Races or Tournaments?

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Outline

The impact of contests on the economy

Historically, contests have had tremendous impact on economic growth.[1] Such an impact is perhaps more evident today, as contests are frequently used by all kinds of organizations (government agencies, philanthropic organizations, commercial firms) that recur to contests for promoting in-house R&D efforts, awarding bonuses or promotions to employees, or crowdsourcing solutions to the most vexing technical problems.[2]

[1] (1) reports evidence that awards offered by the Royal Agricultural Society accelerated agricoltural innovation in the period between 1839 and 1939; (2) documents the impact of the Orteig prize on the development of the aviation sector; XXXX for rail industries; XXXX the longitude prize for navigation in 1860.

[2] XXX, XXX, XXXX

Contest structure: races or tournaments

Despite the diversity of the organizations involved and their goals, it is striking that contest sponsors have come to use very similar competition formats. These are usually either the “race” (e.g., Longitude prize, Orteig Prize, Ansari X-Prize, Netflix prize) where the winner is the first to achieve a particular performance target, or the “tournament” (e.g., Architectural competitions, Golden Carrot Contest, DARPA Grand Challenge, Progressive Insurance Automotive X-Prize) where the winner is the competitor who has achieved the best performance by a fixed deadline.[3]

[3]Other options could be elimination tournaments, round robin, etc. need ref: XXXX

Why race or tournaments?

Contests are cost-effective because help to select most qualified competitors who will carry results without further supervision (3).[4]

Such positive selection, however, crucially depends upon a correct prize structure and appropriate limitations on competition such as "deadlines" or minimum "targets." A poor prize, a short deadline, a difficult target may lead to adverse selection, thereby softening competition in the contest.

The impact of deadlines and targets on competitors will differ whether the contest is a "tournament" or a "race."

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[4] A large literature has focused on the benefits of contests [monitoring costs, insurance, adverse selection, etc.].

Example: prizes to stop the spread of antibiotic resistance

Government cares about “quality” and “time” of solutions

- The UK government launches a tournament called “xxxx” in xxxx.
- The EU commission launches a race called “xxxx” in xxxx.
- Why choosing one or the other? Seem equivalent. But may have different effects on entry, competitors’ payoffs, total effort, etc.

In this paper . . .

We do xxx, yyy, zzzz

Outline

Contest models

Strategic equivalence:

- Contests are "equivalent" to all-pay auctions [ref.]
- Rent-seeking contests are "equivalent" to races [ref.]

However:

- contest designer's payoff may be very different!
- optimal prize structure?
- ...

Outline

Basic setup

Imagine $i = 1, ..., n$ players competing for $k = 1, ..., q$ prizes of value $v_1 \geq v_2 \geq ... \geq v_q$ (normalized $\sum v_k = 1$).

Players simultaneously choose quality y_i and time t_i (y_i/t_i speed).

Each player has an ability a_i drawn at random from a common cdf $F(\cdot)$ with pdf $f(\cdot)$.

The cost function $C(\cdot)$ is Cobb-Douglas:

$$C(a, q, t) = a^{\alpha} y^{\beta} t^{\gamma} \qquad \text{with } \alpha, \gamma \leq -1, \beta \geq 1 \tag{1}$$

- or denoting speed (y/t) by s:

$$C(a, q, t) = a^{\alpha} y^{\beta'} s^{\gamma'} \qquad \beta' = \beta + \gamma, \gamma' = -\gamma. \tag{2}$$

Payoffs

Player i's payoffs:

$$\pi_i = \sum_{k=1}^{q} p_k(y_i, t_i) v_k - C(a_i, y_i, t_i) \tag{3}$$

where $p_k(\cdot)$ is the prob. of winning prize k

Competition

- Let denote a deadline by t_0 and a minimum-quality target by y_0.
- We consider two competitive formats:
  - Race: competition with target where the first to achieve the target wins
  - Tournament: competition with deadline where the best wins

Probability

Let $y_{1:n}, ..., y_{n:n}$ denote the order statistics of the y's. Let denote the corresponding distribution functions by $F_{y_{1:n}}, ..., F_{y_{n:n}}$.

