Races or Tournaments?*

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Abstract

A wide range of economic and social situations are decided by either a race or a tournament. In either situations, agents choose whether and how much to exert some costly effort to increase the probability of being awarded a prize under the uncertainty about the actions of other agents. While a tournament yield outcomes greater than those of a race, the latter prevents unnecessary costs due to an excess of participation. We examine this trade-off empirically. We report the results of a field experiment conducted online where we compare the outcomes — efforts, quality, and diversity of outputs — of three alternative competitive situations motivated by theory: the race, the tournament, and the tournament with a quality requirement.

JEL Classification: xxx; xxx; xxx.

Keywords: xxxx; xxxx xxxx.

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1 Introduction

Organizations and governments have long used prizes to drive innovation and raise workers levels of effort. There are two extreme forms of prize-based competitions: the race, where the timing is important, and the tournament, where the timing does not matter. The growing number of exapmles using both competitions makes questions, when races should be used, and what are the xxxxx.

In this study, we xxxx.

To be sure, the distinction between a race and a tournament has been known since the ancient Greece xxxx. However, understanding the implications is relatively new. This question is also a contest design approach. The design of a contest has received much attention among economists. Economists have a long tradition in studying races of various kinds (patent and arms races), and tournaments (sales tournaments). A large body of works have investigated several aspects of contest design, including the optimal prize structure [XXX], number of competitors [XXX], and the appeal of imposing minimum effort requirements [XXX]. The race vs tournament question may be a bit unusual. Seldom economists have considered the competitive setting being either a race or a tournament as the result of a deliberate choice of contest design. Usually the nature of the competition is seen as exogenous and due to the environemental factors (for example, in a patent race, the type of competition is determined by the legal environment and cannot be easily changed by policy makers). One important exception is Baye and Hoppe (2003) that identifies a strategic equivalence between games of complete information modeled as races and tournaments. This paper is important because connects extends to races the results from a wide literature on the optimal design of a tournament. However, it does not shed light on when it would be desirable to use one or the other.

[This is way our study is relevant] Contests are a very important source of incentives in the economy. In the United States, the government routinely sponsors open programming competitions to solving a variety of issues of public health, education, energy, environment protection, and so on. A large part of which are conducted online (through the web portal Challenge.gov), in the spirt of open innovation initiatives as in xxxx, xxxx, xxxx. Contests are also used extensively in the private sector. A typical use is to expand a firm's innovative capacity by sourcing ideas from outside the boundaries of the organization or in the form of bonuses or promotion mechanisms leading workers to exert higher levels of effort.

To address these questions we proceed in two ways. First, we generalize the well known incomplete information contest model of Moldovanu and Sela (2001). This is enables a direct comparison of equlibrium behaviors under both the race and the tournament within a single theoretical framework. Then, we collect data from a field experiment that we desiged and run to test some of the implications of the theory, and provide policy recommendations.

By this perspective, we are able to show that races cannot be justified simply by the goal of maximizing average effort. And the reason is intuitive. A race awards a prize to first to hit a particular target. Those who will judge the target to hard to achieve will not join the competition and will drop out. On the contrary, those who are able to achieve the target at low costs will not try to exceed the target. As a result, the race is comparable to a competition with fixed "entry costs" or a fixed entry requirement, where agents will decide to either enter and pay a fixed prize, or stay out of the competition. Then, the possible gains in terms of expected revenues from a race are limited to those who would enter the competition and would exert less effort that that required to hit the target. These potential benefits can be obtained under a tournament as well by imposing a a fixed requirement to be eligible for prizes. So, races are not chosen to maximize expected effort of competitors, at least, in the traditional "auction-theoretical" sense.

