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Using auctions to reward tournament winners: theory and experimental investigations

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This article explores theoretical and experimental implications of using auctions to reward winners of research tournaments. This process is a hybrid of the research tournament for a prize and a first-price auction held after the research is complete. The bids in the auction consist of a vector of both quality of the innovation and price. The experimental evidence supports the hypothesis that conducting auctions at the end of research tournaments will generally reduce the sponsor's prize expenditure relative to fixed-prize research tournaments. The potential importance of these results to the U.S. Department of Defense acquisition process is emphasized.

1. Introduction

■ Research tournaments have proved to be excellent mechanisms for stimulating the development of new products and innovations. In 1829, the world was propelled into the age of steam locomotion when George and Robert Stephenson's *Rocket* raced an astonishing 46 kilometers per hour to win the £500 winner's prize at the Rainhill Trials.[1] More recently, research tournaments have been used to build products ranging from better beer packaging[2] and more energy-efficient refrigerators (Langreth, 1994), to high-tech battlefield communication networks (Fletcher, 1993). The U.S. Air Force's "fly before buy" policy for new aircraft and weapons pits defense con-

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[1] Sponsored by the Liverpool and Manchester Railway, the historic Rainhill Trials were used to select an engine for the first-ever passenger railway between two British cities (Day, 1971).

[2] Igor Grbic won $4,500 in Heineken's 2000 beer-packaging design tournament. Heineken's most recent design contest is for the best beer mat (at www.heinekencontest.com).

tractors against each other in high-stakes research competitions often worth billions of dollars,[3] and a much-publicized tournament sponsored by the U.S. Federal Communications Commission (FCC) resulted in the breakthrough development of digital High Definition Television (HDTV). These research tournaments were successful because they mitigated many of the moral hazard problems associated with traditional research and development contracts or other types of research competitions like patent races.

Since the earliest stages of the industrial revolution the engines of economic growth have been fueled by new research and development. But writing effective contracts to induce third parties to engage in research is very difficult. Traditional R&D contracts employ piece-rate compensation schemes in which payments are based on an agent's expenditures or other measurable research efforts. But when it is difficult or expensive for the principal to audit expenditures or monitor research efforts, moral hazard problems can severely reduce the efficiency of piece-rate R&D contracts. An alternative method for alleviating the requirement to monitor agents' efforts is to sponsor a competition and offer a prize to the winner. If the prize is large enough, agents may be induced to conduct research on their own without the need for the sponsor to monitor their efforts or audit their expenditures.

Patent races are the R&D competitions most commonly modelled by economists. Typically, patent races are modelled as winner-take-all contests with the prize going to the first agent to discover a breakthrough innovation or cost-reducing technology as measured by some absolute standard of success.[4] For instance, one of the best-known examples of a patent race in academic circles was the race to prove Fermat's Last Theorem, which mathematicians struggled with for more than 300 years. When Andrew Wiles finally succeeded, circa 1994, he won acclaim not for the elegance of his proof but because he was first. However, this example also illustrates two potential weaknesses in patent race mechanisms. First, patent races can last an unacceptably long time—while science might endure 300 years of effort before achieving success, business will not. Second, patent races may be difficult to enforce if the standard of success is subjective in nature or technically complex to verify. Indeed, although Wiles first claimed success in June 1993, it was not until 1995 that a revised version of his proof gained acceptance within the scientific community; and it was not until 1998 that he was awarded a special silver platter for his work by the International Mathematical Union.[5] The same kinds of verification and enforcement problems can hamper the effectiveness of any R&D contract, with or without competition, whenever success is based on a measurement of innovation quality. If the courts lack the expertise to verify whether an agent has met the required quality threshold, the contract or the competition becomes unenforceable, removing the basic incentive for agents to conduct research.

In contrast to patent race mechanisms and piece-rate contracts, research tournaments can be simple to enforce and do not require expensive monitoring efforts by the principal. In a research tournament, the competition ends at a specific point in time rather than upon reaching a specific innovation quality. Therefore, irrespective of absolute quality measures, the prize in a research tournament is always awarded on a specified date to whichever competitor the sponsor deems to have developed the best innovation at that point in time. To enforce a research tournament contract, courts are required to verify only that the sponsor actually paid the prize. Competitors in a research tournament vie to produce the highest-quality product within a specific period. The *guaranteed* prize payment at a specific moment in time simplifies enforcement and provides the internal inducement for competitors to conduct research without regard to monitoring by the principal. Therefore, research tournaments can be excellent alternatives to patent races or piece-rate contracts when sponsors are sensitive to time schedules, innovation quality is difficult to verify objectively, or monitoring efforts by the principal are expensive.

[3] For example, in 1991 Lockheed won the Air Force's Advanced Tactical Fighter (ATF) fly-off competition with its F-22, earning a production contract worth billions of dollars (Schwartz, 1991).

[4] For some examples of patent race literature, see Dasgupta and Stiglitz (1980), Gilbert and Newberry (1982), and Reinganum (1983).

[5] Information on Wiles and his proof can be found at www.princeton.edu/pr/news/98/q3/0827-wiles.html.

Until recently, research tournaments had largely been overlooked by economists studying R&D mechanisms, but Taylor (1995) showed that an optimally designed tournament can induce an efficient amount of research if the sponsor offers an appropriate prize, limits the number of competitors, and charges each contestant an entry fee to extract the *ex ante* surplus from competing in the tournament. Since Taylor's seminal article, Fullerton et al. (1999) have offered laboratory evidence showing his theory to be generally supported by the behavior of experimental subjects. Fullerton and McAfee (1999) further extended the theory of research tournaments by showing that for a large class of contests the optimal number of competitors is two, and sponsors of tournaments with heterogeneous contestants can identify the best-qualified competitors before the tournament by employing a contestant-selection auction.

Auctions have also been employed in a model by Rob (1986), who proposed using first-price sealed bids to select procurement contractors. In his model the auction sponsor solicits bids from multiple competitors to obtain a low purchase price. Quality is invariant and the procurement auction is held *prior* to conducting the research in Rob's model, so only the winning, low-bid firm engages in research with the goal of discovering manufacturing techniques that reduce production costs.[6]

This article proposes a hybrid R&D contest combining Taylor's research tournament and a first-price auction that is held *after* the research is complete. Like Taylor's model, firms conduct research to improve the quality of their innovation, so competing bids consist of a vector of both quality and price. The first-price auction serves as an alternative reward mechanism in lieu of offering a preannounced, fixed prize. Therefore, the research tournament winner is selected based upon the innovation/price combination that offers the sponsor the greatest total surplus.

There are three main reasons for conducting an auction in lieu of offering a fixed prize for the winner's reward in a research tournament. The first is that bidding provides an additional medium through which contestants can compete. If a contestant's research efforts are only marginally successful, he can still compete at the conclusion of the tournament by lowering his bid price—giving the sponsor the option of flying coach instead of first class. A second reason for hosting an auction in lieu of offering a fixed prize is that in the *absence* of optimal entry fees, the additional competition from an auction will generally induce a greater amount of research effort per prize dollar. This does not imply that an optimally designed auction-style tournament can be more efficient than an optimally designed fixed-prize tournament, since entry fees can be charged to make either tournament economically efficient. But calculating an optimal entry fee in either tournament would require extensive detailed information about contestant capabilities and projected innovation distributions that the sponsor is not likely to know. Therefore the third, and probably the most important, reason for conducting an auction is that it drastically reduces the information burden on the sponsor by eliminating the requirement to calculate a correct prize before the tournament.

Research sponsors may have very limited technical knowledge about the costs of conducting research and the forecast distribution of innovation quality. Yet those are precisely the kinds of details that must be known to calculate the appropriate size of the prize in a fixed-prize tournament. Presumably, if the sponsor has detailed knowledge about the costs of research and distribution of innovations, then the sponsor may also have the expertise to inexpensively monitor research efforts, write traditional R&D contracts, or perhaps even do the research herself. In the absence of such detailed knowledge, however, a research tournament sponsor faces a very complex technical problem of determining the appropriate prize. An auction largely eliminates that problem for the sponsor because it places the information burden for determining bids (prizes) on the competitors. Thus, an auction shifts the "risk" associated with selecting a prize away from the sponsors, who may have little technical information, onto the competitors, who conduct the research and have the best possible information. We show that in the absence of optimal entry fees, an auction generally reduces the sponsor's expected prize expenditure for any given level of innovation quality. Hosting

[6] Rob shows that with a limited number of potential manufacturers, the sponsor can reduce the total cost of procurement by first conducting an educational buy rather than procuring the entire lot in a single buy.

an auction, therefore, becomes a cost-saving alternative whenever a research tournament sponsor has insufficient information to calculate an optimal fixed prize and entry fees.

To test this theory, we conducted a series of economic experiments. The data we collected in these experiments support the hypothesis that prize auctions reduce the cost of sponsoring a tournament when the competition has more than two contestants. Therefore, by following a few straightforward guidelines, even research tournament sponsors with very limited information should be able to host successful research competitions by employing auctions as reward mechanisms.

We begin in Section 2 by recreating Taylor's model of research tournaments and extending his theory to include the use of a postresearch prize auction. This section provides sufficient conditions for auctions to theoretically reduce a sponsor's expected prize payment relative to the size of the fixed prize necessary to generate an equivalent amount of research effort. In Section 3 we offer experimental evidence showing that, in accordance with the theory, auctions generally do reduce the cost of the winner's prize and increase the sponsor's surplus whenever there are several contestants competing in the tournament. Conclusions and recommendations are reserved for Section 4. All proofs are in Appendix A.

