

Racing with uncertainty: a patent race experiment

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Abstract

This paper presents an experimental test of the multi-stage patent race model of Harris and Vickers (Rev. Econ. Stud. 54 (1987) 1). Tied competitors invested more when nearer the end of the race, but this may have followed from a more general and unexplained pattern of a positive correlation between investment and progress in the race. The relationship between investment and gap between competitors was mostly not as predicted. Also, leaders did not invest more than (not heavily lagging) followers, and the race did not approach monopoly as the gap between competitors widened. Overall, the evidence brings only limited support to the theory. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Patent race theory provides a stylized framework to study the incentives to invest in R&D, when competing perfectly rational firms are faced with the prospect that the winner of the race will be awarded patent protection. This paper presents a first precise experimental test of a dynamic model of patent races, specifically Harris and Vickers (1987), HV in what follows: HV represents a synthesis of a substantial body of work on patent races in the 1980s, and thus is a natural model to test.

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Real-world patent races are characterized by *technological uncertainty* and by *dynamic uncertainty*. There is technological uncertainty because, for a given investment in R&D, there is uncertainty in how much and how quickly progress is made in the invention discovery process. There is dynamic uncertainty because, intuitively, the incentives to invest in R&D may change as the race unfolds, according to the position of a firm in the race relative to its competitors and relative to the end of the race.

Early work on patent race theory was focused on investigating the effects of technological uncertainty:¹ the interest was either on the comparative statics of symmetric equilibria (e.g. Dasgupta and Stiglitz, 1980) or on the impact of initial asymmetries (e.g. a monopolist versus a potential entrant; Gilbert and Newbery, 1982) on the incentive to innovate. Only one research progress step is needed for a firm to win the race in these models. Because technological uncertainty is modelled as an exponential process, patent race is ‘memoryless’, i.e. it is not a function of time: firms decide their level of effort once and forever.²

Conversely, dynamic uncertainty alone was the focus of models by Fudenberg et al. (1983) and Harris and Vickers (1985): this research factored out technological uncertainty by having a deterministic relation between investment and progress in the race. Unless additional assumptions are made, a strong result of ‘ ε -preemption’ then obtains: namely, a small ε advantage at the start of the race is sufficient to ensure that the follower drops out from the race immediately.

Neither lack of memory nor ε -preemption appear plausible features of real-world races, and later work tried to combine technological and dynamic uncertainty to achieve a better characterization of patent races (Grossman and Shapiro, 1987; HV). Grossman and Shapiro (1987) have only a two-step patent race, and so their model can be largely considered a special case of HV:³ hence, HV can be considered a particularly suitable benchmark for testing patent race theory. This paper attempts to translate the model presented in Section 4 of HV, displaying both technological uncertainty and a dynamic multi-stage structure, as closely as possible to a laboratory setting.

Econometric evidence on patent race theories has been quite inconclusive. For example, on the one hand Lerner (1997) found support for the relevance of strategic variables in an empirical study of the disk drive industry; on the other hand, Cockburn and Henderson’s (1994) balance from analyzing innovation in the pharmaceutical industry was rather negative for patent race theory. Of course, it is possible that the game structure envisaged by the theory did not accurately map into the real-world settings considered either by Lerner or by Cockburn and

¹ For a review, see Reinganum (1984).

² This is not the case in Reinganum (1982), because in that model there is a Doomsday at which all competition ceases.

³ Grossman and Shapiro’s model is more general in one respect: it allows for positive discounting. HV briefly discuss the effect of discounting in their Section 5.

Henderson. In this context, a laboratory experiment can be especially useful to provide a clean and unambiguous, if minimal, test of the theory (Holt, 1995).

Isaac and Reynolds (1988) tested experimentally the implications of a stochastic model of R&D: they found that, the greater the group size, the greater the aggregate R&D, and, also, the less the appropriability of R&D. However, they were not concerned with the dynamic aspects of patent races. Isaac and Reynolds (1992) investigated R&D spending in the laboratory in a dynamic setting, but without studying the effects of patent protection. Two psychologists, Lopes and Casey (1994), implemented a game experiment vaguely resembling HV's (1987, Section 3) tug-of-war model, but with a fixed number of rounds and an asymmetry in the initial position of the players and in their action space. In their data, the responsiveness of risk-taking behavior to strategic incentives was somewhat less clear than what might have been expectable. However, the correspondence with patent race settings was only very loose, and indeed not sought after by the authors.

In HV's multi-stage race set-up, two players A and B compete for a patent, with value V^+ and W^+ , respectively, values higher than they get if they lose the race. In each round⁴ they decide investment effort rates x and y , respectively. Making suitable technical assumptions, HV showed that the probability of player A winning the round is $x/(x+y)$, and that of B winning is $y/(x+y)$. An isoelastic cost function $c = x^\eta$ (for player A) and $c = y^\eta$ (for player B), for some common $\eta > 1$, relates investment efforts to costs sustained by the players in advance of the race. Whoever wins a certain number of rounds — i.e. has completed the invention discovery process — wins the race and gets the patent.

HV derived a limited set of analytical results for the multi-stage race model, and then looked at the behavior of their model using numerical simulations, for values of η ranging between 1.25 and 12. They discussed particularly the case of a quadratic cost function, i.e. of $\eta = 2$. We focus on some key predictions that will be relevant to assess the usefulness of the model against the experimental data.

Firstly, HV found that leaders make greater investment effort than followers. They derived this analytically for the case in which the leader is close to success, in the sense of having no more than two progress steps m to go, i.e. $m \leq 2$. They also made the same claim more generally on the basis of their numerical simulations. The follower invests more the smaller is the gap, in terms of progress steps, relative to the leader.

Secondly, whenever $m \leq 2$ for the leader, investments are greater when the gap between competitors decreases. More generally, however, the investment of the leader might increase as its lead extends, although once it begins to decline it does so monotonically. From this and the previous result it follows that the correlation

⁴ HV talked about 'stages' rather than rounds. Since in describing the experiment I shall make use of both 'stage' and 'round', and 'stage' will refer to the entire patent race rather than to the single step in the invention discovery process, I am adapting HV's terminology to my later treatment.

between gap and investment should be more negative for followers than for leaders.

Thirdly, when the gap becomes large enough (three or four steps with $\eta = 2$), the leader does almost all of the investment. Fourthly, as previously predicted by Grossman and Shapiro (1987), with $m = 2$ tied players invest less than if they are tied with $m = 1$: in other words, if players are neck-to-neck in the race and they have only one progress step to go, they should invest more than if they are neck-to-neck but have two progress steps to go. Fifthly, HV stressed that in their numerical simulation this result failed to generalize, i.e. it is not always true that with greater m values tied players invest less: nevertheless, the numerical values that they bring for the $\eta = 2$ case suggest only limited non-monotonicity, and so a negative correlation between m and investment of tied players would be consistent with the model.⁵

The experiment presented in this paper was designed to be as similar as possible to the HV multi-stage model, with a quadratic cost function. The experiment provides only limited support for the model. In agreement with the model, with two or fewer progress steps to go, investment increased as the gap between competitors decreased. Also, tied competitors who were more ahead in the race invested more; however, this can be considered as just following from a positive correlation between investment and progress in the race, which is left unexplained by (though it is not inconsistent with) Harris and Vickers. In contrast to the predictions of Harris and Vickers, leaders did not invest more than followers, there was no virtual monopoly by the leader with a sufficiently large gap, and, with the exception noted above, investment did not appear to change as predicted with changes in the gap.

