Races or Tournaments?

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Outline

Contests and economic growth

Historically, awards offered by government

- navigation and cartography (Longitude awards in 1714)
- agricoltural innovation (Royal Agricultural Society awards 1900's)
- aviation industry (Orteig prize in 1919)
- architecture (Thomas Jefferson organized the first US design contest to build the White House in 1790)

Today, contests are common management tool

- Incentives for workers
- Philanthropic initiatives
- Crowdsourcing internal activities to online communities of freelancers

The problem of contest design

How to design a contest? Contest designers need to deal with:

- “Incentive” design problem
  - What is the optimal prize structure?
- “Competition” design problem
  - a “race” competition? or in a “tournament” competition?

Outline

Examples by competition design

- Races
  - Longitude prize (1714 and 2014)
  - Orteig Prize
  - Netflix prize
  - Ansari's X-prize
- Tournaments
  - White House design contest
  - Golden Carrot Contest
  - DARPA Grand Challenges
  - X-Prize Challenges
  - European Commission's Horizon Challenge

Race or tournaments?

- Contest designers face trade-off between "speed" and "quality"
  - Example: seeking solutions to public health care problems (effective and timely)
  - EC's Horizon Challenge (race) and 2014 Longitude Prize (tournament) about antibiotic resistance
- Efficiency concerns
  - Fullerton & MacAfee (1992)'s result that only 2 competitors are optimal
  - Prevent entry:
    - **Minimum quality requirements**
    - **Time deadlines**
  - Example: online platforms running many contests

Prior literature

Theory:

Patent races (e.g., Loury 1979, Nalebuff & Stiglitz 1983); contests (e.g., Lazear & Rosen 1981, Green Stokey 1983, Dixit 1987); debate on prizes or patents for R&D (Wright 1983); strategic equivalence (Baye Hoppe 2003)

Empirical works:

Non-experimental data on contests (e.g., Ehrenberg Bognanno 1990, Knoeber Thurman 1994, Eriksson 1999) and on races (Cockburn Henderson 1994); Laboratory experiments on contests and on races (see Dechenaux et al. 2014).

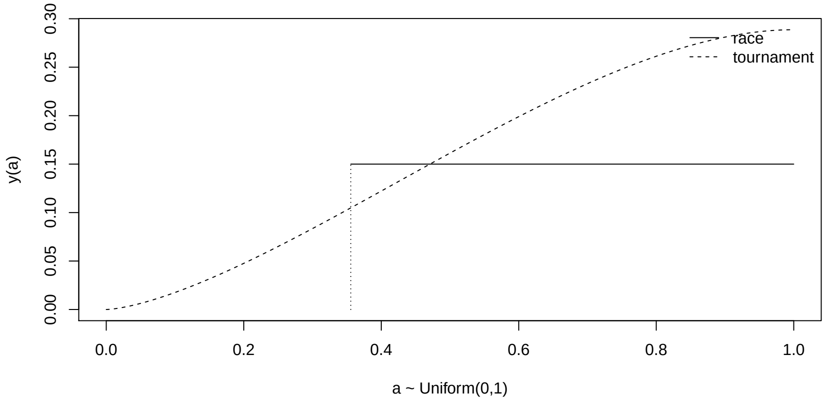
In this paper

- We develop a model to examine contest designer's choice between races and tournaments in one framework
  - The model extends Moldovanu and Sela (2001)'s static "all-pay" contest model
- We design and conduct a field experiment to examine predictions of the model
  - An online community of highly skilled coders with experience in programming competitions

Model's basic setup

- "All-pay" contest with deadline t_0 and target y_0
- Cost: $C(a, q, t) = a^{\alpha} y^{\beta} t^{\gamma} \qquad$ with $\alpha, \gamma \leq -1, \beta \geq 1$
- Payoff: $\pi_i = \sum_{k=1}^{q} p_k(y_i, t_i) v_k - C(a_i, y_i, t_i)$
- Equilibrium in a race: $t^* = b_{\text{race}}(a)$ and $y^* = y_0$
- Equilibrium in a tournament: $y^* = b_{\text{tournament}}(a)$ and $t^* = t_0$

