# Simulations on Correlated Behavior and Social Learning

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**Abstract** We consider a population of agents that can choose between two risky technologies: an old one for which they know the expected outcome, and a new one for which they have only a prior. We confront different environments. In the benchmark case agents are isolated and can perform costly experiments to infer the quality of the new technology. In the other cases agents are settled in a network and can observe the outcomes of neighbors. We analyze long–run efficiency of the models. We observe that in expectations the quality of the new technology may be overestimated when there is a network spread of information. This is due to a herding behavior that is efficient only when the new technology is really better than the old one. We also observe that between different network structures there is not a clear dominance.

## 1 Introduction

We analyze an economy in which agents have to choose between an old technology whose risk is well known, and a new one which not only is risky, but even the probability of success is unknown. They can be thought, as in [4] (inspired by [6]), as farmers that choose between a standard old fertilizer and a new one for which they have only a prior distribution on the quality: henceforth this is the story that we attach to the present exposition. Note that [5] empirically analyzes exactly this situation in a developing country. To be more precise, suppose that each farmer knows that the old fertilizer guarantees a good harvest (payoff normalized to 1) with

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probability $\frac{1}{2}$, and a bad one (payoff normalized to 0) with the same probability. On the other hand the new fertilizer will give a good harvest with a probability $p$ that all farmers suppose to be ex–ante uniformly distributed in the interval $[0,1]$. Let us call $p$ the *quality* of the new technology.

The timing of the theoretical model that we consider is discrete and infinite. Farmers, who have full memory and update their beliefs rationally in the Bayesian way, but are infinitely impatient, will repeatedly make their optimal choices year by year. At the beginning of the first year, in expectations, the two fertilizers are equally good, but each farmer would be happy to try out the new technology and gain more information on the real value of $p$. As time goes on, those farmers that use the old technology and have no information about the new one, will not modify their belief. Those that use the new technology and/or acquire information will get more and more signals on the quality of the new technology and will eventually end up learning which is the best to adopt. However, along any finite step of this process of learning, farmers will find any new information as a valuable good.

We will consider four cases. One in which farmers are isolated and can only perform, on top of past realized harvests, private experiments on the new fertilizer at an exogenously fixed cost $c$ for each of them. Another one in which they are geographically settled on a fixed grid and can, instead of making experiments, observe the last harvest of neighbor farmers who are using the new product, at the same positive cost $c$ for every neighbor they observe. A third one in which they can observe also some farmers which are far away in the grid and not only those in their physical neighborhood. Finally, as fourth case, a completely random directed network. It is clear that in the last three cases case we assume a network structure as the one considered in [3, 8], which has been recently generalized by [2]. Our setup is different from that one because agents, when they adopt the old technology and do not pay the cost of observing neighbors, do not get any information. This is why we do not reach uniformity of behavior in the whole economy, as is obtained and studied in the works cited above.

By simply comparing these four cases some interesting questions arise. The possibility of observing neighbors, which can be thought as the sharing of a local public good [4], will have a positive effect on the expected efficiency of our economy, as modeled in [9] and empirically observed in [5], or will it create a herding effect for which all the agents may select the worst technology just because they imitate their neighbors, as happens for consumers in the case analyzed by [7]? Will there be a different outcome if farmers can observe only their grid–neighborhood, compared to the *globalized* cases in which the network structure has long–range links or is even completely random?

In Section 2 we introduce the model, while the mathematics underlying the analytical results is in the Appendix. We present the different network structures and report the results of computer–based simulations in Section 3. Section 4 concludes.

## 2 The model

Let us start from considering the case in which every single agent is isolated. There are two alternative technologies to choose. One gives a reward either 1 with a probability $k$ (that we will fix to $\frac{1}{2}$) or zero. The other gives the good reward with a probability $p$ that is ex–ante distributed on the interval $P \subseteq (0,1)$, with p.d.f. $f_P(p)$. While the realized $p$ is unknown to agents, the distribution is common knowledge. The timing of each step is the following:

- Stage 0: the decision maker does $n$ trials to learn $p$ at a cost $c > 0$ each. The number of possible trials is bounded by $N$.
- Stage 1: after having observed the outcomes, the project is selected.
- Stage 2: payoffs are realized.