We conclude by observing that the experimental data presents a puzzle from the perspective of patent race theory. It may be due to limitations of the experimental design or it may require further theoretical work.

The structure of the rest of the paper is as follows. Section 2 describes the design of the experiment. Sections 3 and 4 present the univariate and multivariate analysis of the experimental data, respectively. Section 5 concludes.

2. Experiment design

The experiment was made up of nine sessions of four people each, for a total of 36 subjects. Each session lasted 45–60 min. Firstly, subjects were given the instructions and were asked to fill a 10-item multiple-choice questionnaire checking for their full understanding: explanations followed if any answer was

⁵On the basis of the few values provided by HV, with $\eta = 2$ for m between 1 and 10 the Spearman correlation between m and investment should be equal to -0.333 . Unfortunately, the full set of values was not available in either HV or the working paper (Harris and Vickers, 1986).

incorrect. The instructions are reproduced in Appendix A. Then, subjects played in dyads what was labeled as ‘a prize race’ twice, against different people in each stage, and this was publicly known from the beginning. They played the first time for practice and with no actual monetary incentives; the second time they played for real money, and we shall refer to this as ‘the real experiment’. Finally, there was the payment stage: each point earned in the real experiment was worth 1 UK penny. Final winnings averaged 8.86 UK pounds (min = 2.07; max = 16.24; S.D. = 5.19), including a £2 participation fee. The experiment was run on computers:⁶ Fig. 1 has a picture of the computer screen as the subjects saw it during the experiment.

The design of the ‘prize competition’ was identical insofar as possible to the setting of the multi-stage patent race model by HV, and in particular to the numerical simulation discussed in their Section 4, with a quadratic cost function. The ‘prize’ was the same (1000 points, equal to £10) for both subjects, and went to the winner of the patent race. There was no prize whatever if a subject lost the ‘competition’.

At the start of each ‘prize competition’, subjects were given 500 points. They were told that they were competing for a prize of 1000 points: the subject who made 10 progress steps first would win the prize. In order to make progress in the ‘competition’, subjects decided each round how much ‘investment’ they wanted to make; the cost of their investment was equal to the square of the amount invested.⁷ After the competing subjects X and Y made their investments x and y , respectively, the computer determined the probability for X of winning the round as $x/(x + y)$, and that for Y as $y/(x + y)$. If both subjects played 0, it was considered a draw, with neither subject winning the round.⁸ Otherwise, the computer would draw a number randomly, and determine the winner on the basis of the winning probabilities. The winner for the round made one progress step: the winning probability, the investment cost of the other player for the round, who was the winner, and the current updated state of the ‘prize competition’ were communicated to both subjects after they had made the decision for each round. If, after a round, neither subject had yet made 10 progress steps, the ‘competition’ would go on to the next round following the same procedure. If one player achieved 10 progress steps, he or she was declared winner of the competition, awarded 1000 points, and the stage ended. If the ‘prize competition’ was that of the practice stage, subjects could then move on (as soon as the other two subjects in the session had also finished) to the ‘competition’ of the real experiment. In meeting a new

⁶ Instructions and questionnaire were provided on paper. Partitions were used to prevent subjects from seeing each other. Subjects were not allowed to talk to one another during the experiment.

⁷ This corresponded of course to the quadratic cost function. An instruction table enabled subjects to decide investments without having to make computations themselves; also, the experiment program enabled subjects to have the computer determine investment costs for any feasible investment level and for any desired number of times, without committing themselves to any decision.

⁸ This never actually occurred.

Stage 2. For Real Money. Prize Worth 1000 Points

You Are Player 1

Player 1's Score	376	Player 4's Score	398.12
Player 1's Investment Cost This Round	0	Player 4's Investment Cost This Round	0

Progress 1 2 3 4 5 6 7 8 9 10

Player 1

Player 4

OK Cancel View

Fig. 1. Example computer screen of the experimental program. The picture shows what the experimental program may have displayed at the start of a round, for given players, scores and progress. (The screen was in various colors, not in grayscale as in the picture.) The white quadrant displayed how many progress steps the players had made. The investment cost for oneself (i.e. in this example, for Player 1) would be updated as the subjects put numbers in the text box in the left bottom corner and clicked OK or View. The investment cost for the other player would only be updated after *both* subjects made their decisions for the round: in other words, subjects could not see what their competitors had done, before taking their own decision for the round. After the subjects made their decisions, the computers displayed winning probabilities on a pop-up window, and then determined winners and updated scores, progress and investment costs for the round. A label saying 'You win the round!' or 'You lose the round' would appear to the right of the 'You Are Player X' label. Clicking OK on a pop-up window would move the subject to the following round, with the win/lose label disappearing and the round investment costs labels returning to 0.

competitor, subjects did not receive any information about how he or she had played in the practice stage. If the completed 'competition' was that of the real experiment, subjects would move on to the payment stage, get paid, and leave the experimental room one at a time. Final payment was determined as any money left from the 'competition' out of the initial 500 points endowment, plus the eventual prize if any, plus the £2 participation fee.

There was only one structural difference between HV's Section 4 model and the experimental setting: subjects faced a budget constraint (the 500 points given at

the start of the competition), which is missing in the stylized framework of HV.⁹ The presence of a budget constraint was clearly unavoidable for practical reasons. Also, R&D departments in real world companies do face budget constraints, and so one may argue that in this respect the experimental setting is more realistic than the theoretical model.

Nevertheless, it was important to minimize the possibility of distortions that might occur in the (later rounds of the) experiment as a result of a binding budget constraint. In order to achieve this, the instructions stressed the need for subjects to retain enough money to fund the later rounds of the competition in an unconstrained way. The enclosed investment table gave additional guidance on this by providing subjects information on feasible investment levels for 10 and 20 rounds¹⁰ (see Appendix A).

With two exceptions, all participants were (undergraduate or postgraduate) students, and about half of them (49.65%) had participated in prior different experiments. Some of them (19.76%) had some economics background, but only three were postgraduates, and with no special competence in industrial economics.¹¹ There was a slight majority of males (61.51%) and of ‘hard sciences’ students (58.93%). The median age was 23 years, possibly older than that of student samples typically used in experiments (min: 18; max: 32; mean: 23.54; S.D.: 3.84).

3. Univariate analysis

3.1. General results

In the real experiment, with financial incentives and after the practice, the 36 subjects played a total of 582 rounds in dyads, for an average of 16.1 rounds per dyad (min=12, max=19, S.D.=2.42). Subjects were tied in 18.90% of the rounds, and were either leaders or followers in the remaining rounds. With one exception, all subjects varied their investment efforts during their race: the median investment was 4.5, and the mean 4.06, with a S.D. of 2.15. Fig. 2 contains a histogram of the distribution of investment: 95.02% of decisions were concentrated at 7 or below, where 7 was the maximum feasible investment for 10

⁹The experiment had also a procedure to terminate the stage in case of ‘inaction’ (both players investing less than 1 for three consecutive rounds: see the experiment instructions in Appendix A for details). This procedure never had to be used.

¹⁰With no draws, the experiment could last at most 19 rounds (at a minimum, 10).