2. Using auctions to reward research tournament winners: theory

■ For explanatory ease, we retain Taylor's (1995) notation and readers are referred to his article for details not discussed here. By assumption, there are M risk-neutral, homogeneous competitors who compete in a research tournament to win a prize. The tournament lasts T periods, and in each period competitors have a single opportunity to pay research cost C and obtain a single independent draw, x, from the distribution of innovations, $F(x)$, on support $[0, \bar{x}]$. All competitors are assumed to start the tournament with worthless innovations of $x = 0$. Each new innovation is drawn with recall from $F(x)$, allowing each competitor to retain his or her best draw to date. All innovations are privately known until the end of the tournament when competitors deliver their best draw to the sponsor, who evaluates them and awards the prize to the winning contestant.

In Taylor's tournament, the winner is simply the competitor who draws the best innovation before the end of the contest. Taylor's tournament winner is awarded a preannounced fixed prize, P, and all other contestants receive nothing. In accordance with search theory, Taylor showed that the optimal strategy in this tournament is to continue drawing a new innovation in each successive period until one runs out of periods or draws an innovation larger than some "cutoff" value denoted by z. In equilibrium, the optimal, and unique, z-stop value is implicitly defined by the equation[7]

$$P \int_{z}^{\bar{x}} \left[\Phi^{M-1}(y; z, T) - \Phi^{M-1}(z; z, T)\right] f(y)dy - C = 0, \tag{1}$$

where Φ is the date-zero CDF for the value of a firm's best innovation:

$$\Phi(x; zT) = \begin{cases} F^T(x) & x \leq z \\ F^T(z) + [1 - F^T(z)]\dfrac{F(x) - F(z)}{1 - F(z)} & x \geq z. \end{cases} \tag{2}$$

As shown in (1) and (2), a competitor's effort level (implicitly defined by z) is an increasing function of the fixed prize, P, and a decreasing function of the cost of research, C. Additionally, the equilibrium z-stop is also a function of the number of competitors, M, and the length of the tournament, T (i.e., number of research draws permitted).

Instead of announcing a fixed prize, P, research tournament sponsors can alternately conduct a first-price, sealed-bid, "prize" auction at the end of the tournament. In an "auction-style" tournament, each competitor is required to submit his best innovation and a bid, which he will be paid in lieu of the fixed prize if he wins the tournament.[8] Obviously, one benefit of conducting an

[7] Taylor (1995), equation 7.

[8] The bid will usually just take the form of the price the competitor charges for procuring the good.

auction-style tournament is that the sponsor does not have to preannounce a prize value before starting the contest. Furthermore, by requiring bids from competitors, the sponsor gains additional trade space (bid cost versus innovation quality) through which she may be able to increase her total surplus.

In the fixed-prize tournament, if contestant j's best research draw is x_j and the preannounced prize is worth P, then the sponsor's surplus from picking j as the tournament winner is $x_j - P$. In contrast, in an auction-style tournament, if contestant j submits a bid of $B(x_j)$, then j's innovation and bid combination would offer the tournament sponsor a surplus of $x_j - B(x_j)$. The key distinction is that in an auction-style tournament, contestants with lower-quality innovations can maintain their competitiveness at the end of the tournament by submitting smaller bids. Since the winner is ultimately the contestant who submits the innovation and bid *combination* offering the sponsor the largest total surplus, this additional competition can potentially reduce the sponsor's prize cost.

In an auction-style tournament the bidding is a transformation of the standard first-price, independent-private values procurement auction. Therefore, a symmetric, pure-strategy equilibrium bidding function, $B(x)$, for this auction would solve

$$\max_{B(x_i)} B(x_i) \Pr\left[x_i - B(x_i) > x_j - B(x_j)\right] \qquad \forall\, j \neq i.$$

Letting $\Phi(x)$ represent the distribution of the best final innovations drawn by contestants, the unique candidate for a symmetric pure-strategy equilibrium bidding solution in an auction-style tournament would be for each contestant to submit a bid of[9]

$$B(x_i) = \frac{\displaystyle\int_0^{x_i} [\Phi(\xi_i)]^{M-1} d\xi}{[\Phi(x_i)]^{M-1}}. \tag{3}$$

Given equilibrium bidding function (3), the expected surplus to the tournament sponsor $[x_i - B(x_i)]$ is strictly increasing in the quality of the innovation, x_j. Therefore, in equilibrium, conducting a first-price prize auction theoretically will not disturb the relative ranking of competitors by innovation quality. For any given set of innovation discoveries in a tournament, the innovation that would win in a fixed-prize contest should be the same innovation that would win the tournament if an auction were held instead.

When using auctions to reward winners, the optimal research strategy is still a z-stop strategy just as in the fixed-prize tournament, but in an auction-style tournament the z-stop is defined by

$$\int_z^{\bar{x}} \left[B(y)\Phi^{M-1}(y; z, T) - B(z)\Phi^{M-1}(z; z, T)\right] f(y)dy - C = 0. \tag{4}$$

If a fixed-prize tournament and an auction-style tournament have the same number of periods and the same number of competitors employing the same z-stops, then both tournaments must also have identical distributions of winning innovations. Therefore, using (1) and (4) one can compare the expected cost of the winning bid in an auction-style tournament with the cost of the winner's prize in the corresponding "equivalent" fixed-prize tournament. Proposition 1 provides *sufficient* conditions for the expected cost of an auction-style tournament to be less than the cost of the fixed prize necessary to induce an equivalent level of effort.

Proposition 1. When equilibrium bids, $B(x)$, are monotonically increasing in innovation quality, the expected cost of the prize in a first-price, sealed-bid, auction-style tournament is strictly less than the cost of the prize required to elicit the same effort in a fixed-prize tournament.

[9] For a derivation of this equilibrium and a synopsis of auctions, see McAfee and McMillan (1987).

Intuitively, Proposition 1 is appealing because the first-price auction merely places an additional layer of competition onto the tournament *after* research efforts are completed. Therefore, given equivalent z-stops for the two tournaments, one would expect the added auction competition to reduce the sponsor's prize payment. It is logical to believe that contestants with higher-quality innovations will also submit higher bids in equilibrium; therefore, one would imagine that Proposition 1 is always true. But, unlikely as it may seem, mathematically it can happen that over some small range of innovation draws, the equilibrium bids can be inverted—even for a uniform distribution of innovations.[10] Moreover, since checking Proposition 1 involves computing expected bids for every possible innovation draw, it may be technically challenging to verify. Therefore, we offer Proposition 2 as an alternative *sufficient* condition for all distributions of the form $f(x) = \alpha x^{\alpha-1}$, where $\alpha > 0$ and $x \in [0, 1]$.

Proposition 2. For innovation distributions of the form $f(x) = \alpha x^{\alpha-1}$, where $\alpha > 0$ the condition $P \geq B(z)$ is sufficient for the expected cost of the prize in a first-price sealed-bid auction-style tournament to be strictly less than the cost of the prize in a fixed-prize tournament that generates an equivalent level of research effort.

Computationally, Proposition 2 is easier to check than Proposition 1, since it requires the sponsor only to compare the equilibrium bid at the z-stop value against the size of the fixed prize necessary to induce the same z-stop. Numerical analysis of Proposition 2 reveals that in most cases, the sufficient condition is met, so the expected cost of the prize in an auction-style tournament is theoretically smaller than the cost of the prize in an equivalent fixed-prize tournament.[11] Additionally, in the exceptional instances when the sufficient condition of Proposition 2 is not met, the expected cost differences between the two types of tournaments are virtually negligible.

For example, consider the uniform [0, 1] distribution of innovations (corresponding to $\alpha = 1$). For this distribution, the sufficient condition of Proposition 2 reduces to

$$P - B(z) = \frac{z^T(z-1)\left[1 - z^{TM}(M+1-Mz^T)\right]}{(M+1)(z^T-1)\left[z^T - z^{TM}(M+z^T-Mz^T)\right]} - \frac{z}{TM-T+1} > 0.$$

When $P - B(z)$ is positive, the expected prize payment of the auction-style tournament is less than the cost of the fixed prize necessary to induce an equivalent amount of research. In a tournament with $M = 2$ competitors, the least competitive auction bidding environment, this equation further reduces to

$$P - B(z) = \frac{(z-1)[1+2z^T]}{3(z^T-1)} - \frac{z}{T+1}.$$

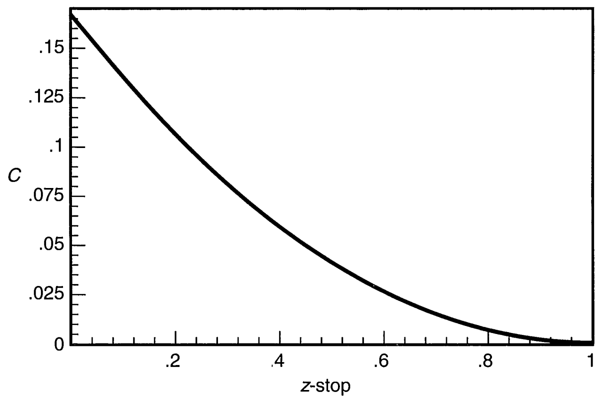
This is positive for all possible values of z when $T \leq 16$, implying that the auction-style tournament is always theoretically cheaper when there are two competitors and fewer than 17 periods of research available during the competition. $P - B(z)$ is also positive for all equilibrium z-stop values in the limit as $T \to \infty$. It is only when the tournament has a finite number of research periods greater than 17 that there is the possibility of the sufficient condition in Proposition 2 not being met. For example, in a tournament with two bidders, a uniform [0, 1] distribution of innovations, and $T = 20$ periods, one can graph the equilibrium z-stop in an auction-style tournament as a function of the cost of research, C, as shown in Figure 1.