Equilibrium performance



Behavioral predictions

1. Competitors will enter a Tournament more than a Race
   - Entry in a tournament is driven by "low-ability" competitors
2. On speed, Race $\succ$ ("dominates") Tournament
   - No clear dominance on performance
3. On performance, Tournament "with Reserve" $\succ$ Tournament and Race

Contest designer's problem

- Maximize expected revenues
- Revenues increase in performance, decrease in time of the winner (y^w, t^w)
- Consider t_0 and y_0 as given
- Expected payoff: $\pi_{cd} = E[y^w - \tau t^w \mid y^w \geq y_0, t^w \leq t_0]$
- Main result: if $\tau \geq \hat{\tau}$, the race should be preferred

Outline

Basic setup

Imagine $i = 1, ..., n$ players competing for $k = 1, ..., q$ prizes of value $v_1 \geq v_2 \geq ... \geq v_q$ (normalized $\sum v_k = 1$).

Players simultaneously choose quality y_i and time t_i (y_i/t_i speed).

Each player has an ability a_i drawn at random from a common cdf $F(\cdot)$ with pdf $f(\cdot)$.

The cost function $C(\cdot)$ is Cobb-Douglas:

$$C(a, q, t) = a^{\alpha} y^{\beta} t^{\gamma} \qquad \text{with } \alpha, \gamma \leq -1, \beta \geq 1 \tag{1}$$

- or denoting speed (y/t) by s:

$$C(a, q, t) = a^{\alpha} y^{\beta'} s^{\gamma'} \qquad \beta' = \beta + \gamma, \gamma' = -\gamma. \tag{2}$$

Payoffs

Player i's payoffs:

$$\pi_i = \sum_{k=1}^{q} p_k(y_i, t_i) v_k - C(a_i, y_i, t_i) \tag{3}$$

where $p_k(\cdot)$ is the prob. of winning prize k

Competition

- Let denote a deadline by t_0 and a minimum-quality target by y_0.
- We consider two competitive formats:
  - Race: competition with target where the first to achieve the target wins
  - Tournament: competition with deadline where the best wins

Probability

Let $y_{1:n}, ..., y_{n:n}$ denote the order statistics of the y's. Let denote the corresponding distribution functions by $F_{y_{1:n}}, ..., F_{y_{n:n}}$.

Then the conditional probability of winning the first prize in a tournament is

$$\Pr(y_i \geq y_{n-1:n-1}) = F_{y_{n-1:n-1}}(y_i) = F(y_i)^{n-1}$$

when $t_i \leq t_0$. And is zero otherwise.

$$\Pr(y_i xxxx) = [1 - F(y_i)]F(y_i)^{n-2}$$

Probability 2

If $a \sim \text{Uniform}(0,1)$, then:

$$p_1(y) = y^{n-1}, \qquad p_2(y) = [1-y]y^{n-2}$$

$$p_1(y)' = (n-1)y^{n-2}$$

$$p_2(y)' = -y^{n-2} + (1-y)(n-2)y^{n-3} = y^{n-3}[(1-y)(n-1) - 1]$$

Contest designer's payoff

Contest designer is risk neutral and wants to max quality while min time of the winner.

Z is the competition format. Let denote the race by $Z = 1$ and the tournament by $Z = 0$. Let denote the winner's actions by (y^w, t^w).

The contest designer's expected payoff:

$$\pi_{cd} = E[y^w - \tau t^w \mid y^w \geq y_0, t^w \leq t_0, R]. \tag{4}$$

Solution concept

We solve the model for its unique symmetric Perfect Bayes Nash Equilibrium (the "equilibrium").

Let denote equilibrium bidding functions with respect to ability by $t(\cdot)$ and $y(\cdot)$.