¹¹Two were Master students in development economics, and one was a research student specialized in income distribution.

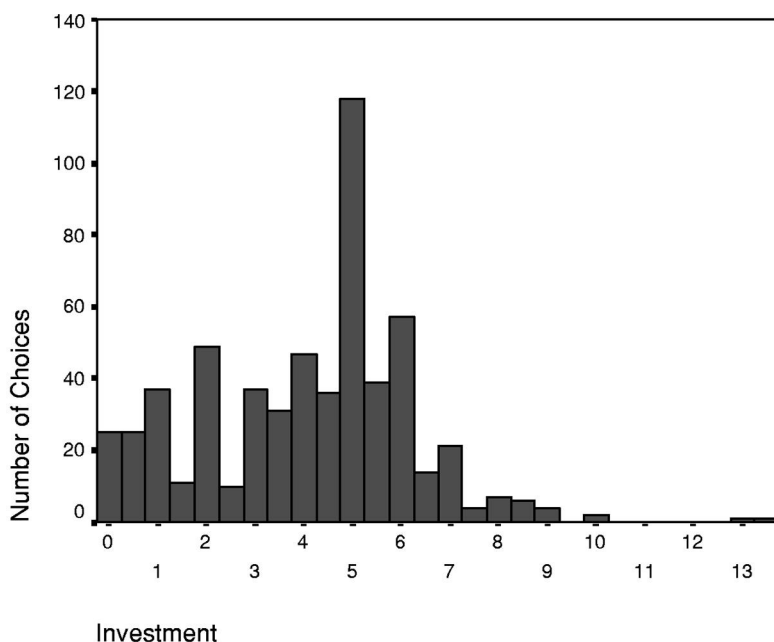


Fig. 2. Histogram of the distribution of investment, full sample ($n=582$).

consecutive rounds; 68.38% of decisions were concentrated at 5 or below, where 5 was the maximum feasible effort for 20 consecutive rounds.

If subjects typically ran out of money in the later part of the experiment, we would expect investments to decrease in the later rounds. However, this is not the case. If we consider the distribution of investment for round 13 onwards (i.e. on average, for the last 25% of the patent races), the median investment increases to 5, while the mean is 4.81. This is significantly higher than 4.01 (in a two-tailed t -test, $t=3.79$, $df=349$, $P<0.0005$).

More generally, there is a positive correlation between investment and round (Spearman's $\rho=0.232$, $P<0.0005$). This suggests that the budget constraint may not have been a problem seriously distorting choices in the later part of the experiment, although this possibility should still be checked on each occasion.

Subjects with an economic or mathematical background invested significantly more (respectively: $\rho=0.085$, $P<0.05$, two-tailed; $\rho=0.111$, $P<0.01$, two-tailed). Older people invested less ($\rho=-0.190$, $P<0.0005$, two-tailed). There was a positive correlation between average amount invested in the practice stage and that invested in the actual experiment¹² ($\rho=0.261$, $P<0.0005$, two-tailed).

¹²This possibly proxies inter-individual differences in risk attitude.

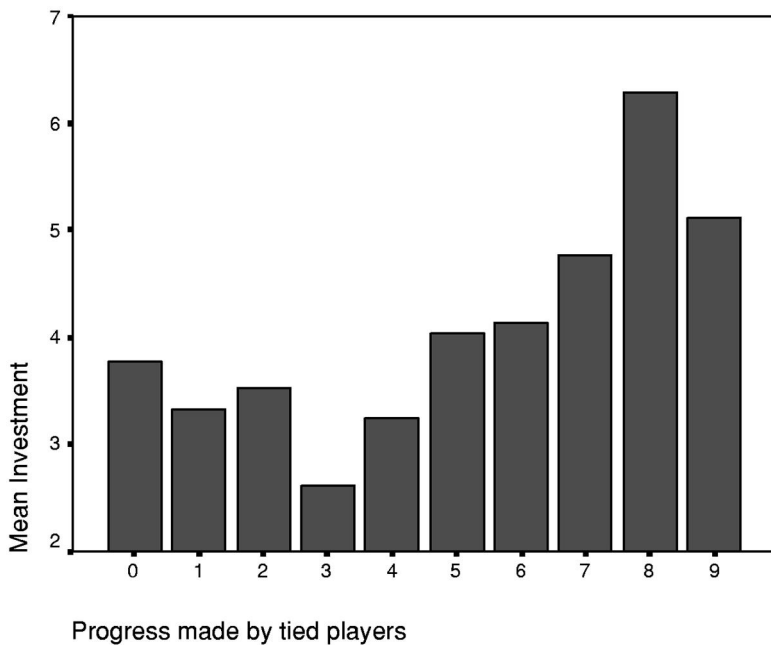


Fig. 3. Mean investment by tied competitors with different levels of progress already made.

3.2. Tied competitors and progress in the race

Fig. 3 shows how investment by tied competitors varies with the number of progress steps already made.

There is a significant positive aggregate relationship between progress and investment (Spearman's $\rho=0.207$, $P<0.02$, one-tailed). This is equivalent to a negative relationship between m and investment, which, as we saw earlier, is consistent with the model with a quadratic cost function.

There is no support for the prediction that subjects invest more if they are tied with one progress step to go than if they are tied with two steps to go: investment was actually higher in the latter case, albeit insignificantly so. However, one cannot exclude in principle the possibility that subjects were more budget-constrained in the case with only one progress step. In order to see whether this was the case, let us construct a binding index (BI) to measure how binding the budget constraint is for a subject: let 'progress' be the number of progress steps completed, and let $BI = \text{score} / (10 - \text{progress})$. In other words, BI is equal to the maximum amount that a subject can possibly spend on average each round if she is to win the race. Intuitively, the greater the score the less binding is a subject's choice; however, the greater the number of progress steps to go, the less a subject

can spend each round, as the money has to last for a greater number of rounds. The greater BI, the less constrained the choice of a subject is. The average BI is 82.504 when players are tied with two steps to go, but just 39.113 when they are tied with one step to go: notwithstanding the small sample size ($n=6$ for each case), the difference is significant ($t=2.365$, $df=10$, $P<0.05$, two-tailed). Hence, tied players with one step to go were more budget-constrained than tied players with two steps to go.¹³

In conclusion, the general finding of a positive relationship between progress and investment for tied competitors is consistent with the HV model, whereas the predictive failure for the cases in which $m=1$ and $m=2$ might be due to the differential impact of the budget constraint.

3.3. *Do leaders invest more than followers?*

A main prediction of HV is that leaders invest more than followers. We first restrict ourselves to the case in which $m \leq 2$, where this result was derived analytically. Fig. 4 depicts the investment of the leader and of the follower for this case, as a function of the gap between leader and follower. While a decline in investment is apparent for both leaders and followers with an increase in the gap size, it is not obvious that leaders are investing more than followers. The average investment of leaders is 4.48, versus an average investment of followers of 4.73, so the reverse is actually true (albeit insignificantly so in a t -test). The only exception from Fig. 4 appears to apply to the extreme case where the gap is as large as 7, but in general leaders do not seem to invest more than followers.

An objection may be that perhaps leaders suffer more from a binding budget constraint than followers, and this would explain the predictive failure of the model in our experimental set-up. I tried to address this objection with two tests. Firstly, a rough test may be to limit the sample to the observations where the leaders had no more than two progress steps to go as before but also a remaining score of at least 100 points. Even so, the average effort of the leader was 4.44, versus 4.53 in the case of followers.