What are races for? We examine a few hypothesis and we provide some examples. First hypothesis is that the sponsor of the race is not primarily interested in total output but also in the time to complete a particular task. In a tournament, this type of preferences can be satisfied by fixing a deadline. Say time within which competitors are asked to provide their efforts. However, assuming competitors have costs from making less time in performing a task and there complementarities in costs, increasing the deadline in a tournament is similar to raising the marginal cost for everyone, which might not be an optimal solution. In a race, by contrast, increasing the deadline will affect entry but, conditional on entry, the time to complete the task will always be less than the deadline. Which means that those with low costs will be mostly affected by the deadline, whereas xxxx. Which may be a superior choice than the tournament.

To fix ideas let consider the following example. The government wants to solve a global public health problem such as "antibiotics resistance." The overuse of antibiotics leads to the phenomenon of "resistance" which is a loss in the power of antibiotics to treat certain infections. This is an increasing threat for public health. The government has the choice of making a contest to engage people in solving this problem. The government has preferences for time in the sense that the government wants to minimize to have the first submission. So, the government fix a requirement in terms of costs of the solution and award a prize to the first to meet this requirement. Example. UK governemet goes for a race. EU xxxx goes for a tournament with a deadline in 2016. (...) The answer to this optimal design question relates to the cost function of agents with respect to “time” and to “effort.” It is hard to say which solution is better. However, it is easier to tell whether you should have one prize or multiple prizes.

There is also a case for efficiency. Consider a platform with many competitions. The platform may want to engage competitors for short period of time provide that solutions are above a certain quality level.

To test our theory we further examine experimental data on competitors making sumibssion in an online computer programming contest. We randomized competitors into 3 groups: 1. race 2. tournament 3. tournament with a quality requirement we study participation, timing of submission and final scores.

We find that, as our theory suggest, participation is higher in the tournament and lower in the race and in the tournament with entry costs. We further find that submission are quicker in a race, whereas are equally distributed at the end of the competition in the the tournament and in the tournament with quality requirement. With respect to final scores, theory predicts as trade-off between a race and a tournament in terms of higher scores vs faster submissions. We do find that scores are higher in the tournament but we do not find a strong trade-off in the sense that race had comparable good quality solutions than the tournament.

2 Literature

This paper is related to the contest theory literature Dixit (1987) Baye and Hoppe (2003), Parreiras and Rubinchik (2010), Moldovanu and Sela (2001), Moldovanu and Sela (2006), Siegel (2009), Siegel (2014). It also relates to the literature on innovation contests Taylor (1995), Che and Gale (2003). And the personnel economics approach to contests Lazear and Rosen (1981), Green and Stokey (1983), Mary et al. (1984).

Empirically, Dechenaux et al. (2014) provide a comprehensive summary of the experimental literature on contests and tourments. Large body of empirical works have focused on sports contests Szymanski (2003). More recently, inside firms (xxx) and online contest (xxxx).

This paper is also related to the econometrics of auctions Paarsch (1992), Laffont et al. (1995), Donald and Paarsch (1996) and more recently Athey et al. (2011), Athey and Haile (2002), and Athey and Haile (2007).

3 The model

def: A contest is a N player game. Players move simultaneously and select a performance variable y_i (the xxxx in performing a task) and a timing t_i (to complete a task), both being nonnegative numbers. Players are ranked according to their choices and a player's rank is given by the function $r_i(y_1, ...y_N, t_1, ..., t_N)$. The top K ranked players (e.g., $r_i < K$) are awarded a prize of value $V_1 > V_2 > ... > V_K$.

The goal for each player is to maximize the expected utility. If the timing is above a given deadline $d > 0$ or the performance is below a certain level $q > 0$, the player gets zero utility. Otherwise, the player is given a rank based on his performance and timing. A contest is xxxx. Ranked .

def: A tournament is a xxxx where players are ranked by their performance level provided that the timing is below a deadline d.

def: A race is a xxxx where players are ranked by their timing provided that the performance is above a threshold level q.

In a tournament, the agent having achieved the highest output quality within the deadline gets the first prize, the agent having achieved the second highest output quality gets the second prize, and so on. In a race, by contrast, the first agent to achieve an output quality of at least $\bar{y}$ within the deadline wins the first prize, the second to achieve the same target gets the second prize, and so on.

