Digging for Golden Carrots: An Analysis of Research Tournaments

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Contracting for research is often infeasible because research inputs are unobservable and research outcomes cannot be verified by a court. Sponsoring a research tournament can resolve these problems. A model is presented in which contestants compete to find the innovation of highest value to the tournament sponsor. The winner receives a prespecified prize. The tournament game has a unique subgame-perfect equilibrium. Free entry is not optimal because equilibrium effort by each researcher decreases in the number of contestants. An optimally designed research tournament balances the probability of overshooting the first-best quality level against the probability of falling short. (*JEL* O32)

> "Electric utilities are turning up the heat in search of a better refrigerator by offering a 30 million dollar reward to the first company to build it. ... The money would go the developer of a refrigerator that would use 25 to 50 percent less electricity and no ... chlorofluorocarbons. ... The utilities asked for bids from manufacturers due in October and promised to pick two semifinalists in December who would build prototypes by next June. The winner would be paid the 30 million ... ."
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> (*New York Times*, 8 July 1992, p. A1)

This article studies incentive mechanisms such as the "golden carrot" research tournament described in the opening excerpt.[1] This type of procurement contest has often been used to induce research on specific projects. For instance, prior to World War II, the U.S. Army Air Corps regularly sponsored prototype tournaments at Dayton to determine which of several competing manufacturers would be awarded a production contract (see Benjamin S. Kelsey, 1982). More recently, William P. Rogerson (1989) performed econometric investigation on 12 major aerospace research contests held by the Department of Defense between 1964 and 1977. He used stock-market data on the contestant firms to estimate the size of the "prize" implicit in each production contract. The average award was estimated to be worth between 10.2 percent and 14.6 percent of the market value of an average contestant firm. The Federal Communication Commission (FCC) recently held a tournament to decide on the American broadcast standard for high-definition television (see *High-Tech Business*, 1989; *The Economist*, 1990; *Broadcasting*, 1993). "The contest is [was] open to anyone with a $200,000 entry fee" (*The Economist*, 1990 p. 58).

While governments are probably the most common sponsors of research tournaments, private corporations and even individuals also conduct high-tech contests of this sort. For instance, Dow and IBM both sponsor annual tournaments in which the winning contestants receive grants to develop their projects for commercial use.[2] In 1959, British industrialist Henry Kremer person-

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[1] The participants in the refrigerator competition christened it the "Golden Carrot Contest."

[2] Dow's Cooperative Research Program is administered through its Corporate R&D Division, and awards renewable annual grants of $30,000. IBM's Cooperative University Research Program awards renewable annual grants of $50,000.

ally sponsored the famous man-powered aircraft contests eventually won by the Gossamer Condor (first to fly a one-mile figure-eight) in 1977 and the Gossamer Albatross (first to cross the English Channel) in 1979.[3] Finally, most competitively awarded academic research grants can also be viewed as research tournaments.

Research tournaments are typically adopted in situations when conventional procurement mechanisms such as competitive spot-markets or bilateral contracts can be expected to perform badly. Generally, the tournament sponsor is a monopsonist that must devise a credible method for committing itself not to exploit its market-power.[4] Lacking such a commitment, researchers, fearing *ex post* rent expropriation, will be reluctant to sink resources into the monopsonist's project.[5] Of course, the most common method for committing to pay for delivery of goods or services is to sign a contract. However, as noted by David C. Mowery and Nathan Rosenberg (1989 p. 82),

> The effectiveness of contracts in the provision of research is undermined by the highly uncertain nature of the research enterprise, the imperfect character of knowledge about a given project,...the difficulty of specifying all contingencies and the uncertainty associated with complex research projects....

Since research inputs are notoriously difficult to monitor, contracts that reimburse researchers for costs are often all but infeasible.[6] On the other hand, since contract courts seldom possess the ability or expertise necessary to evaluate technical research projects, contracts that base compensation on research output are usually impractical as well.[7] A research tournament is a type of incomplete contract designed to overcome precisely these problems.[8] The prospect of winning the tournament provides contestants with incentives to exert research effort. Also, the only role for the courts is to ensure that the specified "prize" (be it production contract, development grant, cash, or other) is awarded to one of the contestants at the conclusion of the contest. It is not even necessary that a court be able to verify the rank order of the final entries, since the sponsor typically has little incentive other than to exchange the prize for the innovation it values most.[9] By sponsoring a research tournament, a monopsonist can credibly commit itself to reward research effort under circumstances in which conventional contracts may not be feasible.[10]

It should be pointed out how a research tournament differs from a standard innovation race, both as an institution and in terms of modeling. The main operational difference is that a tournament rewards the indi-

[3] The Kremer competitions were actually "innovation races" rather than "research tournaments" as is discussed in what follows. See Don Dwiggins (1976) and Morton Grosser (1981).

[4] There is an asymmetry between the situations of selling an object of unverifiable quality to a monopsonist and buying an object of unverifiable quality from a monopolist. The fact that quality cannot be verified creates no difficulty in the case of buying from a monopolist. For more on this, see Alejandro M. Manelli and Daniel R. Vincent (1993).

[5] According to Frederic M. Scherer (1980 p. 458) the Atomic Energy Act of 1946 provided no mechanism for committing the government to pay unbiased compensation for breakthroughs in military uses of atomic energy that under the law could not be patented. He cites two prominent cases: Fermi's breakthrough in the production of radioactive isotopes and Goddard's breakthrough in liquid rocket engines, for which the government made grossly inadequate cash awards *ex post*.

[6] Seminal works on contracting when inputs are unobservable include James A. Mirrlees (1975) and Bengt Holmstrom (1979).

[7] On the verifiability of research outcomes Scherer (1980 p. 458) states, "...estimating the value of inventive contributions is a difficult task, and any bureaucrat council entrusted with the job is bound to make mistakes and perpetrate inequities."

[8] In this respect, a tournament functions much like the internal promotion procedures studied by Lorne H. Carmichael (1983) and James M. Malcomson (1984) in the context of unobservable work effort and unverifiable job performance.

[9] This can be problematic if the sponsor appoints an agent to conduct the tournament on its behalf. The history of government-sponsored procurement tournaments is replete with allegations of corruption and favoritism (see Ronald J. Fox, 1974; Kelsey, 1982).

[10] Steven N. Wiggins (1990) considers several other methods, including costly monitoring and direct ownership, for coping with the moral-hazard and commitment problems mentioned here.

vidual with the best innovation on a specific date, while a race rewards the first individual to discover an innovation of a specific quality. Thus, in a research tournament, the terminal date is fixed, and the quality of innovations varies, while in an innovation race, the quality standard is fixed, and the date of discovery is variable.[11] For instance, the man-powered aircraft contests mentioned above were innovation races rather than research tournaments because Kremer promised to pay 50,000 pounds to the first contestant to fly a specific course. The procurement contests sponsored by the Army Air Corps, by contrast, were tournaments rather than races because prototypes competed head-to-head on a specific date, the winner being the plane that was judged to have performed "best."