A more sophisticated test relies on restricting the sample to leaders with $m \leq 2$ and with a BI value greater than their followers: this ensures that the choices of these leaders were less constrained, if anything, than those of the followers.¹⁴ This does not change the nature of the result: the average effort of these (less

¹³It would be nice to check whether budget constraint concerns were indeed responsible for the predictive failure of the model. Unfortunately, the sample size is too small to investigate this.

¹⁴It is worth noting that any monotonic transformation of the BI function would produce the same sample restriction, for all it matters is that $BI(\text{Leader}) > BI(\text{Follower})$. In particular, using BI indexes that took the possibility of losing bets into account and so used a number higher than 10 on the denominator would not make any difference.

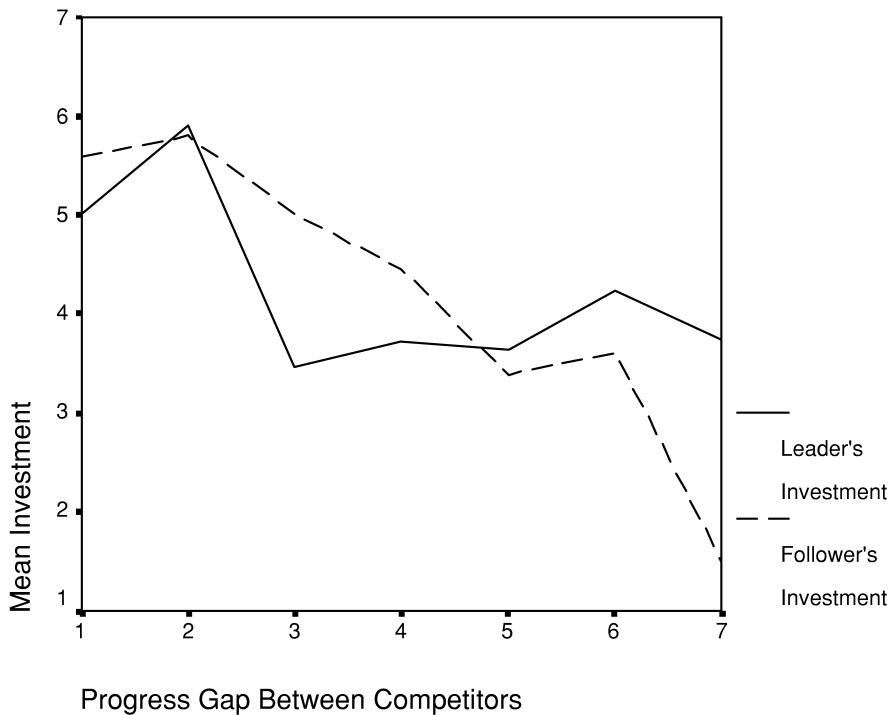


Fig. 4. Investment as a function of the gap between leader and follower, when the leader has two or less progress steps from winning the race.

constrained) leaders was 4.59, versus an average effort of the (more constrained) followers of 4.69.

As we discussed earlier, in the case of a quadratic cost function leaders should invest more than followers in general, even without the restriction $m \leq 2$. Fig. 5 depicts how investment changed with the size of the gap: if in general leaders invested more than followers, we would expect the part of the line to the right of zero to be at a higher level than that to the left of zero.

It is true that there was less investment in the case of heavily lagging followers, i.e. followers who were five or more progress steps behind. We shall discuss this result again below. However, there is no sign of a trend in investment for gaps ranging from -4 to $+7$: as a result, a t -test on the mean effort of leaders (4.019) vs. followers (4.131) is insignificant ($t=0.615$, $df=580$, $P=0.270$ in a one-tailed test). Again, things do not improve if we consider only leaders with a score of at least 100 points (average effort for leaders: 4.106; for followers: 3.968; $t=0.758$, $df=554$, $P=0.278$, one-tailed), or leaders who are less budget-constrained than followers in terms of their respective binding index (average effort for leaders: 4.083; for followers: 3.987; $t=0.488$, $df=477$, $P=0.313$, one-tailed).

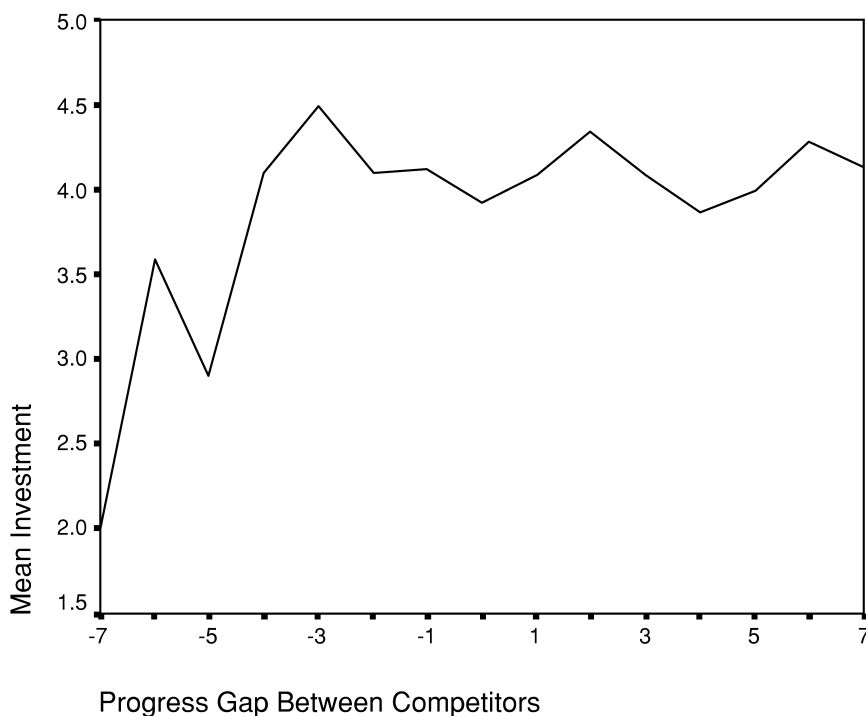


Fig. 5. Investment as a function of the gap between competitors.

In conclusion, in general leaders do not appear to invest more than followers. The only possible exception concerns heavily lagging followers: we shall now see in more detail how investment varies with the gap in progress between competitors.

3.4. Gaps and investment efforts

Let us consider the sample with $m \leq 2$ again: Fig. 4 shows a negative relationship between investment and size of the gap. The relationship is stronger for followers (Spearman's $\rho = -0.404$, $P < 0.0005$, one-tailed), but applies also to leaders (Spearman's $\rho = -0.334$, $P < 0.0005$, one-tailed). Both the direction of the relationship, and the fact that the relationship is stronger in the case of followers than in the case of leaders, are consistent with the HV model. According to the model, as the gap between leaders and followers increases, the followers invest less, while the leader may or may not invest less, though when it starts investing less it does so monotonically.