So, for example, if $C = .1$ (each draw costs a firm 10% of the maximum value of the best-possible innovation), then the auction-style tournament would have an expected equilibrium z-stop of just .2254. In addition to Figure 1, one can also compute the value of the sufficient condition $P - B(z)$ for all possible equilibrium z-stops from zero to one as shown in Figure 2.

[10] Even though there are small ranges of innovations where equilibrium "bid inversion" occurs, the equilibrium probability of winning is still strictly increasing in innovation quality.

[11] One would expect the sufficient condition to be met in tournaments with many periods of research and a low-to-moderate z-stop, because every competitor should draw an innovation larger than z, requiring the equilibrium bid for holding a z-stop-quality innovation to be very small.

FIGURE 1



What is evident in Figure 2, however, is that $P - B(z)$ is positive everywhere except for a very small range of equilibrium z-stops between approximately $z = .94$ and $z = .96$. This means that the expected cost of the winner's prize is theoretically cheaper for auction-style tournaments as long as the equilibrium z-stop is not within that very small range of values. For the equilibrium z-stop to equal .95 requires a per-period research cost of $C = .0007$; it would also require offering a fixed prize of $P = .0446$ to obtain a z-stop of .95. But that is a prize 63 times as large as the cost of conducting research—enough to pay outright for both firms to conduct research every period. Finally, Figure 3 shows the difference between the cost of the fixed prize and the expected cost of the winning bid as a function of the equilibrium z-stop when there are two competitors, $T = 20$ periods of research, and a uniform [0, 1] distribution of innovations.

As Figure 3 shows, the fixed prize is in fact cheaper than the expected cost of the winning bid when the equilibrium z-stop equals .95, but the difference is negligible ($\sim$.0003). The pattern shown in Figure 3 holds for larger values of T. As one increases T above 20, the small z-stop range for which the sufficient condition does not hold gradually shifts upward from .95 to a value of 1.0 in the limit as $T \to \infty$. Moreover, the difference between the costs of the two tournaments throughout this range where the sufficient condition is not met is so small as to be virtually negligible. Thus, from numerical analysis one would conclude in almost all cases with $M = 2$ and a uniform distribution of innovations that the auction-style tournament is theoretically cheaper than an equivalent-effort, fixed-prize contest—but the primary benefit of hosting an auction-style tournament lies in its low information requirements rather than any significant cost advantage.

In Appendix B we provide additional numerical examples showing how the same pattern emerges with more than two competitors and with distributions of innovations that are not uni-

FIGURE 2

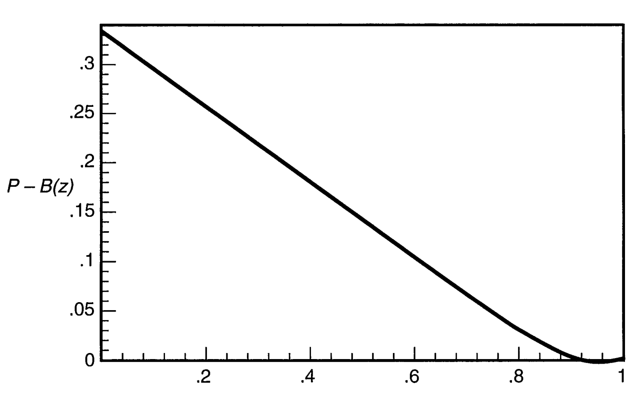
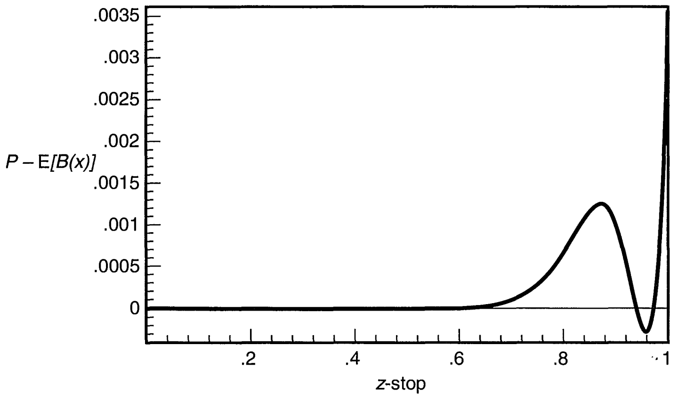


FIGURE 3



form. Using the program *Mathematica* (Wolfram) to check sufficient conditions and calculate the difference in the expected costs between the two types of tournaments, we repeatedly found in tournaments with fewer than 15 periods that the auction-style tournament is always theoretically cheaper. In tournaments with more than 15 periods, there are situations in which the fixed-prize tournament is cheaper, but in these long tournaments the cost differences are almost always negligible between the two different types relative to the value of the innovation—regardless of which tournament style is theoretically less expensive.[12]

Finally, one more favorable characteristic of Proposition 1 is that its proof does not rely on any particular equilibrium bidding function. The size of the tournament prize is a principal choice variable used by Taylor, in addition to entry fees, to create an efficient contest. In an auction-style tournament the sponsor potentially gives up this choice variable. But since the results of Proposition 1 do not depend on any particular bidding function, a tournament sponsor can manipulate the equilibrium z-stop by placing floors and/or ceilings on acceptable bids without sacrificing the potential cost savings of an auction-style tournament. For example, any sponsor who is apprehensive about allowing a completely unregulated bidding competition for fear of inviting excessively large bids could simply place a ceiling on the maximum allowable bid. Such an action would limit the maximum bid payments and could be used to reduce the equilibrium z-stop. Determining a bidding floor or ceiling (much like fixing a reservation price in a seller's auction) can limit the sponsor's risk and help manipulate the z-stop level of effort, yet still places a smaller information burden on the sponsor than having to determine an exact prize amount.

3. Experimental comparison of research tournaments

■ While the discussion above suggests that auction-style tournaments can be theoretically attractive alternatives to conducting fixed-prize research tournaments, the complexity of the equilibrium conditions may make computing z-stops and bids too difficult to work well in practical settings. To examine whether auction-style tournaments perform well in practice, we tested the theory experimentally at the University of New Mexico's computerized experimental economics laboratory.

In a previous experimental study of research tournaments, Fullerton et al. (1999) showed that Taylor's fixed-prize research tournament theory was surprisingly consistent at predicting qualitative changes in the level of effort and the winning innovation. Even though experimental subjects were not sophisticated enough to individually compute and employ the precise optimal z-stop strategy predicted by equation (1), Taylor's theory successfully forecast changes in the collective level of research effort and the value of the winning innovation. Here, however, we report the results of laboratory experiments designed to validate or refute the hypotheses of Section

[12] Additional examples, computed using *Mathematica*, are available upon request from the authors.

2 by comparing the performance of fixed-prize tournaments with auction-style tournaments with the same expected prize value.

Subjects were recruited from undergraduate social science classes. All subjects received a set of written instructions explaining that they would be participating in a market where the task was to decide whether to pay for a draw of a random number in an effort to win a prize. In the laboratory sessions, we referred to a complete multiperiod research tournament as one experimental "round." Subjects participated in numerous rounds during each session, and at the start of each round, every subject was given a new endowment of laboratory currency, labelled "francs," sufficiently large to ensure that he could take a draw in every period without going bankrupt. Subjects started each round with identical draws of $x = 0$, signifying worthless innovations. During each period, subjects were given the opportunity to make a single research draw, generating an innovation value between 1 and 1,000, with each number equally likely. Following each draw, the computer automatically compared the subject's new draw against his best previous draw in that round, retaining the highest innovation until the completion of the round. The cost of each new draw, C, was announced at the beginning of the experiment. Subjects were also told the maximum number of draws that could be taken in each round and the number of other competitors in their group. To prevent collusive behavior, subject groups were mixed before the start of each new round.

In the fixed-prize treatments, at the end of each round a computerized buyer chose the player in each group with the highest draw, [maximum x], and awarded that player the preannounced prize, P. A subject's total payoff at the end of the experimental session, therefore, was equal to the sum of the prizes won in all rounds, plus the sum of the endowments he was given in all rounds, minus the cost of all the research draws he took in all rounds.

In the auction-style tournaments, at the end of each round subjects were asked to submit a sealed bid (B), which was required to be smaller than their best draw, x. A computerized buyer then compared the surplus $[x - B]$ of each player's bid and innovation combination and awarded the player with the largest surplus a prize value equal to his bid, B. Thus, in the auction-style tournament, bids were used in lieu of fixed prizes and the metric used to select the winner was the largest surplus for the sponsor.