Since agents move simultaneously, they do not know the performance of others when deciding their efforts. On the other hand, it is assumed that they know the number of competitors as well as their cost functions to complete the task up to a factor a_i being the agent's private ability in performing the task. Each agent knows his ability but does not know the ability of the others. However, it is common knowledge that abilities are drawn at random from a common distribution F_A that is assumed everywhere differentiable on the support $V \subseteq [0, \infty)$.

It is further assumed that costs are multiplicative

$$C(y_i, t_i, a_i) = c_y(y) \cdot c_t(t) \cdot a_i^{-1}$$

with $c_y(0) \geq 0$, $c_y{}' > 0$, $c_t(d) \geq 0$, and $c_t{}' < 0$.

Each agent is risk neutral and faces the following decision problem

$$\text{maximize} \quad \textstyle\sum_{j=1}^{k} \Pr(\text{ranked } j\text{'th}) V_j - C(y_i, t_i, a_i).$$

3.0.1 Equilibrium in a tournament

We provide here the symmetric equilibrium with one prize and $n > 2$ agents. In appendix XXX, we provide a general formula for $k > 2$ prizes.

Let $y_{1:n} < y_{2:n} < ... < y_{n:n}$ denote the order statistics of the y_j's for every $j \neq i$ and let $F_{Y_{r:n}}(\cdot)$ and $f_{Y_{r:n}}(\cdot)$ denote the corresponding distribution and density for the r'th order statistic.

Proposition 1. *In a tournament, the unique symmetric equilibrium of the model gives, for every $i = 1, ..., n$, the optimal time to completion $t^*(a_i)$ equal to the deadline d and the optimal output quality $y^*(a_i)$ as*

$$y^*(a_i) = V_1 \int_{a_i}^{\infty} f_{Y_{n:n}}(z)dz$$

if $a_i \geq \underline{a}$ (see Moldovanu and Sela, 2001), and equal to zero otherwise.

An important property of is that $y^*(a_i)$ has its upper bound in and lower bound in . Also, equilibrium output quality is monotonic increasing in the agent's ability (see Moldovanu and Sela, 2001). Thus, for every $i = 1, ..., n+1$, the equilibrium expected reward depends only on the rank of his ability relative to the others. Using $F_{A_{r:n}}$ to denote the distribution of the r'th order statistic of abilities gives

$$F_{A_{n:n}}(a_i)V_1 - C(y_i^*, d, a_i).$$

Hence, by setting to zero and solving for the ability, gives the marginal ability $\underline{a}$ as

$$\underline{a} = h(n, V, F_A, C, d).$$

Corollary 1. *Equilibrium behavior in a race*

3.0.2 The expected revenues for the sponsor of the contest

The sponsor of the contest chooses the rules of the competition including prize structure $\{V_j\}_{j=1}^k$, deadline d, target quality q, and competition format (race or tournament). The sponsor maximizes an objective function that is the sum of total quality $Y = \sum_{i=1}^{n+1} Y_i$, time spent $T = \sum_{i=1}^{n+1} T_i$ and prizes paid $V = \sum_{j=1}^{k} p_j V_j$ (with $p_j = 1$ if the prize is awarded and $p_j = 0$ otherwise). Hence, the problem faced by the sponsor is

$$\text{maximize} \quad \int Y - \tau \mathbf{E}T - \mathbf{E}V$$

with the intensity of preferences towards time weighted by $c_t \geq 0$.

3.1 Structural econometric model

4 The field experiment

4.1 The context and experimental design

The field experiment was conducted in the context of an online programming competition run between March 2 and 16, 2016. A programming competition involves contestants writing source code to solve designated problems in a given period of time. These programming competitions are quite common (xxxx) and there are examples where the competition was a tournament (xxxx) — so the timing is not important — but also examples where it was a race such as the popular Netflix Prize where the first competitor team to do xxxx was awarded a $1M prize.