Let us now consider the full sample. As we discussed earlier, there is a drop in investment when the gap becomes sufficiently large, but not otherwise (Fig. 5). I

tried to correlate investment with a few possible measures: ‘gap’ (the value of the gap), ‘positive gap’ (the value of the gap, when positive, else zero), ‘negative gap’ (the absolute value of the gap, when negative, else zero), ‘positive squared gap’ and ‘negative squared gap’. All Spearman correlations are insignificant: however, this may be simply because of the nonparametric nature of the Spearman correlation measure. If we use instead Pearson correlations, the correlation between negative squared gap and investment is significant (Pearson’s $r = -0.071$, $P < 0.05$, one-tailed): it picks up the fall in investment by heavily lagging followers. None of the other values are significant (e.g. the Pearson correlation between gap and investment is only 0.032, $P = 0.218$, one-tailed). Eliminating the subjects with a score less than 100 does not help: the Pearson correlation between investment effort and negative squared gap becomes -0.066 ($P < 0.07$, one-tailed), while all other correlations remain insignificant.

Let us partition the sample according to whether a subject is ‘close’ in the race to the competitor — in the sense of having a (positive or negative) gap of no more than three progress steps — or not. On average $BI = 64.65$ (S.E. = 1.48) for ‘far away’ players, while $BI = 92.947$ (S.E. = 8.48) for ‘close’ players. Therefore, it is possible that, in the full sample, ‘close’ competitors were not able to invest more because they were relatively more budget-constrained than ‘far away’ competitors, and this would explain the ambiguity in the results. In order to check whether this is the case, I truncated the sample of ‘far away’ subjects to those whose BI is lower than 67.61 (i.e. two S.E. above the average BI for ‘close’ subjects): the truncated sample has a mean of 45.4 (S.E. = 1.6). Hence, if anything, the budget constraint is more binding for the ‘far away’ than for the ‘close’ subjects. If the budget constraint hypothesis is correct, we should now be able to see unequivocally more investment on the part of ‘close’ subjects. Unfortunately, this is not the case: the investment of ‘close’ players is just slightly above that of ‘far away’ players (4.125 vs. 3.983), and the difference is insignificant ($t = 0.445$, $df = 534$, $P = 0.329$). Further, all correlations (both Spearman and Pearson) between investment and gap measures are now insignificant. Budget constraint considerations cannot explain the limited predictive success of the HV model in this case.

In conclusion, there is evidence for the prediction that, for $m \leq 2$, investments increase as the gap between competitors decreases. For the more general case, this is not true, except perhaps for heavily lagging followers. Here the puzzle does not really lie with the insignificance of *positive* gaps, because HV were careful to stress that, in the general case, leaders may even increase their investments as the gap increases. Rather, the puzzle lies with the fact, apparent from Fig. 5, that only heavily lagging followers decrease their investment.

3.5. *Is there a virtual monopoly of investment when the gap is large?*

HV predict that, when the gap becomes large enough (three or four steps with $\eta = 2$), the leader does virtually all of the investment. This does not appear to be

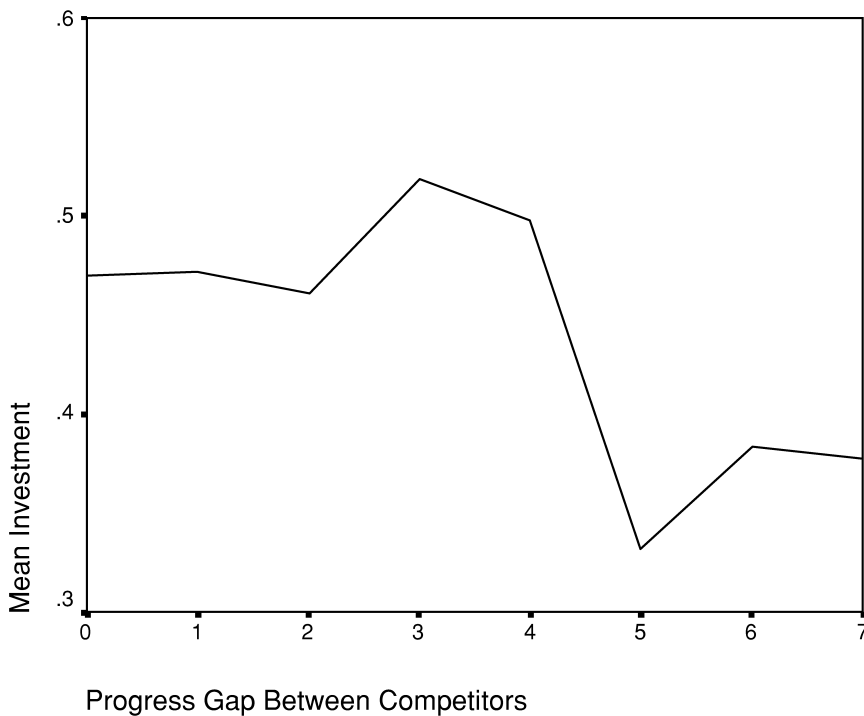


Fig. 6. Investment by the follower as a percentage of overall investment, as a function of the gap between competitors.

true in our sample. Fig. 6 shows the average investment of the follower as a fraction of the sum of the investments of both competitors: the follower appears to contribute 1/3 or more of the aggregate investment. The lowest investment percentage by the follower is when she trails by five progress steps, and is still 34.60%; aggregating the 25 observations with a gap between 5 and 7, the average investment is 36.16%. The lower bound of the 95% confidence interval is 25.74%.

Removing the leaders with a score below 100, or considering only the cases in which the BI of the leaders is higher than the BI of the followers (i.e. leaders are less budget-constrained than the followers), does not change the nature of the results. In the first case, with a sample of 23 observations with a gap between 5 and 7, the lower bound of the 95% confidence interval drops by less than two points, to 24.68%; in the second case, to 24.21%.

One fourth or more is still a sizable fraction of the overall investment: the claim that the leader quickly becomes a virtual 'monopolist' of R&D investment as the lead extends is not supported by the data.

4. Multivariate analysis

4.1. HV predictions in regression models

Our previous tests of the predictions of the HV model have two limitations. Firstly, we ignored the presence of individual-specific effects (e.g. due to differences in risk attitudes). Secondly, only one factor was considered at a time. This section uses regression analysis to address these limitations simultaneously. We deal with the first concern by using random-effects regression models: it is not unreasonable to assume that the individual-specific components are randomly distributed across subjects, as this kind of model assumes (see Greene, 2000). We deal with the second concern by analyzing in a multivariate framework whether the amount invested can be predicted along the lines suggested by HV.

4.1.1. Progress and tied competitors

We discussed in Section 3.2 how HV predicts a positive correlation between investment of tied competitors and progress, i.e. the number of progress steps so far. If we define ‘tie’ as a dummy variable equal to 1 with tied competitors, and to 0 otherwise, we can translate this prediction in that of a positive coefficient on the interaction term $\text{progress} \times \text{tie}$.

4.1.2. Leaders

We use a dummy variable, ‘leader’, equal to 1 if the subject is a leader and to 0 otherwise, as a regressor. HV predicts a positive coefficient on leader.

4.1.3. Positive and negative gaps

As discussed in Section 1, HV predicts that the correlation between gap and investment should be more negative than for leaders. In our regression analysis, we use positive gap, equal (as it will be recalled from Section 3.4) to the size of the gap if positive, and otherwise equal to 0; negative gap, equal to the absolute value of the size of the gap if negative, and otherwise equal to 0; positive square gap and negative square gap, equal to the square of positive gap and negative gap, respectively. Square variables are introduced to allow for non-linearities: HV allows for the relationship between gap and leader’s investment to be positive up to a point. However that may be, HV would predict the overall impact of negative gaps on investment (through negative gap and/or negative square gap) to be negatively signed, and to be more negative than the impact of positive gaps on investment.