In the traditional innovation racing models pioneered by Glen C. Loury (1979) and Tom Lee and Louis L. Wilde (1980), there is a single well-defined invention, whose value is not at issue. There is only a question of when and whether the invention is achieved. The analysis in these models, therefore, naturally focuses on researchers' rates of investment rather than on the ultimate quality of an innovation. The model of research tournaments offered here reverses this emphasis, stressing the eventual quality of the winning invention and suppressing issues involving per-period research intensities. For example, larger prizes in an innovation race reduce the expected amount of time to discovery, while they raise the expected quality of the winning invention in a research tournament.

The model of research tournaments presented in the next section draws upon the sequential-search/R&D literature introduced by Robert E. Evenson and Yoav Kislev (1975), the work on rank-order tournaments and internal labor markets originated by Edward P. Lazear and Sherwin Rosen (1981), and the "contest" models examined by Carl A. Futia (1980), John M. Hartwick (1982), and Rogerson (1982).[12]

Existence and uniqueness of equilibrium play in the tournament is established in Section II. The uniqueness result is somewhat unusual for this type of model, and it is very useful when analyzing the sponsor's tournament design problem in Section III. The instruments available to the sponsor are the number of contestants, an entry fee, and the prize. The ability to limit entry and extract *ex ante* rents with an entry fee allows the sponsor to avoid the "overgrazing" and rent-seeking problems common in models of decentralized innovation races.

Section IV discusses the efficiency of an optimally designed research tournament, relative to the first-best plan that would be adopted in the absence of informational barriers. Because the sponsor cannot commit to stop the contest once an acceptable innovation is discovered, there is a tendency for redundant research to be performed in a tournament. To attenuate this problem, an optimally designed tournament is calibrated so that each contestant exerts less than the efficient level of research effort in equilibrium. While this reduces the probability of redundancy, it introduces the possibility that too little research will ultimately be performed. The main results are summarized in Section V. To save space, straightforward proofs have been abbreviated or omitted. Four technical lemmas and most of the proof of Proposition 2 appear in the Technical Appendix.

I. The Model

Suppose there is a pool of identical risk-neutral research firms standing ready to participate in the following contest.[13] At date

[11] In a seminar, Andy Weiss made the clever observation that a research tournament is like 24 hours at Le Mans (try to go the farthest in the allotted time), while an innovation race is like the Indy 500 (try to be the first to go the allotted distance).

[12] Other seminal works on search theory as applied to R&D include Martin L Weitzman (1979), Kevin Roberts and Weitzman (1981), and Jennifer F. Reinganum (1982, 1983). Path-breaking articles on tournaments include: Jerry R. Green and Nancy L. Stokey (1983), Barry J. Nalebuff and Joseph E. Stiglitz (1983), and Andrei Shleifer (1985).

[13] The actual number of firms in the pool is not critical, provided there are at least two.

$t = 0$ the sponsor "invites" $N \geq 2$ firms to enter the tournament. Each of these firms then decides whether to pay an entry fee $E \geq 0$ in order to compete for a fixed monetary prize $P > 0$.[14] If fewer than two firms enter, the contest is "called off."[15] Otherwise, the prize is awarded on a specified date $T \geq 1$ to the contestant that has produced the innovation of highest value to the sponsor. The contestants are assumed to know the sponsor's money metric utility function over innovations.[16]

The production function for generating innovations is a sequential search process with recall.[17] The distribution of potential innovation values is denoted by F, defined on $[0, b]$ with $F(0) = 0$ and $b \leq \infty$. Also, F has a finite mean and a strictly positive density f everywhere on its domain. The cost (fiduciary plus opportunity) of conducting research in any period is denoted by $C > 0$. To avoid the possibility that contestants become capital-constrained, it is assumed that each firm starts with cash reserves of at least $TC + E$.

If a firm elects to conduct research in period t, it bears the cost C and obtains a single draw $\tilde{x}_{i,t}$.[18] Draws are *statistically independent across time and among firms*. Let $x_{i,t}$ represent the value of the best discovery by firm i by date t. That is, $x_{i,t} = \max\{x_{i,t-1}, \tilde{x}_{i,t-1}\}$ if firm i performed research in period $t-1$, and $x_{i,t} = x_{i,t-1}$ if it did not. A firm may conduct research at the end of each period $t = 0, \ldots, T-1$, and at date T, the firm with the best innovation exchanges it with the sponsor for the prize. If, at the end of the tournament, M firms are "tied" with the best innovation, then the "winner" is randomly selected from among them with equal probability. For simplicity it is assumed that all firms begin with a "worthless" innovation. Also, the scrap value of any innovation not winning the tournament is zero, and there is no discounting by the sponsor or any of the contestants.

The value of its innovations is assumed to be private information to each firm at every date. Furthermore, whether a firm is actively conducting research is unobservable. The sponsor can, however, costlessly and accurately evaluate the array of innovations finally submitted. The number of firms entering the tournament is assumed to be publicly observable. As discussed in the Introduction, the reason for conducting a tournament rather than using a conventional contract is that research effort is private information, and the value of an innovation is not verifiable by a court. A tournament resolves these problems because the act of exchanging the prize for an innovation can easily be verified and enforced by a court. Finally, all of the features of the model listed in this section are assumed to be common knowledge.

II. Equilibrium Play

The environment described in the preceding section gives rise to a dynamic N-player two-stage game of imperfect information. The strategy of each research firm is a sequence of state-dependent decisions. In the initial entry stage of the game, each invited firm decides whether to participate. Then, during the ensuing contest stage, each contestant decides whether to conduct research in each period $t = 0, \ldots, T-1$, given the number of its rivals and the value of its best innovation to date. Any firm choosing not to enter the tournament receives its reserva-

[14] If there were irretrievable setup costs associated with the sponsor's project, it might be necessary to subsidize entry, $E < 0$.

[15] A single entrant would have no incentive to do research, since it could win by remaining idle. Taylor (1993) investigates a common contractual method for inducing a single firm to do research in an environment of unobservable effort and unverifiable performance.

[16] The winner of the golden-carrot refrigerator tournament was determined by a designated formula based on energy efficiency and the number of units produced for retail sale, among other factors.

[17] Search processes are often used to model the production of R&D because they possess many desirable features: stochastic success, diminishing returns, geometrically distributed time to completion, and so forth. For an example, see Lester G. Telser (1982).

[18] The assumption that firms' research intensities are either zero or one is somewhat primitive. Nevertheless, firms are unlikely to be able to alter their scale of operation during the short-run time horizon of a research tournament. So, the zero–one assumption can be viewed as the limiting case of U-shaped short-run average costs. See Brian D. Wright (1983 p. 692) for a similar interpretation.

tion profit of zero. This section constructs the unique subgame-perfect Nash equilibrium (henceforth, just an equilibrium) of this game.

The equilibrium of a research tournament is especially easy to characterize when there is only a single period for conducting research. In this case, the strategies described above reduce to (first) deciding whether to enter and (second) deciding whether to conduct research in the ensuing one-shot subgame.

PROPOSITION 1: *If*

$$\frac{P}{N} - C - E > 0 \tag{1}$$

then the unique equilibrium of a one-shot research tournament is for each invited firm to enter and for each contestant to conduct research.