The competition was hosted on the platform Topcoder.com. Since its launch in 2001, Topcoder.com administers on a weekly basis several competitive programming contests for thousands of competitors from all over the world. Typical assigned problems are data science problems (e.g., classification, prediction, natural language processing) that demand some background in machine learning and statistics. All Topcoder members (about 1M registered users in 2016) can compete and attain a "rating" that provides a metric of their ability as contestants. Other than attaining a rating, the competitors having made the top five submissions in a competition are typically awarded a monetary prize the extent of which depends on the nature and complexity of the problem but is generally between $5,000 and $20,000.

In this study, we worked together with researchers from the United States National Health Institute (NIH) and the Scripps Research Institute (SCRIPPS) to select a challenging problem for the experimental programming competition. The selected problem was based on an algorithm called BANNER built by NIH (Leaman et al., 2008) that uses expert labeling to annotate abstracts from a prominent life sciences and biomedical search engine, PubMed, so disease characteristics can be more easily identified. The goal of the programming competition was to improve upon the current NIH's system by using a combination of expert and non-expert labeling, as described by Good et al. (2014).

The competition was announced on the platform and to all community members via email. A preliminary online registration was required to enroll in the competition, which resulted in xxxx pre-registered participants. Among the xxx pre-registered members, we selected the 299 with previous experience xxxx. This choice was to ensure xxxx.

The competitors were then randomly assigned to separate groups of 10 or 15 people and placed into different "virtual rooms." In each of these rooms, contestants were given access to a private web page listing all other competitors in the same room, a leaderboard updated regularly during the competition, and a common chat that they can use to ask clarifying questions about the contest setup or the problem. The problem was as described above. It was the same for everyone in every virtual room. Competitors had a deadline of 8 days to submit their computer programs. Each submission was automatically scored and feedback in the form of preliminary scores was published regularly on the website via the leaderboard.

These virtual rooms were then randomly assigned to one of three different competitive settings: a race, a tournament, and a tournament with a "reserve score," which is the lowest acceptable by the platform for a submission to be awarded a prize. The experimental design is summarized by the Table XXXX.

Table 1: Experimental design

	Large	Small
Race	60	39
Tournament	60	40
Reserve	60	40

Monetary payoffs were the same in all groups: the first placed competitor was awarded a prize of $1,000, and an additional, consolatory prize of $100 was awarded to the second one. In a race, the first to achieve a score equal to xxxx was placed first. In a tournament, xxxx. And in a tournament with minimum quality requirement, xxxx. Grand prizes of xxxx were awarded to the top xxx in every treatment.

4.2 Data

Individual data about each competitor were obtained from the online web profile on the platform. This profile typically includes when the member registered to the platform, the current rating, the number of past competitions, and so on. Additional personal information, was collected via a mandatory initial and a final survey. In the initial survey, registered competitors were asked basic demographics, including a measure of risk aversion. They were also asked to forecast the number of hours they would be able to spend competing in the next few days (the exact question was: "looking ahead xxxx"). In the final survey, they were asked to look back and tell us their best estimate of the time spent working on the problem.

A total of xxxx registered but only 299 competitors were selected for the challenge;

we excluded those with no past experience on the platform and those with incomplete data on the survey. Signed up competitors were experienced members of the platform: the overall time as registered platform member at the start of the competition ranged between 52.571 and 770.571 weeks. Yet, the direct experience in competing was highly skewed with competitors in the highest 90th percentile having participated in 28.2 more competitions than those in the 10th percentile. Likewise skills as measured by the individual ratings, if there was one, had a skewed distribution with 999 higher points than those in the 10th percentile; see Figure **??**.