4.1.4. Virtual monopoly with large gap

From the size of the significant coefficients on the gap variables, we can also

check whether, for gaps of size three or four, the regression model predicts a virtual monopoly of investment for the leader, as it should according to HV.

We label all the variables discussed so far in this section as ‘HV variables’, since one can make predictions about their coefficients using the theoretical results in HV.

4.1.5. *Other variables*

Other regressors we consider are: ‘progress’, a potentially important variable albeit one for which HV do not infer general predictions from their model; ‘small score’, i.e. a dummy equal to 1 when the subject has less than 100 points, as a proxy for a binding budget constraint;¹⁵ ‘expany’, a dummy equal to 1 if the subject participated in previous economic experiments; some demographical variables, based on age, sex and educational background; some variables based on the practice stage, namely one own and the competitor’s average effort, the number of rounds, the number of rounds with a score below 100 points (NRoundSS), and the number of rounds with a score below 100 points if the subject has lost the practice stage (NRoundSSLoss).

4.2. *Results*

We present the results from five random-effects regressions in Table 1. Model 1 includes all the variables: a Breusch and Pagan LM test for random effects is significant ($\chi^2(1)=163.63$, $P<0.0005$), while a Hausman specification test is insignificant ($\chi^2(7)=8.18$, $P=0.317$), justifying the usage of random-effects models. Models 2 through 4 represent progressively more parsimonious specifications, with insignificant variables removed using F -tests. Model 5 is like model 4 but without ‘progress’: although this leads to a significant drop in goodness-of-fit,¹⁶ it might be useful for better interpreting the results from the other models. This is because there are nontrivial Pearson correlations between ‘progress’ and the HV variables (e.g. $r=0.511$ with positive gap and $r=-0.267$ with negative gap), and so it may be worth looking at the marginal effect of significant HV variables on investment without taking Progress into account.

4.2.1. *Progress in the race and tied competitors*

The univariate analysis in Section 3.2 pointed out a positive correlation between progress of tied competitors and investment. The multivariate analysis shows that

¹⁵ BI was not introduced because it would make the regression biased and inconsistent: BI is computed as $\text{score}/(10-\text{progress})$, and score is equal to $500-(\text{sum of past efforts of the subject})^2$: hence, BI would be correlated with $v(i)$ (and with the past values of $e(i)$). Small score was retained because the linkage with effort is less direct, and it was important to maintain at least some proxy for a more or less binding budget constraint to avoid omitted variable bias.

¹⁶ Indeed, model 5 can explain virtually no within-subject variance in investment.

Table 1
Random-effects regression models on investment

Explanatory variables	Model 1				Model 2				Model 3				Model 4				Model 5			
	Coeff.	S.E.	Prob.	Sign.	Coeff.	S.E.	Prob.	Sign.	Coeff.	S.E.	Prob.	Sign.	Coeff.	S.E.	Prob.	Sign.	Coeff.	S.E.	Prob.	Sign.
Progress	0.32	0.04	0	**	0.313	0.038	0	**	0.325	0.037	0	**	0.316	0.036	0	**				
Progress×tie	−0.083	0.06	0.169		−0.071	0.051	0.162		−0.101	0.046	0.03	*	−0.089	0.046	0.052		0.024	0.047	0.608	
Positive gap	−0.658	0.326	0.044	*	−0.489	0.089	0	**	−0.538	0.08	0	**	−0.511	0.078	0	**	−0.144	0.069	0.037	*
Negative gap	0.219	0.21	0.296		0.245	0.176	0.163													
Positive square gap	0.026	0.049	0.59																	
Negative square gap	−0.056	0.036	0.117		−0.06	0.032	0.063		−0.019	0.013	0.145									
Leader	0.074	0.48	0.877																	
Small score	−0.989	0.403	0.014	*	−0.934	0.402	0.02	*	−0.946	0.402	0.019	*	−0.899	0.402	0.025	*	0.327	0.402	0.415	
Sex	0.586	0.462	0.204		0.767	0.317	0.016	*	0.766	0.308	0.013	*	0.74	0.304	0.015	*	0.572	0.297	0.054	
Age	−0.041	0.054	0.446		−0.043	0.04	0.275													
Economics	1.162	0.922	0.208		1.22	0.757	0.107		1.36	0.72	0.059		1.566	0.698	0.025	*	1.811	0.686	0.008	**
Humanities	1.384	0.954	0.147		1.35	0.783	0.085		1.377	0.754	0.068		1.593	0.73	0.029	*	1.793	0.718	0.012	*
Sciences	0.881	0.906	0.331		1.171	0.711	0.1		1.254	0.678	0.065		1.481	0.652	0.023	*	1.744	0.641	0.007	**
Maths	0.348	0.504	0.491																	
Expany	−0.027	0.431	0.95																	
Avg. invest. in practice stage	0.884	0.245	0	**	0.768	0.141	0	**	0.779	0.135	0	**	0.787	0.134	0	**	0.771	0.131	0	**
Avg. invest. of competitor in pract. stage	−0.079	0.256	0.757																	
N rounds in practice stage	0.028	0.122	0.817																	
NRoundSS	−0.124	0.154	0.423																	
NRoundSSLoss	0.105	0.14	0.453																	
Constant	−0.679	3.072	0.825		−0.44	1.516	0.772		−1.47	0.945	0.12		−1.761	0.913	0.054		−0.948	0.893	0.288	
R^2 — within	0.144				0.143				0.14				0.135				0.009			
R^2 — between	0.554				0.529				0.512				0.522				0.548			
R^2 — overall	0.258				0.253				0.245				0.244				0.163			

Progress, number of progress steps made by a subject. Progress×tie, progress if subject is tied, otherwise 0. Positive gap and negative gap, absolute value of gap between competitors, when this gap is positive (i.e. for leaders) or negative (i.e. for followers), respectively, and 0 otherwise. Positive (negative) square gap, square of positive (negative) gap. Leader, 1 for leaders, 0 for followers. Small score, 1 if a subject has less than 100 points, and 0 otherwise. Sex, 1 for males, 0 for females. Age, age in years. Economics, 1 for students taking an economics-related course, 0 otherwise (similarly for other educational variables). Expany, 1 if a subject had done a previous economic experiment, and 0 otherwise. Avg. invest. (of competitor) in practice stage, average investment (of the competitor) in the practice stage. N rounds in practice stage, number of rounds in the practice stage. NRoundSS, number of rounds in the practice stage in which the subject had less than 100 points. NRoundSSLoss, NRoundSS if the subject lost the practice stage, and 0 otherwise. ** and * stand for significance at the 0.05 and 0.1 levels, respectively.

the relevance of this result has to be qualified: in models 1 through 4, the marginal effect of progress of tied competitors on investment is positive, but *only* through the coefficient on progress; the coefficient on progress \times tie is actually negative, pointing if anything to a smaller effect when competitors are tied, a puzzling result for HV; finally, if we remove progress, the coefficient on progress \times tie is insignificantly different from zero. The results do not appear favourable to HV, though this might be due to the lack of general results on progress in their paper, rather than to a descriptive failure of their model.