The payoff function is denoted by $U(n,d;p,k)$ and the variable $d = 1(d = 0)$ represents the agent's decision to pick the risky (safe) asset:

$$U(n,d;p,k) = k + d(p-k) - cn \ .$$

The agent's objective function to maximize is

$$\max_{d \in \{1,0\}} \left\{ \max_{n \in N} E[U(d,n;p,k)] \right\} \ . \tag{1}$$

Fully Bayesian agents will solve the problem backwards [13, 14]. Given $\hat{n}$ observations collected, the agent chooses the optimal policy $d^*(\hat{n})$ and then selects the optimal sample size. The pair $(d^*, n^*)$ denotes the equilibrium strategy profile.

In the Appendix we fully solve analytically this maximization problem when $f_P(p)$ is a Beta function (which generalizes uniform distributions). We use this result, given in equation (6), to compute numerically the expectations of adopting the new technology if $k = \frac{1}{2}$, $c = 0.004$, $N = 8$ and $p$ ranges between 0 and 0.95. The reasons for which we choose these values for $c$ and $N$ will be explained in next section.

The curve in Figure 1 shows what is the expected probability that the new technology will be adopted by every single farmer at the time that they stop experimenting (and for each of them the probabilities are i.i.d.). On the $x$–axis we have the quality $p$ of the new technology, and the vertical dashed line is the threshold below which the new technology is worst than the old one. Because of the assumptions of the model, the fact that farmers may adopt the new technology even when it is worse (i.e. for quality $p < .5$) can be considered in statistical terms as a type II error. Such errors are not a bad result for the long–run efficiency of our economy, because as long as agents use the new technology they get more and more information on its quality $p$, and will eventually end up dropping it.[1] In terms of long–run efficiency, what should be avoided is the type I error that rejects the new technology even if it

[1] This comes from the fact that all the agents have full information about the old technology. This assumption makes the model analytically tractable.

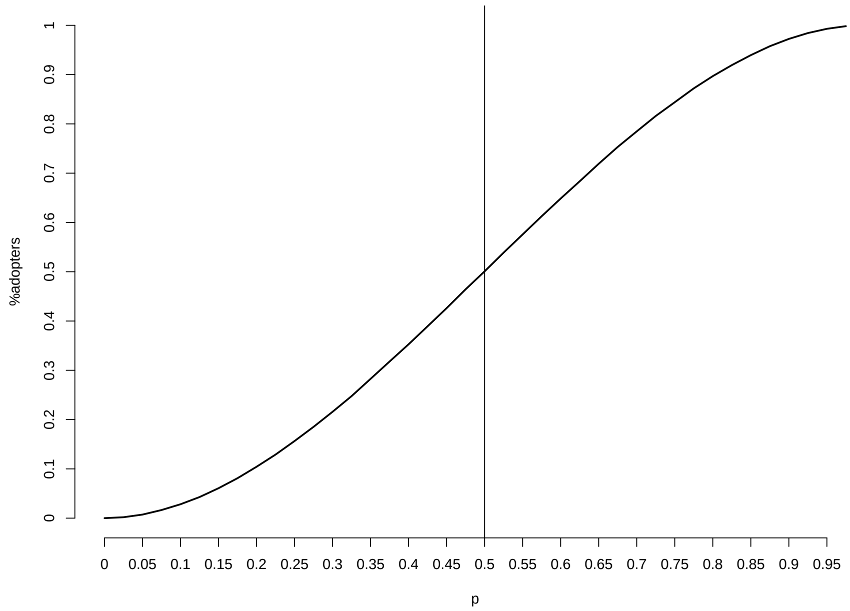


**Fig. 1** On the $x$–axis we have the quality $p$ of the new technology, on the $y$–axis we have the percentage of agents that are adopting it at the time that they stop acquiring extra information. The curve represents the expected probability of adoption of the new technology. Results are based on equation (6), with parameters $k = \frac{1}{2}$, $c = 0.004$ and $N = 8$.

is better than the old one. This may happen frequently when the quality of the new technology is approximately between 0.5 and 0.7.