PROOF:

Suppose $M \in \{2, \ldots, N\}$ firms enter the contest. Then, each contestant has a dominant strategy to do research. To see this, imagine, first, that no contestant does research. Then, each will submit an innovation worth zero and will have expected payoff $P/M - E$. If any contestant does research, while all its rivals are idle, it wins with certainty and receives payoff $P - C - E$. However,

$$\frac{P}{N} - C - E > 0 \Rightarrow \frac{P}{M} > C + E$$

$$\Rightarrow P > \frac{M}{M-1}(C+E)$$

$$\Rightarrow P - C - E > \frac{P}{M} - E.$$

Next, suppose that $m \in \{2, \ldots, M\}$ contestants do research. Then, each of them has expected payoff $P/m - C - E$. If any one of these firms did not do research, its payoff would be $-E$. Thus, (1) insures that doing research is a dominant strategy for all firms entering the contest. Given this, it also insures that entry is a strict best response for each invited firm.

The intuition underlying this result is straightforward. If the prize is large enough so that each invited firm earns positive expected profit by entering and performing research, regardless of its rivals' actions, then the only equilibrium of the tournament involves full participation. If the inequality in (1) is reversed, then the prize will not support full participation, and either some invited firms will decline to enter or contestants will randomize over their research decisions (or both). Of course, a one-shot tournament is somewhat special due to the lack of dynamic considerations during the contest stage. Nevertheless, Proposition 2 below establishes that most of the intuition underlying Proposition 1 carries through in a multiperiod setting, once (1) is amended to account for dynamic research decisions.

Suppose that $M \in \{2, \ldots, N\}$ firms accept the sponsor's invitation to participate in a T-period tournament. Imagine that at some date $t < T$, one of the contestants (call it firm i) has in hand an innovation worth $x \geq 0$. The question is whether this firm should do any further research.

Let $\Pi_i(y)$ be the cumulative distribution function of the best innovation ultimately discovered by any of firm i's rivals. Then, if firm i stops doing research, its expected profit (gross of sunk costs) is $P\Pi_i(x)$, whereas the expected profit from performing one more round of research and then stopping is

$$P\Pi_i(x)F(x) + P\int_x^b \Pi_i(y)\,dF(y) - C.$$

Thus, the net expected profit of an additional round of research is

$$\Delta_i(x) = P\int_x^b [\Pi_i(y) - \Pi_i(x)]\,dF(y) - C. \tag{2}$$

The integral gives the expected increase in profit of discovering an innovation worth more than x. Otherwise, the firm discovers nothing better than x, and expends C in either case. Note that $\Delta_i(x)$ is nonincreasing since the cumulative distribution function $\Pi_i(x)$ is nondecreasing. A standard result from search theory shows that firm i's optimal strategy is to do research until an innovation worth at least z_i is discovered, where

$$(3) \qquad \Delta_i(z_i) = 0$$

(see Steven A. Lippman and John J. McCall, 1976a, b). This important type of research plan will be called a z_i-stop strategy. If $\Delta_i(0) < 0$, then firm i should do no research at all; that is, it should remain idle throughout the contest.

Consider an equilibrium of the research tournament in which contestant i employs a z_i-stop strategy. Then, it is clear that all other contestants $j \neq i$ must follow z_j-stop strategies, since an idle contestant loses the tournament and its entry fee with certainty. Therefore, in equilibrium, either all contestants employ z_i-stop strategies, or none of them does. Moreover, if no contestant uses a z_i-stop strategy, then it must be that $\Delta_i(0) < 0$ for all $i = 1, \dots, M$, which requires P to be sufficiently small. Thus, when the prize is "large enough," an equilibrium of the research subgame is simply a vector of stopping values $(z_1^*, \dots, z_M^*)$. Before characterizing the unique equilibrium vector, two definitions are needed.

Let $K(z_i)$ be the date-zero expected cost of employing a z_i-stop strategy, and note that it is given by

$$(4) \quad (z_i) \equiv C + CF(z_i) + CF^2(z_i) + \cdots + CF^{T-1}(z_i) = C\frac{1-F^T(z_i)}{1-F(z_i)} \qquad z_i \in [0, b].$$

Next, define the date-zero cumulative distribution function for the value of firm i's ultimate innovation under a z_i-stop strategy to be $\Phi(x; z_i, T)$. This distribution is given by

$$(5) \quad \Phi(x; z_i, T) \equiv \begin{cases} F^T(x) & x \le z_i \\ F^T(z_i) + \left[1 - F^T(z_i)\right]\dfrac{F(x) - F(z_i)}{1 - F(z_i)} & x > z_i. \end{cases}$$

If the firm does not obtain an $x > z_i$, it gets the best of T samples, possessing distribution $F^T(x)$. If it gets a draw exceeding z_i, it stops doing research. So, the cumulative distribution function is the conditional distribution of a single draw exceeding z_i, $[F(x) - F(z_i)]/[1 - F(z_i)]$, times the probability, $1 - F^T(z_i)$, of obtaining such a draw, plus the probability, $F^T(z_i)$, of obtaining no draw better than z_i. It is now possible to state the main result of this section.

PROPOSITION 2: *If*

$$(6) \qquad \frac{P}{N} - K(z(N,P)) - E > 0$$

then the unique equilibrium of the research tournament is for all invited firms to enter and for each contestant to adopt the same $z(N, P)$-stop strategy, where the function $z(M, P)$ is defined implicitly by

$$(7) \quad P\int_z^b \left[\Phi^{M-1}(x; z, T) - \Phi^{M-1}(z; z, T)\right] dF(x) - C = 0 \qquad \forall M \in \{2, \dots, N\}.$$

PROOF (sketch):

Lemma A1 (in the Technical Appendix) establishes that $z(M, P)$ exists and is single-valued if and only if $P/M - C \ge 0$, which follows from (6). Suppose that $M \in \{2, \dots, N\}$ firms enter the tournament and that each contestant employs its own z_i-stop

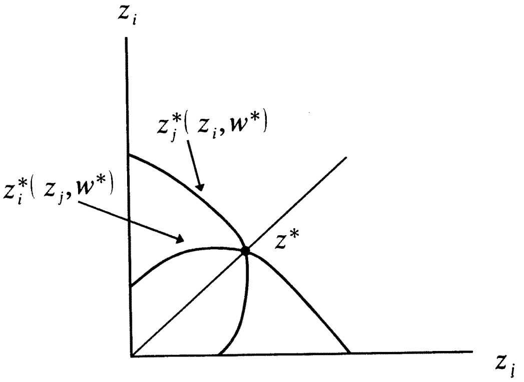

FIGURE 1. BEST-RESPONSE PROJECTIONS FOR ANY TWO CONTESTANTS

strategy. Then, by combining (2), (3), and (5), an equilibrium of the research subgame is any vector $(z_1^*,\dots,z_M^*)$ satisfying the following system:

$$(8)\quad P\int_{z^*}^{b}\left[\prod_{j\neq i}\Phi(x;z_j^*,T)\right.$$
$$\left.-\prod_{j\neq i}\Phi(z_i^*;z_j^*,T)\right]dF(x)$$
$$-C=0 \qquad i=1,\dots,M.$$