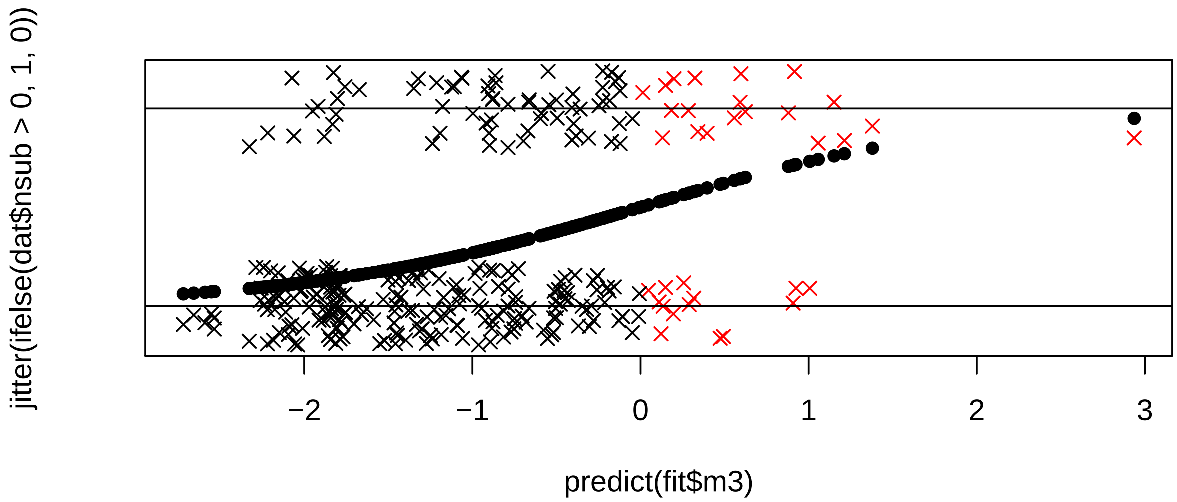
After the two-week submission period, 86 competitors made 1759 submissions overall. The distribution of submissions was rather skewed, with participants in the 90th percentile making 50 more submissions than those in the 10th percentile.

Assuming the decision was independent, to explore the determinants of participation:

$$Pr(y=1) = G(\text{Rating}_i + \text{Experience}_i + \text{T}_i) \tag{1}$$

where $G()$ is logistic.

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: nsub > 0
##
## Terms added sequentially (first to last)
##
##
##                           Df Deviance Resid. Df Resid. Dev Pr(>Chi)
## NULL                                        298        359
## poly(mm.count, deg = 2)    2    21.97       296        337  1.7e-05
## poly(mm.rating2, deg = 2)  2    11.86       294        325   0.0027
## poly(msince2, deg = 3)     3     2.36       291        323   0.5011
## poly(mm.top10, deg = 2)    2     3.93       289        319   0.1405
## poly(badges, deg = 2)      2     1.47       287        317   0.4785
```



This result does not seem to correlate well with the competitor's experience or skills, as the Pearsons's correlation coefficient between the count of past competitions or the rating and the count of submissions is positive but generally low; see Table XXX. Thus, differences in submissions appear idiosyncratic and perhaps related to the way to organize the work rather than systematically associated with underlying differences in experience or skills.

```
##            nsub mm.rating mm.count
## nsub      1.000     0.181    0.178
## mm.rating 0.181     1.000    0.333
## mm.count  0.178     0.333    1.000
```

The timing of submissions was rather uniform during the submission period with a peak of submissions made in the last of the competition. (explain more)