Consider Tournament first.

Maximization problem

- Key observation: $t_i = t_0$ is a (weakly) dominant strategy
- This simplifies the maximization problem to:

$$\max_y \hat{\pi} = \sum_{k=1}^{q} p_k(y)\hat{v}_k - a_i^\alpha y^\beta \tag{5}$$

with $\hat{v}_k$ denoting each prize v_k rescaled by a factor t_0^γ.

First order condition

For each $i = 1, ..., n$, first order conditions are:

$$\sum_{k=1}^{q} p'_k(y)\hat{v}_k = a_i^{\alpha}\beta y^{\beta-1} \tag{6}$$

Solving differential equation

Substituting the equilibrium function $y(\cdot)$ increasing in a_i and with inverse $\phi(\cdot)$, together with a "change of variable" (moving $a_i = \phi(y_i)$ to the lhs):

$$\phi^{-\alpha} \sum_{k=1}^{q} \hat{p}_k'(\phi)\phi' v_k = t_0^{\gamma} \beta y(a)^{\beta-1} \tag{7}$$

Integrating both sides (using the "chain of derivatives" on the lhs):

$$\sum_{k=1}^{q} \hat{v}_k \int_{a_0}^{a} p_k'(x) x^{\alpha} dx + \beta y(a_0)^{\beta-1} = \beta y(a)^{\beta-1} \tag{8}$$

Bidding function

For every $i = 1, ..., n$:

- Time $t(a) = t_0$
- Equilibrium quality y_i for competitor with ability a is given by:

$$y(a) = \left[y(a_0)^{\beta-1} + \frac{1}{\beta} \sum_{k=1}^{q} \hat{v}_k \int_{a_0}^{a} p_k'(x) x^{\alpha} dx \right]^{1/(\beta-1)} \tag{9}$$

with boundary condition $y(a_0) = 0$.

Example

If $a \sim \text{Uniform}(0,1)$ and $q = 2$

First integral:

$$v_1(n-1)\int_0^a x^{(n-2)-\alpha}dx = v_1 \frac{a^{(n-1)-\alpha}}{(n-1)-\alpha}$$

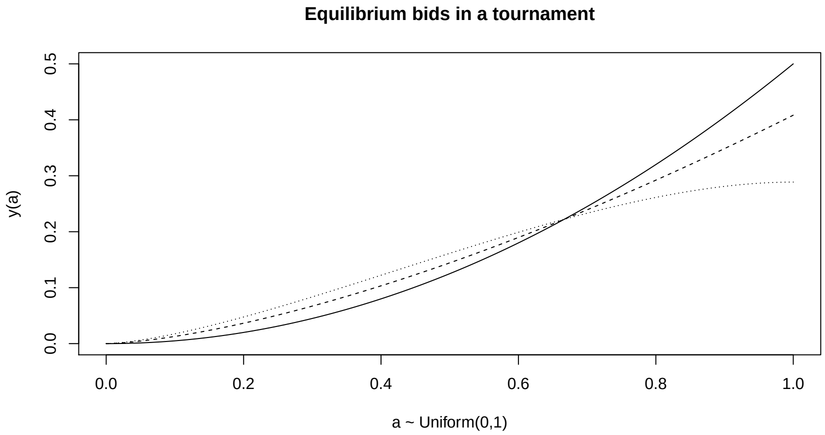
Second integral:

$$v_2 \int_0^a x^{(n-3)-\alpha}[(1-x)(n-1)-1]dx$$

$$= v_2 \frac{a^{-\alpha+n-2}((n-2)(-\alpha+n-1)-a(n-1)(-\alpha+n-2))}{(-\alpha+n-2)(-\alpha+n-1)}$$

Impose normalization: $t_0^{\gamma} = 1$.