Pick any solution to this system and consider two arbitrary firms i and j. Hold fixed the strategies of all other contestants and consider the resulting projection of these two firms' best-response functions to one another's strategies. In the Technical Appendix, it is shown that they are symmetric and cross only once on the 45-degree line, and hence $z_i^* = z_j^*$ (see Fig. 1.) This transforms (8) into (7), and the fact that $z(M,P)$ is single-valued establishes uniqueness of equilibrium play on the research subgames. Finally, consider the entry stage. The foregoing argument shows that in equilibrium each contestant adopts the same $z(M,P)$-stop strategy on a subgame with $M \in \{2,\dots,N\}$ contestants. Therefore, the *ex ante* probability that a contestant facing $M-1$ rivals will win the prize is $1/M$; so, its *ex ante* expected profit is

$$(9)\quad U(M,P)\equiv\frac{P}{M}-K(z(M,P))-E.$$

It is shown in the Technical Appendix that $U(M,P)$ decreases in M, so that (6) insures that all N invited firms enter in equilibrium.

Proposition 2 is a natural extension of Proposition 1. While the dominance property of equilibrium play in the one-shot tournament does not extend to the dynamic contest, uniqueness of the equilibrium fortunately does. Of course, it is not surprising that a symmetric game with identical players possesses a symmetric equilibrium, but the absence of any asymmetric or mixed-strategy equilibria is uncommon.[19] As with Proposition 1, uniqueness here depends on whether the prize is sufficiently large to support full participation. For instance, If $P/N - C < 0$, then multiple equilibria may exist, some of which involve mixing. Moreover, if (6) holds with equality, then the Nth firm is indifferent about entering because its equilibrium expected profit equals its reservation payoff of zero. Thus, there are two pure-strategy equilibria in this case, one in which the Nth firm enters, and one in which it does not. This is similar to the quandary facing the "last" firm to enter a market in long-run competitive equilibrium, since industry profits are positive if it does not enter but zero if it does. It is assumed below that, if (6) binds, all invited firms enter in equilibrium. If the sponsor were worried that only $N-1$ firms would enter, he could raise the prize infinitesimally. The main advantage of uniqueness is that it ensures that the sponsor's tournament design problem is well defined.

III. Tournament Design

Before holding a tournament, the sponsor must decide how many firms to invite, the

[19] For example, Rogerson (1982) limits attention to a symmetric equilibrium but notes that asymmetric equilibria may exist in his model.

size of the prize, and the amount of the entry fee. If a parameterization, (N, P, E), satisfies (6), then all invited firms will enter in equilibrium and conduct research according to the $z(N, P)$-stop strategy defined in (7). The sponsor's expected payoff at the beginning of the tournament is, therefore, the collected entry fees plus the equilibrium expected value of the best innovation net of the prize

$$V = NE + \int_0^b x d\Phi^N(x; z, T) - P$$

where $z = z(N, P)$, and $\Phi^N(x; z, T)$ is the distribution of the best innovation discovered by N contestants, pursuing the equilibrium z-stop strategy [review equation (5)]. It turns out to be easier to work with a transformation which is obtained through integrating by parts

$$(10) \quad V = NE + \int_0^b [1 - \Phi^N(x; z, T)]\, dx - P.$$

Before discussing the tournament design that maximizes (10) subject to the relevant incentive constraints, two results concerning the nonoptimality of an important class of contests is given. For convenience, the integer constraint on the number of firms is ignored.

PROPOSITION 3: *Suppose there is an unlimited number of firms in the contestant pool. Also, imagine that the sponsor sets the prize $P \geq 2C$ and adopts a policy of "free and open entry." That is, he sets $E = 0$ and $N = \infty$. Let $N^{\mathrm{FE}}(P)$ denote the number of firms that do research in the unique pure-strategy free-entry equilibrium outcome. Then*

$$N^{\mathrm{FE}}(P) = \frac{P}{C}$$

and each of these contestants conducts research once and only once because

$$z(N^{\mathrm{FE}}(P), P) = 0.$$

Remark: Any number of firms $M \geq N^{\mathrm{FE}}(P)$ *may enter* the contest in a pure-strategy free-entry equilibrium. The important point is that only $N^{\mathrm{FE}}(P)$ of the entrants will do any research. It is straightforward to check that both active and idle contestants earn expected payoffs of zero and that no contestant has a strict incentive to change unilaterally its research or entry strategy.

The intuition underlying Proposition 3 is easily grasped. When a firm enters the tournament and performs research, it does not account for the externality imposed on the other contestants.[20] Specifically, the more active firms in the tournament, the less likely it is that any one firm will win. For a given prize, entry of active firms continuously erodes the incentive for the other contestants to innovate because $\partial z(m, P)/\partial M < 0$ [see equation (A5) in the Technical Appendix]. A tournament in which each active firm performs a single round of research is unlikely to be optimal, and it may deliver very low expected surplus to the sponsor even when the number of active contestants is large.[21]

The situation is even worse, when mixed-strategy equilibria are considered. Suppose that $M > N^{\mathrm{FE}}(P)$ firms enter a tournament with free and open entry. Then, Proposition 2 indicates that they must randomize over their research decisions in any *symmetric* equilibrium. Let α be the probability with which each contestant conducts research in a symmetric equilibrium. If a contestant deviates by remaining deterministically idle, its expected profit is $(P/M)(1-\alpha)^{M-1}$, since

[20] In a class of related search games, Dale T. Mortensen (1982) shows that such externalities can be eliminated by making the winner compensate the other players.

[21] The Department of Defense uses qualifying auctions to narrow down the set of firms ultimately submitting proposals to develop major weapon systems. An interesting question is whether using an auction to restrict entry into a research tournament might be optimal when there are unobservable asymmetries among the potential contestants. This question could be addressed by combining the model presented here with the auction-R&D mechanism studied by Rafael Rob (1986).

it wins the prize with probability $1/M$ if no other contestant does research. Also, by symmetry, the expected profit from adhering to the equilibrium is $P/M - \alpha C$.[22] So, α is defined to be the smallest positive root of

$$\frac{P}{M}(1-\alpha)^{M-1} = \frac{P}{M} - \alpha C.$$

Using $N^{\mathrm{FE}}(P) = P/C$, this can be rewritten as

$$\alpha M = N^{\mathrm{FE}}(P)\left[1-(1-\alpha)^{M-1}\right]$$

from which it is evident that $\alpha M < N^{\mathrm{FE}}(P)$. In other words, the expected sample size is smaller under a mixed-strategy equilibrium.[23] With this fact in hand, it is straightforward to construct a stochastic dominance argument that proves the following propositions.

PROPOSITION 4: *The expected value of the winning innovation is higher under a pure-strategy free-entry equilibrium than under a symmetric mixed-strategy equilibrium.*

If free entry is not generally desirable, then what is? The sponsor's problem is to maximize (10) (including z among his choice variables) subject to the participation constraint, (6), and the equilibrium constraint, (7). A solution to this problem, (N^*, P^*, E^*, z^*), will be called an optimal tournament.[24] By substituting the (binding) participation and equilibrium constraints into (10), the sponsor's design problem can be simplified tremendously.