Computerization of the experiments allowed for immediate feedback for the subjects to enhance their understanding of the payoff function. After each round, the subjects' computer screens displayed the results of the round and how their payoffs were calculated. Specifically, the subjects were told their own maximum draw (or the maximum surplus in an auction-style contest) and the largest draw (surplus) of the group they were assigned to. If her draw (surplus) was the largest in the group, the subject was informed she had won the prize (bid), and the subject's round balance was calculated as her initial endowment minus the cost of her draws plus the value of the prize (bid). Otherwise the subject's round balance was just her initial endowment minus the cost of her research draws. At the end of the session all francs were converted to dollars at a fixed exchange rate announced prior to the experiment, and the subjects were paid in cash.

By varying the number of contestants, M, and the number of periods per round, T, we covered three basic treatments in our experiments, which we have listed below as treatments 1 through 3 in Table 1. For each treatment we conducted both fixed-prize and auction-style sessions. In Table 1 the theoretical predictions for our fixed-prize sessions are listed first, followed by the theoretical predictions for our auction-style sessions. To test the predictive power of the propositions in Section 2, fixed prizes were selected to be roughly equivalent to the theoretically expected cost of the winning bid in the auction-style tournaments. Since the winning bid and fixed prizes were nearly identical, the predicted difference between the two tournaments is that the auction-style tournaments should theoretically generate greater contestant effort, which is implicitly reflected in the larger predicted winning draws. Similarly, the auction-style tournaments should also generate larger research-to-prize ratios, which are simply the total expected costs of research draws by all contestants divided by the cost of the winner's prize. A research-to-prize ratio less than one implies that the equilibrium expected profit for participating in the tournament is positive.[13]

[13] In theory, risk-neutral contestants would refuse to compete in any tournament in which the research-to-prize ratio exceeded one, since this would imply an expected loss from tournament participation.

TABLE 1 Theoretical Predictions

	Session Treatment Parameters		
	$M = 2, T = 6$ (1)	$M = 3, T = 4$ (2)	$M = 5, T = 2$ (3)
Fixed prize			
Fixed prize	120	103	101
Winning draw	904	906	901
Research-to-prize ratio	.591	.789	.85
Auction			
Expected bid	119	102	101
Winning draw	907	908	902
Research-to-prize ratio	.623	.805	.85

Our baseline treatment was number 2, for which we conducted the most experimental rounds. Table 2 provides summary statistics for the mean and median subject responses in our baseline treatment.

In our baseline treatment, we conducted two fixed-prize and two auction-style sessions, using a total of 66 different subjects. None of the subjects participated in more than one session. Cumulatively, the 66 subjects in treatment 2 participated in 528 auction-style rounds and 500 fixed-prize rounds, taking a total of 7,668 out of a possible 12,336 research draws.[14]

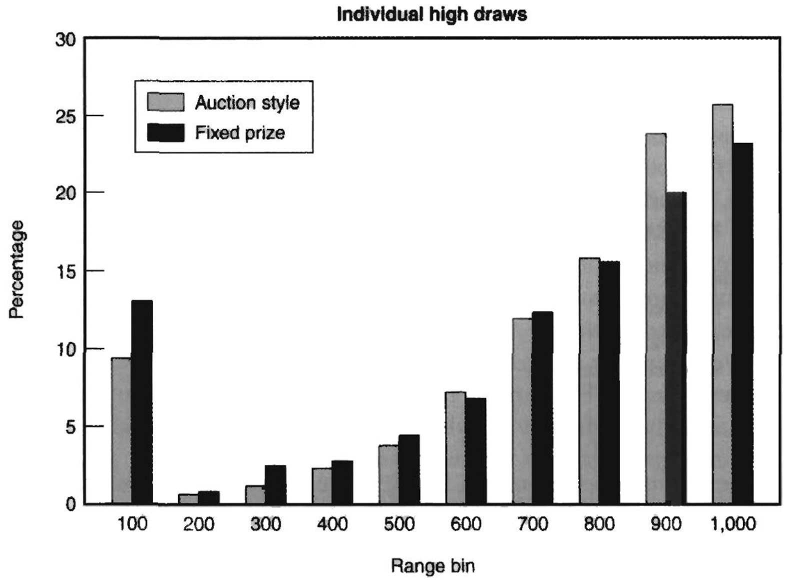
As shown below, the average winning bid payment of just 53.3 in the baseline auction-style experiments was much smaller than the cost of the fixed-prize payment of 103, while the average winning innovation in the auction-style tournament was similar to the fixed-prize tournament (873

TABLE 2 Baseline Treatment 2: Experimental Averages

Fixed-Prize Tournament	Auction-Style Tournament
Theory Predictions	
Fixed prize = 103	Expected bid payment = 102.1
Expected winning draw = 906	Expected winning draw = 907.8
Expected sponsor's surplus = 803	Expected sponsor's surplus = 805.7
Expected research-to-prize ratio = .789	Expected research-to-prize ratio = .805
Expected number of draws per subject per round = 2.68	Expected number of draws per subject per round = 2.74
Experimental Observations	
Fixed prize = 103	Average winning bid payment = 53.3
	Median winning bid payment = 42
Average winning draw = 884	Average winning draw = 873
Median winning draw = 911	Median winning draw = 900
Average sponsor's surplus = 781	Average sponsor's surplus = 819.7
Average research-to-prize ratio = .688	Average research-to-prize ratio = 1.466
Avgerage number of draws per subject = 2.36	Average number of draws per subject = 2.6
Average francs earned per subject = 2784	Average francs earned per subject = 1621
Average dollars earned per subject = $11.14	Average dollars earned per subject = $9.73

[14] 12,336 = 1,028 rounds × 4 periods per round × three contestants per tournament.

FIGURE 4



versus 884).[15,16] Although the cost of the average winning bid in the auction-style tournament was smaller than that in the fixed-prize tournament, the average number of research draws per subject in the former was greater than in the latter. Therefore, the auction-style tournament induced a greater total amount of research than the fixed-prize contest. All these observations in our baseline treatment support the hypothesis that auction-style tournaments induce more research effort per prize dollar.

Figure 4 is a histogram of the percentage breakout of the best draws per contestant per round, which we observed in both the fixed-prize and the auction-style experiments of treatment 2. As shown, in the auction-style tournament, a greater percentage of competitors elected to make at least one draw per round, and the best draws per round of the auction-style contestants were generally larger than the best draws of the contestants in the fixed-prize tournaments. Indeed, the average best draw per round for each contestant in the auction-style competition was 703, while the average best draw per round per contestant was just 656 in the fixed-prize competition.[17]

The most significant empirical issue we face in treatment 2 is reconciling the observation that the auction-style tournament induced more research and larger "best draws" than the fixed-prize tournament, with the seemingly contradictory observation that the average winning innovation in the auction-style tournament was smaller than the average winning fixed-prize innovation. The explanation of this mystery lies in the fact that in 21% of the auction-style rounds, the winner was *not* the contestant with the largest draw in his group.

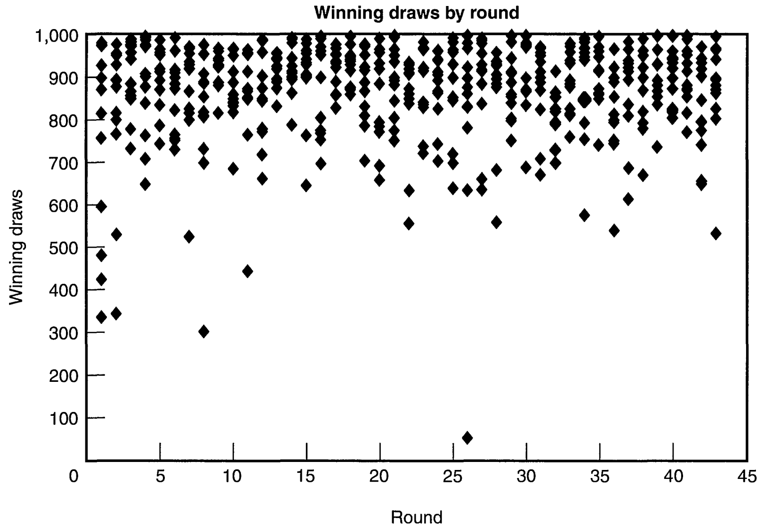
In the auction-style tournament, the winning contestant is the competitor whose innovation and bid *combination* offers the sponsor the largest surplus. Although, in theory, the winning combination should also include the largest draw, in 21% of our experiments the winner did not have the largest draw. Instead, the winning contestant won with a smaller draw by also submitting a much smaller bid to make up for the weaker draw. This ability to trade off lower bids against

[15] Theoretically, obtaining an expected winning innovation of 873 in a fixed-prize tournament would require a prize of 60, which is still larger than the 53.27 average prize payment in the auction-style contest.

[16] Because the subjects in the auction experiments of our base treatment ended up earning so many fewer francs on average, we ultimately offered them a larger exchange rate of .006 dollars per franc (vice .004 dollars per franc) to ensure that they received an adequate payoff for participating in the experiment.

[17] These averages are based on 1,500 (e.g., 500 rounds × 3 contestants per round) observations in the fixed-prize tournaments of treatment 2, and 1,584 observations in the auction-style tournament.