## 3 Simulations on networks

We want to measure and compare the effects of peer imitation and network flow of information. For this reason we consider a new class of environments which have at the individual level the same underlying mathematics, incentives and expected payoffs as the previous one, but is enriched with a new topological structure. We assume now that agents are not isolated but at the same time they cannot make experiments by their own. Instead, they are based in a directed network in which a link from agent $i$ to agent $j$ identifies that $i$ can observe the last harvest (i.e. the outcome of the adopted technology) of agent $j$.

In particular, we will consider three types of networks. The first one is a square torus grid of $200 \times 200$ nodes, where every agent is based on a node and can observe her Moore neighborhood of surrounding 8 nodes. At the end of each time–step she will know wether any one of her 8 neighbors has adopted the old or the new technology. In the latter case she can pay a cost $c$ (which is the same we imposed in previous section for making an experiment) and observe that neighbor. Suppose that agent $i$ has $\ell \leq 8$ neighbors that have used the new technology in the previous time step, then she can, at a cost $c$ per observation, check $n \leq \ell$ of those outcomes (if $n < \ell$ those $n$ are extracted with uniform probabilities). We have chosen the value $c = 0.004$ because, in the simulations described below, we obtain that it makes the threshold $\ell \leq 8$ not more binding than the threshold $N = 8$ that we use in the isolated environment with experiments. We will come back to this point when discussing Figure 3.

The second class of networks that we consider is the one presented in Kleinberg [12], from the original model of [15]. This *small–world* model is very similar in spirit to the *strong and weak ties* literature in sociology that stems from the work of Granovetter [10]. Suppose to start from the grid considered above, but now agents keep fixed only their von Neumann neighborhood of four nodes (left, right, up and below), whereas the other four links are cast at random to four among all the other nodes of the grid, with a probability that is proportional to $D^{-\delta}$, where $D$ is the Euclidean distance on the grid between the original node and the candidate node, and $\delta$ is a non–negative parameter. The resulting network is a directed one, where node $i$ may observe node $j$ although the opposite may not be true if the two nodes are not von Neumann neighbors. The limiting cases of $\delta \to \infty$ and $\delta = 0$ represent respectively the fixed Moore neighborhood considered above, and the case in which the four new links are casted with uniform probabilities across al the other nodes. Once we have a realization of this random formation process, the obtained network will be exogenously fixed and we will run a round of simulations on it. Figure 2 describes the micro-process underlying this network formation process. By now we

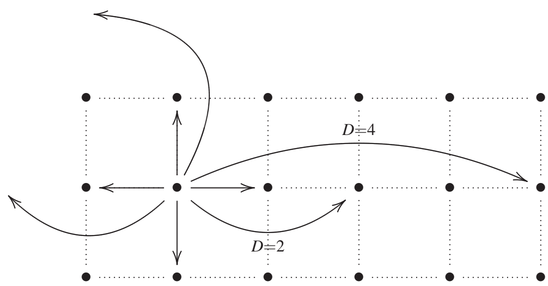


**Fig. 2** Graphical description of the Kleinberg [12] model: a node maintains her 4 von Neumann neighbors, and casts 4 links to other nodes with a probability that is proportional to $D^{-\delta}$. $D$ is the geometrical distance to the target and $\delta$ is a parameter.

will restrict to the case where $\delta = 0$ and we will discuss more on this point below.