PROPOSITION 5: *An optimal tournament solves the following*

$$\text{(11)} \quad \max_{N,z} V = \int_0^b \left[1-\Phi^N(x;z,T)\right]dx - NK(z).$$

Program (11) has a natural interpretation. The integral is just the equilibrium expected value of the best innovation discovered, and $NK(z)$ is aggregate equilibrium expected research costs. This reveals directly that the sponsor receives all of the *ex ante* surplus. He secures the expected value of the best innovation and "pays" the aggregate expected cost.

Remark: Program (11) should not be interpreted too literally. The sponsor does not directly choose the stopping threshold z. Rather, because $z(M,P)$ is monotone increasing in P and decreasing in M, Proposition 5 says that choosing the prize and number of firms to invite *is equivalent to* choosing the unique equilibrium z-stop strategy, provided that (6) is satisfied. Once the solution (z^*, N^*) to (11) is obtained, the optimal values for the prize P^* and entry fee E^* can be recovered by substituting (z^*, N^*) into the binding constraints (6) and (7). In practice, the sponsor first announces the contest parameters (N^*, P^*, E^*), and the contestants then choose $M = N^*$ and $z^* = z(N^*, P^*)$ as a consequence of equilibrium play.

At this point, a few words concerning the exogenously fixed terminal date of the tour-

[22] Note that contestants earn strictly positive expected payoffs under a mixed-strategy free-entry equilibrium, while their expected payoffs are zero in the pure-strategy case. This means that when exactly $N^{\mathrm{FE}}(P)$ firms have entered, each of them prefers entry of an additional firm, and switching to the mixed-strategy equilibrium. I am grateful to Richard Fullerton for this observation and for other interesting comments.

[23] Actually, since $(P/M)(1-\alpha)^{M-1} > 0$ for finite M, entry will continue without bound, and the expected sample size will converge to β, which is defined to be the positive solution to

$$\beta = N^{\mathrm{FB}}(P)(1-e^{-\beta}).$$

[24] If (N^*, P^*, E^*, z^*) maximizes (10) subject to (6) and (7), then it is "optimal" only in the class of tournament mechanisms under consideration (i.e., those defined in Section I). It is possible that other mechanisms exist that deliver higher *ex ante* surplus to the sponsor even under the assumptions of unobservable effort and unverifiable outcomes. See Section IV for details.

nament are in order. The main difficulty is that, without discounting, the sponsor's indirect utility function is increasing in T, so that he prefers the contest to run as long as possible. The rules of the tournament can actually be amended to accommodate an infinite horizon. The sponsor permits the firms to submit their final innovations whenever they wish, and the prize is awarded immediately after the last contestant submits its entry. The key differences under an infinite-horizon contest are that all final innovations exceed $z(N, P)$ in value, because a contestant never runs out of time, and no positive entry fee can be charged because *ex ante* equilibrium rents to contestants are zero. The reason for concentrating on research tournaments with finite terminal dates is that this type of contest is much more common, undoubtedly owing to impatience by tournament sponsors.[25] Assuming a fixed terminal date acknowledges the presence of impatience without introducing the many complications involved with discounting.

In the next section, the existence of an interior solution to (11) is assumed. Ignoring the integer constraint on N, necessary conditions for (z^*, N^*) to be an interior solution are

$$(12) \quad \frac{\partial V}{\partial z} = 0$$

$$\Leftrightarrow \int_{z^*}^{b} \Phi^{N^*-1}(x; z^*, T)[1 - F(x)]\, dx - C = 0$$

and

$$\frac{\partial V}{\partial N} = \int_0^{z^*} - T \ln(F(x)) F^{TN^*}(x)\, dx + \int_{z^*}^{b} - \ln(\Phi(x; z^*, T)) \Phi^{N^*}(x, z, T)\, dx - K(z^*) = 0$$

where the equivalence in (12) is obtained by dividing the first-order condition through by $K'(z)/C$.

IV. Welfare

The first step in analyzing the efficiency of an optimal research tournament is to determine the research policy that would be adopted by the sponsor if there were no informational barriers, the "first-best" plan. In Section I it was assumed that research costs were not observable and that research outcomes were observable by the sponsor, but not verifiable by a court. As will be shown, the sponsor can implement the first-best plan if either one of these assumptions is relaxed.

First, suppose that costs were observable and verifiable.[26] Then, the sponsor could write contracts to reimburse firms for research expenses C on a period-by-period basis, terminating research once an acceptable innovation was discovered. This contract does not require research outcomes to be verifiable by a court, since the sponsor is given the right to stop further research "at will." If there were no transaction or start-up costs, and if the sponsor could costlessly evaluate innovations, then his problem would be equivalent to solving an optimal-sample-size search problem with recall and a fixed time horizon. Suppose that the best innovation discovered by date $t < T$ is worth x. Then, the sponsor, in the first-best world, faces a two-step decision problem. He must first decide whether to stop research and

[25] Wright (1983 p. 732 [footnote 15]) indicates that the chronometer and the process of preserving food through canning were invented under contests of indefinite duration, however. Another example of a research contest with an infinite horizon was Germany's prize for the first (legitimate) proof of Fermat's Last Theorem.

[26] In order for costs to be contractible, they must be not merely observable by the sponsor, but verifiable by a court. Observability is necessary but not sufficient for verifiability.

enjoy terminal utility x. If he decides not to stop, then he must decide how many firms to hire in the current period. This problem is formalized in the following functional equation:

$$(13) \quad V_t^{FB}(x) = \max\left\{x, \max_{N_t} V_{t+1}^{FB}(x) F^{N_t}(x) + \int_x^b V_{t+1}^{FB}(y)\, dF^{N_t}(y) - N_t C\right\} \quad t = 0, \ldots, T-1$$

where the left-hand member within the braces corresponds to stopping research and the right-hand member corresponds to continuing, and where $V_t^{FB}(x)$ is the sponsor's date-t expected utility of adhering to the first-best plan. This type of problem has been studied by several authors (see Samuel Gal et al., 1981; Peter B. Morgan, 1983; Morgan and Richard L. Manning, 1985). The key result needed here is that the solution to (13) is characterized by a unique stationary stopping value z^{FB} and a sequence of integer-valued functions $\{N_t^{FB}(x)\}_{t=0}^{T-1}$. In other words, if an innovation worth $x > z^{FB}$ is discovered, then research is stopped. Otherwise, the sponsor optimally hires $N_t^{FB}(x)$ firms to conduct research in period t.[27]

The structure of this solution suggests how the first-best research plan could be implemented using incentive contracts if costs were not observable but research outcomes were verifiable. Suppose that the best innovation discovered by date $t < T$ is worth $x < z^{FB}$. Then, the sponsor could offer each of $N_t^{FB}(x)$ firms a one-period research contract under which any firm discovering an innovation worth x_i would be paid according to the following scheme:

$$(14) \quad P^{FB}(x_i) \equiv \begin{cases} 0 & x_i \le z^{FB} \\ C/[1 - F(z^{FB})] & x_i > z^{FB}. \end{cases}$$

Since the probability of discovering a profitable innovation under this contract is $1 - F(z^{FB})$, each firm's participation constraint just binds. Of course, the sponsor would stop offering these contracts once an innovation worth at least z^{FB} was discovered. This mechanism does not require costs to be observable, since (14) ensures that performing research is incentive-compatible. On the other hand, it is necessary that outcomes be verifiable by a court, because the terms of payment depend on a direct comparison between the value of a firm's discovery, x_i, and the hurdle value z^{FB}.