FIGURE 5



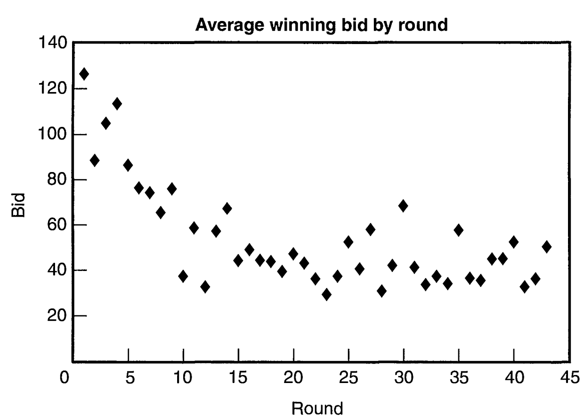
weaker innovations is precisely the kind of competitive behavior that makes the auction-style tournament less expensive and more attractive to the sponsor.

Although the average winning draws were slightly lower in the auction-style tournaments than in the fixed-prize rounds, they were still remarkably strong given the low cost to the sponsor. In Figure 5 we have plotted all the winning draws, by round, in the auction-style sessions of our baseline treatment.[18]

Though there are a few small outliers, 50% of the winning draws in the auction-style tournament were larger than 900, 81% of the winning draws were larger than 800, and 95% of the winning draws were above 670. This is a remarkable performance when one considers that the average cost of those innovations was a prize of just 53 francs.

Presumably, a sponsor hosts a research tournament for the reason that she is unable to do the research herself. Yet if the sponsor could do the research herself and were to spend 60 francs to make six research draws, the expected value of the sponsor's best innovation would still be only 857 less than the average winning draw in the auction-style tournament.[19] In fact, it is

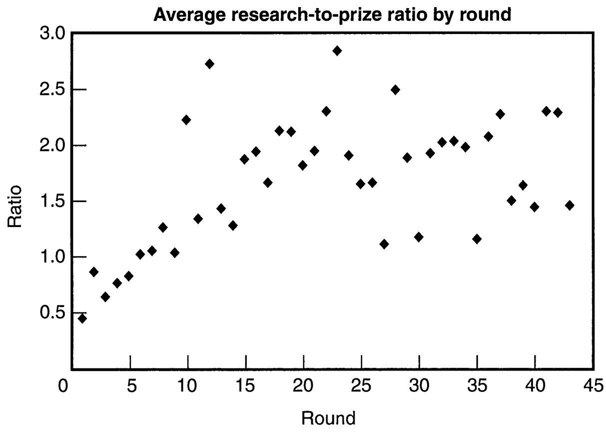
FIGURE 6



[18] We only plot the first 43 rounds because our second baseline auction session only lasted 43 rounds.

[19] Only 47% of the time would the best of six draws be larger than 900, only 74% of the time would it be larger than 800, and only 91% of the time would it be larger than 670.

FIGURE 7



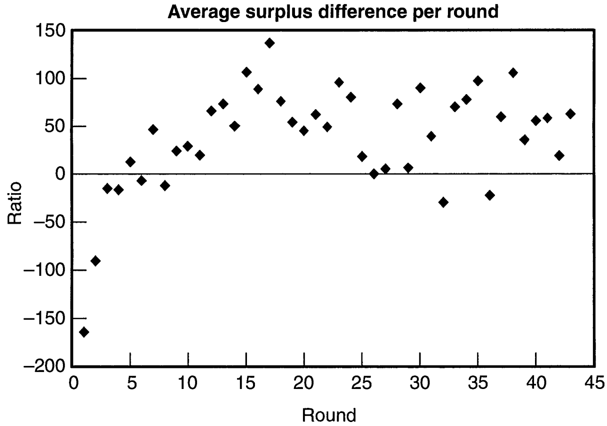
mathematically impossible for the sponsor to take any predetermined number of draws and earn an expected surplus as large as the mean surplus of 820 that we observed in the auction-style experiments of treatment 2. The distribution of winning experimental draws in our auction-style experiments dominates the distribution of the best draws the sponsor could have expected even if she spent more money and conducted all the research herself.

Though the winning innovations shown in Figure 5 remained relatively consistent, in Figure 6 one can see that the winning bids dropped substantially after contestants gained experience.

In the baseline treatment of the auction-style tournament, the subjects clearly became increasingly aggressive in their bidding strategies while they continued to conduct research at about the same level of effort. This bidding effect presented a substantial surplus to the sponsor and generated an average research-to-prize ratio of 1.466. This research-to-prize ratio indicates that contestants cumulatively spent 46% more on research draws than they received in prize payments, so the *ex ante* expected profit for participating in the baseline auction-style round was negative. More surprisingly, Figure 7 shows that as the subjects gained more experience, the research-to-prize ratio increased rather than decreased.

This persistently excessive level of research effort cannot be explained as a Nash equilibrium, yet this type of behavior is commonly observed in laboratory experiments of prize competitions (Davis and Reilly, 1998). Anderson, Goeree, and Holt (1998) show that this type of behavior can be explained using a logit equilibrium that models subjects with bounded rationality.

FIGURE 8



Perhaps the most important measurement of tournament performance from the sponsor's perspective is her surplus. The average surplus in the auction-style sessions was 819.7, whereas it was just 781 in the fixed-prize sessions. In addressing the potential risk to the sponsor, the standard deviation of the winning surplus was also very close for both tournaments (115 for auctions and 107 for fixed-prize contests). In Figure 8, we plot the difference between the average surplus of the auction-style sessions and the average surplus of the fixed-prize sessions by round. Positive values in Figure 8 indicate rounds in which the auction-style tournament generated more surplus for the sponsor than the fixed-prize tournament.

With the exception of the first few rounds before bid stabilization, the auction-style experiments consistently generated greater surpluses than the fixed-prize experiments. In more than 80% of the rounds, the average surplus generated by the auction-style tournament was larger than the average surplus of the fixed-prize tournament. After gaining experience in bidding, the auction-style tournament almost always generated more surplus for the sponsor in our baseline treatment.

Collectively, the evidence of our baseline experimental treatment supports the proposition that auction-style tournaments generate more research per prize dollar than fixed-prize tournaments. After contestants took 7,668 out of 12,336 possible research draws, the mean performance of the auction-style tournament was better than the fixed-prize tournament in the following areas:

(i) more research effort per contestant per tournament (2.6 draws versus 2.36 draws);
(ii) better individual high draws per contestant (703 versus 656);
(iii) better high draws per group (892 versus 884—although the auction's winning draws were lower);
(iv) more research expenditures per prize dollar (1.466 versus .688);
(v) lower prize payments (53 versus 103); and
(vi) larger average surpluses (820 versus 781).

From the sponsor's perspective, in virtually every important measurement of performance the auction-style tournament was preferable to the fixed-prize tournament.

□ **Experimental evidence from other treatments.** To test the robustness of our baseline treatment observations, we ran additional experimental sessions varying the values of M and T. Table 3 presents the mean value observations from these experiments (theoretically expected values are in parentheses adjacent to the mean observations).

On average, the auction-style tournament generated greater surplus for the tournament sponsor than the fixed-prize tournament in each treatment where there were three or more bidders.[20] The auction-style tournament research-to-prize ratios were also larger when there were three or more bidders, implying that the sponsor induced more research per prize dollar by holding the postresearch auction in those treatments.

Based on all our experimental sessions, the only data that were inconsistent with the proposition that auctions generate more research per prize dollar were from treatment 1. We believe the primary reason the auction-style experiments in treatment 1 performed poorly was the greater opportunity for passive research efforts and weak bidding competition when there were only two competitors. In treatment 1, if one of the two contestants was passive it had a potentially huge impact on the outcome of the tournament and occasionally produced extremely large prize payments for low winning innovations. Having only one active researcher and bidder may not be likely to happen in an actual tournament where it is costly for contestants to enter the tournament. However, it does highlight a potential problem in auction-style competitions when there are only two competitors, or when the possibility exists for collusion among bidders. In practice, the sponsor could prevent this from happening by inviting more contestants to participate in her tournament or by placing an upper limit on the maximum allowable bid per contestant.

[20] We conducted only about half as many rounds in treatments 1 and 3 as we conducted in 2.

TABLE 3 **Experimental Results**

	Session Treatment Parameters		
	$M = 2, T = 6$ (1)	$M = 3, T = 4$ (2)	$M = 5, T = 2$ (3)
Fixed prize			
Fixed prize	120	103	101
Winning draw	888 (904)	884 (906)	889 (901)
Surplus	768 (784)	781 (803)	788 (800)
Research-to-prize ratio	.52 (.591)	.68 (.789)	.78 (.85)
Auction			
Expected bid	169 (119)	53 (102)	36.5 (101)
Winning draw	829 (907)	873 (908)	835 (902)
Surplus	660 (788)	820 (806)	798 (801)
Research-to-prize ratio	.33 (.623)	1.46 (.805)	1.83 (.85)

While the average winning draws in the auction-style tournament of treatment 3 were smaller than in the fixed-prize tournament, those draws were more than offset by remarkably low winning bids. In treatment 3, the average winning bid in the auction-style rounds was only about one-third of the predicted bidding equilibrium. These small winning bids resulted in research-to-prize ratios that were significantly larger than one, again implying that the average contestant in these experimental rounds would have earned more money by opting not to conduct any research at all. Obviously, the direct cause of the large research-to-prize ratios was smaller-than-predicted winning bids. Although the total amount of research conducted by contestants in all treatments was relatively close to the amount predicted by theory, the winning bids in treatments 2 and 3 were all substantially smaller than the theoretically predicted winning bids.

