Races or Tournaments?

Andrea Blasco K. J. Boudreau M. Menietti K. R. Lakhani

This version: December 02, 2016

Plan

Known to us for thousands of years

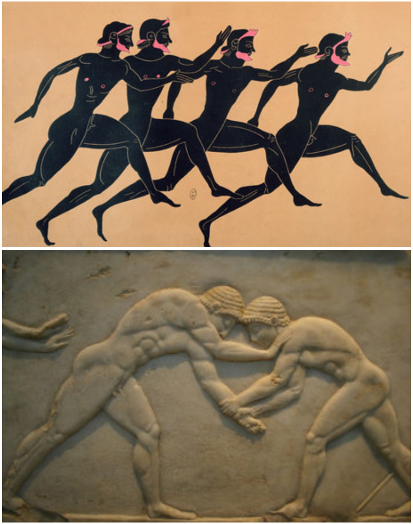


Figure 1:Runners (top) and wrestlers (bottom) c. 500 bc.

Descriptive of many economic situations

Traditional economics:

- Between nations: *arms races, space races, wars*
- Between firms: *patent races, rent seeking contests*
- Inside firms: *sales contests, promotions*

More recently:

- Digital revolution $\rightsquigarrow$ lower costs to set up contests $\rightsquigarrow$ "Race or tournaments" is not exogenous parameter but a choice of contest design

A concrete example

- Problem: UK government and Eu commission want solutions to antibiotic resistance problem
- Open innovation contest approach looks good:
  - long tail distribution of potential solvers
  - cheaper than direct hiring
- EU choice is a "tournament" (The Horizon Prize)
  - the best solution within the next 5 years wins a prize
- UK choice is a "race" (The Longitude Prize)
  - the first to meet a given requirement wins a prize

Why to go with one or the other?

Potential gains of a race

- Get ideas more quickly (speed)
- Stop duplicative efforts once a minimum target has been achieved (efficiency)
- Supplement competition when lacking of intensity (incentives)

Potential costs of a race

- Lower participation
- Lower expected quality

Plan

In this paper

- Extend contest model of Moldovanu and Sela (2001) to have both in one framework
- Design and run experiment to collect data to estimate the model and provide policy recommendations.

Plan

Model highlights

- $k = 1, ..., K$ prizes available of value $V_1 > V_2 > ... > V_K$
- Each agent, $(i = 1, ..., I)$, exerts "multidimensional" efforts
  - time to completion t_i
  - output quality y_i
- Private ability iid $a \sim F$ on support $A \subset [0, \infty)$
- Maximize chances of winning minus costs

$$C(y_i, t_i, a_i) = c_y(y) \cdot c_t(t) \cdot a_i^{-1} \tag{1}$$

with $c_y(0) \geq 0$, $c_y' > 0$, $c_t(d) \geq 0$, and $c_t' < 0$.

Contest design

- Fixed the prize pool, contest designer's problem is

$$\text{maximize} \int y^*(x)dF(x) - \tau \int t^*(x)dF(x),\ \tau \in [0,1]. \quad (2)$$

Choice:

- **Race**: wins the first to achieve at least a quality of $\underline{y}$
- **Tournament**: wins the the highest quality with time to completion less than a deadline d

Equilibrium in a tournament

- Two prizes and $n \geq 3$ agents. Normalize the prize pool $V_1 + V_2 = 1$ and use the percentage $\alpha \geq 1/2$.
- The unique symmetric equilibrium of the model gives, for every $i = 1, ..., n$, the optimal time to completion $t^*(a_i)$ equal to the deadline d and the optimal output quality $y^*(a_i)$ as

$$c_y^{-1}\left[c_y(0) + \frac{1}{c_t(d)}\left(\alpha \int_{a_i}^{\infty} ...dz + (1-\alpha)\int_{a_i}^{\infty} ...dz\right)\right] \quad (3)$$

if $a_i \geq \underline{a}$ (see Moldovanu and Sela 2001), and equal to zero otherwise.