Example 2



Equilibrium bids in a tournament
y(a)
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
a ~ Uniform(0,1)

Bidding function in a race

For every $i = 1, ..., n$ with $a_i \geq \hat{a}$

- Quality $y(a) = y_0$
- Time:

$$t(a) = \left[t(t_0)^{\gamma-1} + \frac{1}{\gamma} \sum_{k=1}^{q} \tilde{v}_k \int_{a_0}^{a} \hat{p}'_k(x) x^{\alpha} dx \right]^{1/(\gamma-1)} \tag{10}$$

with $\tilde{v}_k = v_k / y_0^{\beta}$.

Otherwise, when $a_i < \hat{a}$, and $y(a) < y_0$.

Zero profit

The zero profit condition for the marginal player:

$$\sum p_k(y_0, t_0)v_k = \hat{a}^{\alpha} y_0^{\beta} t_0^{\gamma} \tag{11}$$

Hence, the marginal ability is pinned down:

$$\hat{a} = \left[\sum p_k(y_0, t_0)v_k / y_0^{\beta} t_0^{\gamma}\right]^{1/\alpha} \tag{12}$$

Example 2

If $a \sim \text{Uniform}(0,1)$, then ZPC

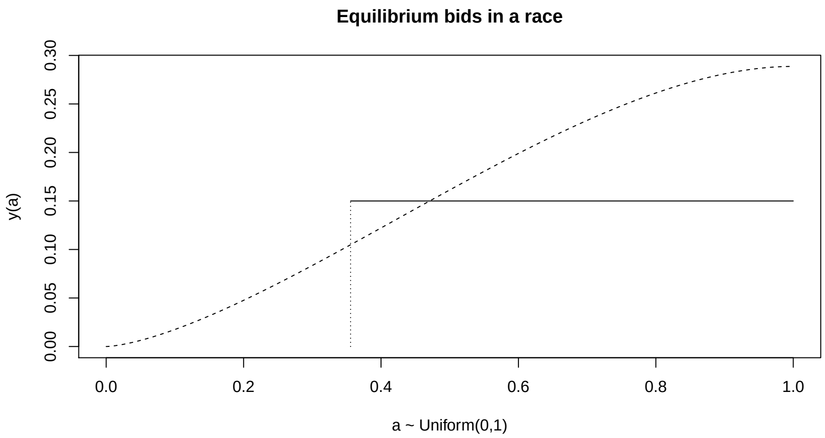
$$p_1(y) = y^{n-1}, \qquad p_2(y) = [1-y]y^{n-2}$$

$(t_0^\gamma = 1)$

$$\pi_i = v_1 p_1(a) + v_2 p_2(a) - a^\alpha y_0^\beta$$

$$(ZPC) \qquad v_1 a^{n-1} + (1-v_1)[1-a]a^{n-2} - a^\alpha y_0^\beta = 0$$

Example



Equilibrium bids in a race
y(a)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.2
0.4
0.6
0.8
1.0
a ~ Uniform(0,1)

Outline

The context: Topcoder.com

Recruit 229 competitors on Topcoder for eight-day programming competition

Three key factors:

- Platform members are sophisticated competitors
- Observe measures of skills
- Rich data analytics about performance and timing

The contest

- Total prize purse $41000
  - Grand prizes of $6000 across competition styles
  - Room prizes of $1000 and $100 for 1st and 2nd
- Task solving Named Entity Recognition Problem in medical research