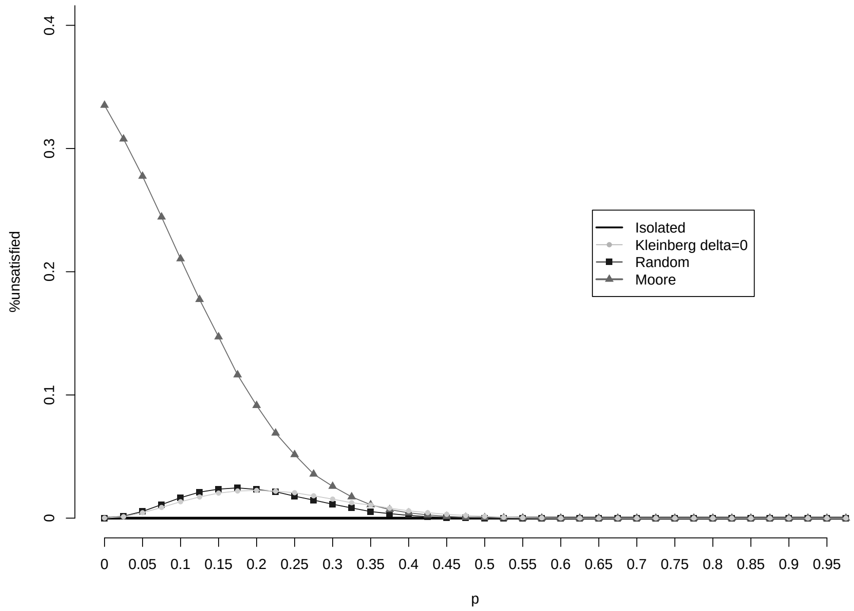


**Fig. 3** Results based on 20 iterated simulations with $k = \frac{1}{2}$, $c = 0.004$ and $N = 8$. $x$–axis shows the quality $p$. $y$–axis shows the mean percentage of agents that would have made experiments but are constrained by the threshold of 8 (isolated case) or by the opportunities of making observations (networks).

Finally, as a benchmark limiting case, we will consider a completely directed regular random network of out–degree 8, among the 40000 agents that represent our population. That is a network in which every node can observe 8 other nodes, but these are drawn at random, with replacement, uniformly and independently, in the whole set of agents.

First of all we compare with the isolated case our three network structures: the Moore neighborhood on the torus grid, the Kleinberg small–world model with $\delta = 0$, and the completely random network. Figure 3 shows the percentage of agents that, under the four cases, are constrained by the threshold imposed to the number of experiments, in the isolated case, or by the number of surrounding neighbors who are still using the new technology (in the remaining network cases). It should be noted that, as we are mostly interested in type I errors that happen for quality $p > 0.50$, the choice of $c = 0.0004$ and $N = 8$ makes the constrains of the model not binding for the results.

Figures 4 and 5 show the main comparison: the percentage of adopters of the new technology at the time that all agents stop experimenting or observing others' outcomes, leaving the society without any new information. They both represent the

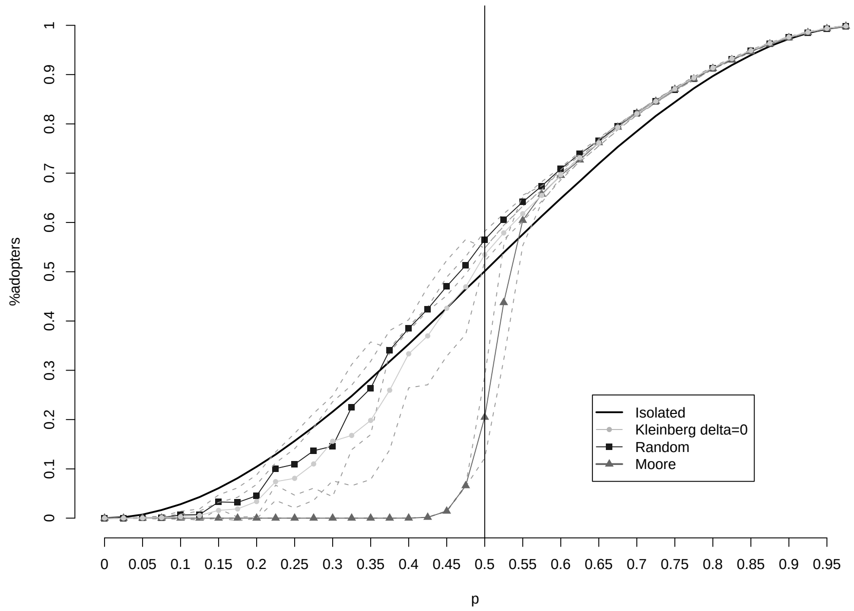


**Fig. 4** Results of the simulations with $k = \frac{1}{2}$, $c = 0.004$ and $N = 8$, for the four cases under consideration. On the $x$–axis is $p$. On the $y$–axis are average results, and confidence interval of variances, for the percentage of agents that are adopting the new technology at the time that they stop acquiring extra information.