Equilibrium properties

- Equilibrium output quality is monotonic increasing in the agent's ability (see Moldovanu and Sela 2001).
- Thus, for every $i = 1, ..., n+1$, the equilibrium expected reward $R(a_i)$ depends only on the rank of his ability relative to the others

$$R(a_i) \equiv \alpha F_A(a_i)^n + (1-\alpha)n[1 - F_A(a_i)]F_A(a_i)^{n-1} \tag{4}$$

- This gives the functional relationship between the $\underline{a}$ and the parameters

$$\underline{a} = h(n, \alpha, F_A, C_0) \tag{5}$$

Equilibrium in a Race

The unique symmetric equilibrium of the model gives, for every $i = 1, ..., n$, the optimal quality $y^*(a_i)$ equal to the minimum requirement $\underline{y}$ and the optimal time to completion $t^*(a_i)$ as

$$c_t^{-1}\left[c_t(d) + \frac{1}{c_y(0)}\left(\alpha\int_{a_i}^{\infty} ...dz + (1-\alpha)\int_{a_i}^{\infty} ...dz\right)\right] \qquad (6)$$

if $a_i \geq \underline{a}_{\text{race}}$, and equal to zero otherwise.

- Marginal type as

$$\underline{a}_{\text{race}} = h(n, \alpha, F_A, C_{\underline{y}}) \qquad (7)$$

with $C_{\underline{y}} \equiv c_y(\underline{y})c_t(d)$ instead of C_0.

A comparison

Proposition (participation)

All else equal, participation is higher in a tournament compared to a race.

Proposition (ex-post 'efficiency')

All else equal, there always exists a subset of types $\tilde{A} \subset A$ for which the output quality is higher in a race compared to a tournament.

$\implies$ races may dominate tournaments even if time to completion is unimportant ($\tau = 0$)!!!

A comparison, optimal design

Proposition (Optimal design)

Given α *and for* $\tau > 0$*, the contest designer's optimal choice of a race and a tournament depends on the curvature of the cost function.*

$\rightsquigarrow$ When costs are linear, a tournament allows too many entrants with low quality output and full time to completion. When costs are sufficiently convex, entry of these inefficient types is prevented in a tournament.

$\rightsquigarrow$ Companion paper on the optimal α^* in preparation

A comparison, tournaments with entry requirements

- Many studies on minimum entry requirements in contests (and in auctions)

Corollary

All else equal, if $\tau = 0$, then a tournament with sufficiently high entry requirement dominates a race.

$\rightsquigarrow$ entry requirement may raise $\underline{a}$ to $\underline{a}_{\text{race}}$ $\rightsquigarrow$ reducing participation.

Plan

Example: nonlinear cost functions

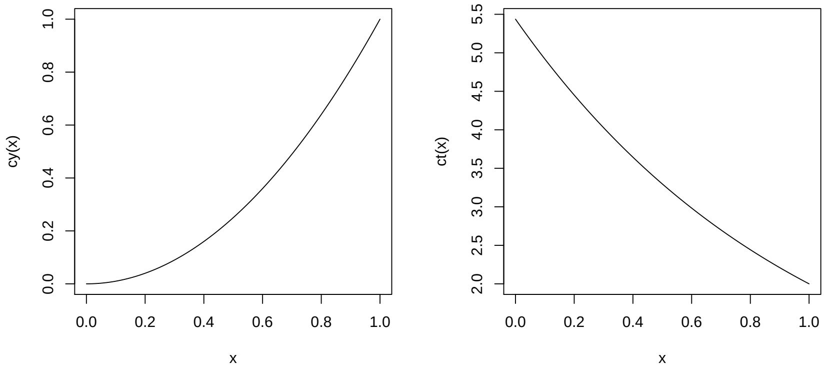


Figure 2:Example of non-linear cost functions with respect to output quality (left panel) and time to completion (right panel).

Example: Log-Normal ability distribution

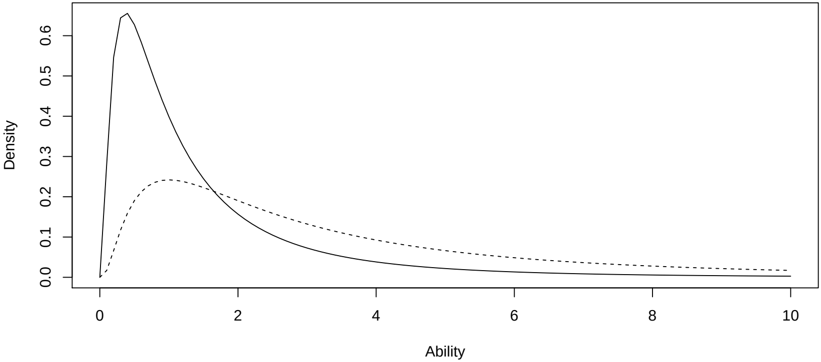


Figure 3:Private abilities are iid from the log-normal with mean 0 (——) and 1 (- - -).