Anderson, Goeree, and Holt (1998) show that in a logit equilibrium with boundedly rational players, the extent of excessive effort or rent dissipation increases in the number of players, just as we observed in our experiments. There are several factors that could collectively explain why competitors spent too much on research relative to the size of the winner's prize and behaved as if they had bounded rationality:

(i) underestimating the size of the profits required to sustain equilibrium;
(ii) sunk costs;
(iii) small marginal costs of research.

The first possible explanation is that some of the participants simply underestimated the size of the prize required to sustain a positive-profit equilibrium. This mistake can easily be made, as the following example illustrates: Suppose five contestants are competing in a tournament and each spends 20 francs on research (i.e., they each take two draws). At the conclusion of the contest, if the winning contestant earns 50 francs—a sum worth 2.5 times more in prize money than he spent on research—he may feel very good about continued participation in the tournament. The idea of winning 50 francs on an investment of 20 francs ostensibly seems like a very profitable strategy. But the group *collectively* spent 100 francs on research, so the research-to-prize ratio was actually two, implying a negative expected profit for contestants. Notably, when we looked at all the winning bids in the auction-style sessions of treatments 2 and 3, the winner rarely was observed earning a prize larger than the collective research expenditures of his group. In treatment 2, the winning bidder's prize was larger than the collective research expenditures of his group only 25% of the time. In treatment 3, the winning bid was larger than the collective research expenditures of the group only 16% of the time.

Another explanation for the large research-to-prize ratios is that all research costs are already sunk at the time of the bidding in an auction-style tournament, since subjects do not submit their bids until after they have completed all of their research. Therefore, competitors who do not draw large innovations may be tempted to submit very low bids to recoup at least a fraction of their sunk research costs. Although this explanation can be unravelled by backward induction, experiments have often shown that subjects are not particularly good at backward induction. In theory, contestants should have had the foresight to know that other contestants will have the same postresearch incentive to reduce their bids and elect to either reduce their level of research or choose not to conduct research at all. However, we did not observe a significant amount of learning by subjects that was manifest by reduced research in later rounds.

One final explanation for the large research-to-prize ratios could be the small marginal cost of making a research draw. Since each draw only cost 10 francs, subjects may have been more willing to risk that small sum of laboratory currency to "win" the tournament. To test this hypothesis, we conducted two additional experimental sessions using three competitors per group, one period per round, a cost per draw of 25 francs, and a uniform distribution of innovations truncated at the bottom at 525. Therefore, this treatment roughly simulated our baseline treatment but instead of allowing subjects to make incremental draws at a cost of 10 francs each, in this treatment subjects had to decide whether to make a single draw at a cost of 25 francs. In this excursion, the average research-to-prize ratio we observed was .99, which was an improvement over our baseline treatment value of 1.466. However, the predicted ratio for this treatment was only .63, so we still observed a research-to-prize ratio substantially larger than predicted. Therefore, although small incremental research costs may "lure" some subjects into doing more than the optimal amount of research, our data were inconclusive as to whether this is a significant factor in the large research-to-prize ratios we observed in our experiments. Regardless of the cause, the large ratios in the auction-style sessions only add further support to the hypothesis that auction-style competitions are likely to generate more research per prize dollar.

4. Conclusions

■ The theoretical and experimental results reported in this article support the hypothesis that prize auctions will generally reduce a sponsor's prize expenditure in research tournaments. For innovation distributions of the form $f(x) = \alpha x^{\alpha-1}$, a sufficient condition for auctions to reduce the cost of the prize payment is that the equilibrium z-stop bid is smaller than the prize required to induce an equivalent z-stop in a fixed-prize tournament. Numerical analysis suggests this sufficient condition is met in the vast majority of research scenarios, but for long tournaments the projected differences in prize payments between the two tournament types is usually very small. Therefore, the primary reason for hosting an auction probably remains the reduction in information required of the sponsor to host an auction-style tournament, rather than preannouncing a winner's prize.

Our experimental evidence generally supports the theoretical propositions put forth in this article, except for the case in which there were only two bidders. With two competitors, the occasional tendency of one competitor to opt out of conducting any research or to bid very passively resulted in some large prize payments that ultimately reduced the average surplus earned by the sponsor in the auction experiments. However, in the baseline experimental treatments that had three competitors, the auction-style research tournament performed better than the fixed-prize tournament in measures of total surplus, total research, largest innovation per group per round, and total research per prize dollar. On average, the auction-style tournament also generated more surplus for the sponsor in sessions employing five bidders. Thus, we believe there is substantial theoretical and experimental evidence favoring auction-style tournaments whenever the sponsor has limited access to information about contestants and their abilities. Tournament sponsors should exercise caution with an auction-based reward system, however, whenever there are only two competitors or there is a significant possibility for collusion among bidders. In those circumstances, the sponsor can reduce her risk by placing a maximum bid restriction on the auction, similar to the actions one would take by including a reservation price in a traditional seller's auction.

The Department of Defense (DoD) is probably the most prolific sponsor of research competitions in the United States. Since convening the Packard Commission in 1986, the DoD has strongly encouraged "fly-before-buy" prototype competitions in the acquisition of weapon systems (U.S. President's Blue Ribbon Commission, 1986). These prototype competitions resemble research tournaments in many ways, and anecdotal evidence suggests that they may help contain the costs of research and reduce the number of late specification changes required in the development process.[21] Unfortunately, the potential savings from DoD research competitions may not have been fully realized in the past because weapon system designs were often forced to conform to overly stringent military specifications and fixed quality standards rather than allowing for flexible tradeoffs between price and performance called for in a research tournament and auction setting.[22] Since 1995, however, the DoD has sought to abolish the use of excessively detailed military specifications by implementing a policy that treats cost as an independent variable (CAIV).[23] This should help improve the efficiency of DoD acquisitions, although there are other government-unique constraints on DoD weapon system acquisition programs that may limit the ability of the government to strike the best possible bargain for its research dollar.[24]

Of course, there is no reason to limit the sponsorship of research tournaments and auctions to large weapon system procurement programs. Research tournaments could be easily sponsored on one of the growing number of procurement auction Internet web sites. A search using the terms "reverse auction" or "procurement auction" on any one of the leading Internet search engines today yields a handful of web sites and testimonies of savings garnered by employing auctions for business procurement. For example, Newcorn (2000) reports that Simmons saved $417,000 in thirty minutes by conducting a reverse auction to purchase polyethylene film for wrapping finished mattresses. Seffers (2000) reported that the Army's use of a reverse auction resulted in an approximate 50% savings on computer equipment. While these web sites appear to cater predominantly to "off-the-shelf" procurement needs, it would be a natural extension to use the same methods for research tournaments.[25] Indeed, this may soon be the future of reverse auctions on the Internet, as proposed in a recent article (Rector, 2000) in *Washington Technology*:

> But what if the reverse auction technique was used simply as one step in the e-acquisition process, ... In other words, what if the government posted an electronic solicitation... performed a streamlined evaluation of offerors' proposals based on non-price evaluation factors, conducted an online reverse auction to obtain the offerors' best prices, then made a 'best value' award decision based on both price and non-price factors? Such an approach would appear to provide the government with the best of both worlds in commercial-item acquisition: flexibility to select the best-value solution, coupled with assurances that the lowest possible prices had been obtained. In fact, an e-acquisition approach of this type may well supplant the Multiple Award Schedule program in the coming years, because the MAS program has the same objective—best value at lowest cost—but probably could not be as efficient or as ruthlessly cost-effective as an auction-based, electronic acquisition.

The theoretical and experimental results presented in this article suggest that the procurement mechanism proposed by Rector is likely to produce outstanding results in terms of research effort, product quality, and price, eventually making research tournaments themselves another growing Internet industry.

Appendix A

■ Proofs of Propositions 1 and 2 follow.

Proof of Proposition 1. We begin with a lemma that shows a relationship between fixed-prize tournaments and auction-style tournaments.

---

[21] See Easterbrook (1991) for a discussion comparing the high cost of developing the B-2, awarded without a fly-off competition, with the development costs of the F-22, awarded with a fly-off competition.

[22] One should note, however, that the price paid for DoD items is often cost-based, which requires auditing and monitoring efforts. Complying with audit and monitoring requirements is expensive and defeats the primary economic purpose for hosting a tournament, reducing or eliminating the requirement for monitoring.

[23] The CAIV policy was originally established by memorandum from Deputy Under Secretary of Defense (Acquisition and Technology) Paul Kiminski in a December 1995 letter on "Reducing Life Cycle Costs for New and Fielded Systems." For details on how CAIV is currently being implemented, see DoD Instruction 5000.2-R (U.S. DoD, 1999).

[24] For example, one widely studied practice that limits the effectiveness of DoD procurement programs is its well-known willingness, for security reasons, to renegotiate contracts long after entering sole-source procurement.

[25] Note that if the winning bidders in these auctions undertake additional research following award of the contract to reduce production costs, then they are already engaged in efforts closely resembling Rob's model.