result over the same 20 iterations of computer based simulations on different realizations of the networks described by the different models. Everything is repeated as quality $p$ ranges from 0 to 0.95 with a step of 0.025. Figure 4 considers a larger interval for quality and reports confidence interval of variances, Figure 5 focuses on a smaller interval and only on mean values. In both figures the black bold curve represents the isolated case from Figure 1.

Before commenting the result we will further investigate the model of Kleinberg. The theoretical analysis performed in [12] shows that in this model there is a threshold at $\delta = 2$, for the behavior of the agents located in the network, with respect to the flow of information. Kleinberg proves that, for values of $\delta > 2$, the behavior is analogous to the fixed torus grid of Moore neighborhoods; instead, for values of $\delta < 2$, the system behaves as a completely random network. We have not checked this prediction for our specific model, which has a similar structure of information flow, as we rely the analytical results in [12]. We can now generalize and consider essentially three different cases, from Figures 4 and 5: the benchmark isolated case; the Moore neighborhood on the torus grid; and the random network. The Kleinberg model behaves as the fixed grid for $\delta > 2$, and as the random network for $\delta < 2$.

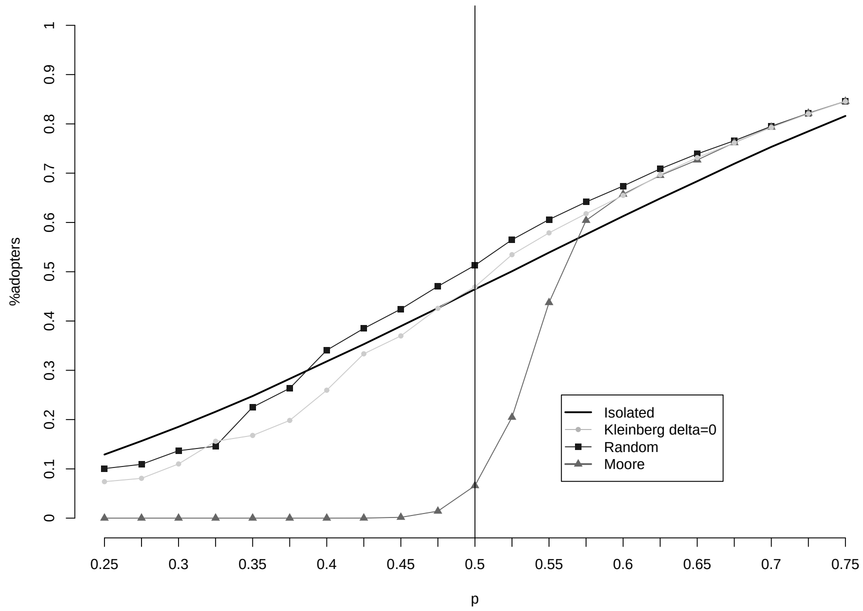


**Fig. 5** Results of the same simulations shown in Figure 4. The range for the *x*–axis is reduced and only mean values are reported on the *y*–axis.