Example: outcomes

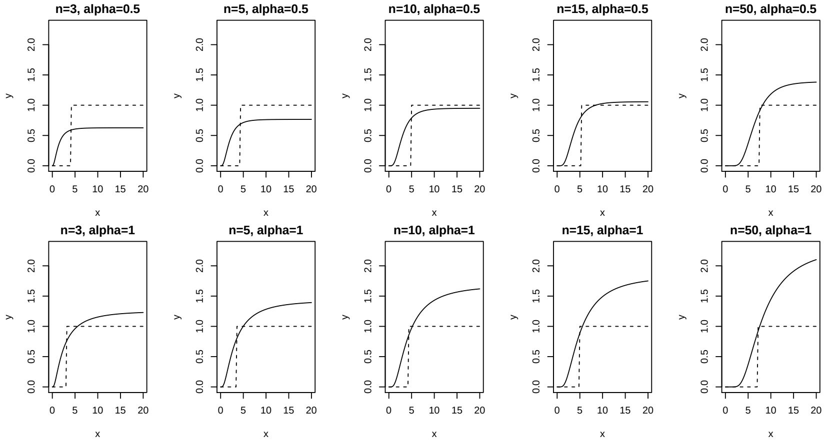


Figure 4:Equilibrium bidding functions of quality in a tournament (—) and in a race (- - -) with abilities drawn from a log Normal distribution. The number of competitors and the fraction of prize pool to the winner α is at the top of each panel.

Plan

Model Estimation

- Goal: simulate the model under different conditions
- Estimation objectives:
  - ability distribution
  - entry costs C_0 or $C_{\underline{y}}$
- Laffont, Ossard, and Vuong (1995)'s econometrics of auctions

Basic setup

- We observe (y_i, t_i, x_l) from $l = 1, ..., L$ contests and x_l vector of contest l characteristics
- Ability from $a_{il} \sim F_A(\cdot|\theta, x_l)$ where F is known up to θ.
- Consider entry: $y_i > 0$ when $a_i > \underline{a}$, and $y_i = 0$ otherwise.

$$\Pr(y_i = 1|\theta, x_l) = F_A(\underline{a}|\theta, x_l). \tag{8}$$

- Estimation with Maximum Likelihood

$$Likelihood = F_A(\underline{a}|\theta, x_l)^e [1 - F_A(\underline{a}|\theta, x_l)]^{n-e} \tag{9}$$

with $e = \sum y_i$.

Differences with auctions

- No "simultaneous" actions
  - $\rightsquigarrow$ observed y_i possibly truncated at the top
- No directly observed time to completion
  - last submission can be earlier
  - first submission can be later
- Competitors are not ex-ante equal, although it is difficult to predict the winners
- Prize structure may be more complex than what described in the model (learning, reputation, etc.)

Plan

Need of experimental data

- Observational data are rare:
  - No contests on the same problem
  - Different incentive structure, etc.
  - Competition type usually matches problem
  - Agents self-select
- Modeling needs iid data
- "Traditional" experiment without model may not be helpful

Experimental design

	Competition	Room size
1	Race	N=10
2	Tournament	N=10
3	Reserve	N=10
4	Race	N=15
5	Tournament	N=15
6	Reserve	N=15

Table 1:Experimental design

Implementation details

- Pre-registration of 5 days
- Competition of 9 days
- Payoffs
  - Room prize: \$1000 to winner of the room, \$200 second placed
  - Grand prize: \$4000 to winner of the group
- Initial and final survey
  - demographics
  - looking ahead / looking back
  - risk aversion
- Platform data

Our data

- >400 pre-registered
  - 299 enrolled in the experiment (not newly registered members)
- 86 (28%) participated by making a submission
- 1,759 code submissions
- 16 passed the target of the race (8 on the final scores)