Remark: This discussion highlights the general principle that, in order for the first best to be unobtainable in this type of environment, it is necessary both that inputs and outputs be unverifiable. These informational barriers are common in research environments, especially when the research objective is very technical or unique. It is worth noting, however, that both input-based and output-based contracts are used in certain research settings. For instance, personal-injury lawyers often conduct research on a contingency basis, since the value of their discoveries, the size of settlements or damage awards, is easily verified.

The rest of this section identifies and discusses the differences between the first-best research plan embodied in the solution to (13) and the optimal research tournament of the previous section.

PROPOSITION 6: $z^* < z^{FB}$.

PROOF:

Lemma A4 (in the Technical Appendix) says that z^{FB} is defined implicitly by

[27] Under an infinite horizon without discounting, the first-best number of firms $N^{FB}(x)$, is always 1, for $x \le z^{FB}$. This is because the sponsor does not care how long it takes to discover an acceptable innovation, but he would like to avoid the waste associated with discovering multiple innovations exceeding the stopping value. A finite horizon creates an environment of impatience under which some parallel research, $N_t^{FB}(x) > 1$, is generally desirable, however.

$$\int_{z^{\mathrm{FB}}}^{b}[1-F(x)]\,dx-C=0.$$

Consider the value for z^* defined implicitly in (12), when $N^*=1$

$$\int_{z^*}^{b}\Phi^0(x;z^*,T)[1-F(x)]\,dx-C=0.$$

Obviously, $z^{\mathrm{FB}}=z^*$ in this case. However, $N^*>1$ in a tournament, so the proof will be complete if it can be shown that z^* decreases as N increases. Implicitly differentiating (12) with respect to N, and rearranging terms gives equation (15), below. The numerator of the fraction in (15) is positive, while Lemma A2 implies that the denominator is negative.

Some care is needed in interpreting this result because z^{FB} and z^* play somewhat different roles in the solutions to their respective optimization problems. The value z^{FB} is a global stopping threshold. That is, under the first-best plan, the sponsor stops research *by all firms* the first time *any single firm* discovers an innovation worth at least z^{FB}. This type of global stopping criterion is not feasible under the tournament mechanism. Instead, research by all firms is stopped under an optimal tournament only after *each and every firm* discovers an innovation worth at least z^*.

The sponsor would be better off *ex ante* if he could implement a scheme with a global stopping criterion, but this type of mechanism always depends on the verifiability of research outcomes. In other words, it is not credible for the sponsor to announce that he will stop the contest and award the prize to the first contestant submitting an innovation worth at least z^{FB}. Once this contest is in progress, the sponsor never has an incentive to stop it before date T, because he does not bear the marginal cost of continued research. Absent verification by a court, the sponsor possesses a weakly dominant strategy to claim always that any innovation submitted prior to T does not beat the hurdle value. Knowing this, the contestants will be concerned only with their date-T innovations, which transforms the contest back into a T-period tournament.

Remark: It may be helpful to think of the first-best stopping criterion as a target and an optimally designed tournament as a strategy for shooting an arrow in a gusting tail-wind. If the sponsor aims directly at the target, $z^*=z^{\mathrm{FB}}$, he will almost certainly overshoot the mark, which corresponds to multiple firms discovering innovations worth more than z^{FB}. Proposition 6 confirms this intuition by indicating that the sponsor's best chance for hitting the target is to aim short, $z^*<z^{\mathrm{FB}}$. This analogy can also be used to understand why z^* decreases in N as shown in (15). If one thinks of N as the speed imparted to the arrow, then it is clear that the sponsor must adjust his aim downward to compensate for a more forceful shot. Technically, the expected value of the winning innovation is the expected value of the first-order statistic possessing distribution $\Phi^N(x;z^*,T)$. If N is increased, the expected value of the first-order statistic increases along with the probability of obtaining multiple draws exceeding z^{FB}. Therefore, it is necessary to reduce z^* in order to maintain the same expected target. The optimal number of contestants N^* depends critically on the time horizon, since a short horizon forces the sponsor to substitute parallel for sequential research. That is, N^* is large when T is small.

While an optimal tournament is designed to emulate the first-best research plan *ex ante*, it is unlikely to achieve this goal

$$(15)\quad \frac{dz^*}{dN}=\frac{\displaystyle\int_{z^*}^{b}-\ln(\Phi(x;z^*,T))\Phi^{N-1}(x;z^*,T)[1-F(x)]\,dx}{\displaystyle\int_{z^*}^{b}(N-1)\Phi^{N-2}(x;z^*,T)\Phi_2(x;z^*,T)[1-F(x)]\,dx-F^{T(N-1)}(z^*)[1-F(z^*)]}$$

ex post. The sponsor reduces but does not eliminate the potential for overshooting by setting $z^* < z^{FB}$. Moreover, calibrating the tournament in this way introduces the possibility of a research shortfall. If all N^* contestants discover innovations with values in the interval (z^*, z^{FB}) before date T, all research will stop in equilibrium, because each firm employs the z^*-stop strategy. Note, however, that the first-best plan calls for research to continue in this case, since no innovation worth more than z^{FB} has been discovered. When designing the optimal tournament, the sponsor must balance the probability of overshooting (stopping too late) against the probability of a shortfall (stopping too soon). Since *ex post* inefficiency involves improper conditional stopping, the following result should come as no surprise.[28]

PROPOSITION 7: *Suppose the sponsor is restricted to a one-period horizon, $T = 1$. Then, so long as $N_0^{FB}(0) \geq 2$, the optimal research tournament implements the first-best plan.*

This result accords with the main finding of Lazear and Rosen (1981) that established the efficiency of nondynamic rank-order tournaments with risk-neutral contestants. Proposition 7 also lends yet another perspective on George J. Stigler's (1961, 1962) celebrated fixed-sample-size search model. By choosing the optimal prize and entry fee, it is possible to induce the efficient number of firms to conduct research in a one-shot contest.

At first glance, Proposition 7 seems to provide a method for implementing the first-best research plan in a multiperiod setting too. Rather than devise one multiperiod contest, a series of one-shot tournaments could be sponsored with the number of contestants chosen optimally in each round, and the series would be terminated as soon as an innovation worth more than z^{FB} is discovered. Unfortunately, this mechanism will usually not work as described.