For values of $p < 0$ we can have type II errors of adopting the new technology when it is actually worse. In such cases the correlated structure of the fixed torus grid creates a herding behavior that is beneficial to agents in the short–run. However, as the agents in the long–run experience the real quality $p$ of the new technology, this is not so important for the long–run efficiency of our model. For high values of $p$, more or less above 0.7, all the network structures are slightly more efficient (even in the long–run in this case) than the isolated case: again herding has a positive effect independently on correlations. There is finally an interesting interval when the the new technology is better than the old one but the difference is not strong, i.e. approximately for values of $0.5 < p < 0.70$. In this case the random network still performs better than the isolated case: the long range links provide reinforced information that promotes the spread of the new technology. On the other hand, in the torus grid where information is acquired only in the local neighborhood, and is therefore highly correlated, the network structures performs very badly. This happens because there is a risk of bad initial realizations of the new technology: herding behavior will make the rest and the new technology will be discarded on the basis of few correlated pieces of information.

## 4 Discussion

We study a model of flow of information concerning the unknown quality $p$ of a new technology that may be adopted instead of a known old one with quality $k = 0.50$. This flow happens on different network structures, and we compare them with a benchmark case in which information is obtained through isolated experiments. The flow of information through a network is characterized by herding effects that may be positive or negative for the long–run efficiency of the model. When the quality of the new technology is actually worse than the old one, then in terms of lung–run efficiency our model predicts that, independently on the different assumptions, the agents will end up learning the true quality in any case (i.e. type II errors sooner or later will be discovered and abandoned). When instead the new technology is actually better than the old one, the agents may end up discarding it before they have an accurate enough prediction. What is the effect of a network structure with respect of the risks of a similar type I error to occur?

Our simulations show that the flow of information through the network has a positive effect when the quality of the new technology is really much better than the quality of the old one (a $p > .7$ compared to $k = 0.5$). When instead this difference is not so strong, then a random network that avoids the possibility of correlated information is still more efficient. In this latter case we find that instead a fixed grid, in which the flow of information is correlated in the local neighborhood of the agents, performs much worse than the benchmark case in which agents have only the possibility to make isolated experiments on their own.

As the random network always perform as least as well as the grid, for $p > 0.50$ it is clear that an optimal planner should always incentivate a highly uncorrelated network with respect to a geographically constrained one. What if the costs of shifting from one network to the other are high and only partial changes can be implemented? Analyzing the Kleinberg model [12], which is continuous in a single parameter $\delta$, and has as one of the extremal cases the fixed grid, we obtain at the other extremum the same behavior of the random network. Theoretical results in [12] show that this model has actually only two types of behaviors, depending on the value of $\delta$ above or below 2. A small value of $\delta$ describes the situation in which long–range links, that jump across the fixed geographical proximity, are frequent and break the possibility of correlated behaviors. This point is related to a huge literature in sociology based on the seminal argument of Granovetter [10], who argues exactly that long–range *weak ties*, as he calls them, help enormously in the spread of information across the society.

We hope that the present work could be a starting point for future investigations. An interesting enrichment could be the possibility of letting the agent endogenously create a market and trade their information. Another one could be that of making their utilities depend also on the number of adopters of the new technology, through the kind of externalities analyzed in [11].

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## Appendix

The agent's objective function to maximize is given in equation (1). We model each single experiment as a random variable $X \in \{0,1\}$ with $Pr(X = 1|p) = p$. Let us denote by $x_n$ the vector of realizations out of $n$ trials. Hence, by Bayes' rule, we obtain the posterior probability, given $x_n$,

$$f_{P|X_1,\ldots,X_n}(p|x_n) = \frac{f_{X_1,\ldots,X_n|P}(x_n|p) f_P(p)}{\int_P f_{X_1,\ldots,X_n|P}(x_n|p) f_P(p) dp} \quad . \tag{2}$$

Assuming that $P = (0,1)$ and $f_P(p)$ follows a Beta distribution

$$f_P(p) = \frac{\Gamma(\alpha_0 + \beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)} p^{\alpha_0 - 1}(1-p)^{\beta_0 - 1} \quad ,$$

with $\alpha_0 > 0$ and $\beta_0 > 0$ known[2]. We can rewrite the denominator of (2), the probability to observe $x_n$ realizations given the priors, as

$$\int_P f_{X_1,\ldots,X_n|P}(x_n|p) f_P(p) dp = \int_0^1 p^{\Sigma x_i}(1-p)^{n-\Sigma x_i} \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)} p^{\alpha_0-1}(1-p)^{\beta_0-1} \quad .$$