Using a z-stop strategy, the date-zero CDF for the value of a firm's best innovation drawn is

$$\Phi(x; z, T) = \begin{cases} [F(x)]^T & x \leq z \\ [F(z)]^T + [1 - F^T(z)]\dfrac{F(x) - F(z)}{1 - F(z)} & x > z. \end{cases}$$

This CDF has a density of

$$\phi^L(x) = T[F(x)]^{T-1} f(x) \qquad x \leq z$$
$$\phi^U(x) = \frac{1 - F^T(z)}{1 - F(z)} f(x) \qquad x > z.$$

Lemma A1. If a subset of equilibrium bids for innovations greater than z are increasing, then $P > B(z)$ for a fixed-prize tournament and an auction-style tournament with identical $z > 0$.

Proof. Assume $B(z) \geq P$. From (1),

$$P \int_z^{\bar{x}} \left[\Phi^{M-1}(y; z, T) - \Phi^{M-1}(z; z, T)\right] f(y)dy - C = 0.$$

Substitute $B(z)$ for P:

$$\int_z^{\bar{x}} \left[B(z)\Phi^{M-1}(y; z, T) - B(z)\Phi^{M-1}(z; z, T)\right] f(y)dy - C \geq 0$$
$$y > z \Rightarrow B(y) > B(z) \Rightarrow \int_z^{\bar{x}} \left[B(y)\Phi^{M-1}(y; z, T) - B(z)\Phi^{M-1}(z; z, T)\right] f(y)dy - C > 0.$$

From (4),

$$\int_z^{\bar{x}} \left[B(y)\Phi^{M-1}(y; z, T) - B(z)\Phi^{M-1}(z; z, T)\right] f(y)dy - C = 0.$$

This is a contradiction. Therefore $P > B(z)$. *Q.E.D.*

For Lemma A1, if a maximum-bid restriction forces firms with innovations greater than the z-stop to bid the maximum allowable bid, this would just imply that $P = B(z)$. However, Proposition 1 would still hold with a strict inequality as long as there was some innovation less than the z-stop with an equilibrium bid that is less than the maximum allowable bid. Now, to complete the proof of Proposition 1, equate the left-hand sides of (1) and (4), which both equal zero, and rearrange:

$$\int_z^{\bar{x}} [P - B(y)]\Phi^{M-1}(y; z, T) f(y)dy = \int_z^{\bar{x}} [P - B(z)]\Phi^{M-1}(z; z, T) f(y)dy.$$

From Lemma 1,

$$P > B(z) \Rightarrow \int_z^{\bar{x}} [P - B(y)]\Phi^{M-1}(y; z, T) f(y)dy$$
$$\Rightarrow P \int_z^{\bar{x}} \Phi^{M-1}(y; z, T) f(y)dy > \int_z^{\bar{x}} B(y)\Phi^{M-1}(y; z, T) f(y)dy.$$

Multiply $f(y)$ on both sides by $[1 - F^T(z)]/[1 - F(z)]$ (becoming $\phi^U(y)$ because $y > z$) to get

$$P \int_z^{\bar{x}} \Phi^{M-1}(y; z, T)\phi(y)dy > \int_z^{\bar{x}} B(y)\Phi^{M-1}(y; z, T)\phi(y)dy$$
$$P > B(z) > B(x),\ \forall\, x < z \Rightarrow P \int_0^{z} \Phi^{M-1}(y; z, T)\phi(y)dy > \int_0^{z} B(y)\Phi^{M-1}(y; z, T)\phi(y)dy.$$

Add this inequality to the previous inequality and multiply everything by M to get

$$P \int_0^{\bar{x}} M\Phi^{M-1}(y; z, T)\phi(y)dy > \int_0^{\bar{x}} B(y)M\Phi^{M-1}(y; z, T)\phi(y)dy.$$

The left side reduces to P and the right side is the expected cost of the auction-style tournament. ($M\Phi^{M-1}(y; z, T)\phi(y)$ is just the largest order statistic pdf of M draws.) *Q.E.D.*

Lemma A1 and Proposition 1 do not specify any particular bidding formula; instead they use the generic notation of $B(x)$. Though the proofs are written with the assumption that the bids are strictly increasing, upon inspection it is apparent that Proposition 1 will still hold as long as the bids are nondecreasing and there is at least one subset of innovations where bids are increasing.

Proof of Proposition 2. We begin by assuming $P > B(z)$ and $f(x) = \alpha x^{\alpha-1}$ for all $\alpha > 0$. For $x \leq z$,

$$B(x) = \frac{\int_0^x \xi^{\alpha T(M-1)} d\xi}{x^{\alpha T(M-1)}} = \frac{\alpha}{\alpha T(M-1)+1},$$

which is strictly increasing in x. The rest of the proof follows identically from the proof of Proposition 1 above.

In Lemma A2, we prove there is a unique, symmetric z-stop for the basic auction-style tournament just as there was a unique z-stop for Taylor's fixed-prize tournament. Since we are considering a particular type of auction, this proof relies on the specific bidding equilibrium described in equation (3).

Lemma A2. The auction-style tournament has a unique symmetric z-stop equilibrium.

Proof. Substituting Φ into the optimal bid function, (3), the equilibrium bids are

$$B(x_i) = \frac{\int_0^{x_i} [\Phi(\xi_i)]^{M-1} d\xi}{[\Phi(x_i)]^{M-1}} = \begin{cases} \dfrac{\int_0^{x_i} [F^T(\xi_i)]^{M-1} d\xi}{[F^T(x_i)]^{M-1}} & x \leq z \\[2ex] \dfrac{\int_0^{z} [F^T(\xi_i)]^{M-1} d\xi + \int_z^{x} [G(\xi_i)]^{M-1} d\xi}{[G(x_i)]^{M-1}} & \text{for } x > z, \end{cases}$$

where

$$G(\xi) = \left[[F(z)]^T + [1 - F(z)^T] \frac{F(\xi) - F(z)}{1 - F(z)} \right].$$

Substitute the bids above into the z-stop equation, (1), and reduce. Following Taylor, define the real valued function

$$\Delta(z) = \int_z^{\bar{x}} \int_z^x [G(\xi)]^{M-1} d\xi f(y) dy - C. \tag{A1}$$

We must show $\Delta'(z) < 0$, implying that $\Delta(z) = 0$ at most once.

$$\Delta'(z) = \int_z^{\bar{x}} \left[-[G(z)]^{M-1} + (M-1) \int_z^x [G(\xi)]^{M-2} \left\{ TF^{T-1}(z) - \frac{1 - F^T(z)}{1 - F(z)} \right\} \left(\frac{1 - F(\xi)}{1 - F(z)} \right) f(z) d\xi \right] f(x) dx.$$

Using a proof by induction, it can be shown that

$$\left\{ TF^{T-1}(z) - \frac{1 - F^T(z)}{1 - F(z)} \right\} \leq 0,$$

which implies $\Delta'(z) < 0$. *Q.E.D.*

Appendix B

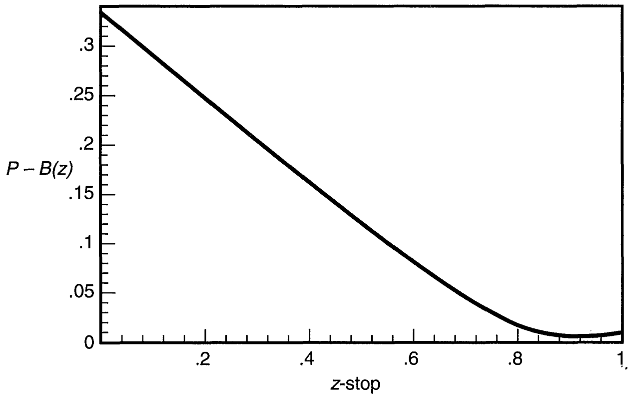
■ In this Appendix we provide numerical examples demonstrating the implications of Proposition 2 and provide the results of a number of nonparametric statistical tests we performed.

□ **Numerical analysis of auctions and fixed-prize tournaments.** Consider a generic distribution on the support [0, 1] of the form $f(x) = \alpha x^{\alpha-1}$ for all $\alpha > 0$. Based on Proposition 2, if $P > B(z)$, then the expected cost of the auction-style tournament will be less than the fixed prize required to induce an equivalent amount of research effort for these distributions.

First, consider an example with $\alpha = 1$ (uniform distribution), $M = 2$, $T = 2$. Then $P - B(z)$ reduces to just

$$P - B(z) = \frac{(z-1)[1 + 2z^T]}{3(z^T - 1)} - \frac{z}{T+1} = \frac{1 + z^2 - z}{3} > 0 \qquad \forall\, z \in [0, 1].$$

FIGURE B1



So, by Proposition 2 we know that regardless of the equilibrium z-stop, the auction-style tournament prize is less than the size of the fixed prize required to induce the same z-stop. For $M = 2$, as $T \longrightarrow \infty$, then

$$P - B(z) \longrightarrow \frac{1-z}{3} > 0 \qquad \forall\, z \in [0, 1).$$

Therefore, in the limit as firms have an unlimited opportunity to discover an innovation at least as good as the z-stop, the auction-style tournament again reduces the expected size of the final prize. For moderate tournament lengths of $T \geq 17$, there are small ranges of equilibrium z-stops where $P < B(z)$, as illustrated in the example from Section 2.

Consider another example with $\alpha = .5$. This density is shaped similarly to an exponential density on the interval from 0 to 1. However, the results are quite similar to those of the uniform distribution. With $M = 2$, Proposition 2 holds everywhere for $T < 25$. Figures B1 and B2 are for $T = 20$, just as in the example above.