4.2.2. *Leaders*

The coefficient on leader is insignificant, and the variable can be easily dropped out using *F*-tests. Exactly as in the univariate analysis, and contrarily to HV, there is no evidence that leaders invested more than followers.

4.2.3. *Positive and negative gaps*

Models 4 and 5 give the clearest picture of the effects of progress gaps on investment: positive squared gap, negative gap and negative squared gap have been dropped on the way because they are insignificant.¹⁷ Against HV, we are not able to find a significant negative correlation between gap investment for followers, nor a larger one for followers rather than leaders: to the contrary, there is a significant negative coefficient on positive gap, and, as shown by model 5, not all of it can be explained by covariations between progress and positive gap.

4.2.4. *Virtual monopoly with large gap*

In the light of the poor performance of negative gap and negative squared gap in the multivariate analysis, the prediction of a virtual monopoly by the leader with a sufficiently large gap relative to the follower is also not corroborated. This is easy to see for models 4 and 5, where there are no remaining gap variables from the perspective of the follower. The best case that can be made for a virtual monopoly by the leader comes from model 3, which retains negative square gap as one of its regressors.¹⁸ Table 2 uses model 3's coefficients to estimate the marginal impact of a gap on investment, taking into account not only the gap variables but also the extra progress made by the leader.

¹⁷Negative squared gap is significant at a very liberal 0.15 level in model 3. On the one hand, it might capture the dip in investment for very large negative gaps, as displayed by Fig. 5. On the other hand, however, if one drops progress from model 3, negative squared gap becomes entirely insignificant ($\beta = -0.0003$, S.E. = 0.014, $P = 0.98$).

¹⁸See the previous footnote for a discussion of this.

Table 2
Model 3's effect of progress gap on the investment by leaders and followers

Absolute size of gap	Net effect for follower	Net effect for leader
0	0	0
1	−0.019	−0.212
2	−0.077	−0.425
3	−0.172	−0.637
4	−0.307	−0.85
5	−0.479	−1.062
6	−0.69	−1.275
7	−0.939	−1.487

The effect is estimated on the basis of model 3 in Table 1, taking the coefficients on progress, positive gap and negative square gap into account. Note that the coefficient on negative square gap is significant only at the 0.15 significance level. Model 3 is used because it gives a best chance to the HV predictions on the effects of progress gaps, not because it is claimed to be the best specification.

Even for gaps as large as seven progress steps, Table 2 predicts greater investment by the followers than by the leaders.

4.2.5. Other results

A small score means a binding budget constraint and, unsurprisingly, led to less investment. Subjects who were not economics, humanities or hard sciences students (e.g. non students, or non economic social scientists) invested less. The 'sex' dummy is equal to 1 for males, so the positive coefficient on sex suggests that males invested significantly more. This may be in agreement with Powell and Ansic's (1997) experimental finding that women tend to be more risk-averse. Similarly, the significant coefficient on the average investment in the practice stage probably proxies for individual-specific differences in preferences towards risk-taking.

4.3. Implications

HV's performance is quite disappointing in our multivariate analysis, and possibly more so than in our univariate analysis in Section 3. It is an open issue what the effect of progress *in general* should be in a patent race model such as that envisaged by HV; however, model 5 testifies to the fact that progress cannot be blamed for, say, our inability to find an independent progress \times tie effect. It is possible that, along the lines of what we conjectured in Section 3.2, a binding budget constraint may reduce the size of the coefficient on progress \times tie. Nevertheless, the multivariate analysis makes this slightly less plausible, since small score *is* at least a partial proxy for a binding budget constraint.

5. Conclusions

This paper presents an experimental test of Harris and Vickers' (1987) multi-stage patent race model, a model combining both technological and dynamic uncertainty. Only limited support for the model was found.

In the experiment, with two or less progress steps to go on the part of the leader, investment increased as the gap between competitors decreased. In addition, tied competitors who were more ahead in the race invested more: while this is in agreement with the model, it can also be considered as following from a more general finding of a positive correlation between investment and progress in the race; this more general correlation is not explained by — though is not inconsistent with — Harris and Vickers.

Other predictions of the model did not appear to find support in the data. Followers did not display a negative correlation between investment and gap, nor a more negative correlation than for leaders. The only possible exception, emerging from the univariate but not so clearly from the multivariate analysis, was in the case of heavily lagging followers: however, even they contributed at least one fourth or more of the investment. Therefore, in contrast to the theoretical prediction, it was not true that a virtual monopoly was achieved as the lead extended. It was also not true that leaders generally invested more than followers.

These results obviously present a puzzle for economists who believe in patent race theory as a suitable approximation to real-world patent races. There might be at least two answers to the puzzle (and possibly more).

Firstly, there may be intrinsic limitations in the experimental design preventing it from being a perfect test of the theory. We tried to control throughout for the necessary presence of a budget constraint in our design. By allowing for random effects and individual-specific characteristics, in the multivariate analysis we also controlled for interindividual psychological differences, that might make more difficult the mapping of our experimental data to the theoretical framework. Nevertheless, no claim is here being made of having provided a definitive answer to these delicate issues of correspondence between theory and evidence.

Still, since decision-makers in R&D departments might be different in, say, risk preferences, and budget constraints do exist for companies in the real world, this lack of correspondence might suggest a need to develop the theory further. One can only wonder whether, for example, further developments in the modelling of patent races would be able to explain the strong positive correlation between investment and progress in the race, not only on the part of tied competitors.

Secondly, it is quite possible that the degree of rationality that we expect of economic agents in behaving according to the optimal strategy may simply be too large, and that this may explain the predictive failure of the model. This may be true even if we admit that optimal play in a patent race is to be considered a matter of implicit rather than explicit knowledge (e.g. Zizzo, 2000). Explicit knowledge is abstract knowledge that subjects can explicitly reason on (e.g. that of a physicist

able to analyze formally the dynamics of a game of billiards); implicit knowledge is a set of behavioral patterns and skills that a subject has acquired, for example by reinforcement learning, without any need for explicit reasoning (e.g. that of a champion billiard player). It is clear that the explicit knowledge required to handle patent races optimally is daunting, in the light of the difficulty of the dynamic programming exercise; however, it may very well be the case that the optimal strategy is also complex enough so as to prevent subjects from conforming to it using their implicit set of bounded-rational rules.¹⁹

It is left for future research to adjudicate among explanations of our puzzling results. This might require further experimental work. It might also require, on the one hand, more modelling research in the dynamics of multi-stage stochastic patent races, and, on the other hand, an attempt to flesh out a bounded-rational model that was suitable to analyze data from experimental patent races.²⁰

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Appendix A. Experiment instructions

A.1. General guidelines

In this experiment you will use the computer to read information and make decisions. You will be asked to make decisions by entering a number in the cell on the bottom-left corner of the screen and clicking some buttons. To input or change numbers, click the mouse pointer in the cell. You will then be able to type or erase numbers in the cell using the keyboard. Please always remember to type numbers as digits (say, 50) rather than as letters (say, fifty). You can give commands to the computer by clicking on the buttons at the appropriate times. To press a button, click on it with the mouse pointer.

IMPORTANT: please do not try to exit the experiment program even temporarily. You are not allowed to speak to any of the participants in the experiment at any time. You are also not allowed to get up from your seat before the end of the

¹⁹For a case where this holds, in the context of static normal-form game play, see Zizzo and Sgroi (2000).