Then the conditional probability of winning the first prize in a tournament is

$$\Pr(y_i \geq y_{n-1:n-1}) = F_{y_{n-1:n-1}}(y_i) = F(y_i)^{n-1}$$

when $t_i \leq t_0$. And is zero otherwise.

$$\Pr(y_i xxxx) = [1 - F(y_i)]F(y_i)^{n-2}$$

Probability 2

If $a \sim \text{Uniform}(0,1)$, then:

$$p_1(y) = y^{n-1}, \qquad p_2(y) = [1-y]y^{n-2}$$

$$p_1(y)' = (n-1)y^{n-2}$$

$$p_2(y)' = -y^{n-2} + (1-y)(n-2)y^{n-3} = y^{n-3}[(1-y)(n-1)-1]$$

Contest designer's payoff

Contest designer is risk neutral and wants to max quality while min time of the winner.

Z is the competition format. Let denote the race by $Z = 1$ and the tournament by $Z = 0$. Let denote the winner's actions by (y^w, t^w).

The contest designer's expected payoff:

$$\pi_{cd} = E[y^w - \tau t^w \mid y^w \geq y_0, t^w \leq t_0, R]. \tag{4}$$

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Solution concept

We solve the model for its unique symmetric Perfect Bayes Nash Equilibrium (the "equilibrium").

Let denote equilibrium bidding functions with respect to ability by $t(\cdot)$ and $y(\cdot)$.

Consider Tournament first.

Maximization problem

- Key observation: $t_i = t_0$ is a (weakly) dominant strategy
- This simplifies the maximization problem to:

$$\max_y \hat{\pi} = \sum_{k=1}^{q} p_k(y)\hat{v}_k - a_i^\alpha y^\beta \tag{5}$$

with $\hat{v}_k$ denoting each prize v_k rescaled by a factor t_0^γ.

First order condition

For each $i = 1, ..., n$, first order conditions are:

$$\sum_{k=1}^{q} p'_k(y)\hat{v}_k = a_i^{\alpha} \beta y^{\beta-1} \tag{6}$$

Solving differential equation

Substituting the equilibrium function $y(\cdot)$ increasing in a_i and with inverse $\phi(\cdot)$, together with a "change of variable" (moving $a_i = \phi(y_i)$ to the lhs):

$$\phi^{-\alpha} \sum_{k=1}^{q} \hat{p}'_k(\phi)\phi' v_k = t_0^{\gamma} \beta y(a)^{\beta-1} \tag{7}$$

Integrating both sides (using the "chain of derivatives" on the lhs):

$$\sum_{k=1}^{q} \hat{v}_k \int_{a_0}^{a} p'_k(x) x^{\alpha} dx + \beta y(a_0)^{\beta-1} = \beta y(a)^{\beta-1} \tag{8}$$

Bidding function

For every $i = 1, ..., n$:

- Time $t(a) = t_0$
- Equilibrium quality y_i for competitor with ability a is given by:

$$y(a) = \left[y(a_0)^{\beta-1} + \frac{1}{\beta} \sum_{k=1}^{q} \hat{v}_k \int_{a_0}^{a} p'_k(x) x^{\alpha} dx \right]^{1/(\beta-1)} \tag{9}$$

with boundary condition $y(a_0) = 0$.

Example

If $a \sim \text{Uniform}(0,1)$ and $q = 2$

First integral:

$$v_1(n-1)\int_0^a x^{(n-2)-\alpha}dx = v_1\frac{a^{(n-1)-\alpha}}{(n-1)-\alpha}$$

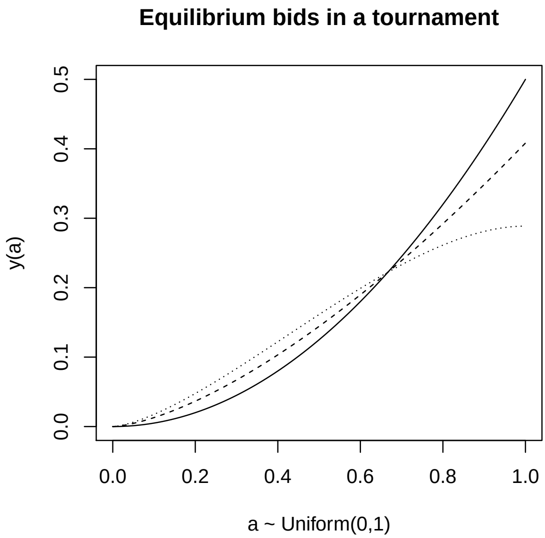
Second integral:

$$v_2\int_0^a x^{(n-3)-\alpha}[(1-x)(n-1)-1]dx$$

$$= v_2\frac{a^{-\alpha+n-2}((n-2)(-\alpha+n-1)-a(n-1)(-\alpha+n-2))}{(-\alpha+n-2)(-\alpha+n-1)}$$

Impose normalization: $t_0^\gamma = 1$.