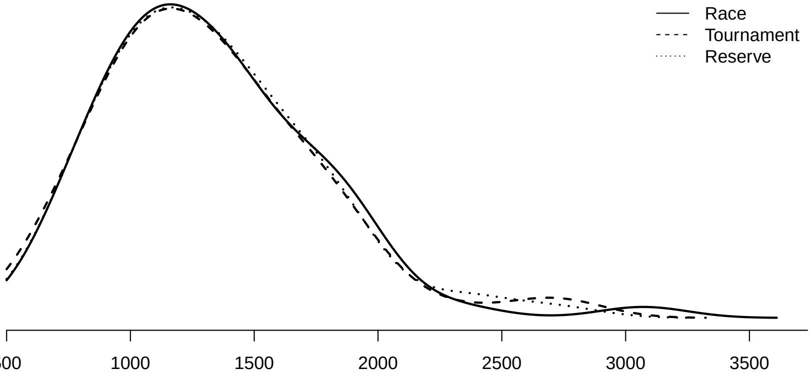
Experimental design

- 8 day submission phase
- Split into 24 rooms of 10 and 15 competitors
- 3x2 experimental design (Race, Tournament, Reserve) x (10, 15)

Data

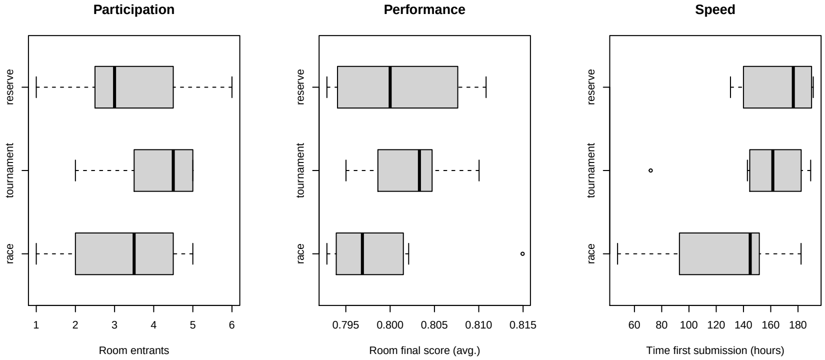
	Mean	Median	St.Dev.	Min	Max	Obs.	P-value
year	2009.9	2010	4	2001	2015	299	0.596
rating	1322.4	1278	425	593	3071	205	0.989
registrations	17.6	9	23	1	161	299	0.626
submissions	7.2	2	12	0	91	299	0.867
lpaid	8.4	8	3	3	14	139	0.791
nwins	0.3	0	2	0	27	299	0.370
ntop10	1.6	0	5	0	64	299	0.273
risk	6.4	7	2	1	10	279	0.958
hours	31.3	24	25	0	192	277	0.995
male	1.0	1	0	0	1	276	0.404
timezone	2.1	2	5	-8	10	277	0.389
grad	0.5	0	1	0	1	278	0.208
below30	0.7	1	0	0	1	278	0.503

Skill rating distribution

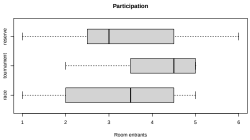


Outline

Distribution of room outcomes

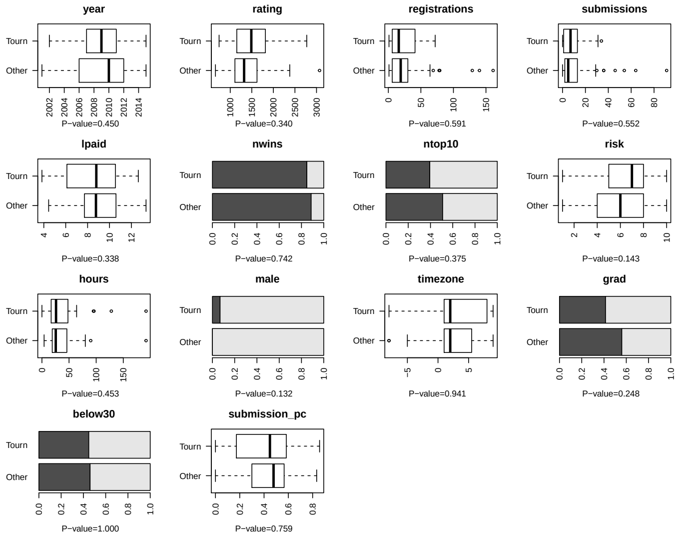


Greater participation in the Tournament




```
##
##  Welch Two Sample t-test
##
## data:  submit by treatment != "tournament"
## t = 1.4889, df = 18.166, p-value = 0.07684
## alternative hypothesis: true difference in means is
## 95 percent confidence interval:
##  -0.1333372        Inf
## sample estimates:
## mean in group FALSE  mean in group TRUE
##              4.1250              3.3125
```