Awarding a prize at an intermediate stage of the research process conveys information about the relative quality of the discovery made by the winner. Only if the sponsor can ensure that each contestant begins each round of play on an equal footing will this information not present incentive-compatibility and participation problems.[29] For example, if the contestant pool is infinite, the sponsor could implement the first-best research program by conducting a series of one-shot tournaments in which no firm ever competed in more than a single contest. However, there are reasons, even in this extreme example, why the multiperiod tournament might be preferable. For instance, if it is costly for the sponsor to evaluate innovations, then the series of one-shot contests may be less desirable, since it forces him to appraise every project after each round. Determining the precise conditions under which a multiperiod research tournament constitutes an optimal second-best mechanism is an interesting and challenging topic for future study.

V. Conclusion

This article has examined the procurement institution of research tournaments. As noted in the Introduction, such contests have long been used in business, government, and academia for eliciting inventive activity, although they have received little analytic attention to date. In practice, a research tournament is usually adopted by a monopsonist as a method for committing itself to reward agents in an environment of unobservable research inputs and unverifiable research outcomes.

Section I presented a formal model of a research tournament which was analyzed in Sections II, III, and IV. Section II showed that each contestant's best response in the tournament game is to employ a stopping

[28] A second potential source of inefficiency—not considered here—is that it is not possible in a tournament to select optimally the number of firms doing research from period to period.

[29] See Rogerson (1982) for a discussion of incumbency advantages in a series of decentralized one-shot contests.

strategy under which it stops doing research once it discovers an innovation worth more than some critical value. When the prize is large enough to support full participation, a symmetric research tournament has a unique subgame-perfect equilibrium in which each invited firm enters and each contestant employs the same stopping strategy.

The sponsor's tournament design problem was analyzed in Section III. It was shown that a policy of free and open entry is generally not optimal because high levels of entry give rise to low levels of research effort in equilibrium. Thus, under an optimally designed tournament, the sponsor restricts participation and extracts all expected surplus by taxing contestants through an entry fee. Because equilibrium research effort by each firm increases with the size of the prize and decreases with the number of contestants, there is a one-to-one correspondence between choosing the optimal prize and entry fee and choosing the optimal number of contestants and equilibrium effort level. This simplifies the sponsor's contest design problem tremendously, and it facilitated comparison with the first-best research plan in Section IV.

An optimally designed research tournament is calibrated to hit the first-best target level of research in expected value, through it is unlikely to achieve this aim *ex post*. Due to the lack of a global stopping criterion, there is a natural tendency for overshooting the target in a tournament. Recognizing this, the sponsor sets the equilibrium stopping threshold in the tournament below the first-best level. This introduces the potential for a research shortfall, in which all research stops too soon, but reduces the probability of overshooting, that is, stopping research too late. Under a one-period horizon, conditional stopping criteria are not applicable, and an optimally designed research tournament is *ex post* efficient. While the model of research tournaments offered here is obviously quite stylized, it captures many of the salient features of actual research contests and provides a point of departure for analyzing models with more institutional details or empirical investigations.

TECHNICAL APPENDIX

LEMMA A1: *Let $Z(M,P)$ be the set of all points $z(M,P)$ satisfying (7). Then, $Z(M,P)$ contains at most one element, $z(M,P)$, and $Z(M,P)$ is nonempty if and only if $P/M \geq C$. Moreover, if $P/M = C$ then $Z(M,P) = \{0\}$.*

PROOF:

Define the real-valued function $\Delta(z)$ by

$$\Delta(z) = P\int_z^b [\Phi^{M-1}(x;z,T) - \Phi^{M-1}(z;z,T)]\,dF(x) - C.$$

Then, $Z(M,P)$ is defined by

$$Z(M,P) = \{z \in \mathbb{R}^+ \,|\, \Delta(z) = 0\}.$$

First it is shown $\Delta'(z) < 0$, which implies that $\Delta(z) = 0$ at most once. Begin by writing

$$\Delta'(z) = -P[F^{T(M-1)}(z) - F^{T(M-1)}(z)]f(z)$$

$$+ P\int_z^b [(M-1)\Phi^{M-2}(x,z,T)\Phi_2(x,z,T) - T(M-1)F^{T(M-1)-1}(z)f(z)]\,dF(x).$$

Therefore, $\Delta'(z) < 0$ if

$$\Phi_2(x,z,T) \leq 0 \quad \forall x \in [0,b]$$

which is established in Lemma A2 below. To obtain the rest of the proof, note that

$$\Delta(0) = P\int_0^b \Phi^{M-1}(x,0,T)\,df(x) - C$$

$$= P\int_0^b F^{M-1}(x)\,dF(x) - C$$

$$= \frac{P}{M}F^M(x)\Big|_{x=0}^{b} - C = \frac{P}{M} - C.$$

Also, $\Delta(b) = -C$, so $\Delta(z) = 0$ only if $P/M - C \geq 0$, and $P/M - C = 0$ implies $Z(M,P) = \{0\}$.

LEMMA A2: $\Phi_2(x;z,T) \leq 0 \quad \forall x \in (z,b]$.

PROOF:

Suppose $x \geq z$, and begin with $T = 1$:

$$\Phi(x;z,1) = F(x) \Rightarrow \Phi_2(x;z,1) = 0.$$

Now consider $T = S$, where S is any positive integer. Then the result will follow inductively if it can be shown that

$$\Phi_2(x;z,S) \geq \Phi_2(x;z,S+1).$$

Direct calculation of these partials and some straightforward algebra then yield the result.

LEMMA A3: $\Phi_{21}(x;z,T) > 0$ *for* $x \in [z,b]$ *and* $T > 1$.

PROOF:

Using Young's theorem, differentiate first with respect to x:

$$\Phi_1(x;z,T) = \frac{1-F^T(z)}{1-F(z)}f(x) = f(x)\sum_{t=0}^{T-1}F^t(z).$$

This is obviously positive and increasing in z.

LEMMA A4: *If the best innovation discovered by* $t < T$ *is worth* z^{FB}, *then the first-best plan calls for hiring a single firm to conduct research*:

$$N_t^{\text{FB}}(z^{\text{FB}}) = 1 \qquad t = 0,\ldots,T-1.$$

PROOF:

Note that $V_T^{\text{FB}}(x) = x$, and use (13) to write

$$\text{(A1)} \qquad V_{T-1}^{\text{FB}}(x) = \max\left\{x,\ \max_{N_{T-1}} xF^{N_{T-1}}(x) + \int_x^b y\,dF^{N_{T-1}}(y) - N_{T-1}C\right\}$$

$$= \max\left\{x,\ \max_{N_{T-1}} x + \int_x^b [1 - F^{N_{T-1}}(y)]\,dy - N_{T-1}C\right\}$$

where the second line follows from integrating by parts. When the sponsor has an innovation worth z^{FB} "in hand," he is indifferent between stopping research and hiring the optimal number of firms. Therefore, z^{FB} equates the elements within the braces of (A1). Moreover, the stationarity of z^{FB} implies that this equality holds for all t:

$$z^{\mathrm{FB}} = \max_{N_t} z^{\mathrm{FB}} + \int_{z^{\mathrm{FB}}}^{b} [1 - F^{N_t}(y)]\, dy - N_t C$$

$$\Leftrightarrow \max_{N_t} \int_{z^{\mathrm{FB}}}^{b} [1 - F^{N_t}(y)]\, dy - N_t C = 0.$$