```
scores$submax <- ave(scores$subid, scores$id, FUN=max)
par(mfrow=c(2, 1), mar=c(4,4,2,2))
plot(subid==1 ~ as.POSIXct(subts), data=scores, type='h', yaxt='n'
    , xlab='', ylab='', main='Dispersion time first submission')
plot(subid==submax ~ as.POSIXct(subts), data=scores, type='h'
    , yaxt='n', xlab='', ylab='', main='Dispersion time last submission')
```

Scores: xxxx

5 Empirical analysis

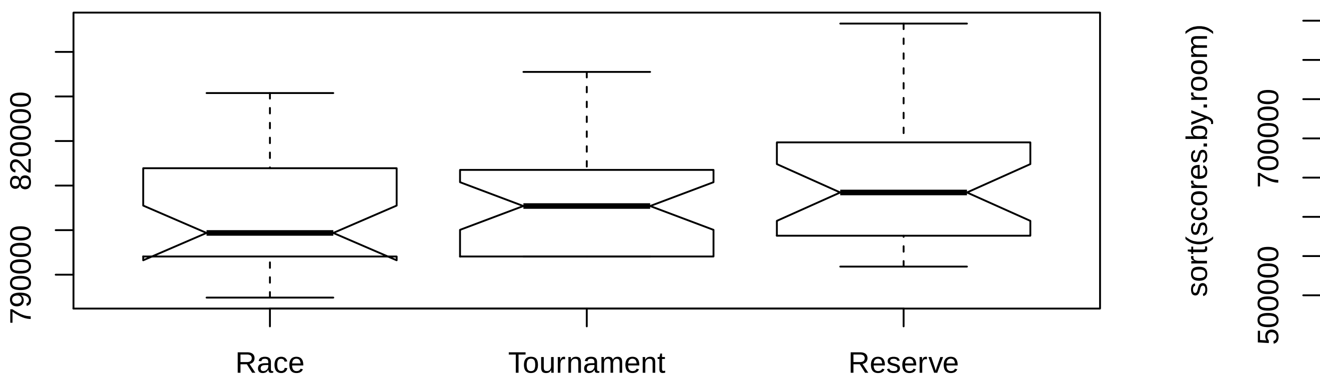
5.1 Treatment differences

Difference in participation by treatments are show in Table XX.

We find no differences in the room size.

Ex-post

```
## Warning in bxp(structure(list(stats = structure(c(784858.58, 794084.93,
## some notches went outside hinges ('box'): maybe set notch=FALSE
```



Timing: early vs late

Using a Chi-square test of independence, we find no significant differences in participation rates associated with the assigned treatments (p-value: 0.000419); see Table XX.

Further, we model participation rates as a logistic regression. We use a polynomial of third degree for the count of past competitions to account for non-linear effects of experience; and we use an indicator for whether the competitor had a win or not. Also, taking into account differences in ability, participation rates are not significantly different.

5.2 Estimation results

Participation to the competition by treatment is shown in Figure **??**. Participation here is measured by the proportion of registered participants per treatment who made any submission during the eight-day submission period. Recall that competitors may decide to enter into the competition and work on the problem without necessarily submitting. In a tournament, for example, competitors are awarded a prize based on their last submission and may decide to drop out without submitting anything. However, this scenario seems unlikely. In fact, competitors often end up making multiple submissions because by doing so they obtain intermediate feedback via preliminary scoring (see Section XXX for details). In a race, competitors have even stronger incentives to make early submissions as any submission that hits the target first wins.

`Table xxx`

We find that the propensity to make a submission is higher in the Tournament than in the Race and in the Tournament with reserve, but the difference is not statistically significant (a Fisher's exact test gives a p-value of `round(fisher.test(nsub.tab)$p.val,3)`). As discussed in Section XXX, we may not have enough power to detect differences below 5 percentage points. However, we find the same not-significant result in a parametric regression analysis of treatment differences with controls for the demographics and past experience on the platform; see Table **??**. Adding individual covariates reduces variability of outcomes, potentially increasing the power of our test. In particular, Table **??** reports the results from a logistic regression on the probability of making a submissions. Column 1 reports the results from a baseline model with only treatment dummies. Column 2 adds demographics controls, such as the age, education, and gender. Column 3 adds controls for the past experience on the platform. Across all these specifications, the impact of the treatment dummies (including room size) on entry is not statistically significant.

5.3 Simulation results

6 Empirical analysis

6.1 Estimation results

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$$fig : entry$$

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6.2 Simulation results

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