Hence, by denoting $\sum x_i \equiv y$, we obtain an explicit form for the posterior distribution, that is the p.d.f of $p$ after having observed the data:

$$f_{P|X_1,\ldots,X_n}(p|x_n) = \frac{p^{\alpha_0+y-1}(1-p)^{\beta_0+n-y-1}}{\int_0^1 p^{\alpha_0+y-1}(1-p)^{\beta_0+n-y-1} dp} \quad . \tag{3}$$

Following [1], we can simplify (3) to

$$f_{P|X_1,\ldots,X_n}(p|x_n) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1} \quad . \tag{4}$$

As a result, the posteriors of $p$ follow the Beta distribution as well but with reassigned constants. Relabeled, here, as $\alpha = \alpha_0 + y$ and $\beta = \beta_0 + n - y$. The conditional mean of $p$ is

$$E[p|X_1,\ldots,X_n] = \frac{\alpha}{\beta+\alpha} = \frac{\alpha_0 + y - 1}{\alpha_0 + \beta_0 + n - 1}$$

and the variance,

$$VAR[p|X_1,\ldots,X_n] = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}$$

This formulation is convenient since now we can set $\alpha_0 = y_0 + 1$ and $\beta_0 = n_0 - y_0 + 1$ and interpret the initial prior distribution as the posteriors of p with $y_0$ positive realizations being observed out of $n_0$ trials. It generalizes our model to the case in which

[2] It encompasses the case of the Uniform distribution for $\alpha_0 = \beta_0 = 1$

an agent has already observed some data and decides how much new observations to acquire:

$$f_{P|X_1,\ldots,X_n}(p|x_n) = \frac{\Gamma(n_0+1+n)}{\Gamma(y_0+1+y)\Gamma(n_0-y_0+1+n-y)} p^{y_0+y}(1-p)^{n_0-y_0+n-y} \quad .$$

Finally, for a given sample size $\hat{n}$, the objective function is easily rewritten as

$$\max_d \; E[U(d;\hat{n},p,k)] = k + d\left(\frac{y_0+y}{1+n_0+\hat{n}} - k\right) \quad , \tag{5}$$

leading to the following optimal policy:

$$d^* = 1 \text{ if } \frac{y_0+y}{1+n_0+\hat{n}} \geq k \quad ; \quad d^* = 0 \text{ otherwise} \quad .$$

It is equivalent to define a threshold

$$\bar{x} = k(n+n_0+2) - x_0 - 1$$

to obtain

$$\max_{n\in N} U = \frac{\binom{n_0}{x_0}(n_0+1)}{(n+n_0+1)}\left[\sum_{x=0}^{\bar{x}-1} k\frac{\binom{n}{x}}{\binom{n+n_0}{x+x_0}} + \sum_{x=\bar{x}}^{n} \frac{x_0+x+1}{n_0+n+2}\frac{\binom{n}{x}}{\binom{n+n_0}{x+x_0}}\right] - cn$$

Given this solution we are able to solve also the sample size's problem:

$$\begin{aligned}
\max_{n\in N} U &= \sum_{y=0}^{n} E\left[U(d^*(n),n;p,k)|X_1,\ldots,X_n\right] f_{X_1,\ldots,X_n}(x_n) - cn \\
&= \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)} \sum_{y=0}^{n} \max\left[\frac{\alpha}{\alpha+\beta},\, k\right] \int_0^1 p^{\alpha-1}(1-p)^{\beta-1} dp - cn \\
&= \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)} \sum_{y=0}^{n} \max\left[\frac{\alpha_0+y}{\alpha_0+\beta_0+n},\, k\right] \frac{\Gamma(\alpha_0+y)\Gamma(\beta_0+n-y)}{\Gamma(\alpha_0+\beta_0+n)} - cn \quad .
\end{aligned} \tag{6}$$

We base on equation (6) the numerical computations of the expected behaviors of our model.

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