Platform experience and skills

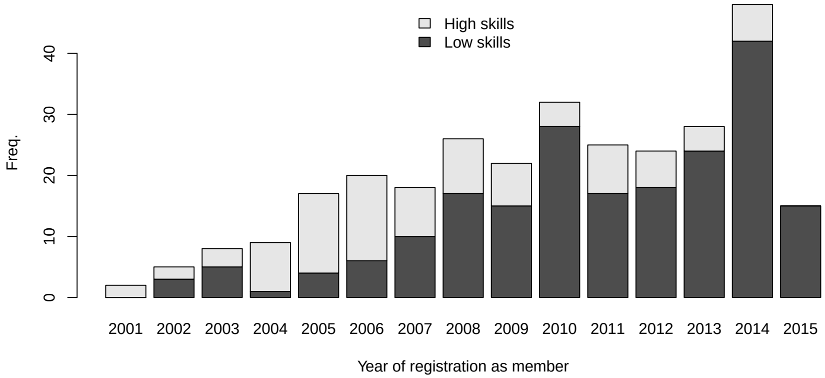


Figure 5:Signed-up competitors by the year of registration to the platform and if the competitor was ranked top 10 in one or more past competitions (high skill) or not (low skill).

Entry and platform experience

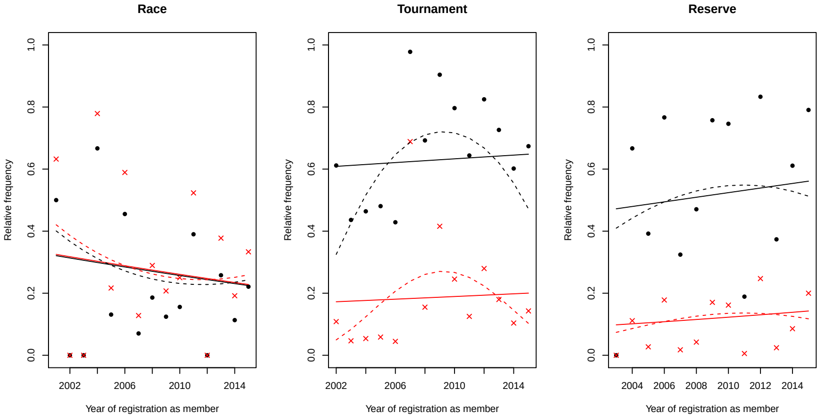


Figure 6:Association between the probability of participating, individual experience, and skills by treatment. Dots show relative frequency of participants with high (dark dots) and low (red crosses) skills. Curves show the conditional probability predicted by a Probit model that is linear (—) or quadratic (- - -) in experience.

Scores

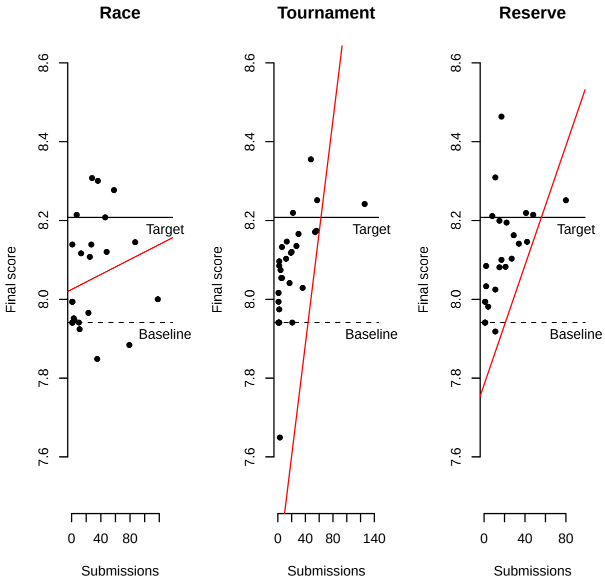


Figure 7:Association between scores and submissions by treatment.

Timing of submissions

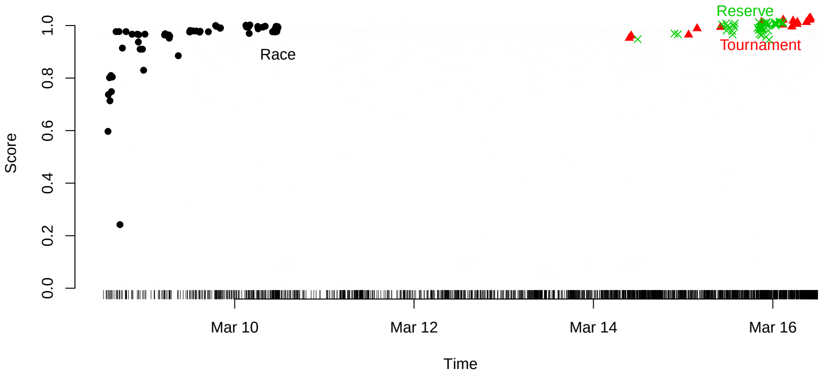


Figure 9:Scores and timing of the group winners.