²⁰For a recent bounded-rational perspective on the theory of the firm, see Furubotn (2001).

experiment. If you have a query that the instructions are unable to solve, please raise your hand and we shall do our best to solve it with a low voice. Failure to comply with these instructions will imply the termination of the experiment and the loss of all gains. Your patience is much appreciated.

After you read these instructions, you should fill the short enclosed questionnaire to check your understanding of the instructions. The only purpose of the questionnaire is to make sure that everything is clear before you start the experiment. Please do not start completing the questionnaire before having read and understood the instructions.

The experiment itself is divided into two stages, a practice stage and a ‘real’ stage. The two stages are identical in almost all respects: they each constitute a *prize competition*, in a sense that will be discussed shortly. There are only two differences between the two stages:

1. *you play the competition against a different player in each stage*: player 1 is matched with player 3 in the practice stage, but with player 4 in the real stage; player 2 is matched with player 4 in the practice stage, but with player 3 in the real stage. *Nothing of what you do in the practice stage will be told to your future opponent*, and similarly you will learn nothing about the practice stage choices of your future opponent;

2. *the real stage is played for real money*, namely each point you have at the end of the real stage is converted into 1 UK penny in the payment phase immediately afterwards; so, if you have 1230 points, it means that you won 12 pounds and 30 pence. Instead, *the practice stage is played only for practice, not for real money*.

At the end of the experiment, you will be earning whatever amounts you have from the real stage in the experiment, plus £2 of participation token.

A.2. *The prize competition*

In the prize competition, you and the other player compete for a prize of 1000 points (i.e. £10 in the real stage). In order to win the prize, you have to make a certain number of ‘progress steps’. Your progress and that of the other player is shown and updated round after round in the white quadrant in the screen. At the start of the competition the quadrant shows no progress on the part of both you and the other player. As you make progress, the computer will display coloured blocks according to the number of progress steps you have made, and similarly for the other player. So at any point in the game both you and the other player will know what your respective progress is in the competition.

The prize is won by whoever makes 10 progress steps first. How do you make progress? At the start of the competition, *you and the other player receive 500 points each*. Every round, you decide how much investment you want to make,

and you put this number in the box to the bottom left of the screen. You have to pay for the investment you do with your money. *The cost of your investment is equal to the square of the amount invested*, i.e. put it differently, $\text{cost} = (\text{investment}) \times (\text{investment})$. But don't worry: you need not make any computations yourself! Enclosed with these instructions you will find a table, that tells you how much your investment would cost you. Also, if you put a number in the box and you click View, the computer will tell you exactly how much the amount would cost you — and nothing else will happen; in other words, as long as you click View, you can just compute the cost of each possible investment, without taking any actual decision!

When you are happy with your investment, click OK and then OK again to confirm (if you click OK only once, and you are unhappy with the choice, you can click Cancel to cancel it). The computer will tell you to wait, in order to check whether the other player has made his or her choice. Please wait *at least* 10 seconds between clicks on the message box telling you to wait, because too frequent clicks may cause computer crashes.

After the computers learn the investment choices by you and the other player, they will compute your probability of winning the round by dividing the amount you invested by the sum of the amounts invested by you and the other player. For example, if you invested 2 and the other player invested 3, you will have a $2/(2+3) = 2/5 = 40\%$ chance to win the round.

What this means is that *the greater the amount you invest relative to the amount invested by the other player, the greater the probability of winning the round*. You will be told what is your probability of winning the round, and then the computer will determine the winner randomly according to this probability. *Whoever wins the round makes a progress step*.

You will then need to decide your investment for the following round, and the competition will continue in the same fashion until either you or the other player has made 10 progress steps and won the prize.

A.3. *Additional rules and tips*

In general, you are free to invest whatever amount you want each round, or no amount at all. If you invest nothing, and the other player invests something, he or she wins the round with probability 100%, i.e. for certain. If neither of you invests anything, the round is a draw, and neither player makes any progress.

The two following restrictions apply to your freedom of choosing any investment you want.

1. *Inaction*. If neither you nor the other player invest at least 1 point for three consecutive rounds, the competition is terminated, and the prize is lost.

2. *Budget*. You can invest as much as you like . . . as long as you have the money to invest! This does not only imply a restriction on the amount you can

invest each round. It also implies that *you should use your money carefully in order not to run out of money, or be left with very little money, in the later rounds of the competition*. If you run out of money, you can only invest zero points, and will be asked to do so to ensure the speedy completion of the experiment.

Any competition lasts at least 10 rounds, but more typically it will last *between 15 and 20 rounds*. You should make sure that you have enough funds for the later parts of the competition, in order for your choices not to be constrained — *ideally, for at least 20 rounds*. (Of course, you may still want to invest a relatively high amount in some particular round). The last two columns of the investment table are meant to help your planning.

If you have any doubt or query at any point in the experiment, please raise your hand and we shall be more than happy to help you.

Many thanks for participating in this experiment, and good luck!

Investment table

Investment	Cost	Number of points necessary to sustain this cost for 10 rounds	Number of points necessary to sustain this cost for 20 rounds
0	0	0	0
0.5	0.25	2.5	5
1	1	10	20
1.5	2.25	22.5	45
2	4	40	80
2.5	6.25	62.5	125
3	9	90	180
3.5	12.25	122.5	245
4	16	160	320
4.5	20.25	202.5	405
5	25	250	500
5.5	30.25	302.5	xxx
6	36	360	xxx
7	49	490	xxx
8	64	xxx	xxx
9	81	xxx	xxx
10	100	xxx	xxx
11	121	xxx	xxx
12	144	xxx	xxx
13	169	xxx	xxx

14	196	xxx	xxx
15	225	xxx	xxx
16	256	xxx	xxx
17	289	xxx	xxx
18	324	xxx	xxx
19	361	xxx	xxx
20	400	xxx	xxx
21	441	xxx	xxx
22	484	xxx	xxx
23	xxx	xxx	xxx

Investment: *this is the amount that you can decide to invest each round.* You can decide to invest also fractions that are not in the table, for example 3.75 or 2.3 points.

Cost: *the cost is equal to the square of the amount invested.* Put it differently, cost of investment = investment \times investment, i.e. investment multiplied by itself. For example, if you invest 2, the cost is $2 \times 2 = 4$ points.

You must have enough points to be able to fund the entire competition. Since the competition will usually last between 10 and 20 rounds, you should make sure that you have enough funds for the later parts of the competition — *ideally, for at least 20 rounds.* (Of course, you may still want to invest a higher than usual amount in some particular round). The last two columns are meant to help your planning.

Number of points necessary to sustain this cost for 10 rounds: this tells you how many points you need to spend to sustain the corresponding cost for 10 rounds. In our example above, with an investment of 2, the cost per round is 4, so the cost needed for 10 rounds is 40.

Number of points necessary to sustain this cost for 20 rounds: this tells you how many points you need to spend to sustain the corresponding cost for 20 rounds. In our example above, with an investment of 2, the cost per round is 4, so the cost needed for 20 rounds is 80. The rows corresponding to costs sustainable for 20 rounds are highlighted in bold in the table. xxx means that the number of points required is too high, i.e. it is above 500 points.

Please do not hesitate to ask if anything is unclear.

You can now start completing the questionnaire.

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