Let M^{FB} be the positive integer solving the maximization problem. Then, the last line above implies

$$\text{(A2)} \qquad \int_{z^{\mathrm{FB}}}^{b} \left[1 - F^{M^{\mathrm{FB}}}(y)\right] dy = M^{\mathrm{FB}} C.$$

Let $B(M)$ be the expected benefit from hiring M firms to conduct research in state z^{FB}:

$$B(M) \equiv \int_{z^{\mathrm{FB}}}^{b} [1 - F^{M}(y)]\, dy.$$

Then, (A2) says that the expected benefit from hiring the optimal number of firms is equal to the costs (otherwise the sponsor would not be indifferent about stopping). However, note that

$$B''(M) = \int_{z^{\mathrm{FB}}}^{b} -[\ln(F(y))]^2 F^{M}(y)\, dy < 0.$$

Therefore, if $M^{\mathrm{FB}} > 1$ it is possible to reduce the number of firms conducting research, raising the marginal expected benefit, and leaving marginal costs unchanged, contradicting the optimality of M^{FB}.

PROOF OF PROPOSITION 2 (the rest of the story):

Let $(z_1^*, \dots, z_M^*)$ be any equilibrium of the research subgame. Assume without loss of generality that $i < j$, and define the vector $\mathbf{w}^*$ and the probability distribution $\gamma(x; \mathbf{w}^*)$ as follows:

$$\mathbf{w}^* \equiv (z_1^*, \dots, z_{i-1}^*, z_{i+1}^*, \dots, z_{j-1}^*, z_{j+1}^*, \dots, z_M^*)$$

$$\gamma(x; \mathbf{w}^*) \equiv \prod_{k \neq i,j} \Phi(x; z_k^*, T).$$

Substitute these definitions into (8):

$$P\int_{z_i^*}^{b} \left[\Gamma(x; \mathbf{w}^*)\Phi(x; z_j, T) - \gamma(z_i^*; \mathbf{w}^*)\Phi(z_i^*; z_j, T)\right] dF(x) - C = 0.$$

Next, differentiate this with respect to z_j, recalling that z_i^* is a function of z_j:

$$\text{(A3)} \qquad P\int_{z_i^*}^{b} \Bigg\{ \gamma(x; \mathbf{w}^*)\Phi_2(x; z_j, T) - \gamma(z_i^*; \mathbf{w}^*)\Phi_2(z_i^*; z_j, T)$$
$$- \left[\gamma_1(z_i^*, \mathbf{w}^*)\Phi(z_i^*; z_j, T) + \gamma(z_i^*; \mathbf{w}^*)\Phi_1(z_i^*; z_j, T)\right] \frac{\partial z_i^*}{\partial z_j} \Bigg\}\, dF(x) = 0.$$

There are two cases to consider. First, suppose $z_i^* > z_j$. Then, (A3) can be rewritten as

$$\frac{\partial z_i^*}{\partial z_j} = \frac{\int_{z_i^*}^{b}\left[\gamma(x;\mathbf{w}^*)\Phi_2(x;z_j,T) - \gamma(z_i^*;\mathbf{w}^*)\Phi_2(z_i^*;z_j,T)\right] dF(x)}{[1-F(z_i^*)]\left[\gamma_1(z_i^*;\mathbf{w}^*)\Phi(z_i^*;z_j,T) + \gamma(z_i^*;\mathbf{w}^*)\Phi_1(z_i^*;z_j,T)\right]}.$$

This expression is positive because the denominator is positive (since it involves only probability distributions and their first derivatives), and the numerator is also positive, because Lemma A3 implies

$$\Gamma(x;\mathbf{w}^*)\Phi_2(x;z_j,T) > \gamma(z_i^*;\mathbf{w}^*)\Phi_2(z_i^*;z_j,T)$$

for $x > z_i^*$.

Next, suppose $z_i^* < z_j$. Then, (A3) can be rewritten as

$$\frac{\partial z_i^*}{\partial z_j} = \frac{\int_{z_j}^{b}\Gamma(x;\mathbf{w}^*)\Phi_2(x;z_j,T)\, dF(x)}{[1-F(z_i^*)]\left[\gamma_1(z_i^*;\mathbf{w}^*)F^t(z_i^*) + \Gamma(z_i^*;\mathbf{w}^*)TF^{T-1}(z_i^*)f(z_i^*)\right]}.$$

This expression is negative, however, because the denominator is positive and the numerator is negative by Lemma A2. Therefore, i's best-response function, $z_i^*(z_j,\mathbf{w}^*;M,P)$, is increasing in z_j until it hits the 45-degree line, and it decreases thereafter. The same analysis can be applied to $z_j^*(z_i,\mathbf{w}^*;M,P)$ to show that it increases in z_i until hitting the 45-degree line and decreases thereafter. Finally, the fact that $z_i^*(z,\mathbf{w}^*;M,P)$ is the same function as $z_j^*(z,\mathbf{w}^*;M,P)$ implies that the best-response functions cross only when $z = z_i^* = z_j^*$ (review Fig. 1). However, this means that all M contestants must use the same $z(M,P)$-stop strategy in any equilibrium of the research subgame. Hence, (8) is equivalent to (7), and Lemma A1 establishes uniqueness.

For the second part of the proof, it must be shown that $U(M,P)$ decreases in M. Differentiate (9) to get

$$U_1(M,P) = -\frac{P}{M^2} - K'(z(M,P))z_1(M,P). \tag{A4}$$

Now, perform the integration in (7) and rearrange terms to get

$$\frac{P}{M} - K(z(M,P)) = P\left[F^{T(M-1)}(z(M,P)) - \frac{M-1}{M}F^{TM}(z(M,P))\right].$$

Differentiate this with respect to M, and solve to get

$$z_1(M,P) = -\frac{(P/M^2)[1-F^{TM}(z(M,P))]}{PT(M-1)f(z(M,P))[F^{T(M-1)-1}(z(M,P)) - F^{TM-1}(z(M,P))] + K'(z(M,P))}. \tag{A5}$$

Substituting (A5) into (A4) shows that $U_m(M,P) < 0$ if and only if

$$-\frac{P}{M^2} + K'(z(M,P))\left\{\frac{(P/M^2)[1-F^{TM}(z(M,P))]}{PT(M-1)f(z(M,P))[F^{T(M-1)-1}(z(M,P)) - F^{TM-1}(z(M,P))] + K'(z(M,P))}\right\}$$
$$< 0$$
$$\Leftrightarrow \frac{P}{M^2}\{PT(M-1)f(z(M,P))[F^{T(M-1)-1}(z(M,P)) - F^{TM-1}(z(M,P))] + K'(z(M,P))\}$$
$$> K'(z(M,P))\frac{P}{M^2}[1-F^{TM}(z(M,P))]$$
$$\Leftrightarrow PT(M-1)f(z(M,P))[F^{T(M-1)-1}(z(M,P)) - F^{TM-1}(z(M,P))]$$
$$> -K'(z(M,P))F^{TM}(z(M,P))$$

which follows because the left side of the last line is positive and the right side is negative.

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