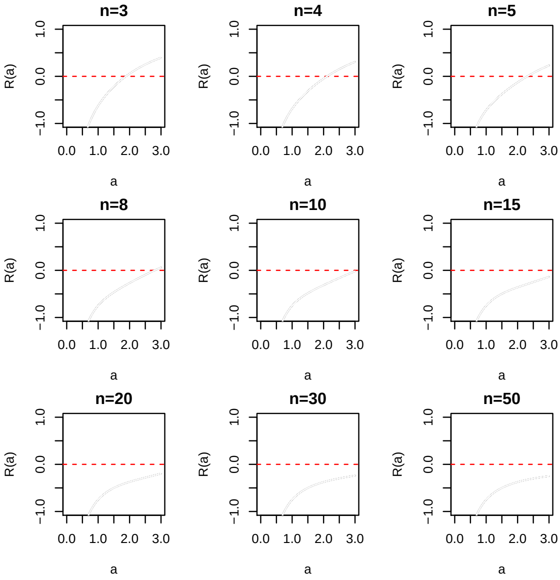
Timing of submissions

	Race	Tournament	Target
1	...1.1111	111	111
2	...1111.1	.11.11111	11
3	111......	111111111	111..11..
4	1111..111	.11111...	..1111111
5	..1.11111	1..111111	11111
6	.1111....	...111111	11
7	111111111	11111111.	....11.11
8	111111.1	.....11.1	...11.111
9	.11111111	.11.....1	1...1.111
10	..1.1111.	..111.111	.11.11111

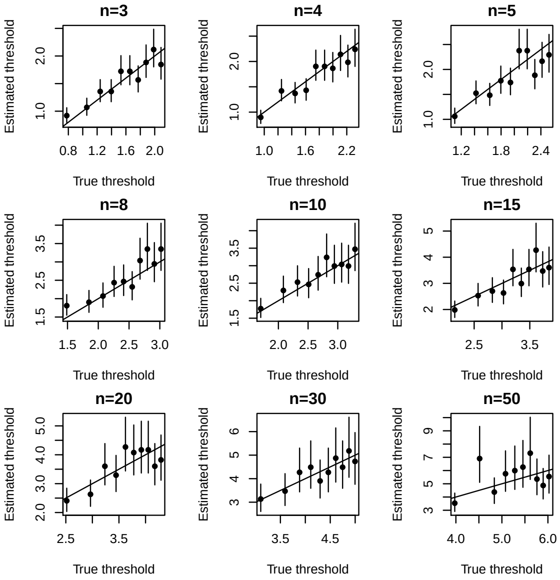
Table 2:Daily frequency of submissions made. Each line indicates the submissions made by one competitor during the 9-day submission period by treatment. The '1' indicates that the competitor made at least one submission in that day, whereas the '.' indicates no submissions. Observations are ordered by the competitor's final score in decreasing order.

Plan

Simulate structural model



Simulate estimation



Structural analysis of the data

- Assumption:
  - ability is distributed $a \sim F(|\theta, x_l) = \text{log-N}(0, 1)$

	Large	Small
Race	1.86	1.92
Tournament	1.69	1.38
Reserve	1.69	2.13

Table 3:Estimated marginal type ($\alpha = 1$).

Estimated entry costs

- Recall: zero-profit condition $\implies \underline{a} = h(\alpha, n, F_A, C_0)$
- $\implies$ entry cost $C_0 = f(\alpha, n, F_A, \hat{\underline{a}})$

	Large	Small
Race	1.78	9.94
Tournament	0.80	1.25
Reserve	0.80	16.64

Table 4:Estimated entry costs as % of prize pool.

Plan

Summing up

Data consistent with theory

- Races: had the problem solved in the first 3 days
- Tournament w/target: had highest quality
- Tournament: lowest entry costs

To do next:

- Allow distribution F to vary
- Estimate more complex prize structure ($\alpha \neq 1$)
- Deal with truncation/censoring issues

Laffont, Jean-Jacques, Herve Ossard, and Quang Vuong. 1995. "Econometrics of First-Price Auctions." *Econometrica* 63 (4). Econometric Society: 953–80.

Moldovanu, Benny, and Aner Sela. 2001. "The Optimal Allocation of Prizes in Contests." *The American Economic Review*. JSTOR, 542–58.