L aboratório
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C omputação
C ientífica

COMOD
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Relatório de Atividades

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1 Fisher Information

Let $f(x;\theta)$ be the probability density function of the random variable $\mathbb{X}$ conditioned on the parameter θ. The Fisher information measures the amount of information that an observation of $\mathbb{X}$ carries about the unknown parameter θ. The partial derivative of the natural logarithm of the likelihood function is called **score** (S):

$$\mathsf{S}(x,\theta) = \frac{\partial}{\partial\theta}\log\left[f(x;\theta)\right]. \tag{1}$$

Fisher information is defined as the variance of the score S:

$$\begin{aligned}\mathcal{I}(\theta) = \mathsf{E}[\mathsf{S}^2|\theta] &= \mathsf{E}\left[\left(\frac{\partial}{\partial\theta}\log\left[f(x;\theta)\right]\right)^2\middle|\theta\right] \\ &= \int_{\mathbb{R}}\left(\frac{\partial}{\partial\theta}\log\left[f(x;\theta)\right]\right)^2 f(x;\theta)\,\mathrm{d}x\end{aligned}. \tag{2}$$

If $\log\left[f(x;\theta)\right]$ is twice differentiable with respect to θ and under certain regularuty conditions, Fisher information can be written as

$$\mathcal{I}(\theta) = \mathsf{E}\left[-\frac{\partial^2}{\partial\theta^2}\log\left[f(x;\theta)\right]\middle|\theta\right]. \tag{3}$$

Let $\mathbb{X}$ be a scalar Gaussian random variable *i.e.* $\mathbb{X}\sim\mathbb{N}(\mu,\sigma^2)$. Then the probability density function is parameterized by the parameters μ and σ:

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]. \tag{4}$$

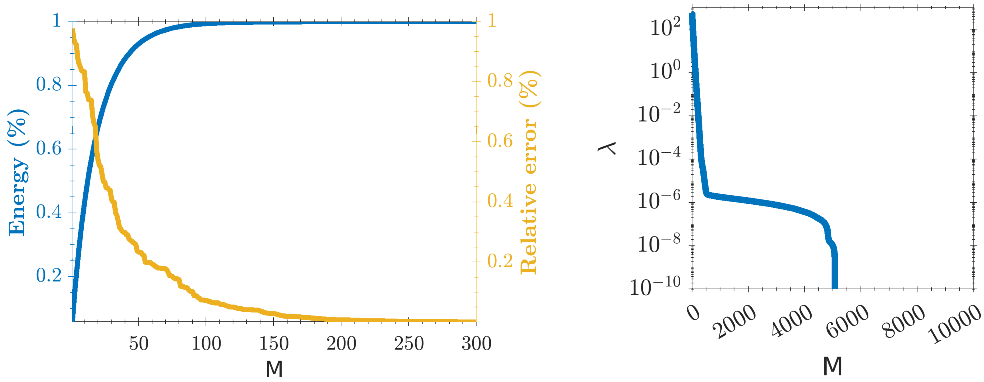
Substituting Eq. (4) in Eq. (3) where $\theta=\mu$ or σ we can compute de Fisher information for a Gaussian variable as:

1 Fisher Information

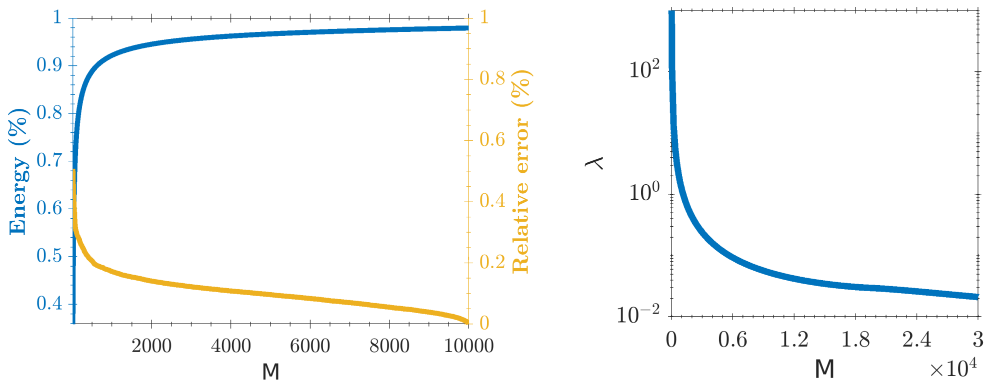
$$\begin{aligned}
\mathcal{I}(\mu) &= \mathsf{E}\left[-\frac{\partial^2}{\partial\mu^2}\left[\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right)-\frac{1}{2}\left(\frac{\mathbb{X}-\mu}{\sigma}\right)^2\right]\Big|\mu\right] \\
&= \mathsf{E}\left[-\frac{\partial}{\partial\mu}\left(-\frac{\mathbb{X}-\mu}{\sigma^2}\right)\Big|\mu\right] \\
&= \mathsf{E}\left[\frac{1}{\sigma^2}\right]=\frac{1}{\sigma^2}
\end{aligned} \tag{5}$$

$$\begin{aligned}
\mathcal{I}(\sigma) &= \mathsf{E}\left[-\frac{\partial}{\partial\sigma}\left[-\frac{1}{\sigma}+\frac{(\mathbb{X}-\mu)^2}{\sigma^3}\right]\Big|\sigma\right] \\
&= \mathsf{E}\left[-\frac{1}{\sigma^2}+\frac{3(\mathbb{X}-\mu)^2}{\sigma^4}\Big|\sigma\right] \\
&= -\frac{1}{\sigma^2}+\frac{3\sigma^2}{\sigma^4}=\frac{2}{\sigma^2}
\end{aligned} \tag{6}$$

2 KLE


Energy (%)
Relative error (%)
M
λ
M

(a)


Energy (%)
Relative error (%)
M
λ
M
$\times 10^4$

(b)

REFERENCES