No evidence of skill-based selection



Deal with noise in performance

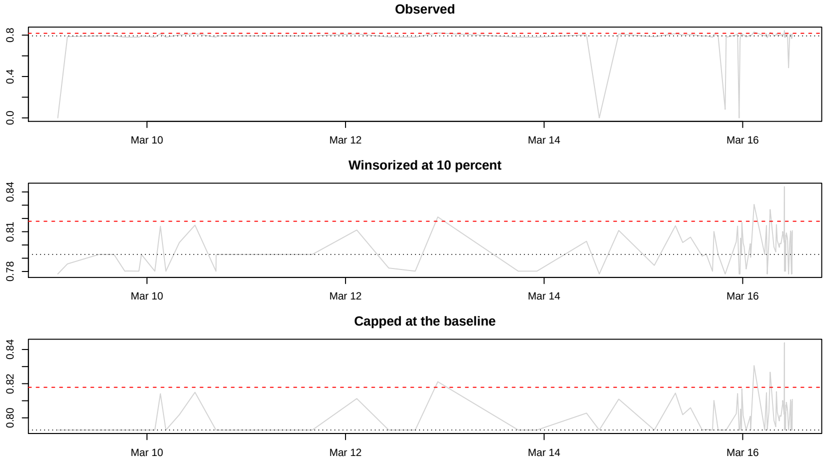
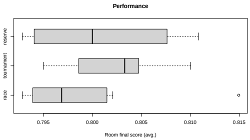


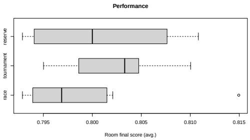
Figure: Scores over time

No evidence of higher performance in Reserve



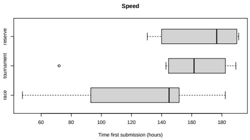

```
##
##  Welch Two Sample t-test
##
## data:  final.cap by treatment == "reserve"
## t = -0.072322, df = 12.406, p-value = 0.9435
## alternative hypothesis: true difference in means is
## 95 percent confidence interval:
##  -0.006620624  0.006193726
## sample estimates:
## mean in group FALSE  mean in group TRUE
##           0.8006715           0.8008850
```

No evidence of difference in performance between Tournament and Race




```
##
##  Welch Two Sample t-test
##
## data:  final.cap by treatment
## t = -1.0592, df = 12.08, p-value = 0.3102
## alternative hypothesis: true difference in means is
## 95 percent confidence interval:
##  -0.009927344  0.003429224
## sample estimates:
##       mean in group race mean in group tournament
##                0.7990470                0.8022961
```

Speed was higher in the Race




```
##
##  Welch Two Sample t-test
##
## data:  firstsub by treatment == "race"
## t = 1.958, df = 10.805, p-value = 0.03827
## alternative hypothesis: true difference in means is
## 95 percent confidence interval:
##  2.818276      Inf
## sample estimates:
## mean in group FALSE  mean in group TRUE
##            160.7990            126.1204
```

To summarize

1. Participation was higher in the Tournament
   - It was not driven by low-skill competitors
2. No evidence of a difference in performance
3. Competitors worked faster in a race

Interpretation:

- Taken together, though competitors seemed to “like” tournaments more, they “worked” more in a race.
- Tournament with reserve does not seem to be better than tournament

Outline

Modeling individual behavior

Identification of causal effect of competition on individual behavior is problematic

1. Actions are correlated, violating one key assumption of Rubin's potential outcomes causality model
2. Censoring (entry/exit decisions are only partially observed)
3. Dynamics

Under our model, however, we have:

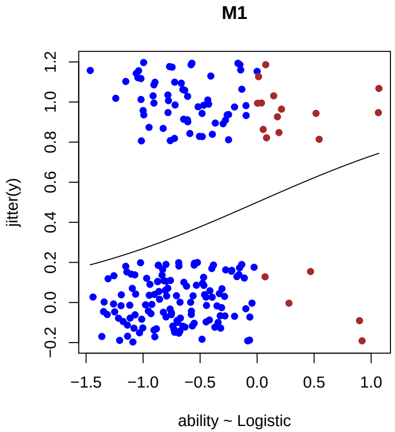
$$y = 1 \iff \text{ability} > a_0$$

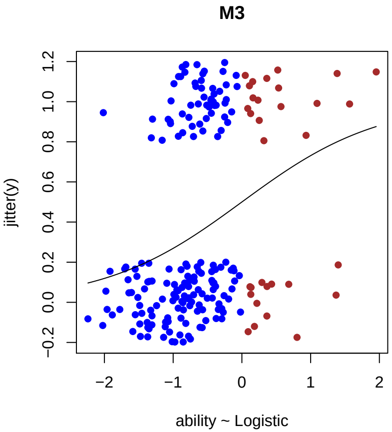
$\leadsto$ "single-index" models have nice structural interpretation.

Entry decision

	(1)	(2)	(3)	(4)
TREATMENTTOURNAMENT	0.32	0.32	0.30	0.30
	(0.31)	(0.37)	(0.37)	(0.38)
TREATMENTRESERVE	0.04	0.20	0.20	0.29
	(0.32)	(0.37)	(0.37)	(0.38)
RATING.100		0.10^{***}	0.11^{***}	0.12^{***}
		(0.04)	(0.04)	(0.04)
HOURS.IMP			0.02^{**}	0.02^{**}
			(0.01)	(0.01)
TIMEZONE.IMP				−0.01
				(0.03)
GRAD.IMP				−0.32
				(0.32)
BELOW30.IMP				-0.63^{*}
				(0.32)
MALE.IMP				−0.39
				(0.98)
RISK.IMP				0.01
				(0.07)
CONSTANT	-1.03^{***}	-2.15^{***}	-2.74^{***}	−1.98
	(0.23)	(0.56)	(0.63)	(1.29)
Observations	299	205	205	205
Log likelihood	−178.75	−129.19	−126.50	−124.21
Akaike information criterion	363.49	266.38	263.00	268.43
Notes:	$^{***}p < .01$; $^{**}p < .05$; $^{*}p < .1$			

