$$

and

$$\begin{aligned} EV^{1,1} \;&=\; \int_0^{\hat{x}_1} \left(\bar{x}\cdot(2-\bar{x})\frac{\Pi^m}{2} - c\cdot\bar{x}\right) d\bar{x} + \\ &\quad + \int_{\hat{x}_1}^1 \left(\hat{x}_1\cdot(2-\hat{x}_1)\frac{\Pi^m}{2} - c\cdot\hat{x}_1\right) d\bar{x} \end{aligned} \tag{27}$$

As before, I can define a level of rival's R&D such that the slope of (8) vanishes

$$\hat{x}_0 \equiv \frac{EV^{1,0} - c}{EV^{1,0} - EV^{1,1}} \quad . \tag{28}$$

The rest of the proof is analogous to the previous lemma. **Q.E.D.**

Proof of Lemma 3.

In a weak patent regime when firms have symmetric technological knowledge the equilibrium R&D investments depend again on a cut-off function

$$\hat{x}^{w_1} \equiv 1 - \frac{8c}{(1-\delta)\cdot\alpha^2} \ . \tag{29}$$

And in an analogous manner to the analysis conducted before, we have the reported solutions. **Q.E.D.**

Lemma 5. *Let's denote by x^* the leader's equilibrium effort and by y^* that of the follower. In the second period, if only one firm has succeeded in the* *initial invention and the follower has chosen not to cooperate, then*

(i) if $\frac{\Pi^d - c}{\beta \Pi^d} > \bar{x} > \frac{\beta \Pi^d - c}{\beta \Pi^d}$ *there is a unique equilibrium where* $x^* = \bar{x}$ *and* $y^* = 0$*,*

(ii) if $\bar{x} < \frac{\beta \Pi^d - c}{\beta \Pi^d}$ *there is a unique equilibrium* $x^* = y^* = \bar{x}$*,*

(iii) if $\bar{x} > \frac{\Pi^d - c}{\beta \Pi^d}$ *there is no symmetric equilibria and the unique "most cooperative" equilibrium, i.e., each firm exerts positive effort, is such that* $x^* = \frac{\beta \Pi^d - c}{\beta \Pi^d}$ *and* $y^* = \frac{\Pi^d - c}{\beta \Pi^d}$*.*

Proof of Lemma 5.

Suppose that firms have asymmetric knowledge, e.g., $\{s_1^i > s_1^j\}$;

As a first step, I define the level of equilibrium leader's effort such that the follower's slope vanishes, and vice versa:

$$\hat{x}_1' \equiv \left(\beta - \frac{c}{\beta \Pi^m} \right) \tag{30}$$

$$\hat{y}_1' \equiv \left(1 - \frac{c}{\Pi^m} \right) \frac{1}{\beta} \tag{31}$$

Note that $\hat{y}_1' > \hat{x}_1'$ for $\beta < 1$. The main difference with the analysis conducted before, is that now there are no symmetric equilibria. Hence, we must defined a new selection criterion and we use the definition of "most cooperative" equilibria. Specifically, firms are plying a "most cooperative equilibria" if both are exerting a strictly positive level of effort; Thus, when there are multiple equilibria we will select the "most cooperative"; Finally, we proceed as we did previously to obtain the reported solutions. **Q.E.D.**

Proof of Lemma 4.

At the second period, firms are always symmetric. After averaging for all possible upper limit's realizations, I obtain the following expected payoff function

$$EV^{1,1} = \int_0^{\hat{x}^{w_1}} \left[1-(1-\bar{x})^2\right]\Pi^d - c\cdot\bar{x}\; d\bar{x} + \int_{\hat{x}^{w_1}}^{1} (\Pi^d - c)\; d\bar{x} \tag{32}$$

$$EV^{1,0} = \int_0^{\hat{x}^{w_1}} \left[\bar{x}(\Pi^d - c)\right]\; d\bar{x} + \int_{x^{w_1}}^{1} (\Pi^d - c)\; d\bar{x} \tag{33}$$

$$EV^{0,1} = \int_0^{\hat{x}^{w_1}} (\Pi^d - c)\; d\bar{x} + \int_{\hat{x}^{w_1}}^{\frac{\hat{x}^{w_1}}{\beta}} \bar{x}\Pi^d\; d\bar{x} + \int_{\frac{\hat{x}^{w_1}}{\beta}}^{1} \bar{x}\Pi^d\; d\bar{x} \tag{34}$$

Therefore it can be defined a new threshold $\hat{x}^{w_0}$ and so, the remaining parts of the proof are analogous to earlier lemmas. **Q.E.D.**.

Proof of Proposition 3.

In a strong regime, I suppose that followers are able to offer the type of contract described in the text. Therefore their payoff function becomes

$$U_1^f = x_1^f(1-y^l)\Pi^m - c\cdot x_1^f\ . \tag{35}$$

However, given the structure of the contract, leaders are not going to reduce their equilibrium R&D effort and so, $x_1^{l^*} = \bar{x}_1$. This implies that a follower will exert positive effort if $(1-\bar{x}^l)\Pi^m \geq c$, therefore if $\bar{x} < 1-c/\Pi^m$, otherwise it sets R&D effort to zero. This possibility turns into a the following cut-off at the first period, and so the expected probability to innovate μ^{sl} is computed and can be contrasted with the previous result. **Q.E.D.**

10. Appendix B

As a first step consider the product market as a monopoly. If all steps of innovation are accomplished, and assuming null marginal cost in production, the equilibrium quantity produced is $q^m = \frac{\alpha}{2} = p^m$. Thus, $\Pi^m = \frac{\alpha^2}{4}$. And so it is a matter of simple algebra to compute the social surplus in this case $S^m = \frac{\alpha^2}{8}$.

Then, suppose that the product market is a duopoly. Now, the corresponding equilibrium quantity is arguably greater than that under monopoly: $q^d = (1+\gamma)q^m$ with $\gamma \geq 0$. Therefore, $p^d = \alpha - \frac{\alpha}{2}(1+\gamma)$.

Hence, by the definition of duopoly profits, i.e., $\Pi^d \equiv (1-\delta)\Pi^m/2$, we obtain $\gamma = (1-\delta)^{1/2}$. Finally, by substituting the resulting γ into the function of social surplus under duopoly, this turns out to be equation (22).