In Figure B1 it can be seen that Proposition 1 holds for all equilibrium values of z-stops. This is also reflected in Figure B2, which shows the difference between the expected winning bid cost as a function of the z-stop and the size of the prize required to obtain the same z-stop in a fixed-prize tournament. Note that having an exponentially shaped function appears to increase the difference in costs between the fixed prize and the expected winning bid payment. Nevertheless, relative to the value of the expected winning innovation, the difference is still negligible.

Finally, consider an example with $\alpha = 3$. This creates an entirely different-shaped distribution of innovations where high-value innovations are discovered more frequently, yet once again the results are similar to those of the examples above. Figures B3 and B4 are for $M = 2$ and $T = 20$, just as in the previous examples.

Although one cannot make universal statements about expected cost differences that apply to all distributions, using the program *Mathematica* the authors numerically checked a wide variety of values for M, T, and α. These results, available upon request from the authors, consistently showed that for virtually all plausible parameters and z-stops the auction-style tournament performs at least as well as the fixed-prize tournament with respect to the cost of the final prize.

□ **Statistical hypothesis testing: baseline treatment 2.** To test our claims about the relative performance of the auction-style tournament compared to the fixed-prize tournament, we conducted a number of nonparametric tests that we

FIGURE B2

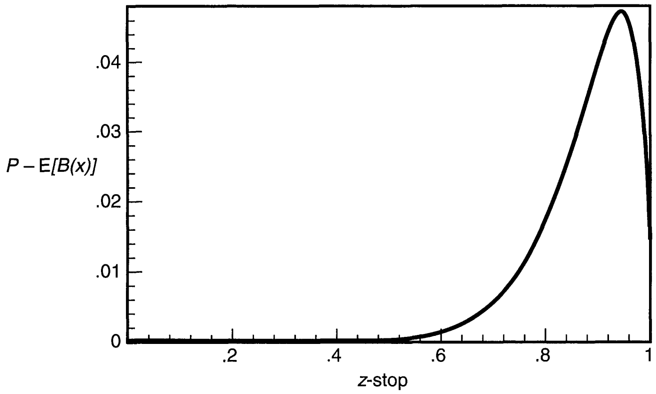
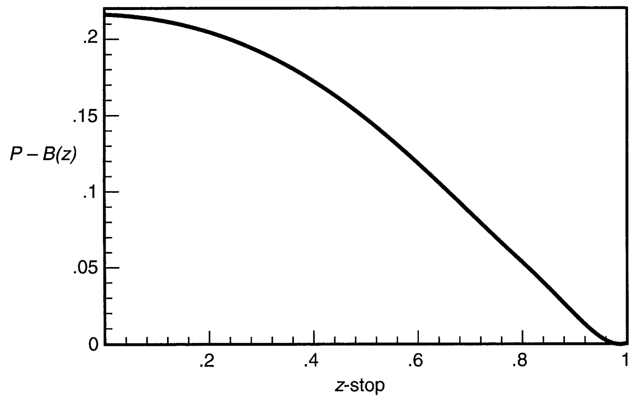


FIGURE B3



report here. We used the Wilcoxon Signed-Ranks Test for Paired Observations (DeGroot, 1989) to compare the average winning draw and average surplus per round. We conducted this test because comparisons by round allow us to control for learning effects that may cause subjects to adjust their strategies as they gain more experience. The test is constructed by computing the difference between the paired observations from each type of tournament. It is assumed that these differences form a symmetric distribution with respect to some point θ. So the test allowed us to compare the auction-style surplus and winning draws per round with the fixed-prize surplus and winning draws per round. The hypotheses we tested were:

Hypothesis 0. $\theta \leq 0$.

Hypothesis 1. $\theta > 0$.

The distribution of the statistic, S_n, created with the Wilcoxon Signed-Ranks Test converges to a standard normal distribution as $n \longrightarrow \infty$. Our test results for the baseline treatment 2 were as follows:

Winning surplus. $S_n = 4.11$ (reject Hypothesis 0, implying that the auction-style tournament generated more surplus than the fixed-prize tournament at any level of significance).

Winning draws. $S_n = -1.195$ (failed to reject Hypothesis 0 at the .05 level of significance, implying that the fixed-prize tournament could not definitively be said to produce higher winning draws than the auction-style tournament).

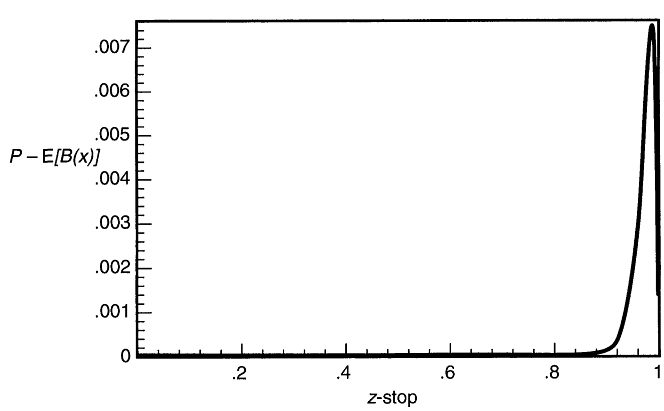
Thus, using the Wilcoxon Signed-Ranks Test on the data from our baseline treatment, we cannot say with statistical significance that the fixed-prize tournament generates larger winning innovations than the auction-style tournament, but the auction-style tournament generated a larger surplus for the sponsor.

The Wilcoxon-Mann-Whitney Ranks Test (DeGroot, 1989) is similar to the previous test, except it does not pair the observations by round. As a result, the "learning effects" get lost in the rankings. However, this test uses the entire set of data when comparing the distributions generated by the two tournament types. Here we tested the following hypotheses:

Hypothesis 0. $f(x) = g(x)$.

Hypothesis 1. There exists some constant $\theta \neq 0$, such that $f(x) = g(x - \theta)$.

FIGURE B4



Our results were as follows:

Winning surplus. $S_n = 7.4$ (rejects the null hypothesis at any level of significance and implies that the auction-style tournament generates more surplus for the sponsor).

Winning draws. $S_n = -1.94$ (rejects the null hypothesis at the .05 level of significance and implies that the fixed-prize tournament generates larger winning draws).

High draws. $S_n = 4.39$ (rejects the null hypothesis at any level of significance and implies that the auction-style tournament generates larger draws per contestant).

Analysis: baseline treatment. These tests statistically describe what we claimed in the body of the article. First, the auction-style tournament generated more surplus and more research (larger draws) per contestant than the fixed-prize tournament. On the other hand, the statistical inference is mixed on the winning draws comparison. When we pair observations by rounds, we cannot make any definitive statistical claim about the size of the winning innovations, but when we disregard the by-round comparisons, the statistic suggests that the fixed-prize tournament's winning draws were larger at a .05 level of significance. Regardless of which statistic one chooses to believe, the effect of the bidding was to increase the level of surplus generated for the sponsor above what would have been achieved in the absence of bidding. Thus, the baseline treatment data support the hypothesis that auctions reduce the cost of inducing research in a tournament.

□ **Statistical hypothesis testing: other treatments.** We also conducted nonparametric statistical tests to verify our claims about the relative performance of the auction-style tournament compared to the fixed-prize tournaments in treatments 1, 3, and 4. The tests we conducted were all based on the Wilcoxon-Mann-Whitney Ranks Test (DeGroot, 1989). Again, for this test we tested the following hypotheses:

Hypothesis 0. $f(x) = g(x)$.

Hypothesis 1. There exists some constant $\theta \neq 0$, such that $f(x) = g(x - \theta)$.

Treatment 1:

Winning surplus $S_n = -5.39$ (rejects the null hypothesis at any level of significance and implies that the fixed-prize tournament generated more surplus for the sponsor).

Winning draws. $S_n = -3.65$ (rejects the null hypothesis at almost any level of significance and implies that the fixed-prize tournament generated larger winning draws).

High draws. $S_n = .15$ (fails to reject the null hypothesis, and suggests that the highest draws per contestant per round were similar in the two tournaments).

Treatment 3:

Winning surplus. $S_n = 2.32$ (rejects the null hypothesis at a .05 level of significance and implies that the auction-style tournament generates more surplus for the sponsor).

Winning draws. $S_n = -3.15$ (rejects the null hypothesis at almost any level of significance and implies that the fixed-prize tournament generated larger winning draws).

High draws. $S_n = -3.55$ (rejects the null hypothesis and suggests that the fixed-prize tournament generated higher draws per contestant due to the larger winning prizes).

Analysis: other treatments. Collectively, these tests statistically mirror our claims in the article. The auction-style tournament generated about the same level of high draws per round in treatment 1, but less aggressive bidding with only two bidders resulted in larger surpluses in the fixed-prize tournament. In our laboratory auction-style tournaments with only two contestants, the passive bidding behavior of one contestant had a significant impact on the outcome of the tournament, which resulted in lower surpluses.

In treatment 3, as reported in the article, due to the extremely competitive bidding with five contestants, the auction-style tournament generated more surplus than the fixed-prize tournament. But because the winning bids were small, the auction-style tournament did not generate high draws or winning draws as large as those in the fixed-prize tournament. However, the sponsor substantially increased her surplus by trading off the lower prize payments for slightly smaller innovations. Thus, in treatment 3 the auction-style tournament sessions generated more total surplus and induced more research per prize dollar than the fixed-prize tournament sessions.

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