Model's fit is ok

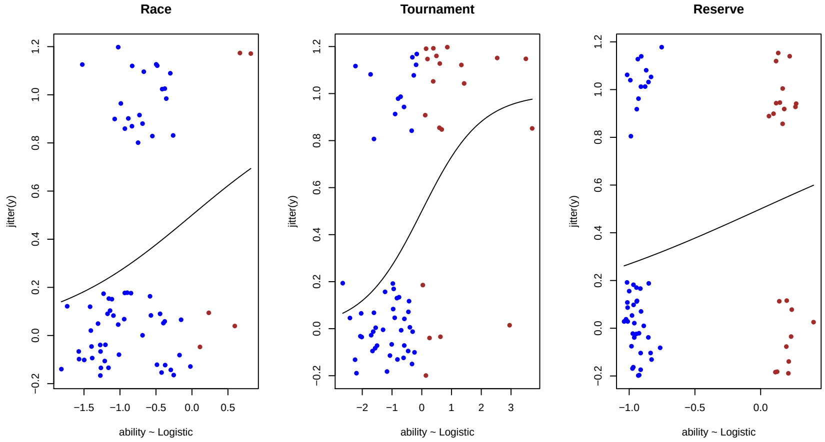




Entry decision across competition styles

	All	Race	Tourn	Rese
RATING.100	0.11^{***}	0.11^{*}	0.21^{***}	0.02
	(0.04)	(0.07)	(0.08)	(0.06)
HOURS.IMP	0.02^{**}	0.02	0.03^{**}	0.0002
	(0.01)	(0.01)	(0.01)	(0.01)
BELOW30.IMP	-0.54^{*}	-0.04	-0.63	-1.10^{**}
	(0.31)	(0.55)	(0.57)	(0.55)
CONSTANT	-2.22^{***}	-2.67^{**}	-3.92^{***}	-0.05
	(0.62)	(1.15)	(1.23)	(1.11)
Observations	205	68	69	68
Log likelihood	-125.30	-40.57	-37.87	-42.51
Akaike information criterion	258.60	89.14	83.75	93.02
Notes:	$^{***}p < .01$; $^{**}p < .05$; $^{*}p < .1$			

Model's fit


Race
Tournament
Reserve
jitter(y)
ability ~ Logistic
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−1.5
−1.0
−0.5
0.0
0.5
−2
−1
0
1
2
3

Incorporating Scores

Individual scores are censored $\rightsquigarrow$ OLS is problematic.

We examine "production speed" y_i ($= \text{score}_i / t_i$) at a given point in time.

Then, our data's likelihood is (e.g., Tobit):

$$\mathcal{L} = \prod_{i=1}^{N} \Pr(Y \geq 0)^{1-I(y_i)} \times \Pr(Y = y_i)^{I(y_i)}.$$

Under the model's equilibrium, this becomes

$$\mathcal{L} = \prod_{i=1}^{N} [1 - F(a_{0,i})]^{1-I(y_i)} \times f(b(\text{ability}_i) = y_i)^{I(y_i)}.$$

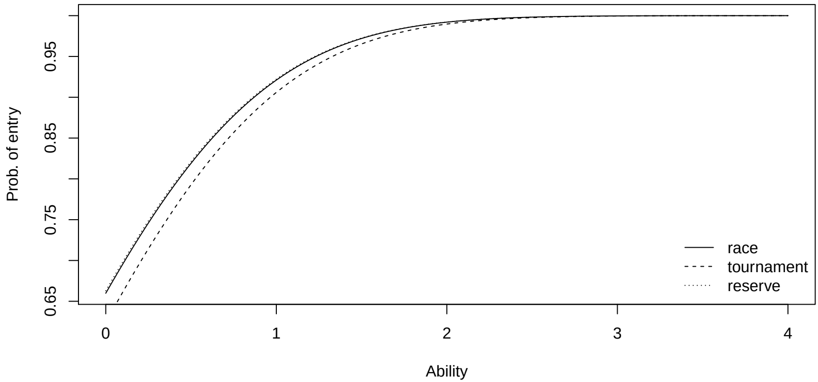
Estimation

- We use parametric F known up to a vector θ, that we estimate from the data.
- Compare against Tobit model (our main benchmark)
- Alternatively, replace F with skill rating's $\hat{F}$ (our second benchmark)

Benchmark

	Tobit		*normal*	
	(1)	(2)	(3)	(4)
TREATMENTTOURNAMENT	0.10	0.02	0.03	0.02
	(0.16)	(0.16)	(0.05)	(0.06)
TREATMENTRESERVE	-0.01	0.07	-0.002	0.02
	(0.16)	(0.16)	(0.05)	(0.06)
RATING.100		0.05^{***}		0.02^{***}
		(0.02)		(0.01)
HOURS.IMP		0.01^{***}		0.003^{**}
		(0.003)		(0.001)
CONSTANT	-0.41^{***}	-1.06^{***}	0.21^{***}	-0.11
	(0.14)	(0.28)	(0.04)	(0.10)
Observations	299	205	299	205
Log likelihood	-212.65	-152.89	-114.80	-82.37
Notes:	***p $<$ .01; **p $<$ .05; *p $<$ .1			

Estimated probability of entry



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References I