Example 2


Equilibrium bids in a tournament
y(a)
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
a ~ Uniform(0,1)

Outline

Bidding function

For every $i = 1, ..., n$ with $a_i \geq \hat{a}$

- Quality $y(a) = y_0$
- Time:

$$t(a) = \left[t(t_0)^{\gamma-1} + \frac{1}{\gamma} \sum_{k=1}^{q} \tilde{v}_k \int_{a_0}^{a} \hat{p}'_k(x) x^{\alpha} dx \right]^{1/(\gamma-1)} \tag{10}$$

with $\tilde{v}_k = v_k / y_0^{\beta}$.

Otherwise, when $a_i < \hat{a}$, and $y(a) < y_0$.

Zero profit

The zero profit condition for the marginal player:

$$\sum p_k(y_0, t_0)v_k = \hat{a}^\alpha y_0^\beta t_0^\gamma \tag{11}$$

Hence, the marginal ability is pinned down:

$$\hat{a} = \left[\sum p_k(y_0, t_0)v_k / y_0^\beta t_0^\gamma\right]^{1/\alpha} \tag{12}$$

Example 2

If $a \sim \text{Uniform}(0,1)$, then ZPC

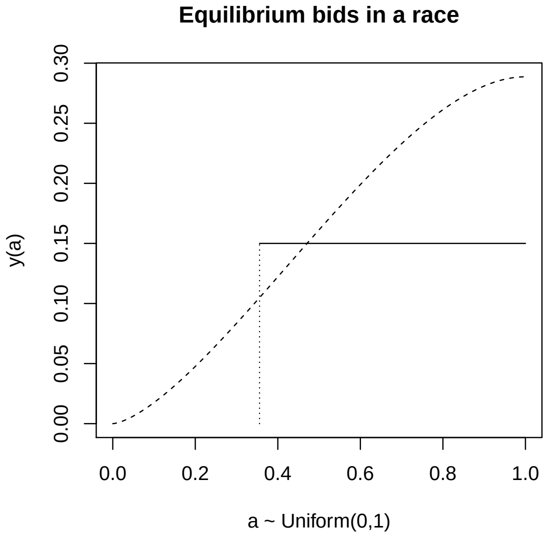
$$p_1(y) = y^{n-1}, \qquad p_2(y) = [1-y]y^{n-2}$$

$(t_0^\gamma = 1)$

$$\pi_i = v_1 p_1(a) + v_2 p_2(a) - a^\alpha y_0^\beta$$

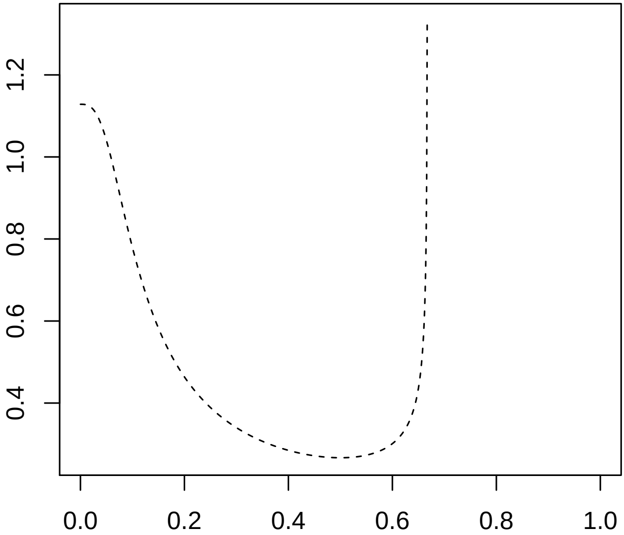
$$(ZPC) \qquad v_1 a^{n-1} + (1-v_1)[1-a]a^{n-2} - a^\alpha y_0^\beta = 0$$

Example



Equilibrium bids in a race
y(a)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.2
0.4
0.6
0.8
1.0
a ~ Uniform(0,1)

Time



Outline

Comparing bidding functions

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Differences

1. Not payoff equivalent for those at the bottom of the ability distribution.
2. Ex-post quality there is no dominance
3. Race ex-post dominates tournament wrt time
4. Even seemingly similar competitions lead to behavior very different

Outline

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Data

Outline

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References I

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3. Fullerton RL, McAfee RP (1999) Auctionin Entry into Tournaments. *Journal of Political Economy* 107(3):573–605.